EFFECT OF INCOHERENT SCATTERING ON SHOT NOISE CORRELATIONS IN THE QUANTUM HALL REGIME

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We have studied the effect of incoherent scattering in a Hanbury Brown and Twiss situation with electrons in edge states of a three-terminal conductor submitted to a strong perpendicular magnetic field. The modelization of incoherent scattering is performed by introducing an additional voltage probe through which the current is kept equal to zero which causes voltage fluctuations at this probe. It is shown that inelastic scattering can lead in this framework to positive correlations, whereas correlations remain always negative for quasi-elastic scattering.

1 Introduction

The effect of incoherent scattering on transport and noise in mesoscopic structures has been of interest in a number of previous works (for review see [1,2]). In our work [3] we examine how it affects the noise correlation in a 3-terminal structure in the Integer Quantum Hall regime.

To demonstrate the reality of edge states [4] (despite their small contribution to the overall density of states) the possibility of creating a non-equilibrium population [5] has been crucial. Several experiments have investigated the equilibration of edge states selectively populated with the help of various contacts by transport measurements[6,7,8,9]. A natural problem is to investigate the effect of inter-edge state scattering on the noise on such structures.

Recently, Hanbury Brown and Twiss (HBT) experiments studying current correlations using partially degenerate stream of fermions were performed[10,11], starting from the idea [12] of using the edge states in a conductor submitted to a strong magnetic field. A Y-structure is discussed in Ref. [13] and a HBT experiment without a magnetic field has also been realized[14]. Here we are interested in the experimental arrangement of Oberholzer *et al.* [11] which is depicted in figure 1, with the only difference that the magnetic field, in this experiment, was adjusted in such a way that only one spin-degenerate edge state (filling factor $\nu = 2$) carries the current, and not two, as it is represented on the figure. Contact 1, which is at a potential eV above the potentials of the two other contacts, acts as a carrier source and contacts 2 and 3 as detectors for the beams of electrons. The incident beam is splitted at two quantum point contacts (QPC) which can be tuned by applied voltages. In the following we will denote by T_1 and $R_1 = 1 - T_1$ the transmission and reflection coefficients at QPC at contact 1, and introduce T_3 and R_3 for QPC at contact 3. The quantity of interest in this experiment, revealing statistical properties of the carriers, is the correlation $S_{23} = \langle \Delta I_2 \Delta I_3 \rangle$ between the current at the two contacts. To be precise the noise spectrum is defined as $S_{\alpha\beta}(\omega) 2\pi\delta(\omega+\omega') = \langle \Delta\hat{I}_\alpha(\omega)\Delta\hat{I}_\beta(\omega') + \Delta\hat{I}_\beta(\omega')\Delta\hat{I}_\alpha(\omega)\rangle$ with $\Delta\hat{I}_\alpha(\omega) = \hat{I}_\alpha(\omega) - \langle\hat{I}_\alpha(\omega)\rangle$, where $\hat{I}_\alpha(\omega)$ is the Fourier transform of the current operator at contact α. (For a recent presentation of the formalism and notations, see Ref. [2]). The zero frequency limit will be denoted: $S_{\alpha\beta} = S_{\alpha\beta}(\omega = 0)$. Following the scattering approach, we construct the scattering matrix for the system to calculate the noise correlation [15,2]:

$$S_{\alpha\beta} = \frac{2e^2}{h} \int \mathrm{d}E \sum_{\gamma,\lambda} \mathrm{Tr}\left\{ A^{\alpha}_{\gamma\lambda} A^{\beta}_{\lambda\gamma} \right\} f_\gamma (1 - f_\lambda) \,, \tag{1}$$

where f_α is the Fermi-Dirac distribution at the contact α; we consider the case of zero temperature. The matrices A's are related to the on-shell s-matrix: $A^{\alpha}_{\gamma\lambda} = \delta_{\alpha\gamma}\delta_{\alpha\lambda} - s^{\dagger}_{\alpha\gamma}(E)s_{\alpha\lambda}(E)$.

After having constructed the s-matrix for the arrangement of figure 1 with one edge state we can compute the correlations between the currents at contacts 2 and 3, which is eventually found to be [11]:

$$S_{23} = -\frac{2e^2}{h}|eV|R_3T_3T_1^2 \, . \tag{2}$$

The R_3T_3 factor is the usual partition factor due to the separation of the electron beam at QPC 3. The T_1^2 term is due to the fact that the processes leading to correlations between contacts 2 and 3 involve two electrons transmitted through the QPC 1, which happens with probability T_1^2. Compared to the initial experiment by Henny *et al.* [10] the experiment by Oberholzer *et al.* [11] introduces fluctuations in the incident beam and thus the current correlations are not determined already from current conservation alone.

