

## Short-time growth of a Kardar-Parisi-Zhang interface with flat initial conditions

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(Received 6 August 2012; published 31 October 2012)

The short-time behavior of the  $(1 + 1)$ -dimensional Kardar-Parisi-Zhang (KPZ) growth equation with a flat initial condition is obtained from the exact expressions for the moments of the partition function of a directed polymer with one end point free and the other fixed. From these expressions, the short-time expansions of the lowest cumulants of the KPZ height field are exactly derived. The results for these two classes of cumulants are checked in high-precision lattice numerical simulations. The short-time limit considered here is relevant for the study of the interface growth in the large-diffusivity or weak-noise limit and describes the *universal* crossover between the Edwards-Wilkinson and the KPZ universality classes for an initially flat interface.

DOI: [10.1103/PhysRevE.86.041151](https://doi.org/10.1103/PhysRevE.86.041151)

PACS number(s): 05.40.–a, 05.20.–y, 05.70.Np

### I. INTRODUCTION

The growth of interfaces in the presence of noise can be classified into several universality classes. When the growth rate does not depend on the slope of the interface, the growth process falls in the simplest Edwards-Wilkinson (EW) class. When instead the growth rate is slope dependent (e.g., in the so-called lateral growth), the process falls into the class defined by the nonlinear continuum Kardar-Parisi-Zhang (KPZ) equation [1,2]

$$\partial_t h = \nu \nabla^2 h + \frac{1}{2} \lambda_0 (\nabla h)^2 + \xi(x, t), \quad (1)$$

where  $h(x, t)$  is the interface height,  $\nu$  the diffusivity, and  $\lambda_0$  the strength of the slope-dependent growth (with  $\lambda_0 = 0$  giving the EW model).  $\xi(x, t)$  is the stochastic noise, chosen as a centered Gaussian with short-range correlations:

$$\overline{\xi(x, t) \xi(x', t')} = R_\xi(x - x') \delta(t - t'), \quad (2)$$

where  $\int_x R_\xi(x) dx = D$ . Concerning the space dependence, the most usual choice is to take uncorrelated random disorder, i.e.,  $R_\xi(x - x') = D \delta(x - x')$ .

In one spatial dimension, the KPZ universality class shows a fairly robust anomalous scaling exponent [3] for the fluctuations of the height of the interface  $h(x, t) \sim t^{1/3}$  and indeed such anomalous behavior at large time has been proved for several discrete solvable models [4–7] which are believed to belong to the KPZ class. However, this exponent is only one of the facets of the universality of the KPZ equation: further universal information is encoded in the full probability distribution function (PDF) of these fluctuations, but their exact calculation is an extremely difficult task which is complicated by the fact that, even after a long time, the system keeps some memory of the initial conditions. Remarkably, these initial conditions can be classified in a few subclasses, each leading to a distinct universal result for the statistics of the height field at large time [8,9]. Impressive theoretical progress has been recently achieved and has led to exact solutions directly of the continuum KPZ equation for a wedge (or droplet) [10–13] and flat [14,15] and stationary [16] initial conditions. In the first two cases the PDF of the height  $h(x, t)$  at a given point converges at large time to the

so-called Tracy-Widom Gaussian unitary ensemble (GUE) and Gaussian orthogonal ensemble (GOE) universal distributions [17], for droplet and flat initial conditions, respectively. Further impetus to the field has been given by recent experiments on turbulent liquid crystals [18,19] in which these two long-time predictions have been confirmed with high accuracy.

In the literature, much emphasis has been given to the long-time limit, mainly because of the connection with random matrix theory valid for all models belonging to the KPZ class. However, in the general case the scale of the fluctuations heavily depends on the microscopic details of the model: for instance, the exact values of the mean and the variance of the height fluctuations are known only for a few solvable discrete models. On the contrary, the limit of high diffusivity or weak noise allows complete determination of the scale of the fluctuations as a function of only three parameters  $\lambda_0$ ,  $\nu$ , and  $D$ . All the other microscopic details such as the disorder correlations or the lattice effects are relevant only at very short times,  $t < t_f$ . In particular the above mentioned exact solutions for the KPZ height distributions are valid for arbitrary times  $t > t_f$  in the limit of high diffusivity.

Indeed, these solutions can be expressed in terms of Fredholm determinants with rather complicated kernels, from which it is not always easy to extract the limiting behavior for long and short times. It is then interesting to obtain, by simpler means, the small-time behavior in an explicit form, and to confirm it in numerical simulations. This has been achieved in the case of the droplet initial conditions [11,20], and the aim of this paper is to present a similar result in the case of the flat initial condition. As discussed in more detail below, there are generically three time regimes:

(i) a nonuniversal very-short-time regime  $t \sim t_f$  where the growth depends on the short-scale details of the system [e.g., small deviations from the flat initial condition, the precise form of  $R_\xi(x)$ , etc., . . .];

(ii) a short-time regime  $t_f \ll t \ll t^*$  where the crossover from the EW to the KPZ regime takes place;

(iii) a large-time regime  $t \gg t^*$  where KPZ scaling holds.

In the high-diffusivity limit, since  $t^*$  is fixed to be very large, the height distribution can be exactly computed for all times  $t \gg t_f$ . Conversely, in the low-diffusivity limit, the height

distribution depends on the microscopic details of the system for all times and, when  $t \rightarrow \infty$ , these nonuniversal details affect only the typical scale of the height fluctuations, which are Tracy-Widom distributed.

The paper is organized as follows. In the next section we discuss the mapping of the KPZ equation to the directed polymer. In Sec. III, we report the small-time expansion of the moments of the partition function of the directed polymer (DP) obtained in Ref. [15] and we check them in numerical simulations. In Sec. IV we calculate analytically the small-time expansion of the connected moments of the height field and in Sec. V we check them by numerical simulations. Three Appendices contain some more technical calculations.

## II. MAPPING TO THE DIRECTED POLYMER

Via the Cole-Hopf transformation, the KPZ equation (1) can be mapped onto the directed polymer in a random environment which is an equilibrium statistical physics problem [1,21,22]. A growth starting from a droplet initial condition is mapped onto a fixed-end-point polymer, while a flat initial surface translates to a directed polymer with one end point fixed and the other free [15]. Indeed, the canonical partition function of a directed polymer  $x(\tau)$  at temperature  $T$  in a random environment is defined in the continuum by the path integral

$$Z(x, t | y, 0) = \int_{x(0)=y}^{x(t)=x} Dx e^{-\lambda_0 \int_0^t d\tau [(1/2)(dx/d\tau)^2 + V(x(\tau), \tau)]}, \quad (3)$$

and maps to the KPZ equation after the identifications

$$\frac{\lambda_0}{2\nu} h = \ln Z, \quad 2\nu = T, \quad \lambda_0 \xi(x, t) = -V(x, t). \quad (4)$$

