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Short-time relaxation of a driven elastic string in a random medium

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We study numerically the relaxation of a driven elastic string in a two-dimensional pinning landscape. The relaxation of the string, initially flat, is governed by a growing length $L(t)$ separating the short steady-state equilibrated length scales from the large length scales that keep memory of the initial condition. We find a macroscopic short time regime where relaxation is universal, both above and below the depinning threshold, different from the one expected for standard critical phenomena. Below the threshold, the zero-temperature relaxation towards the first pinned configuration provides an experimentally convenient way to access all the critical exponents of the depinning transition independently.

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The study of the dynamics of elastic interfaces in disordered media is relevant for diverse experimental situations ranging from magnetic[1–3] or ferroelectric[4,5] domain walls, contact lines of liquid menisci on a rough substrate,[6] to crack propagation.[7,8] A fundamental problem is the response of these systems to external fields which pull the elastic interface with a force f.

Considerable progress has been made in understanding the steady-state dynamics under the applied force. At zero temperature, the system is pinned by the disorder and the velocity of the interface remains zero up to a critical force f_c. Above f_c the system undergoes a depinning transition[9–13] and moves with a nonzero average velocity. Fisher viewed the depinning transition as a critical phenomenon[14]: the driving force f plays the role of the control parameter, the mean velocity v is the order parameter vanishing at f_c with an exponent β, and the divergent correlation length $\xi_f \sim (f-f_c)^{-\nu}$ can be defined from the velocity-velocity correlation function.[13] The analogy of the depinning transition with standard critical phenomena has been, however, recently challenged. By studying the low-temperature limit of the steady-state motion it was found that no divergent length scale exists below threshold.[15]

The nonsteady dynamics, although experimentally relevant, has received less attention. Recently, Schehr and Le Doussal[16] investigated this regime for an interface initially flat by analyzing two-time correlation functions, as $f \to f_c^+$. Their functional renormalization group calculations show that the transient dynamics displays universal behavior. Below threshold, numerical studies of the zero-temperature relaxation towards the pinned state have identified a length scale ξ_f diverging with the exponent ν at f_c.[17,18] In Ref. 15 it was shown that this length ξ_f does not affect steady-state properties, but describes transient processes (deterministic avalanches triggered by thermally activated events) during the steady-state low-temperature motion for $f<f_c$, and it is ultimately related to the vanishing of the density of metastable states as one approaches f_c. The $f=0$ relaxation towards equilibrium at finite temperature has been studied both numerically[19–23] and analytically.[24,25]

In the present paper we analyze the transient dynamics of an interface pulled with a finite force close to f_c and show that it is a powerful method to extract the critical properties of the depinning transition. Indeed, the analysis of the relaxation dynamics has been used extensively to study equilibrium critical phenomena.[26] The basic idea behind this dynamic approach is the existence of a growing length $L(t)$. Let us use for simplicity the example of the Ising model: the system is prepared in the ground state, characterized by a global magnetization $m=1$, and at $t=0$ it is annealed at a temperature T, close to the critical point T_c. The global magnetization m relaxes to its equilibrium value following a time evolution controlled by $L(t)$: for lengths below $L(t)$ the system is equilibrated, while for lengths larger that $L(t)$ the system keeps memory of the $t=0$ initial condition. After a microscopic time, the relaxation is governed by the dynamical exponent z and $L(t) \sim t^{1/z}$ before reaching the equilibrium correlation length. In this macroscopic time regime scaling arguments lead to a universal behavior for the relaxation of the order parameter, although the system is far from its equilibrium state.[26] Analogously, if we assume the presence of such a growing length $L(t)$ in the transient regime of an initially flat driven elastic interface, the scaling form for the relaxation of the velocity is given by

$$v(t,f) = \xi_f^{-\beta/\nu} F(L(t)/\xi_f), \tag{1}$$

where the function $F(s) \sim s^{-\beta/\nu}$ for small s. When $s \gtrsim 1$, $F(s) \sim$ const for $f>f_c$, in order to get the steady-state velocity $v \sim (f-f_c)^{\beta}$; for $f<f_c$, where the order parameter is zero at $T=0$, $F(s)$ must be modified to take into account the exponential decay of the velocity. In standard phase transitions, a scaling form equivalent to Eq. (1) describes, in general, the evolution of the order parameter. In this paper we show that the scaling form (1) describes the relaxation near depinning, but in a nonstandard way. While in standard critical phenomena ξ_f represents the correlation length on each side of the transition, for depinning ξ_f represents the steady-state correlation length only above threshold but a purely transient correlation length below threshold, absent in the steady-state geometry of the line.[15] With this identification of the relevant lengths the analysis of the transient gives access to all the