In the following we will be interested in the case where the distance between the QPC's is long so that incoherent processes may occur along this edge. If only one edge state is populated, phase breaking processes are of little interest since S_{23} measures in this case quantum-statistical properties of the carriers revealed by the separation of the beam at QPC 3, and what happens before this separation is of little importance. In contrast, when several edge states are populated, inter-edge state scattering along one boundary may cause a redistribution of the carriers between the edge states and thus modify the noise properties of each channel. This is the situation that has interested us [3]. We will not take into account the spin degeneracy which only would multiply the conductances and the noise by a factor 2.

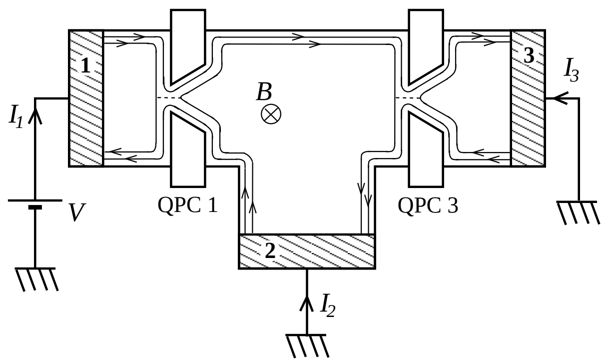

Figure 1: Experimental arrangement of [11]. The two edge states are populated at contact 1. One of those channels is perfectly transmitted the other being partially transmitted at QPC 1. The transmission may be tuned by an applied gate voltage. QPC 3 splits the partially degenerate beam and distributes the current to contacts 2 and 3 where it is measured. Without inter-edge state scattering, the perfectly transmitted channel does not manifest itself on the correlations.

2 Coherent inter-edge state scattering

To describe coherent scattering between edge states we introduce the probability ε that an electron, starting in one of the two channels, is scattered into the other edge state when it travels between the two QPC's. After having constructed the s-matrix, formula (1) gives [3]:

$$S_{23}^{\mathrm{coh}} = -\frac{2e^2}{h}|eV|R_3\Big[(1-\varepsilon)^2T_3T_1^2 + \varepsilon^2T_3 + \varepsilon(1-\varepsilon)(1-2R_3T_1+T_1^2)\Big] \, . \tag{3}$$

Let us now discuss two limiting cases. If $\varepsilon = 0$ (no inter-edge state elastic scattering) we fall back to the previous situation (2) since the outer edge state is noiseless. Another interesting case occurs for $T_3 = 0$ (see figure 3). Then the two edge states are perfectly separated at QPC 3. The first edge state flows towards contact 3 whereas the second goes to contact 2. This situation is particularly interesting since it allows us to study separately the noise spectrum of each edge state after having travelled along the upper edge of the conductor. In the absence of scattering between edge states, the currents remain uncorrelated, whereas if redistribution of charges occurs between edge states, it may induce correlations. For coherent scattering, we find: $S_{23}^{\text{coh}} = -\frac{2e^2}{h}|eV|\varepsilon(1-\varepsilon)R_1^2$. The only separation of the beam that can cause correlations occurs along the edge with usual partition factor $\varepsilon(1-\varepsilon)$; the factor R_1^2 ensures that, if the currents in the two channels are noiseless before exchanging carriers along the upper edge, they remain noiseless and uncorrelated.

3 Incoherent inter-edge state scattering

We modelize the presence of incoherent scattering by adding a fictitious contact at the edge along which incoherent scattering is expected to occur [16,1,2]. Let us remark that the only place where it can have some influence on the correlations S_{23} is the edge between the two QPC's where we have introduced it (see figure 2). Since the current is obviously conserved along the edge, we have to impose that the current I_4 through this contact is zero at any time. The consequence is that the potential μ_4 at this probe is fluctuating.

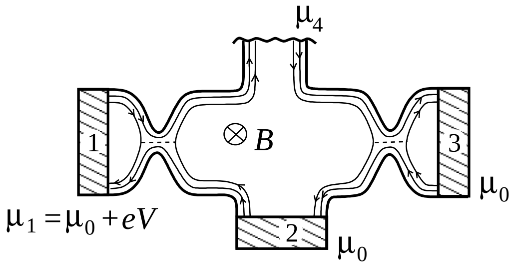

Figure 2: Modelization of phase breaking processes along the upper edge: introduction of a fictitious contact.