A Gaussian noise  $\xi(x, t)$  corresponds to a random potential  $V(x, t)$  which is a centered Gaussian with correlator  $\overline{V(x, t)V(x', t')} = R_V(x - x')\delta(t - t')$  with  $R_V(x) = \lambda_0^2 R_\xi(x)$ . The white noise in the KPZ equation corresponds in polymer language to disorder with  $\delta$  correlations,

$$\overline{V(x, t)V(x', t')} = \bar{c}\delta(t - t')\delta(x - x'), \quad \bar{c} = D\lambda_0^2. \quad (5)$$

This mapping is valid in the bulk and does not depend on the KPZ initial condition which translates into conditions for the end points of the polymer. For the KPZ equation with flat initial condition, one should consider the partition sum with one fixed end point (at  $x$ ) and another free (at  $y$ ) [14,15], resulting in the partition function

$$Z(x, t) = \int_{-\infty}^{\infty} dy Z(x, t | y, 0). \quad (6)$$

The recent analytical progress has been made possible by the calculation of the moments  $\overline{Z(x, t)^n}$  of the DP partition sum. By replicating the partition function  $Z(x, t)$ , the DP is mapped [23] onto the quantum mechanics of a bosonic system of  $n$  particles interacting with an attractive  $\delta$ -function potential, i.e., the celebrated Lieb-Liniger model [24]. This model is integrable via the Bethe ansatz and the eigenstates are known for both repulsive [24] and attractive interactions [25], which is the case of our interest. The moments can be expressed as a sum over these eigenstates [14,15] (generically labeled by  $\mu$

in the following),

$$\overline{Z(x, t)^n} = \sum_{\mu} \frac{\Psi_{\mu}^*(x, \dots, x)}{\|\mu\|^2} e^{-tE_{\mu}} \times \int_{-\infty}^{\infty} \prod_{j=1}^n dy_j \Psi_{\mu}(y_1, \dots, y_n), \quad (7)$$

in terms of the many-body wave function  $\Psi_{\mu}(y_1, \dots, y_n)$  and of the eigenenergies  $E_{\mu}$  of the state  $\mu$ . In the infinite system the eigenstates are easily enumerated, being organized in clusters of bound particles, called strings. The norms of the states  $\|\mu\|$  and the equal-point wave functions have simple expressions [26] and lead to a time-dependent PDF starting from a droplet initial condition [11,12]. The integral over the  $y_i$  in Eq. (7), necessary to treat the flat initial condition, is more delicate but was handled in Refs. [14,15], leading to the moments  $\overline{Z(x, t)^n}$  for arbitrary  $n$ . From these the moment's generating function at all times has been written in terms of a Fredholm Pfaffian [14,15] (the square root of a Fredholm determinant). This allowed a proof that the PDF of  $\ln Z(x, t)$ , i.e., of the height field  $h(x, t)$ , converges at large times to the GOE Tracy-Widom distribution.

Here we follow a different route. We recall in the next section the exact expressions for the lowest moments  $n = 2, 3, 4$  and from them we extract the small-time cumulants of  $\ln Z$ , i.e., of the KPZ height field.

## III. Moments $\overline{Z^n}$ AND THEIR SMALL-TIME BEHAVIOR

For a flat initial condition, the one-point distribution of  $Z(x, t)$  does not depend on  $x$  because of translational invariance. Thus in the following, we simply denote

$$Z \equiv Z(x, t). \quad (8)$$

Since we are dealing with an initially flat interface we must have  $\overline{Z^n} = 1$  at  $t = 0$  (which is a nontrivial condition in terms of the Bethe ansatz). Taking the average of Eq. (3) over the Gaussian disorder gives the mean partition function

$$\overline{Z(x, t | y, 0)} = \frac{1}{\sqrt{2\pi T t}} e^{-(x-y)^2/2Tt} e^{[R_V(0)/2T^2]t}, \quad (9)$$

and so from the integral in Eq. (6) we have

$$\overline{Z} = e^{v_0 t} \quad \text{with} \quad v_0 = \frac{R_V(0)}{2T^2}. \quad (10)$$

To eliminate this nonuniversal (self-energy) contribution, it is convenient to define

$$z = Z/\overline{Z}, \quad (11)$$

which by construction satisfies  $\overline{z} = 1$  at all times. (Note that in Ref. [15] the self-energy contribution was omitted, but we indicate it here explicitly for later purposes. What is called  $Z$  in Ref. [15] is thus  $z$  here.) This will be useful later for comparison with the numerical simulation of lattice models.

All results for the continuum DP and KPZ models are expressed in terms of a dimensionless parameter

$$\lambda = \left(\frac{t}{4t^*}\right)^{1/3} \quad \text{with} \quad t^* = \frac{2T^5}{\bar{c}^2} = \frac{2(2\nu)^5}{D^2\lambda_0^4}, \quad (12)$$

where, in the language of the DP,  $t^*$  is the crossover time scale between the Brownian diffusion at small time (i.e.,  $\lambda < 1$ ) and the glassy large-time behavior (i.e.,  $\lambda > 1$ ). Within the context of the growth model,  $t^*$  is the crossover scale between Edwards-Wilkinson and KPZ regimes. Note that a spatial crossover scale can also be defined as  $x^* = \sqrt{\nu t^*} = T^3/\bar{c} = (2\nu)^3/D^2\lambda_0^2$ . Both scales become very large in the large-diffusivity limit or, equivalently, in the weak-noise limit.

It is important to recall that for any given microscopical model with a cutoff (e.g., a lattice model) there are additional time and space scales. The easiest example is the same continuum KPZ equation (or DP model) with disorder correlated over a nonzero correlation length  $r_f$ , i.e.,  $R_\xi(x)$  is a function decaying on a scale  $r_f$ . Then, it is easily shown [11] that if  $x^* \gg r_f$  one can replace  $R_\xi(x) \rightarrow D\delta(x)$  in which  $D = \int dx R_\xi(x)$ . More generally, the condition for the existence of the universal short-time regime studied here is that  $t^*$  and  $x^*$  must be much larger than any characteristic microscopic scale—generically called  $r_f$  and  $t_f$  here—such as the lattice spacing for a lattice model. Note also that if the initial condition is not perfectly flat on scales of the order of  $r_f$ , this will also not affect any result as long as  $x^* \gg r_f$ . Of course, the very short time and space regime with  $t \leq t_f$  and  $x \leq r_f$  is nonuniversal.

