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Effect of Incoherent Scattering on Shot Noise Correlations in the Quantum Hall Regime

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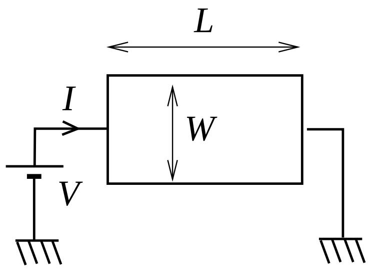
0. Foreword : Mesoscopic physics

• Macroscopic conductor :



$$\vec{j} = \sigma \vec{E} \qquad \text{(Ohm's law)}$$

$\frac{I}{W} = \sigma \frac{V}{L} \Rightarrow$ Conductance : $\boxed{G = \frac{I}{V} = \sigma \frac{W}{L}}$

Until when the formula remains valid if one diminishes W, L ?

• Mesoscopic physics :

concerned with intermediate scales at which quantities
of interest are sensitive to quantum interference

micro $\ll$ meso $\ll$ macro

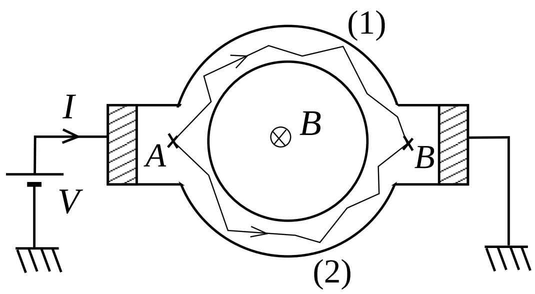
Electrons feel : - elastic collisions (impurities, defects,...)
$\Rightarrow$ elastic mean free path ℓ_e
- inelastic collisions (e-ph, e-e,...)
$\Rightarrow$ phase coherence length $L_\varphi(T)$

mesoscopic regime : $\boxed{L \lesssim L_\varphi(T)}$

(Ex. : $L_\varphi(T) \sim 10\mu m$ in AsGa at $T = 50mK$)

• A mesoscopic effect : Aharonov-Bohm oscillations of G

(Webb & Washburn, 1986)



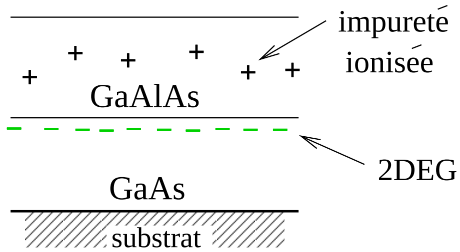
$$\begin{aligned} G &\sim |\mathcal{A}(A \to B)|^2 \\ &= |\mathcal{A}_{(1)} + \mathcal{A}_{(2)}|^2 \\ &= \left| |\mathcal{A}_{(1)}| e^{i\frac{e}{\hbar}\phi_1 + i\chi} + |\mathcal{A}_{(2)}| e^{i\frac{e}{\hbar}\phi_2} \right|^2 \end{aligned}$$

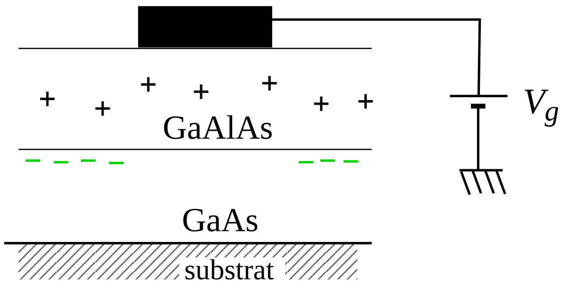
$$\mathcal{A}_{\mathcal{C}} \propto \exp i\frac{e}{\hbar} \int_{\mathcal{C}} d\vec{r} \cdot \vec{A}$$

$$G(\Phi) \simeq g_0 + g_1 \cos\left(2\pi \frac{\Phi}{\phi_0} + \chi\right)$$

where $\Phi = \phi_1 - \phi_2$ is the magnetic flux through the ring

• Mesoscopic Samples : 2d electron gas (2DEG)