critical exponents of the transition. To make the study more transparent we consider only the case of strictly zero temperature, in which the interface relaxes deterministically towards a final pinned configuration without reaching any steady state below threshold, thus probing transient deterministic dynamics only. We also restrict ourselves here to the simple case of a string with short-range elasticity moving in a two-dimensional random landscape, but our method naturally applies to higher-dimensional systems as well.

The string is described by a single-valued function $u(x,t)$, which measures its transverse displacement u from the x axis at given time t. The equation of motion is given by

$$\partial_t u(x,t) = \partial_x^2 u(x,t) + F_p(u,x) + f, \tag{2}$$

where $F_p(u,x)$ is the pinning force with correlations $\overline{F_p(u,x)F_p(u',x')} = \Delta(u-u')\delta(x-x')$, the overbar represents the average on the disorder realization, and $\Delta(x)$ is a short-ranged function. In our simulations we have set this range equal to 1, analyzed system sizes up to $L=2048$, and results were averaged from 2000 to 10 000 disorder realizations. For a fixed force $f<f_c$ we let evolve an initially flat line $u(x,0)=0$, up to its final pinned configuration, which can be detected by a very efficient algorithm in polynomial time.[27] The geometrical properties of the line can be described using the averaged structure factor, which for a general configuration is defined as

$$S_f(q,t) = \overline{\left\langle \left| \frac{1}{L}\int_0^L dz u(x,t) e^{-iqx} \right|^2 \right\rangle}, \tag{3}$$

where L is the length of the line, $q=2\pi n/L$, with $n=1,\ldots,L-1$. For a self-affine line we have $S(q)\sim q^{-(1+2\zeta)}$, thus yielding the roughness exponent ζ. In Fig. 1(a) we show the structure factor of the pinned line $S_f(q)\equiv S_f(q,\infty)$ for different values of $f<f_c$. Two regimes can be identified: at small length scales, the geometry of the line becomes self-affine and it is characterized by the depinning roughness exponent $\zeta\approx 1.25$.[34] At large length scales $S_f(q)$ reaches a plateau which represents the memory of the initial flat condition. As shown by the perfect collapse in the inset, $S_f(q)$ is governed by a single length ξ_f, given by the crossover between these two regimes. In Fig. 1(b) we see that the crossover length increases with the force, diverging with the depinning exponent ν at a finite value, identified with the threshold f_c. Small deviations from this behavior are observed at very small forces and whenever the crossover length approaches the system size L. Using the finite-size analysis of Fig. 1(b) we can easily extrapolate the value of the critical force for the infinite system and identify the crossover length with ξ_f. This length corresponds to the size of the minimal string rearrangement needed to reach a metastable configuration from the flat one, and its divergence is due to the vanishing density of metastable states approaching f_c from below. ξ_f can be associated with the divergent length found in Refs. 17 and 18 and with the size of the deterministic avalanches in the steady-state motion.[15] In the following we show that although ξ_f does not affect steady-state properties below threshold,[15] it affects the transient relaxation.