We now recall the results of Ref. [15] for the four lowest moments, together with their small-time (i.e., small- $\lambda$ ) behavior:

$$\begin{aligned} \overline{z^2} &= e^{2\lambda^3} [1 + \operatorname{erf}(\lambda^{3/2}\sqrt{2})] \\ &= 1 + 2\sqrt{\frac{2}{\pi}}\lambda^{3/2} + 2\lambda^3 + \frac{8}{3}\sqrt{\frac{2}{\pi}}\lambda^{9/2} + O(\lambda^6), \end{aligned} \quad (13)$$

with (note the misprint in Ref. [15] for the definition of the error function)  $\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z dt e^{-t^2}$ ,

$$\begin{aligned} \overline{z^3} &= 4e^{8\lambda^3} - 2e^{2\lambda^3} - 2e^{8\lambda^3} \operatorname{erfc}(\lambda^{3/2}2\sqrt{2}) + e^{2\lambda^3} \operatorname{erfc}(\lambda^{3/2}\sqrt{2}) \\ &= 1 + 6\sqrt{\frac{2}{\pi}}\lambda^{3/2} + 14\lambda^3 + 40\sqrt{\frac{2}{\pi}}\lambda^{9/2} + O(\lambda^{11/2}), \end{aligned} \quad (14)$$

with  $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$ , and finally

$$\begin{aligned} \overline{z^4} &= 8e^{20\lambda^3} - 8e^{8\lambda^3} - 4e^{20\lambda^3} \operatorname{erfc}(3\sqrt{2}\lambda^{3/2}) \\ &\quad - 4e^{8\lambda^3} [-2\operatorname{erfc}(2\lambda^{3/2}) + e^{12\lambda^3} \operatorname{erfc}(4\lambda^{3/2})] \\ &\quad + 48 \int_0^\infty dx \frac{\left(\frac{2x+1}{\sqrt{4x+1}} - \frac{\sqrt{2x+1}}{x+1}\right) e^{-8\lambda^3 x}}{4\pi[4x(x+3)+5]} \end{aligned} \quad (15)$$

$$\begin{aligned} &= 1 + 12\sqrt{\frac{2}{\pi}}\lambda^{3/2} + \left(44 + \frac{24}{\pi}\right)\lambda^3 \\ &\quad + 8(21 + 8\sqrt{2})\sqrt{\frac{2}{\pi}}\lambda^{9/2} + O(\lambda^6), \end{aligned} \quad (16)$$

where the series expansion of the integral is performed in Appendix A.

Before embarking on the calculation of the cumulants of  $\ln Z$ , we now report the results of numerical simulations for the determination of  $\overline{z^n}$  for  $n = 2, 3$ . As explained in more detail in Sec. V, the simulations are performed for a directed polymer on a square lattice. We also consider the high-temperature limit, which ensures that all details of the lattice become irrelevant and the results can be expressed as functions of the single

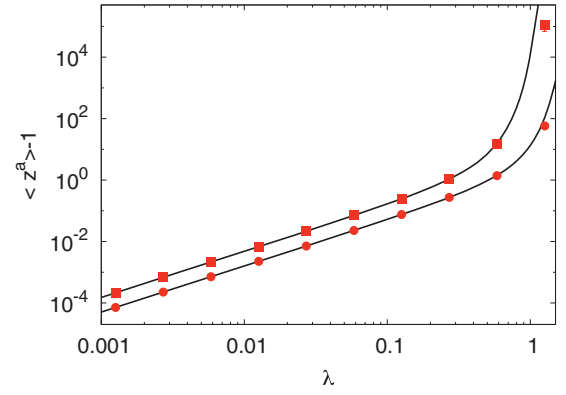


FIG. 1. (Color online) From top to bottom the moments  $\overline{z^3} - 1$  (solid line, red squares) and  $\overline{z^2} - 1$  (solid line, red circles) for different values of  $\lambda$ . Solid lines correspond to the analytical predictions in Eqs. (13) and (14). Averages are performed over  $15 \times 10^6$  samples of size  $t = 512$  and  $\bar{c} = 1$ . There are no adjustable parameters.

parameter  $\lambda$ . The procedure and the identification of  $\lambda$  on the lattice have been introduced already in Refs. [11,27] and are described again in Sec. V. We report the numerical data for  $\overline{z^n}$  in Fig. 1 which are found to be in excellent agreement with our analytic predictions up to  $\lambda \approx 0.6$ , while some deviations at larger  $\lambda$  are evident. These deviations are caused by the undersampling due to the growing importance of the tails in the distribution of  $z$ , and will be properly explained in Sec. V.

#### IV. CUMULANTS OF $\ln Z$ AT SMALL TIME

From the above formulas for  $\overline{z^n}$  and following the procedure described in Appendix B, we obtain the small- $\lambda$  (i.e., small-time) expansion of the first four cumulants of the free energy:

$$\begin{aligned} \overline{\ln z} &= -\sqrt{\frac{2}{\pi}}\lambda^{3/2} + \left(\frac{5}{3} - \frac{6}{\pi}\right)\lambda^3 \\ &\quad + \left(\frac{106}{3} - 16\sqrt{2} - \frac{40}{\pi}\right)\sqrt{\frac{2}{\pi}}\lambda^{9/2} + O(\lambda^6), \end{aligned} \quad (17)$$

$$\begin{aligned} \overline{(\ln z)^2} &= 2\sqrt{\frac{2}{\pi}}\lambda^{3/2} + \left(\frac{20}{\pi} - 6\right)\lambda^3 \\ &\quad + \left(\frac{176\sqrt{2}}{3} + \frac{512}{3\pi} - \frac{412}{3}\right)\sqrt{\frac{2}{\pi}}\lambda^{9/2} + O(\lambda^6), \end{aligned} \quad (18)$$

$$\begin{aligned} \overline{(\ln z)^3} &= \frac{8(\pi - 3)\lambda^3}{\pi} \\ &\quad + \left(248 - 96\sqrt{2} - \frac{352}{\pi}\right)\sqrt{\frac{2}{\pi}}\lambda^{9/2} + O(\lambda^6), \end{aligned} \quad (19)$$

$$\overline{(\ln z)^4} = \left[64\sqrt{2} + \frac{320}{\pi} - 192\right]\sqrt{\frac{2}{\pi}}\lambda^{9/2} + O(\lambda^6), \quad (20)$$

and of course  $\overline{(\ln Z)^p} = \overline{(\ln z)^p}$  for  $p \geq 2$ . As explained in Appendix B, in order to compute the next term  $O(\lambda^6)$  in the small-time expansion, or the fifth and higher cumulants, we would need the fifth moment  $\overline{z^5}$  that we did not analyze here, but which is in principle known [15]. A simple check can be performed on these formulas, namely, one can compute the

series expansion in  $\lambda^{3/2}$  of  $\exp[\sum_{p=1}^4 \frac{n^p}{p!} \overline{(\ln z)^{p^c}}]$  and check that the expansion of all  $\overline{z^n}$  for  $n = 1, 2, 3, 4$  given above is reproduced up to order  $o(\lambda^{9/2})$ . Although this procedure also allows the derivation of Eqs. (17)–(20) by adjusting the coefficients of the series expansion in  $\lambda^{3/2}$ , the method described in Appendix B is more systematic.

