Comment on "Interaction-induced dephasing of Aharonov-Bohm oscillations"

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In Ref.[1], Ludwig & Mirlin (LM) have studied the decay of Aharonov-Bohm (AB) harmonics of the conductance of a weakly disordered metallic ring of perimeter L, due to decoherence from electron-electron interaction. They obtained the result [eq. (16)]

$$\langle\delta g_n^2\rangle_{\rm LM} \sim \left(\frac{L_T}{L}\right)^{7/2}\left(\frac{\nu_0 D}{L}\right)^{3/4}\mathrm{e}^{-n\,S_{\rm inst}} \tag{1}$$

where $S_{\rm inst} = \frac{C_\gamma}{C_0}\,\frac{\pi}{8}\,(\frac{L}{L_N})^{3/2} \sim \frac{L^{3/2}T^{1/2}}{\nu_0^{1/2}D}$. The parameter C_γ involves the ratio of the perimeter and the length of the connecting wires ($\gamma=0$ corresponds to infinitely long connecting wires). The ratio C_γ/C_0 varies monotonously between 1 and $C_1/C_0 = 1/\sqrt{2}$. ν_0 is the density of states and D the diffusion constant. $L_T=\sqrt{D/T}$ is the thermal length. We have introduced the Nyquist length $L_N=(\nu_0 D^2/T)^{1/3}$. We argue here that, in eq. (1), the prefactor $\propto T^{-7/4}$, is incorrect.

Path integral formulation.– The starting point of LM is the path integral formulation introduced in Ref.[2]. The AB harmonics $\langle\delta g_n^2\rangle$ can be cast in the form :

$$\langle\delta g_n^2\rangle \sim \frac{D^2}{R^4T}\int_0^\infty \mathrm{d}t\int \mathrm{d}\Theta_1\mathrm{d}\Theta_2\int_{\theta_1(0)=\Theta_2}^{\theta_1(t)=\Theta_1}\mathcal{D}\theta_1\int_{\theta_2(0)=\Theta_2}^{\theta_2(t)=\Theta_1}\mathcal{D}\theta_2$$
$$\times\mathrm{e}^{-\int_0^t\mathrm{d}t'\left[\frac{R^2\dot\theta_1^2}{4D}+\frac{R^2\dot\theta_2^2}{4D}+V(\theta_1,\theta_2)\right]} \equiv \frac{D^2}{R^4T}\times\mathcal{I}_n \tag{2}$$

with the constraint that the pair of paths, put end to end, have a winding n around the ring (the fields $\theta_{1,2}(t')$, $t'\in[0,t]$, live inside the ring). $R=L/(2\pi)$ is the radius. The potential $V(\theta_1,\theta_2)$ accounts for decoherence due to electron-electron interaction. The path integral is then estimated by semiclassical arguments. As noticed by LM the dominant classical paths satisfy $\theta_1^{cl}(t')=-\theta_2^{cl}(t')$ with initial and final conditions $\Theta_1=0$ and $\Theta_2=\pi$. Then for $n=1$ "the problem is reduced to that of a particle of mass $m=R^2/D$ tunneling with zero energy in the potential $V(\theta)=V(\theta,-\theta)=\frac{4RT}{\nu_0 D}\theta(1-\frac{\theta}{\pi})$" with $\theta\in[0,\pi]$. To simplify we consider here the limit of long connecting wires ($\gamma=0$) since the temperature dependence of the prefactor is not expected[1] to depend on γ. They obtained $\mathcal{I}_n\propto\mathrm{e}^{-nS_{\rm inst}}$ where $S_{\rm inst}$ is the action given above. Our statement is that LM have not used properly the semiclassical method to estimate the remaining prefactor. We first recall their estimation.

LM's prefactor.– LM wrote that the prefactor have three origins. (A) Fluctuations around classical paths : each path integral brings a factor $\sqrt{m\omega}$ where $\omega\sim\sqrt{T/(\nu_0 R)}$. This gives a factor $m\omega\sim S_{\rm inst}$. (B) Fluctuations around optimal initial and final points lead to a factor $S_{\rm inst}^{-1}$. (C) Fluctuations around the optimal time leads to a factor $t_{\rm opt}/\sqrt{S_{\rm inst}}$ where $t_{\rm opt}\sim\omega^{-1}\sim\sqrt{\nu_0 R/T}$ is the time needed by the instanton to cross the quadratic barrier. $\mathcal{I}_n$'s final prefactor is therefore[1] $\frac{t_{\rm opt}}{\sqrt{S_{\rm inst}}}\sim\frac{\nu_0^{3/4}D^{1/2}}{T^{3/4}R^{1/4}}$ which leads to (1) after multiplication by $D^2/(R^4T)$.