Let us now analyze the time evolution of the line towards

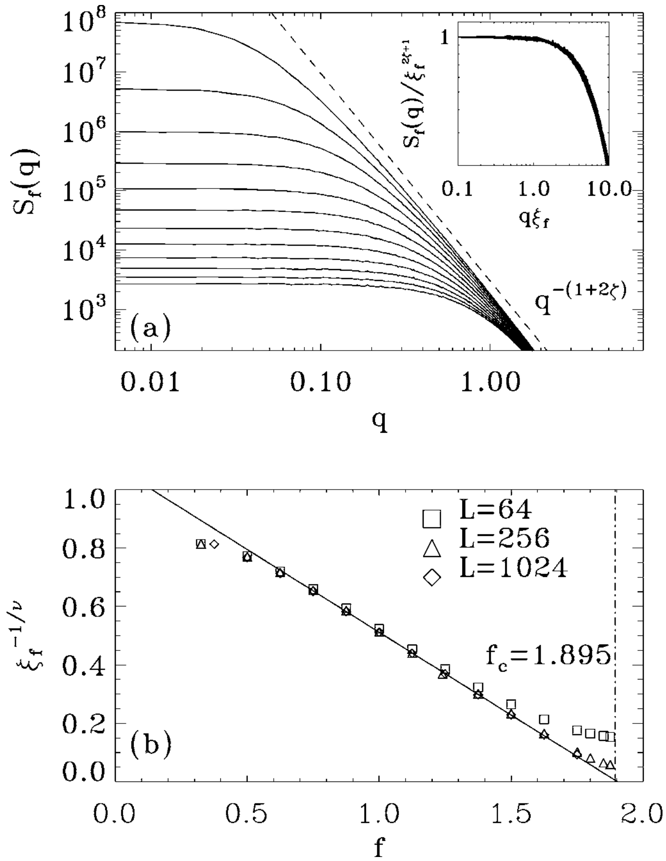

FIG. 1. (a) Structure factor $S_f(q)$ of the pinned configuration obtained by relaxing an initially flat line for different forces $f=0.325,\ldots,2.0$, increasing from the bottom to the top curve. Inset: collapse using the crossover length ξ_f, and the depinning roughness exponent $\zeta=1.25$. (b) Finite size study of ξ_f vs f. The solid line is a fit to $\xi_f\sim(f_c-f)^{-\nu}$, where $\nu=1.33$. The dashed-dotted line indicates the extrapolated value of f_c.

the final pinned configuration for $f<f_c$ and to the sliding steady state for $f>f_c$. For this purpose we solve numerically Eq. (2) by using a second-order Runge-Kutta method. In Fig. 2(a) we show the typical evolution of $S_f(q,t)$ for a force $f=1.80$. Once again, two roughness regimes are observed, one corresponding to the memory of the flat initial configuration and the other to the depinning roughness $\zeta=1.25$. As we can see in the inset, a growing length scale, identified with $L(t)$, governs the time evolution. In Fig. 2(b) we show the time evolution of $L(t)$ for forces above and below the threshold. We can distinguish three regimes for the evolution of $L(t)$. After a first microscopic time regime where the line is practically flat, we find a macroscopic short time regime where the growth of $L(t)$ is controlled by the depinning dynamical exponent $z\sim 1.5$, as $L(t)\sim t^{1/z}$. This result shows that the depinning transition is characterized by a universal short-time relaxation. The crossover to the third regime occurs when $L(t)\sim\xi_f$, after which we can distinguish the relaxation above or below threshold. For $f<f_c$, $L(t)$ saturates to ξ_f, the characteristic length of the final pinned configuration. For $f>f_c$, $L(t)$ continues to grow as $L(t)\sim t^{1/2}$. The thermal dynamical exponent $z=2$ is produced by the finite velocity,