For short times, the dominant term in the PDF is the variance  $(\ln z)^{2^c}$  which increases as  $t^{1/2}$ . Using  $\ln Z = \lambda_0 h / (2\nu)$ , one finds

$$\overline{h^2}^c = D \sqrt{\frac{t}{2\pi\nu}} + O(\lambda_0^2 t). \quad (21)$$

Hence the first term of the expansion of  $\overline{h^2}^c$  is independent of  $\lambda_0$ , the coefficient of the nonlinear growth in the KPZ equation. It corresponds to the Edwards-Wilkinson Gaussian scaling regime  $\delta h \sim t^{1/4}$ , also found in Appendix C; cf. Eq. (C3), where we derive the leading short-time behavior for the average height and variance using perturbation theory directly on the KPZ equation.

The third and fourth cumulants behave as  $t$  and  $t^{3/2}$ , respectively, suggesting that the fourth cumulant is subdominant and that the first corrections to the EW Gaussian scaling are given by the third cumulant as  $\delta h \sim t^{1/3}$ , which is the form of the KPZ scaling. Indeed, it is interesting that the third cumulant is linear in  $t$  at both short and long times (but with an amplitude going from  $\frac{8(\pi-3)}{\pi} = 0.360563$  to  $\mu_3^{\text{GOE}} = 0.598268$ ; see below).

From the above we can compute the skewness

$$\begin{aligned} \gamma_1 &= \frac{\overline{(\ln Z)^3}^c}{[\overline{(\ln Z)^2}^c]^{3/2}} = \frac{2^{3/4}(\pi-3)}{\sqrt[4]{\pi}} \lambda^{3/4} \\ &\quad + \frac{\pi(67-48\sqrt{2}+9\pi)-86}{(2\pi)^{3/4}} \lambda^{9/4} + O(\lambda^{15/4}) \\ &= 0.178865 \dots \lambda^{3/4} + 0.0138 \dots \lambda^{9/4} + O(\lambda^{15/4}), \end{aligned} \quad (22)$$

which at short times scales as  $\gamma_1 \sim t^{1/4}$ , and the kurtosis

$$\begin{aligned} \gamma_2 &= \frac{\overline{(\ln Z)^4}^c}{[\overline{(\ln Z)^2}^c]^2} = (40-24\pi+8\sqrt{2}\pi) \sqrt{\frac{2}{\pi}} \lambda^{3/2} + O(\lambda^3) \\ &= 0.115565 \dots \lambda^{3/2} + O(\lambda^3), \end{aligned} \quad (23)$$

which scales as  $t^{1/2}$ .

Now we recall that at large time one can write [14,15]

$$\frac{\lambda_0 h}{2\nu} = \ln Z = v_\infty t + \lambda \eta_t, \quad (24)$$

such that  $\eta_t$  converges to the GOE Tracy-Widom (TW) distribution  $\lim_{t \rightarrow \infty} \text{Prob}(\eta_t < s) = F_1(s)$ . The skewness and kurtosis thus converge for large times to their GOE values

$$\gamma_1 \rightarrow \gamma_1^{\text{GOE}} = 0.29346452408 \dots, \quad (25)$$

$$\gamma_2 \rightarrow \gamma_2^{\text{GOE}} = 0.1652429384 \dots, \quad (26)$$

consistent with a crossover for  $\lambda \approx 1.6 \pm 0.3$ . The amplitude of the (nonfluctuating) linear term is nonuniversal,  $v_\infty = v_0 - \bar{c}^2/12 = v_0 - D^2 \lambda_0^4/12$ , where  $v_0 = \frac{R_V(0)}{2T^2} = \frac{\lambda_0^2 R_\xi(0)}{8\nu^2}$  is the amplitude at short time (after the very-short-time regime  $t \gg t_f$ ). Note that the difference  $v_\infty - v_0$  is universal. At large

time one also has that  $\overline{\ln z} = \overline{\ln Z} - \ln \bar{Z} = \lambda \mu_1 - D^2 \lambda_0^4 t/12$  is universal, where  $\mu_1^{\text{GOE}} = -1.2065335745820 \dots$  is the mean of the TW distribution, while  $\overline{(\ln Z)^c} \rightarrow \lambda^2 \mu_2$  where  $\mu_2^{\text{GOE}} = 1.607781034581 \dots$  is the variance of the TW distribution.

## V. NUMERICAL RESULTS

Numerical simulations are performed for the square lattice model depicted in Fig. 2. Directed paths grow along the diagonals of the lattice with only (0,1) or (1,0) moves (hard constraint condition), starting at (0,0) and with the second end left free. With each site of the lattice is associated an independent and identically distributed random number  $V(x,t)$  (here we use a Gaussian distribution with variance equal to 1). The time coordinate is given by  $t = i + j$  and the space coordinate by  $x = (i - j)/2$  (see Fig. 2). The partition sum over all paths  $\gamma_t$  growing from (0,0) up to time  $t$  is defined as

$$Z(t) = \sum_{\gamma_t} \exp \left[ -\beta \sum_{(x,\tau) \in \gamma_t} V(x,\tau) \right]. \quad (27)$$

The partition function satisfies the following transfer matrix recurrence relation implemented in our simulation:

$$Z_{x,t+1} = (Z_{x-1/2,t} + Z_{x+1/2,t}) e^{-\beta V_{x,t+1}}, \quad (28)$$

with  $Z_{x,0} = \delta_{x,0}$ . The free end partition function is computed by summing over all end points  $Z(t) = \sum_x Z(x,t)$ . To avoid numerical instabilities we divide all partition functions at fixed  $\tau$  by the biggest one and record its logarithmic value. As in the model in the continuum, on the lattice also  $Z(x,t)$  grows exponentially in time, as can be seen by averaging the sum over all possible paths:

$$\overline{Z(t)} = \sum_{\gamma_t} \prod_{x \in \gamma_t} \overline{e^{-\beta V(x)}} = 2^t e^{\beta^2 t/2}. \quad (29)$$

For this reason we work numerically with the ratio  $\ln(z) = \ln(Z/\bar{Z})$ , which remains small, but exhibits strong fluctuations.

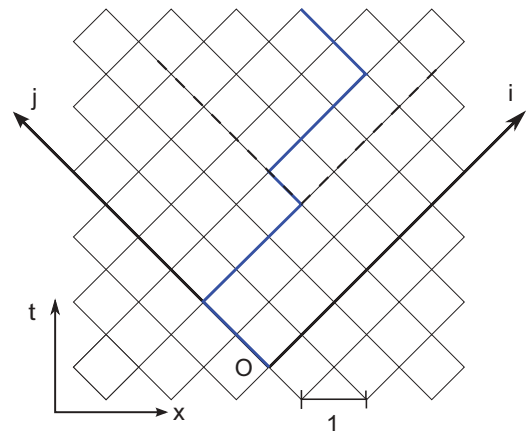


FIG. 2. (Color online) Sketch of the directed polymer model analyzed in numerical simulations. The blue solid line corresponds to a polymer growing over the square lattice under the hard constraint condition.