Path integral for series of quadratic barriers.– We show that the points (A) and (C) lead to a wrong result when applied to the simpler calculation of the path integral

$$C(n,0)=\int_0^\infty\mathrm{d}t\int_{\chi(0)=0}^{\chi(t)=n}\mathcal{D}\chi\,\mathrm{e}^{-\int_0^t\mathrm{d}\tau\left[\frac14\dot\chi^2+\mathcal{V}(\chi)\right]} \tag{3}$$

where the field χ lives on $\mathbb{R}$ and $\mathcal{V}(\chi)$ is a periodic potential (to mimic the ring) given by $a\chi(1-\chi)$ for $\chi\in[0,1]$. If $a\gg1$ we can compute the path integral with semiclassical approximation. For $n=1$, the result is dominated by the instanton crossing the barrier in a time $t_{\rm opt}=\pi/(2\sqrt{a})$ with the action $S_{\rm inst}=\frac{\pi}{8}\sqrt{a}$. It is well known that semiclassical method must be used with care when initial and final points are turning points (where energy= $\mathcal{V}(\chi)$). In this case, in the neighbourhood of the turning points, the semiclassical solution must be matched to the exact solution for a linear potential (given by Airy functions). We obtain[4] :

$$C(1,0)\simeq-\frac{\sqrt{3}\mathrm{Ai}(0)}{4\mathrm{Ai}'(0)}\,a^{-1/3}\,\mathrm{e}^{-S_{\rm inst}} \tag{4}$$

Now we compare this expression with the result obtained by following LM's arguments. According to points (A) and (C) (here we do not integrate over initial and final points), the prefactor should be $\sqrt{m\omega}\,t_{\rm opt}/\sqrt{S_{\rm inst}}$ where $m=1/2$ and $\omega=2\sqrt{a}$, therefore LM's arguments yield the incorrect result $C(1,0)\sim a^{1/4}\frac{a^{-1/2}}{a^{1/4}}\mathrm{e}^{-S_{\rm inst}}=a^{-1/2}\mathrm{e}^{-S_{\rm inst}}$.

Correct AAS and AB amplitudes for $\gamma\simeq0$.– It was shown in Ref.[3] that the relation $\mathcal{I}_n=\frac{L^2}{2D}C(n,0)$ is exact for $\gamma=0$. In this case harmonics of the Al'tshuler-Aronov-Spivak (AAS) oscillations is precisely given by $\langle\Delta g_n\rangle\sim-C(n,0)$ with $a=(L/L_N)^3$. This leads to

$$\langle\Delta g_n\rangle\sim\frac{L_N}{L}\,\mathrm{e}^{-n\frac{\pi}{8}(\frac{L}{L_N})^{3/2}}\qquad\text{for } L\gg L_N \tag{5}$$

AAS oscillations are related to the AB oscillations by $\langle\delta g_n^2\rangle \sim (\frac{L_T}{L})^2\langle\Delta g_n\rangle$. Therefore, for $L \gg L_N$:

$$\langle\delta g_n^2\rangle \;\sim\; \left(\frac{L_T}{L}\right)^{8/3}\left(\frac{\nu_0 D}{L}\right)^{1/3} \mathrm{e}^{-n\frac{\pi}{8}(\frac{L}{L_N})^{3/2}} \qquad (6)$$

The expected temperature dependence of AB harmonics is $\langle\delta g_n^2\rangle \sim T^{-4/3}\mathrm{e}^{-nL^{3/2}T^{1/2}}$. Results (5,6) hold for $\gamma \lesssim 11(\frac{L}{L_N})^{3/2}$ as shown in Ref.[7].

Effect of the connecting arms.– Note that LM's path integral (2) is an approximation since it neglects the possibility for the trajectories to explore the arms. This approximation is reasonable for $L_N \ll L$. Therefore in LM's calculation, the presence of the connecting arms appears through their parameter γ entering in $V(\theta_1, \theta_2)$ only. As a consequence the path integral they consider is very similar to the one for an isolated ring (moreover exactly the same for $\gamma = 0$).

The leads can have several effects : (*i*) In the regime $L_N \gg L$, winding trajectories spend most time in the arms, which modifies strongly the winding properties and affects decoherence[3,5]. (*ii*) In the regime $L_N \ll L$, short arms are responsible for additional temperature-dependent preexponential factors for two reasons. First, if arms are shorter that the phase coherence length ($l_a \ll L_N$), winding trajectories feel the reservoirs[3,5]. This cannot be taken into account within LM's scheme of approximations. Second, as pointed by LM, short leads ($l_a \ll L$) induce inhomogeneous cooperon inside the ring which also brings some additional L_N dependent factor [their eq. (A.1), factor (B) above]. However, as it was discussed in Ref.[7], this can only occurs in situations rather irrealistic experimentally, if one wants to avoid the first effect (since it should conciliate $L_N \ll L$, $l_a \gg L_N$ and $l_a \ll L$). As a conclusion, results (5,6) for $\gamma \simeq 0$ seem more appropriate in practice.

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4 Here the potential present cusps at $\chi = 0$ and $\chi = 1$, however we obtain similar a dependence for a periodised potential or a single quadratic barrier. In this latter case, $\mathcal{V}(\chi) = a\chi(1-\chi)$ for $\chi \in \mathbb{R}$, one finds $C(1,0) \simeq -\frac{\pi}{2}[\mathrm{i\,Ai}(0) + \mathrm{Bi}(0)]^2 a^{-1/3}\mathrm{e}^{-\frac{\pi}{8}\sqrt{a}}$ for the retarded Green's function.

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6 Note that the correct dependence in the lengths of the arms (denoted l_a and l_b, both $\gg L$) are obtained[3] by multiplying (5) by $L^2/(l_a+l_b)^2$ and (6) by $L^4/(l_a+l_b)^4$.

7 C. Texier, Effect of connecting wires on the decoherence due to electron-electron interaction in a metallic ring, preprint cond-mat arXiv:0707.2916.