Creep Motion of an Elastic String in a Random Potential

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We study the creep motion of an elastic string in a two-dimensional pinning landscape by means of Langevin dynamics simulations. We find that the velocity-force characteristics are well described by the creep formula predicted from phenomenological scaling arguments. We analyze the creep exponent μ and the roughness exponent ζ. Two regimes are identified: when the temperature is larger than the strength of the disorder, we find $\mu \approx 1/4$ and $\zeta \approx 2/3$, in agreement with the quasi-equilibrium-nucleation picture of creep motion; on the contrary, when lowering the temperature enough, the values of μ and ζ increase, showing a strong violation of the latter picture.

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Understanding the physical properties of disordered elastic systems is a challenging question relevant to a host of experimental situations. Indeed, such a situation is realized in many different systems, ranging from periodic ones, such as vortex lattices [1–3], charge density waves [4], and Wigner crystals [5], to interfaces, such as magnetic [6–8] or ferroelectric [9] domain walls, fluid invasion in porous media [10], contact lines of liquid menisci on a rough substrate [11], and domain growth [12]. The competition between disorder and elasticity in these systems leads to unique physical properties and, in particular, to glassy behavior. One particularly important question is how the system responds to an external force (magnetic or electric field for domain walls, current for vortices, etc.). At zero temperature, because of disorder the system is pinned and the velocity of the elastic structure remains zero up to a critical force F_c. At finite temperature, however, the barriers to motion due to pinning can always be passed by thermal activation, and one expects, thus, a finite response at finite force. Although it was initially believed that the response was linear [13], it was subsequently proposed [14,15] that, because of the glassy nature of the disordered system, no linear response would exist. The slow dynamics of the system for $F \ll F_c$, so-called creep, is expected to be controlled by thermally activated jumps of correlated regions over the pinning energy barriers separating different metastable states. By adding some strong assumptions on this physical picture of the motion, elegant scaling arguments were used [16,17] to infer the small F response, leading to the ''creep formula''

$$V(F) \sim \exp\left[-\frac{U_c}{T}\left(\frac{F_c}{F}\right)^{\mu}\right], \tag{1}$$

where U_c is an energy scale, and μ is a characteristic exponent that can be obtained from the relation

$$\mu = \frac{D - 2 + 2\zeta}{2 - \zeta}, \tag{2}$$

where D is the dimensionality of the elastic system, and the exponent ζ is the *equilibrium* roughness exponent of the *static* system. The above formulas are, indeed, derived under the assumption that the movement is so slow that static properties can be used. Relation (2) is remarkable since it links the statics with the nonlinear transport of a disordered elastic system.

Going beyond the simple scaling arguments or checking for such a law has proved to be very challenging. Although a sublinear response was clearly seen in various systems [1] with good agreement with (1), a precise determination of the exponent was clearly more difficult. Relation (2) has been confirmed experimentally only for magnetic domain walls [6] (see also [18] for vortices). On the theoretical side the phenomenological predictions of (1) and (2) have been derived by a functional renormalization group (FRG) calculation [19,20], starting directly from the equation of motion, and are valid in an $\epsilon = 4 - D$ expansion. This calculation confirmed the phenomenological hypothesis made in the scaling derivation and the validity of (2) up to the lowest order in ϵ. Although the velocity found in the FRG calculation was identical to the one of the scaling derivation, important differences were also found, notably on the characteristic sizes involved in the motion.

In spite of these results, the physical picture of creep motion is still very phenomenological, and many important questions remain open. A systematic study of the temperature (or disorder strength) dependence of the creep response is particularly lacking, both from experiments and theory. Moreover, from the theoretical point of view, the experimentally very relevant case of low dimensional interfaces (where thermal effects are expected to be very important), like elastic lines describing domain walls in thin films, possesses a difficult problem to tackle analytically, since the FRG [20,21] can hardly be used in $D = 1$. Such studies are quite crucial given the recent experimental results on creep in magnetic [6–8] and ferroelectric [9] systems. Numerical simulations are a valuable alternative theoretical tool to address this open issue. In this respect, creep simulations of elastic strings ($D = 1$) have been done in the past [22,23], but given the limited range of

velocities available, they were neither systematic nor conclusive about the validity of (1) and (2).

In this work we used a Langevin dynamics method to study the velocity-force (VF) characteristics and the dynamic roughness ζ of an elastic string in a random potential. The range of velocities we can explore allows a precise check of the creep law. Although we find that the creep law does describe well the data, we also find that the equilibrium hypothesis for ζ is not verified. This leads at low temperatures or strong disorder to creep and roughness exponents that become larger than the predicted values $\mu = 1/4$ and $\zeta = 2/3$, respectively.