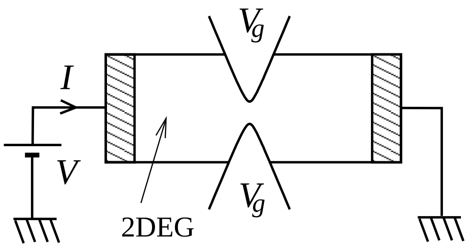




Metallic gate $\Rightarrow$ local control of electron density

• Quantification of conductance (van Wees *et al.*, 1988)

$\rightarrow$ Transmission through constriction is adjustable by tuning V_g



quantum of conductance : $G_0 = \frac{e^2}{h}$

Noise

- Transport : averaged current $\langle I \rangle = GV$

- Noise : current fluctuations $\boxed{S = \langle \Delta I^2 \rangle}$

origin of fluctuations : thermal noise or/and shot noise
Shot noise :

Classical theory : $S = 2e\langle I \rangle$ (Poisson)
Quantum theory for one channel with transmission T :
$S = 2e\langle I \rangle (1 - T)$

- Measure of the fractional charge excitations in the FQHE

→ 2DEG submitted to strong magnetic field

→ fractional quantum Hall effect (FQHE) regime :

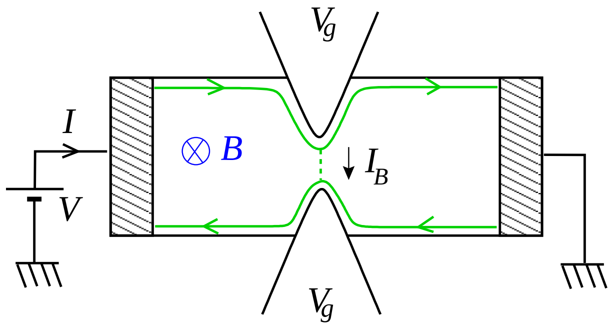
$\sigma_{xy} = \nu \frac{e^2}{h}$ with $\nu = 1/3,\ 2/5,\ 2/3, \ldots$

(Tsui, Störmer & Gossard, 1982)

→ Excitations have fractional charges

(Laughlin, 1983)

→ Current carried by edge states



For $\nu = 1/3$
(Saminadayar *et al.*, 1997)
(de-Picciotto *et al.*, 1997)
For weak backscattering,
shot noise is :

$$S = 2\langle \Delta I^2 \rangle = 2e^* \langle I_B \rangle$$

measurement of $\boxed{e^* = \frac{e}{3}}$

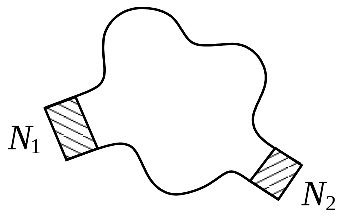
- Universality

a. Chaotic cavities

$S = 2e\,\langle I\rangle\,\frac{N_1 N_2}{(N_1+N_2)^2} + \text{(weak loc.)}$

for symmetric cavity : $S = 2e\,\langle I\rangle\,\frac{1}{4}$

(Nazarov, 1995)
Experiment : (Oberholzer *et al.*, 2000)