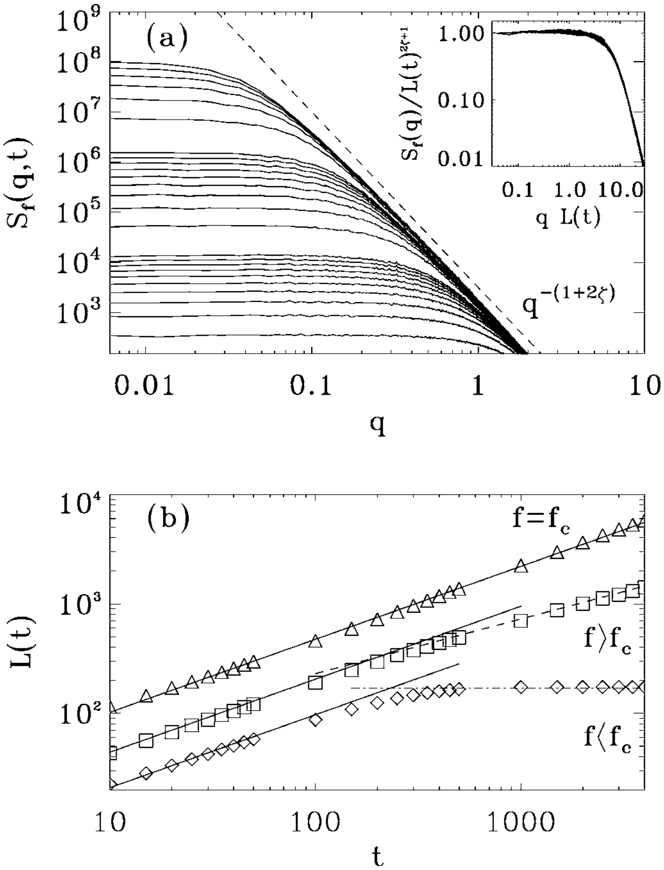

FIG. 2. (a) Typical evolution of the structure factor $S_f(q,t)$ during the relaxation at $f=1.80$, calculated at different times, increasing from the bottom to the top curve, obtained by using lines of size $L=1024$. Inset: collapse using the crossover length $L(t)$. (b) Growth of $L(t)$ for $f=f_c$ (△), $f=1.75<f_c$ (□), and $f=2.05>f_c$ (◇). The lines indicate the power law behaviors in the different regimes: $L(t)\sim t^{2/3}$ (solid lines), $L(t)\sim t^{1/2}$ (dashed line), and $L(t)\sim cte$ (dash-dotted line). The data for each force have been vertically shifted for clarity.

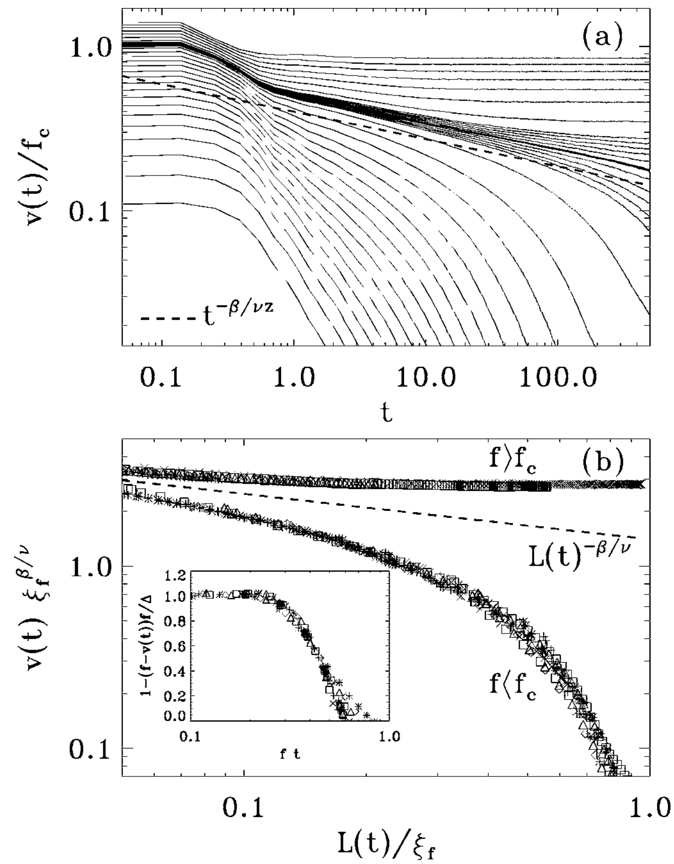

FIG. 3. (a) Relaxation of the velocity $v(t,f)$ for different forces around the critical force, from $f=0.5$ to $f=2.50$. The dashed line indicates the power-law behavior $t^{-\beta/\nu z}$, where β, ν, and z are critical exponents. (b) Collapse of all forces using the scaling of Eq. (1) based on the existence of a growing length $L(t)$ and a second static length ξ_f associated with the density of metastable states. The dashed line indicates the power-law behavior $L(t)^{-\beta/\nu}$. The inset shows the scaling in the microscopic time regime, as predicted by the perturbation theory [Eq. (4)].

which makes disorder act as an effective thermal noise above the length scale ξ_f.[13] Finite-size effects are expected if one of the two important length scales ξ_f and $L(t)$ become of the order of the system size L.