We study the creep motion of an elastic string in two dimensions driven through a random potential. The string is described by a single valued function $u(z, t)$, which measures its transverse displacement u from the z axis at a given time t. We therefore exclude overhangs and pinched-off loops that eventually could be produced in the motion of domain walls. Assuming a linear short-range elasticity and a purely relaxational dynamics, the phenomenological Langevin equation describing the motion (per unit length) is given by

$$\gamma \partial_t u(z, t) = c\partial_z^2 u(z, t) + F_p(u, z) + F + \eta(z, t), \quad (3)$$

where γ is the friction coefficient, c is the elastic constant, F is the driving force, and $F_p(u, z) = -\partial_u U(u, z)$ is the pinning force derived from the disordered potential $U(u, z)$. The stochastic force $\eta(z, t)$ ensures a proper thermal equilibration (at $F = 0$) and satisfies $\langle \eta(z, t)\rangle = 0$, $\langle \eta(z, t)\eta(z', t')\rangle = 2\gamma T\delta(z - z')\delta(t - t')$, with $\langle \cdots \rangle$ denoting thermal average. The sample to sample fluctuations of the random potential are given by

$$\overline{[U(u, z) - U(u', z')]^2} = \delta(z - z')R^2(u - u'), \quad (4)$$

where the overline denotes the average over disorder realizations. In this work we consider a random-bond type of disorder, characterized by a short-ranged correlator $R(u)$, of range r_f and strength $R(0)$. A physical realization of this kind of disorder is, for instance, the random anisotropy for magnetic domain walls [6].

To solve numerically Eq. (3) we discretize the string along the z direction, $z \to j = 0, \ldots, L - 1$, keeping $u_j(t)$ as a continuous variable. A second order stochastic Runge-Kutta method [24] is used to integrate the resulting equations. To model a continuous random potential satisfying (4), we generate, for each j, a cubic spline $U(u_j, j)$ passing through M regularly spaced uncorrelated Gaussian random points, with zero mean and variance $R(0)^2$ [25]. Moreover, the random potential satisfies periodic boundary conditions,

$$U(u_j + M, j) = U(u_j, i + L) = U(u_j, j), \quad (5)$$

which defines a finite sample of size (M, L).

We are interested in the VF characteristics and the dynamic roughness exponent ζ in the creep regime for different values of T and disorder strength $R(0)$. The center of mass $X(t)$ moves with the average velocity,

$$V(t) = \overline{\left\langle \frac{d}{dt} X(t) \right\rangle}. \quad (6)$$

In the long time limit the string achieves a stationary state with $V(t) = V(F)$. The roughness exponent ζ is then obtained from the average structure factor,

$$S(q) = \overline{\left\langle \left| \frac{1}{L} \sum_{j=0}^{L-1} u_j(t) e^{-iqj} \right|^2 \right\rangle}, \quad (7)$$

where $q = 2\pi n/L$, with $n = 1, \ldots, L - 1$. From dimensional analysis we know that for small q, $S(q) \sim q^{-(1+2\zeta)}$. Fitting our numerical data with this function we get ζ.

We simulate systems of sizes $L = 64, 128, \ldots, 1024$ and $M = 2L$, with $c = r_f = \gamma = 1$. In this Letter we show the results for $L = 512$, where finite-size effects are negligible. We take $R(0) = 0.12, 0.30$ and temperatures ranging from $T = 0.8R(0)$ to $3.5R(0)$. We start each simulation with a flat initial configuration ($u = 0$) at the force $F = F_c/10$, and then decrease slowly F in steps of $\Delta F = 0.01F_c$ up to $F/F_c \approx 0.01$. F_c is calculated to high precision for each disorder realization using a fast-convergent algorithm [25]. The properties of interest have to be calculated when the stationary state is achieved. In practice we let the string complete two turns around the system for the initial force and one turn for the following forces. After this equilibration we estimate numerically (6) and (7) approximating the average over disorder and thermal realizations by a single time average over one turn. In the inset of Fig. 1 we demonstrate that this criterion ensures the relaxation of $V(t)$.

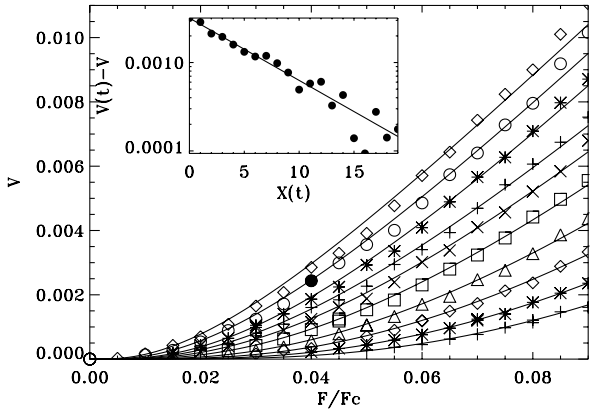

FIG. 1. VF characteristics for $R(0) = 0.30$. Curves correspond to $T = 0.24, 0.26, \ldots, 0.42$ from bottom to top. Solid lines are fits of the creep law (1) with U_c and μ as fitting parameters. Contrary to the naive creep prediction, the optimal fit parameter μ is temperature dependent. The inset shows the relaxation of $V(t)$ to its stationary value V, corresponding to the ● symbol in the main figure.