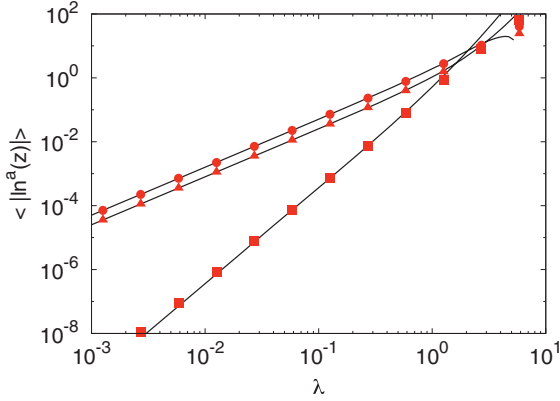


FIG. 3. (Color online) From top to bottom, the cumulants ( $15 \times 10^6$  samples)  $\overline{\ln^2 z}^c$  (solid line, red circles),  $-\overline{\ln z}^c$  (solid line, red triangles), and  $\overline{\ln^3 z}^c$  (solid line, red squares) for  $t = 512$ . The solid lines are the analytical predictions in Eqs. (17)–(19) up to  $O(\lambda^{9/2})$ , with  $\bar{c} = 1$ . There are no adjustable parameters.

In the limit of high  $T$ , the statistical fluctuations of  $z$  depend only on the unique dimensionless variable

$$\lambda = \left( \frac{\bar{c}^2 \kappa t}{8T^5} \right)^{1/3}, \quad (30)$$

which is the lattice version of Eq. (12). Note that the scaling  $T \rightarrow T/\kappa, \bar{c} \rightarrow \bar{c}/\kappa^2$  allows one to go from the discrete model variables Eq. (30) to the continuous model variables Eq. (12).

In the high-temperature regime, the parameters  $\kappa$  and  $\bar{c}$  can be computed explicitly [27]. Indeed  $\bar{c}^2$  is just the variance of the uncorrelated random numbers. Instead  $\kappa$  can be extracted from the model without disorder, for which the polymer behaves like a particle diffusing on a one-dimensional lattice ( $x$  being the particle position at time  $t$ ). The mean square displacement of the particle is given by  $\langle x^2(t) \rangle_T = Tt/\kappa$ . Within the normalization used in this paper, we have  $\kappa = 4T$ .

Using this algorithm, we have numerically computed the cumulants on the square lattice at high temperature and compared them with the analytic predictions in Eqs. (17)–(19). The data for  $\overline{\ln^n z}^c$  for  $n = 1, 2, 3$  are reported in Fig. 3. The

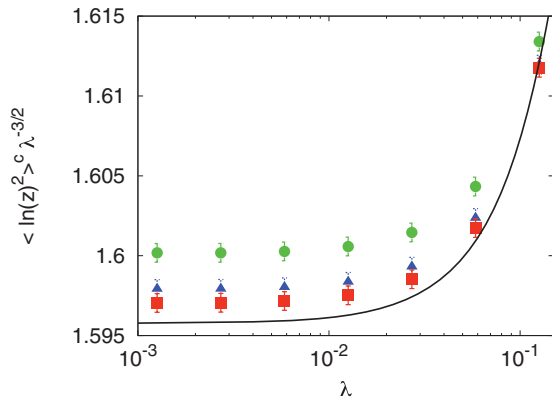


FIG. 4. (Color online) Finite-size effects for  $\overline{\ln^2 z}^c$ . Solid line, analytical prediction Eq. (18). Numerical data, from top to bottom,  $t = 128$  (green circles),  $t = 256$  (blue triangles),  $t = 512$  (red squares). Averages are performed over  $15 \times 10^6$  samples with  $\bar{c} = 1$ .

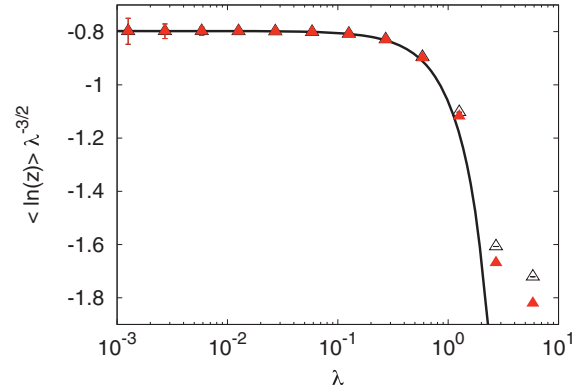


FIG. 5. (Color online) Finite-size effects for  $\overline{\ln z}^c$ . Solid line, analytical prediction Eq. (17). Numerical data, from top to bottom,  $t = 256$  (empty triangles),  $t = 512$  (full triangles). Averages are performed over  $15 \times 10^6$  samples with  $\bar{c} = 1$ . The large error bars when  $\lambda < 10^{-2}$  are due to the vanishing value of  $\overline{\ln z}^c$  as  $\lambda \rightarrow 0$ .

agreement with the analytical predictions is excellent, which is even more impressive when we consider that these figures are produced *without any fit parameter*. In Figs. 4 and 5 we show in more detail the convergence to the theoretical value for a fixed value of  $\lambda$  as a function of polymer length. The increase of the polymer length  $t$  is equivalent to heating up the system, hence approaching the universal prediction of the high-temperature regime.

The analytical predictions for the moments of  $z$  are exact for all  $\lambda$ . However, we can see in Fig. 1 that precision is quickly lost above the threshold  $\lambda \sim 1$ . This is due to the fact that, for large  $\lambda$ , typical values of  $z$  strongly differ from the average value  $\bar{z} = 1$ . The moments of  $z$  are then dominated by rare occurrences of very large  $z$  induced by the presence of heavy tails. This is shown in Fig. 6 where we see that, as  $\lambda$  grows, the mode of the distribution quickly goes to 0 while the tail becomes fatter. A simple example of this peculiar behavior lies in the log-normal probability distribution which is characterized by an exponentially small typical value and a

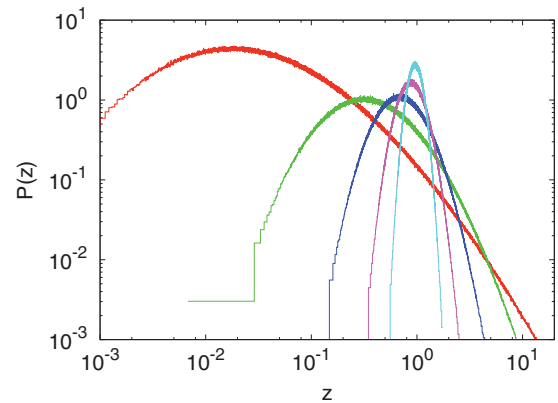


FIG. 6. (Color online)  $P(z)$  with  $z = Z/\bar{Z}$  for  $\lambda = 1.26, 0.58, 0.27, 0.126$ , and  $0.058$ , from left to right. Histograms are obtained from numerical simulations with  $t = 512$ ,  $\bar{c} = 1$ , and  $15 \times 10^6$  samples. When  $\lambda$  is very small,  $P(z)$  is self-averaging. When  $\lambda$  grows, a heavy tail is developed and  $z_{\text{typ}} \ll \langle z \rangle = 1$ .