We consider two models of incoherent scattering: (*i*) a model of inelastic scattering [16] which is defined by imposing that the total current through the fictitious contact vanishes $I_4 = 0$ and that the average distribution function at this contact is a Fermi distribution. (*ii*) A model of quasi-elastic scattering [1] where not only the total current $I_4 = \int \mathrm{d}E\, j_4(E)$ vanishes but moreover the contribution to the current of the states of energy E vanishes $j_4(E) = 0$; it follows that the average distribution function at contact 4 is not the equilibrium Fermi distribution.

In both models the fact that I_4 vanishes on average gives information on the average distribution function at the fictitious contact and imposing that the fluctuations vanish also, $\Delta I_4 = 0$, relates the intrinsic fluctuations of the current to the potential fluctuations $\delta\mu_4$.

Finally we obtain the following results (for details on the derivation, see [3]), for the quasi-elastic case:

$$S_{23}^{\text{qe}} = -\frac{e^2}{h}|eV|\frac{R_3}{4}\left[2T_3(1+T_1)^2 + (1+T_3)R_1^2\right], \tag{4}$$

and for the inelastic case:

$$S_{23}^{\text{in}} = -\frac{e^2}{h}|eV|\frac{R_3}{2}\left[2T_3(1+T_1) - (1+T_3)R_1T_1\right] \tag{5}$$

leading to the possibility of positive correlations (let us recall that correlations in the presence of coherent scattering only are always negative due to the fermionic nature of carriers [15]). If $T_3 = 0$, we get $S_{23}^{\text{in}} = +\frac{e^2}{h}|eV|\frac{R_1T_1}{2}$. This result means that this modelization of inelastic scattering gives the possibility for a fluctuation of the potential μ_4 to inject in a correlated way in the two channels some current.

Note that the occurence of positive correlation S_{23}^{in} might be destroyed by two factors [3]: (i) large transmission T_3, (ii) temperature effects since we should recover at temperature $kT \gg |eV|$ the fluctuation-dissipation theorem.

On table 1 we summarize the various results for the noise correlations obtained for $T_3 = 0$ (see figure 3). To compare the coherent case with the incoherent cases, we set $\varepsilon = 1/2$ since the two models of incoherent scattering equilibrate the current in the two channels.

Nature of scattering	$\tilde{S}_{23}$	$\tilde{S}_{22}$	$\tilde{S}_{33}$
no scattering	0	$2R_1T_1$	0
coherent ($\varepsilon = 1/2$)	$-\frac{R_1^2}{2}$	$\frac{1+T_1}{2}R_1$	$\frac{1+T_1}{2}R_1$
quasi-elastic	$-\frac{R_1^2}{4}$	$\frac{1+3T_1}{2}\frac{R_1}{2}$	$\frac{1+3T_1}{2}\frac{R_1}{2}$
inelastic	$+\frac{R_1T_1}{2}$	$\frac{R_1T_1}{2}$	$\frac{R_1T_1}{2}$

Table 1: Noise and correlation for $T_3 = 0$ depending on the nature of inter-edge state scattering. $S_{\alpha\beta} = \frac{e^2}{h}|eV|\tilde{S}_{\alpha\beta}$.

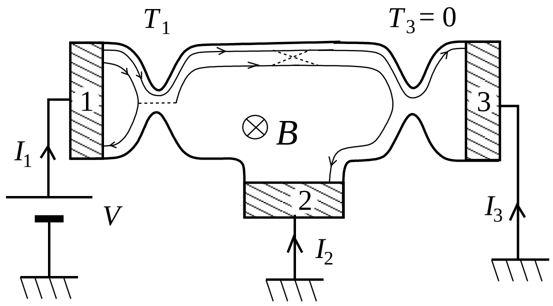

Figure 3: Flow of the average current when $T_3 = 0$.

4 Conclusion

Transport measurements [6,7,8] have provided information on the presence of inter-edge state scattering. Here we have shown that the study of noise correlation may give some information, not only on the existence, but moreover on the nature of inter-edge state scattering: if $T_3 = 0$, in the absence of inter-edge state scattering the correlation S_{23} vanishes, for elastic scattering and for quasi-elastic scattering it is negative and for inelastic scattering it is positive.

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