In the steady state the observables are controlled by the correlation length ξ_f. In particular, the mean velocity, the order parameter of the depinning transition, is given by $v\sim\xi_f^{-\beta/\nu}$. However, in the transient regime, $L(t)$ plays a crucial role in the relaxation of all the observables. In particular, in Fig. 3(a) we see that the three growth regimes previously found for $L(t)$ correspond to three regimes of $v(t,f)$. At the shortest times the line moves freely, $v\sim f$, since pinning forces average to zero for a flat line. The first nonzero corrections to this free-flow behavior can be therefore obtained by using a second-order perturbation theory in the disorder. This gives

$$v(t,f)\sim f-\frac{\Delta(0)-\Delta(ft)}{f}. \qquad (4)$$

The inset of Fig. 3(b) shows that (4) holds up to times $t<r_f/f$, where $r_f=1$ is the correlation length of the disorder. The first stage of microscopic relaxation of $v(t,f)$ thus gives access to the shape of the disorder correlator $\Delta(u)$ for $u<r_f$. The data obtained at longer times verify the scaling relation proposed in Eq. (1), as we show in Fig. 3(b). This collapse shows that the velocity is controlled by the ratio $L(t)/\xi_f$. When $L(t)\ll\xi_f$ the growing length $L(t)$ is the only relevant scale in the problem and replaces ξ_f in the steady-state relation $v\sim\xi_f^{-\beta/\nu}$. This leads to $v(t,f)\sim L(t)^{-\beta/\nu}$, as is observed in the initial slope of the collapsed curves in Fig. 3(b), with $\beta\sim0.33$. Subsequently, for $f>f_c$ the scaling function tends to a constant while for $f<f_c$ has a fast decay. A similar relaxation of the velocity was found in other disordered models.[28,29]

In conclusion, we have identified the relevant static and dynamical lengths $L(t)$ and ξ_f controlling the dynamics of relaxation of an elastic line under an applied force. We have shown that Eq. (1) describes well the universal relaxation of the velocity observed for $t>1/f$ and f close to the threshold. In contrast to standard critical phenomena, below threshold the static length ξ_f does not represent any steady-state correlation length. The proposed scaling relationship for the time relaxation can be used to get all the critical exponents of the depinning transition independently: we can get ν from the characteristic length ξ_f of the pinned configuration below threshold [or from the crossover to the thermal growth of $L(t)$ above threshold], z from the evolution of $L(t)$, and β from the relaxation of the velocity. This is best accomplished close to the depinning threshold since ξ_f is large and the macroscopic short time regime longer. As for equilibrium critical phenomena, this is a convenient method for both numerics and experiments, since it avoids the problem of equilibrating a sample in the steady state near the depinning threshold. When $L(t)$ becomes of the order of the correlation length ξ_f all the observables approach their steady-state value for $f>f_c$ or their value at the final pinned configuration for $f<f_c$ exponentially fast. This study is therefore relevant for experimental situations whenever T is low enough to assure well-separated time scales for deterministic and thermally activated motion.

Finally, we point out that the experimental characterization of interfaces pinned below f_c, prepared under an unusual protocol should be a convenient tool to investigate depinning, since the geometry of these interfaces yields an independent measure of the exponents ζ and ν, as shown in Fig. 1. This feature is important, as these exponents are usually related. The so-called statistical tilt symmetry (STS) is present whenever the elastic force is a linear function of the deformation field $u(x)$. In general, the STS relation depends on the range of the elastic interactions: for a short-ranged elasticity, as in Eq. (2), the relation reads $\nu=1/(2-\zeta)$, while for the long-ranged interactions, expected for crack propagation in solids or for the contact line of liquids, the relation becomes $\nu=1/(1-\zeta)$. Recent experiments in these two systems[7,30] gave roughness exponents ζ that are systematically bigger than the one computed by numerical simulations.[27] In order to explain these deviations the presence of nonlinear elastic corrections able to change the universal behavior of the depinning transition[27,31,32] has been invoked. The measurement of an eventual violation of the statistical tilt symmetry relation from the study of interfaces pinned below f_c could be thus used as a smoking gun test for this hypothesis.