Typical VF characteristics obtained in the simulations are shown in Fig. 1. In the whole range of temperature and pinning strength analyzed we find that the VF curve can be well fitted by the creep formula (1) with U_c and μ as fitting parameters. We thus confirm the predicted stretched exponential behavior, and rule out the original proposal of Ref. [13]. However, contrary to the naive creep relation (2), we find that both fitting parameters, and not only U_c as predicted in [21,26], can depend on temperature. Note that although (1) provides an excellent fit, our data can also be fitted with a power law. This behavior for $V(F)$ was proposed supposing either high temperatures [27] or logarithmic growth of the energy barriers [28]. However, even if a power law fit of our data is in principle possible, it leads to strong temperature dependent exponents in severe contradiction with the predicted value [29]. We conclude that a power law fit is not an adequate fit of the data.

Analysis of various values of disorder and temperature shows essentially two different regimes of creep motion. To investigate further these regimes, we show in Fig. 2(a) the VF characteristics for two values of disorder and temperature representative of each regime. For the small disorder case we get the exponent $\mu = 0.26 \pm 0.01$ which is compatible with the predicted theoretical value $\mu = 1/4$, obtained from (2) using the equilibrium roughening exponent $\zeta_{\mathrm{eq}} = 2/3$ [30,31]. The situation is quite different for the strong disorder case, where although the fit with the creep formula (1) is still excellent, the value of the exponent $\mu = 0.36 \pm 0.01$ is now clearly in excess with respect to the predicted theoretical value.

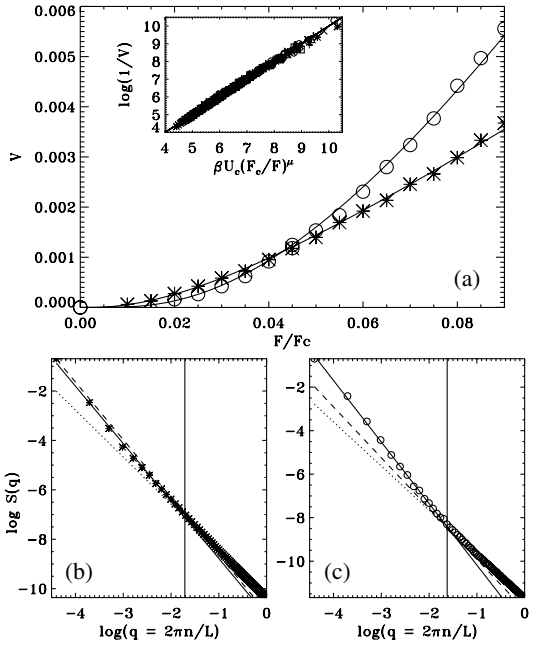

FIG. 2. (a) VF characteristics: (i) $R(0) = 0.12$, $T = 0.24$ (*); (ii) $R(0) = 0.30$, $T = 0.30$ (○). Solid lines are the fitting curves using (1). The inset shows $\log(1/V)$ vs $\beta U_c(F_c/F)^{\mu}$ for all T, $R(0)$, using their respective fitting parameters $\mu(T)$ and $U_c(T)$. (b) [(c)] Structure factor $S(q)$ at $F/F_c = 0.02$, for (i) [(ii)]. Solid lines are fitting curves for small qs. We extract $\zeta = 0.67 \pm 0.05$ for (b) and $\zeta = 0.9 \pm 0.05$ for (c). Dashed (dotted) lines correspond to the reference value $\zeta = 2/3$ ($\zeta_T = 1/2$). The vertical lines indicate the approximate location of the crossover from the thermal to the random manifold scaling.