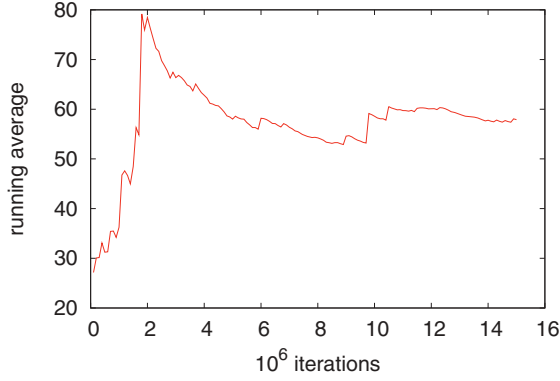


FIG. 7. (Color online) Estimator of  $\overline{z^2} - 1$  given by  $M_N = (1/N) \sum_{i=1}^N z_i^2 - 1$ , where  $z_i$  is the rescaled partition function of a single disorder realization and  $N$  is the number of realizations (iterations), for  $\lambda = 1.26$ . The sudden variations around  $10 \times 10^6$  samples shows that one *single* event contributes to a finite fraction (around 10%) of the whole sum.

heavy tail  $\sim e^{-\ln^2 z}$ . Here, for large  $\lambda$ , the heavy tail behaves as  $e^{-\ln^{3/2} z}$  with the exponent  $3/2$  corresponding to the Tracy-Widom asymptotic behavior. In practice, because of this tail, the moment estimators converge extremely slowly, even for important sampling. An example is shown in Fig. 7, where we estimate  $\overline{z^2} - 1$  for  $\lambda = 1.26$  with  $N = 15 \times 10^6$  samples. While the prediction from Eq. (18) is  $\overline{z^2} - 1 = 109.023$ , we found a value around 60 from numerical simulations. This discrepancy is explained by the central limit theorem which predicts fluctuations of order  $\sim (\overline{z^4}/N)^{1/2}$ . Using Eq. (20), we see that  $\overline{z^4}$  grows very fast in  $\lambda$  and would require  $N = 10^{14}$  samples for us to have a good estimation of  $\overline{z^2}$  for  $\lambda = 1.26$ .

## VI. CONCLUSION

In this paper we have studied the stochastic KPZ equation with flat initial conditions and extracted from the results of Ref. [15] the short-time behavior of the connected moments of the distribution of the height field at a given point. In this way, we have been able to probe universality specifically with respect to the introduction of a short scale in the noise correlations, or a discretization in the DP model. A wider domain of investigation, ranging from step bunching instabilities in crystal growth to ballistic deposition (see, e.g., Ref. [5]) and going beyond the goals of this work, is to prove the universality in a broader sense (including, e.g., change in the nonlinear KPZ term, as in Ref. [28], or biased diffusion current, as in Ref. [29]).

The importance of the results presented here stems from the proof of the existence of a short-time *universal* regime which describes the crossover from the Edwards-Wilkinson to the KPZ growth and which can be observed when the diffusivity is large or the noise is weak. We have compared our analytical predictions to high-precision numerical simulations of a discrete model, which shows how this universality arises. Apart from the theoretical interest *per se*, these predictions, valid for all times, should be useful also in future experiments in which the parameters of the growth could be varied and

controlled in a more refined way so as to easily access this universal crossover.

## APPENDIX A: EXPANSION OF AN INTEGRAL

We need to compute the small- $\lambda$  expansion of the integral

$$I = \int_0^\infty dx f(x) e^{-8\lambda^3 x}, \quad (\text{A1})$$

where  $f(x)$  and its large- $x$  expansion are

$$\begin{aligned} f(x) &= 48 \frac{\left(\frac{2x+1}{\sqrt{4x+1}} - \frac{\sqrt{2x+1}}{x+1}\right)}{4\pi(4x(x+3)+5)} \\ &= \frac{3}{\pi} x^{-3/2} - \frac{3(21+8\sqrt{2})}{8\pi} x^{-5/2} + O(x^{-7/2}). \end{aligned} \quad (\text{A2})$$

It is convenient to write

$$\begin{aligned} I &= \int_0^\infty dx f(x) + \int_0^\infty dx \frac{3}{\pi x^{3/2}} (e^{-8\lambda^3 x} - 1) \\ &\quad + \int_0^\infty dx f_1(x) (e^{-8\lambda^3 x} - 1), \end{aligned} \quad (\text{A3})$$

with  $f_1(x) = f(x) - 3x^{-3/2}/\pi$ . Two integrals are easily done, giving

$$I = 1 - 12\sqrt{\frac{2}{\pi}} \lambda^{3/2} + \int_0^\infty dx f_1(x) (e^{-8\lambda^3 x} - 1), \quad (\text{A4})$$

and the remaining integral is  $O(\lambda^3)$ . This can again be written as

$$\begin{aligned} &\int_0^\infty dx f_1(x) (e^{-8\lambda^3 x} - 1) \\ &= (-8\lambda^3) \int_0^\infty dx x f_1(x) + \int_0^\infty dx f_1(x) (e^{-8\lambda^3 x} - 1 + 8\lambda^3 x) \\ &= \left(44 + \frac{24}{\pi}\right) \lambda^3 + \int_0^\infty dx f_1(x) (e^{-8\lambda^3 x} - 1 + 8\lambda^3 x), \end{aligned} \quad (\text{A5})$$

where the remaining integral is now  $O(\lambda^{9/2})$  and can be split again as

$$\begin{aligned} &\int_0^\infty dx f_1(x) (e^{-8\lambda^3 x} - 1 + 8\lambda^3 x) \\ &= \int_0^\infty dx \left[ -\frac{3(21+8\sqrt{2})}{8\pi} x^{-5/2} \right] (e^{-8\lambda^3 x} - 1 + 8\lambda^3 x) \\ &\quad + \int_0^\infty dx f_2(x) (e^{-8\lambda^3 x} - 1 + 8\lambda^3 x), \end{aligned} \quad (\text{A6})$$

with

$$f_2(x) = f_1(x) + \frac{3(21+8\sqrt{2})}{8\pi} x^{-5/2}. \quad (\text{A7})$$

Thus, putting together the three pieces, we have

$$\begin{aligned} I &= 1 - 12\sqrt{\frac{2}{\pi}} \lambda^{3/2} + \left(44 + \frac{24}{\pi}\right) \lambda^3 \\ &\quad - 8(21+8\sqrt{2}) \sqrt{\frac{2}{\pi}} \lambda^{9/2} + O(\lambda^6). \end{aligned} \quad (\text{A8})$$

### APPENDIX B: FROM THE MOMENTS OF $Z$ TO THE MOMENTS OF $\ln Z$

In general the knowledge of the moments  $\overline{z^n}$  for some low integer  $n$  does not allow one to extract much information about the cumulants  $(\overline{\ln z})^{n^c}$ . In the present case, however, for small time (small  $\lambda$ ),  $z$  is concentrated around its mean value  $\bar{z} = 1$  and this allows one to obtain the behavior of the cumulants at small times.