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[1] S. Lemerle, J. Ferré, C. Chappert, V. Mathet, T. Giamarchi, and P. Le Doussal, Phys. Rev. Lett. **80**, 849 (1998).
[2] T. Shibauchi, L. Krusin-Elbaum, V. M. Vinokur, B. Argyle, D. Weller, and B. D. Terris, Phys. Rev. Lett. **87**, 267201 (2001).
[3] F. Cayssol, D. Ravelosona, C. Chappert, B. Fenne, and J. P. Jamet, Phys. Rev. Lett. **92**, 107202 (2004).
[4] T. Tybell, P. Paruch, T. Giamarchi, and J. M. Triscone, Phys. Rev. Lett. **89**, 097601 (2002).
[5] P. Paruch, T. Giamarchi, and J. M. Triscone, Phys. Rev. Lett. **94**, 197601 (2005).
[6] S. Moulinet, A. Rosso, W. Krauth, and E. Rolley, Phys. Rev. E **69**, 035103(R) (2004).
[7] K. J. Måløy and J. Schmittbuhl, Phys. Rev. Lett. **87**, 105502 (2001).
[8] L. Ponson, D. Bonamy, and E. Bouchaud, Phys. Rev. Lett. **96**, 035506 (2006).
[9] O. Narayan and D. S. Fisher, Phys. Rev. B **46**, 11520 (1992).
[10] T. Nattermann, S. Stepanow, L. H. Tang, and H. Leschhorn, J. Physiol. (Paris) **2**, 1483 (1992).
[11] P. Chauve, P. Le Doussal, and K. Wiese, Phys. Rev. Lett. **86**, 1785 (2001).
[12] A. Rosso, A. K. Hartmann, and W. Krauth, Phys. Rev. E **67**, 021602 (2003).
[13] O. Düemmer and W. Krauth, Phys. Rev. E **71**, 061601 (2005).
[14] D. S. Fisher, Phys. Rev. B **31**, 1396 (1985).
[15] A. B. Kolton, A. Rosso, T. Giamarchi, and W. Krauth, Phys. Rev. Lett. **97**, 057001 (2006).
[16] G. Schehr and P. Le Doussal, Europhys. Lett. **71**, 290 (2005).
[17] O. Narayan and A. A. Middleton, Phys. Rev. B **49**, 244 (1994).
[18] L. W. Chen and M. C. Marchetti, Phys. Rev. B **51**, 6296 (1995).
[19] A. Barrat, Phys. Rev. E **55**, 5651 (1997).
[20] H. Yoshino, Phys. Rev. Lett. **81**, 1493 (1998).
[21] A. B. Kolton, A. Rosso, and T. Giamarchi, Phys. Rev. Lett. **95**, 180604 (2005).
[22] S. Bustingorry, J. L. Iguain, C. Chamon, L. F. Cugliandolo, and D. Dominguez, cond-mat/0603503 (unpublished).
[23] S. Bustingorry, L. F. Cugliandolo, and D. Dominguez, Phys. Rev. Lett. **96**, 027001 (2006).
[24] L. F. Cugliandolo, J. Kurchan, and P. Le Doussal, Phys. Rev. Lett. **76**, 2390 (1996).
[25] L. Balents and P. Le Doussal, Phys. Rev. E **69**, 061107 (2004).
[26] B. Zheng, in *Computer Simulation Studies in Condensed-Matter Physics XVII*, edited by D. P. Landau, S. P. Lewis, and H.-B. Schüttler (Springer-Verlag, Heilderberg, 2005).
[27] A. Rosso and W. Krauth, Phys. Rev. E **65**, 025101(R) (2002).
[28] C. Lee and J. M. Kim, Phys. Rev. E **73**, 016140 (2006).
[29] S. Zapperi, J. J. S. Andrade, and J. MendesFilho, Phys. Rev. B **61**, 14791 (2000).
[30] S. Moulinet, C. Guthmann, and E. Rolley, Eur. Phys. J. E **8**, 437 (2002).
[31] P. Le Doussal, K. J. Wiese, E. Raphael, and R. Golestanian, Phys. Rev. Lett. **96**, 015702 (2006).
[32] M. Adda-Bedia, E. Katzav, and D. Vandembroucq, Phys. Rev. E **73**, 035106(R) (2006).
[33] G. Leaf, S. Obukhov, S. Scheidl, and V. Vinokur, J. Magn. Magn. Mater. **241**, 118 (2002).
[34] Previous studies have found two regimes but with an incorrect force dependent ζ exponent in the rough regime (Ref. 33).