To understand in more detail the nature of the two regimes, we calculate the roughness exponent ζ using the structure factor (7). Quite generally, one can predict that the short distance behavior of an elastic string is dominated by thermal fluctuations ($\zeta_T = 1/2$). On the other hand, because of the finite velocity, the quenched disorder acts effectively as a thermal noise at the largest length scale. Thus, in this case, the expected exponent is also $\zeta_V = 1/2$ [32]. Finally, at intermediate length scales, the physics is determined by the competition between disorder and elasticity. In particular, in our simulations we verified that the Larkin length [33] is negligible. Therefore, a random manifold scaling, characterized by a nontrivial roughness exponent, takes place. In Figs. 2(b) and 2(c) we show the structure factor for the two cases analyzed in Fig. 2(a). As predicted, we get $\zeta \sim \zeta_T = 1/2$ for large q. At a certain scale we observe a crossover between the thermal and the random manifold scaling. The location of this crossover decreases as T ($R(0)$) is increased (decreased). We can also observe that the second velocity-controlled crossover is not achieved in our finite-size simulation due to the very slow dynamics. Interestingly, for the small disorder case, the random manifold scaling gives $\zeta = 0.67 \pm 0.05$, in excellent agreement with the equilibrium value $\zeta_{\mathrm{eq}} = 2/3$, while a much higher roughness exponent $\zeta = 0.9 \pm 0.05$ is found for the strong disorder case. These results are consistent with the previous ones for the creep exponent μ. This conclusion holds for the whole range of temperature and disorder strength analyzed, as we can see in Fig. 3(a). We find that the relevant parameter to define the two regimes is $T/R(0)$. Moreover, we notice that, although the values of ζ and μ depart from the equilibrium values, relation (2) still seems to hold, within the error bars for the two exponents. This is highly nontrivial since Eq. (2) is derived from a calculation of the barriers in an equilibrium situation.

We finally discuss the behavior of the barriers $U_c(T)$, shown in Fig. 3(b). The observed linear dependence agrees with the phenomenological prediction for the so-called high temperature regime [21,26]. This regime corresponds to $T/R(0) \gg 1$ where most (but not all) of our data lie. Note, however, that [21,26] predict also a strong temperature renormalization of the critical force F_c, which is replaced by $F_c f(T)$ in (1). We find that the resulting effective barrier $f(T)^{\mu} U_c(T)$ is completely inconsistent with our data using the predicted $f(T)$, so the physics of this regime clearly still need to be explained theoretically. We note that [22] also finds a renormalization of $f(T)$ much weaker than the predicted in [21,26]. We finally

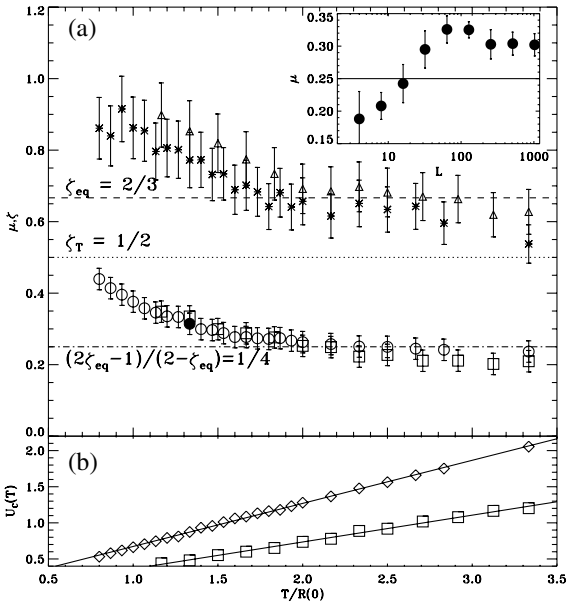

FIG. 3. (a) Roughness exponent, $\zeta(T)$, and creep exponent, $\mu(T)$, vs T. $\zeta(T)$ (*) and $\mu(T)$ (○) correspond to $R(0) = 0.30$ and $\zeta(T)$ (△), and $\mu(T)$ (□) correspond to $R(0) = 0.12$. The dashed line gives the equilibrium roughness exponent $\zeta_{\rm eq} = 2/3$, and the dotted line the purely thermal roughness $\zeta_T = 1/2$. In the inset we show a finite-size analysis of μ for the ● symbol. (b) Effective energy barriers $U_c(T)$ vs T for $R(0) = 0.30$ (◇) and $R(0) = 0.12$ (□).

note that the observed strong temperature dependence is peculiar to low dimensional walls ($d = 1, 2$), where thermal fluctuations lead to unbounded displacements, contrary to what happens in higher dimensions.

In conclusion, we have found two regimes of creep motion. The first one occurs when the temperature is larger than the strength of the disorder, giving $\mu \sim 1/4$ and $\zeta \sim 2/3$ as predicted by assuming a quasiequilibrium nucleation picture of the creep motion. This implies that the domain wall has time to reequilibrate between hops, being the underlying assumption behind (2) essentially satisfied. The second regime occurs for temperatures smaller than the strength of the disorder, and is characterized by anomalously large values of both exponents. This clearly shows that in this regime the domain wall stays out of equilibrium, and that the naive creep hypothesis does not apply. Note that the measured roughness exponent is intermediate between the equilibrium value and the depinning value $\zeta_{\rm dep} = 1.26 \pm 0.01$ [25]. The fact that the thermal nucleation, which is the limiting process in the creep velocity, is followed by depinning like avalanches was noted in the FRG study of the creep [20]. Whether such avalanches and the time it would take them to relax to equilibrium is at the root of the observed increase of the exponent is clearly an interesting but quite complicated open question.

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