Let us write  $z = 1 + u$  with  $\bar{u} = 0$  and compute its connected moments. In order to lighten the notation we introduce the notation  $\mu_p \equiv \overline{z^p}^c$ . Using the expressions for  $\overline{z^{n^c}}$  in the main text,  $\overline{u^{n^c}}$  are given by

$$\begin{aligned} \mu_2 &= \overline{u^2} = \overline{z^2} - 1 \\ &= 2\sqrt{\frac{2}{\pi}}\lambda^{3/2} + 2\lambda^3 + \frac{8}{3}\sqrt{\frac{2}{\pi}}\lambda^{9/2} + O(\lambda^6), \end{aligned} \quad (\text{B1})$$

$$\begin{aligned} \mu_3 &= \overline{u^3} = (\overline{z^3} - 1) - 3(\overline{z^2} - 1) \\ &= 8\lambda^3 + 32\sqrt{\frac{2}{\pi}}\lambda^{9/2} + O(\lambda^6), \end{aligned} \quad (\text{B2})$$

$$\begin{aligned} \mu_4 &= \overline{u^4} = \overline{z^4} - 1 - 4(\overline{z^3} - 1) + 6(\overline{z^2} - 1) - 3(\overline{z^2} - 1)^2 \\ &= 64\sqrt{2}\sqrt{\frac{2}{\pi}}\lambda^{9/2} + O(\lambda^6). \end{aligned} \quad (\text{B3})$$

Given the above trend it is reasonable to assume that  $\overline{z^{p^c}} = \overline{u^{p^c}} = O((\lambda^{3/2})^{p-1})$ . Based on this assumption, we want to construct a systematic series expansion of  $(\overline{\ln z})^{n^c}$  in powers of the cumulants of  $z$ . The reasoning is the following. First we write

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{r^n}{n!} (\overline{\ln z})^{n^c} &= \ln \overline{z^r} = \ln \overline{(1+u)^r} \\ &= \ln \left( 1 + \sum_{k=1}^{\infty} \frac{r(r-1)\cdots(r-k+1)}{k!} \overline{u^k} \right). \end{aligned} \quad (\text{B4})$$

Expanding the right-hand side in powers of  $r$ , we obtain formally each  $(\overline{\ln z})^{n^c}$  as an (infinite) series of the moments  $\overline{u^k}$ . The moments  $\overline{u^k}$  can themselves be expressed as functions of the cumulants  $\mu_p$  by writing

$$\overline{e^{wu}} = 1 + \sum_{m=2}^{\infty} \frac{w^m}{m!} \overline{u^m} = \exp \left( \sum_{p=2}^{\infty} \frac{w^p}{p!} \mu_p \right), \quad (\text{B5})$$

and identifying them order by order in  $w$ . We can now replace  $\mu_p \rightarrow a^{p-1} \mu_p$  where  $a$  is to be set to unity at the end. This replacement allows us to keep track of the order in  $\lambda^{3/2}$  of each cumulant. Using MATHEMATICA, it is now simple to first generate the expansion (B5), truncate it to a given order in  $a$ , and then insert it in Eq. (B4). During this process, we see that, e.g.,  $\overline{u^4} = O(a^2) = \overline{u^3}$ , i.e., in Eq. (B4) one must keep a few more orders compared to Eq. (B5). Since we have not computed  $\overline{z^5} = O(a^4)$ , we can get our cumulants only up to

order  $a^3$ . Doing so we obtain

$$\begin{aligned} \overline{\ln(z)} &= -\frac{a\mu_2}{2} + a^2 \left( \frac{\mu_3}{3} - \frac{3\mu_2^2}{4} \right) \\ &\quad + a^3 \left( -\frac{5\mu_2^3}{2} + 2\mu_3\mu_2 - \frac{\mu_4}{4} \right) + O(a^4), \end{aligned} \quad (\text{B6})$$

$$\begin{aligned} \overline{\ln(z)^2}^c &= a\mu_2 + a^2 \left( \frac{5\mu_2^2}{2} - \mu_3 \right) \\ &\quad + a^3 \left( \frac{32\mu_2^3}{3} - 8\mu_3\mu_2 + \frac{11\mu_4}{12} \right) + O(a^4), \end{aligned} \quad (\text{B7})$$

$$\begin{aligned} \overline{\ln(z)^3}^c &= a^2 (\mu_3 - 3\mu_2^2) \\ &\quad + a^3 \left( -22\mu_2^3 + 15\mu_3\mu_2 - \frac{3\mu_4}{2} \right) + O(a^4), \end{aligned} \quad (\text{B8})$$

$$\overline{\ln(z)^4}^c = a^3 (20\mu_2^3 - 12\mu_3\mu_2 + \mu_4) + O(a^4). \quad (\text{B9})$$

Setting  $a = 1$  and replacing the  $\mu_p$  by their actual values above, we find the result given in the text.

### APPENDIX C: SHORT-TIME PERTURBATION THEORY FOR THE KPZ EQUATION

As a further final check, we recover here the leading short-time behavior for the first and second cumulants of the height directly from the perturbative expansion of the KPZ equation. We start from the second cumulant, which is easier. The KPZ equation can be studied in perturbation in  $\lambda_0$ , which is equivalent to short time. This is clear from the definition of  $\lambda$  in Eq. (12) which gives the perturbative parameter  $\lambda^{3/2} \propto \sqrt{t/t^*} \propto \sqrt{t}\lambda_0^2$ . We can write  $h = h^{(0)} + h^{(1)} + \dots$  where  $h^{(n)} = O(\lambda_0^n)$ . With the flat initial condition, the lowest order is just the Edwards-Wilkinson result, which in Fourier space is

$$h_{q,t}^{(0)} = \int_0^t dt_1 e^{-\nu q^2(t-t_1)} \xi_{q,t_1}; \quad (\text{C1})$$

this leads to the variance

$$\overline{h_{q,t}^{(0)} h_{q',t}^{(0)}} = 2\pi \delta(q+q') \int_0^t dt_1 e^{-2\nu q^2 t_1} \tilde{R}_\xi(q), \quad (\text{C2})$$

where  $\tilde{R}_\xi(q)$  is the Fourier transform of the noise correlator  $R(x)$ , assumed to be of range  $r_f$  in space. Then simple algebra gives

$$\begin{aligned} \overline{h^{(0)}(x,t)^2} &= \int \frac{dq}{2\pi} \int_0^t dt_1 e^{-2\nu q^2 t_1} \tilde{R}_\xi(q) \\ &= \int \frac{dq}{2\pi} \tilde{R}_\xi(q) \frac{1 - e^{-2\nu q^2 t}}{2\nu q^2} = \frac{D}{\sqrt{\nu}} \sqrt{\frac{t}{2\pi}}, \end{aligned} \quad (\text{C3})$$

where the last equation is valid for  $t \gg r_f^2/\nu$ , i.e., away from the (nonuniversal) very-short-time regime. Here  $D = \tilde{R}_\xi(q=0) = \int dx R(x)$  is the only memory of the short-scale details and thus for  $t \gg t_f$  one can set  $\tilde{R}_\xi(q=0) = D$ , i.e., we have a  $\delta$  correlator in space. Using the correspondence  $\ln Z = \lambda_0 h / (2\nu)$  and  $\lambda^{3/2} = D \lambda_0^2 (t/8)^{1/2} / (2\nu)^{5/2}$  one recovers exactly the leading term in Eq. (21).

The discussion of the average height  $\bar{h}$  is more subtle because we need to retain  $R_\xi(x)$  in an essential way, as there are nonuniversal contributions. For a flat initial condition,  $\overline{h(x,t)}$

is  $x$  independent; hence the following equation is exact at all times:

$$\partial_t \bar{h} = \frac{\lambda_0}{2} \overline{(\nabla h)^2}. \quad (\text{C4})$$

At small time we can use

$$\partial_t \bar{h} = \frac{\lambda_0}{2} \overline{(\nabla h^{(0)})^2} = \frac{\lambda_0}{4\nu} \int_q \tilde{R}_\xi(q) (1 - e^{-2\nu q^2 t}). \quad (\text{C5})$$

Splitting this term in two pieces and integrating each of them separately over time, we obtain

$$\bar{h} = \frac{\lambda_0 R(0)}{4\nu} t - \frac{\lambda_0}{4\nu} \int \frac{dq}{2\pi} \tilde{R}_\xi(q) \frac{1 - e^{-2\nu q^2 t}}{2\nu q^2}. \quad (\text{C6})$$

One recognizes the same integral entering the second moment, and so for  $t \ll t^*$  we have

$$\bar{h} = \nu_0 t - \frac{\lambda_0}{4\nu} \overline{h^2}, \quad (\text{C7})$$

which indeed reproduces exactly, for  $t \gg t_f$ , the leading *negative* correction in Eq. (17). The first term linear in time is, however, always present, and nonuniversal. The same exact term arises in the DP, and corresponds to the multiplicative contribution to the moments  $\overline{Z^n} = e^{tn(1/2T^2)R_\nu(0)} \equiv e^{tn(\lambda_0^2/8\nu^2)R_\xi(0)}$  arising from the equal replica (self-energy) terms after averaging the replicated partition sum. Although usually dropped, these terms are present and correspond to an additive (nonuniversal) nonrandom correction  $\frac{\lambda_0^2}{8\nu^2} R_\xi(0)t$  to  $\ln Z$ .

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- [1] M. Kardar, G. Parisi, and Y. C. Zhang, *Phys. Rev. Lett.* **56**, 889 (1986).
- [2] A.-L. Barabasi and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, Cambridge, 1995); J. Krug, *Adv. Phys.* **46**, 139 (1997).
- [3] D. A. Huse, C. L. Henley, and D. S. Fisher, *Phys. Rev. Lett.* **55**, 2924 (1985).
- [4] K. Johansson, *Commun. Math. Phys.* **209**, 437 (2000).
- [5] M. Prahofer and H. Spohn, *Phys. Rev. Lett.* **84**, 4882 (2000); *J. Stat. Phys.* **108**, 1071 (2002); **115**, 255 (2004).
- [6] J. Baik and E. M. Rains, *J. Stat. Phys.* **100**, 523 (2000).
- [7] P. L. Ferrari, *Commun. Math. Phys.* **252**, 77 (2004).
- [8] I. Corwin, [arXiv:1106.1596](https://arxiv.org/abs/1106.1596).
- [9] P. L. Ferrari and H. Spohn, [arXiv:1003.0881](https://arxiv.org/abs/1003.0881)
- [10] T. Sasamoto and H. Spohn, *Phys. Rev. Lett.* **104**, 230602 (2010); *Nucl. Phys. B* **834**, 523 (2010); *J. Stat. Phys.* **140**, 209 (2010).
- [11] P. Calabrese, P. Le Doussal, and A. Rosso, *Europhys. Lett.* **90**, 20002 (2010).
- [12] V. Dotsenko, *Europhys. Lett.* **90**, 20003 (2010); *J. Stat. Mech.* (2010) P07010; V. Dotsenko and B. Klumov, *ibid.* (2010) P03022.
- [13] G. Amir, I. Corwin, and J. Quastel, *Commun. Pure Appl. Math.* **64**, 466 (2011).
- [14] P. Calabrese and P. Le Doussal, *Phys. Rev. Lett.* **106**, 250603 (2011).
- [15] P. Le Doussal and P. Calabrese, *J. Stat. Mech.* (2012) P06001.
- [16] T. Imamura and T. Sasamoto, *Phys. Rev. Lett.* **108**, 190603 (2012); *J. Phys. A* **44**, 385001 (2011).
- [17] C. A. Tracy and H. Widom, *Commun. Math. Phys.* **159**, 151 (1994).
- [18] K. A. Takeuchi and M. Sano, *Phys. Rev. Lett.* **104**, 230601 (2010); K. A. Takeuchi, M. Sano, T. Sasamoto, and H. Spohn, *Sci. Rep. (Nature)* **1**, 34 (2011).
- [19] L. Miettinen, M. Mylly, J. Merikoski, and J. Timonen, *Eur. Phys. J. B* **46**, 55 (2005).
- [20] S. Prolhac and H. Spohn, *Phys. Rev. E* **84**, 011119 (2011).
- [21] M. Kardar and Y.-C. Zhang, *Phys. Rev. Lett.* **58**, 2087 (1987); T. Halpin-Healy and Y.-C. Zhang, *Phys. Rep.* **254**, 215 (1995).
- [22] J. M. Burgers, *The Non-Linear Diffusion Equation* (Reidel, Dordrecht, 1974); J. Bec and K. Khanin, *Phys. Rep.* **447**, 1 (2007).
- [23] M. Kardar, *Nucl. Phys. B* **290**, 582 (1987).
- [24] E. H. Lieb and W. Liniger, *Phys. Rev.* **130**, 1605 (1963).
- [25] J. B. McGuire, *J. Math. Phys.* **5**, 622 (1964).
- [26] P. Calabrese and J.-S. Caux, *Phys. Rev. Lett.* **98**, 150403 (2007); *J. Stat. Mech.* (2007) P08032.
- [27] S. Bustingorry, P. Le Doussal, and A. Rosso, *Phys. Rev. B* **82**, 140201 (2010).
- [28] J. Krug and H. Spohn, *Phys. Rev. A* **38**, 4271 (1988).
- [29] A. Pimpinelli, V. Tonchev, A. Videcoq, and M. Vladimirova, *Phys. Rev. Lett.* **88**, 206103 (2002).