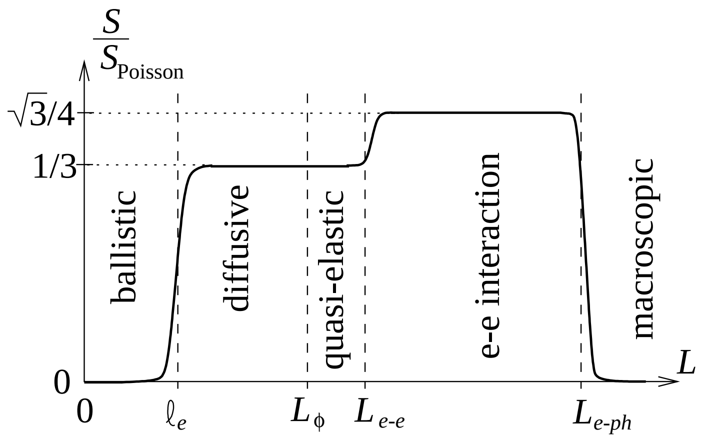
b. Disordered conductors

various approches : ⋆ random matrices (Beenakker & Büttiker, 1992)
⋆ Boltzmann-Langevin approach (Nagaev, 1992)
⋆ diagrammatic (Altshuler, Levitov & Yakovets, 1994)

→ For a disordered wire (with elastic scattering) : $S = 2e\,\langle I\rangle\,\frac{1}{3} + \text{(weak loc.)}$

→ weak localization & universal fluctuations
(de Jong & Beenakker 1992), (Macêdo & Chalker 1994)

→ Effect of $e-e$ interactions (if $\ell_{e-e} \ll L \ll \ell_{e-ph}$) : $S = 2e\,\langle I\rangle\,\frac{\sqrt{3}}{4}$
(Nagaev, 1995)

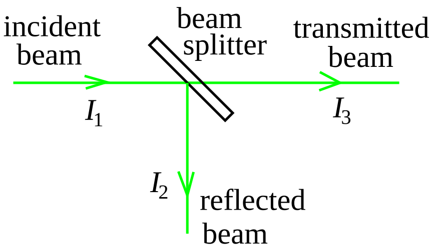


Experiment : (Henny *et al.*, 1999)

1. Introduction - Motivations

⊛ Hanbury-Brown and Twiss (HBT) experiments with fermions

- What is a HBT experiment ?



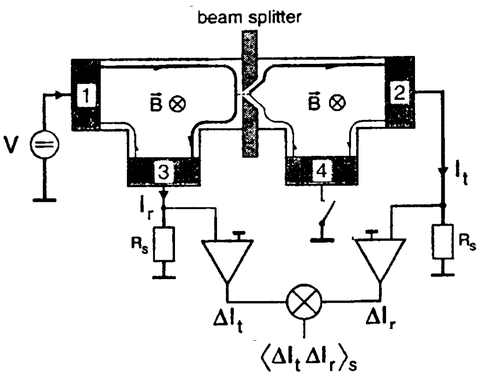
→ measure of correlation $S_{23} = \langle \Delta I_2 \Delta I_3 \rangle$
→ $S_{23} > 0$ for bosons and $S_{23} < 0$ for fermions for coherent scattering.

- Use a QPC to inject single mode electrons
  Oliver *et al.* Science **284** 299 (1999)

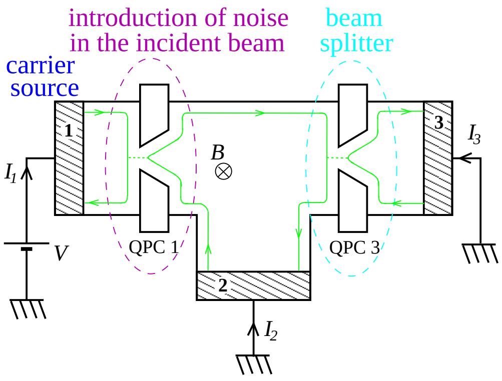
- Use edge states of a conductor in the quantum Hall regime

→ Fully degenerate incident stream

Henny *et al.* Science **284** 296 (1999)



→ Partially degenerate incident stream
Oberholzer *et al.* Physica E **6** 314 (2000)



⊛ If several edge states are populated : equilibration of populations might occur due to inter-edge states scattering

Transport Measurements :

- van Wees *et al.* 1989
- Komiyama *et al.* 1990
- Alphenaar *et al.* 1990

No equilibration for 2 edge states

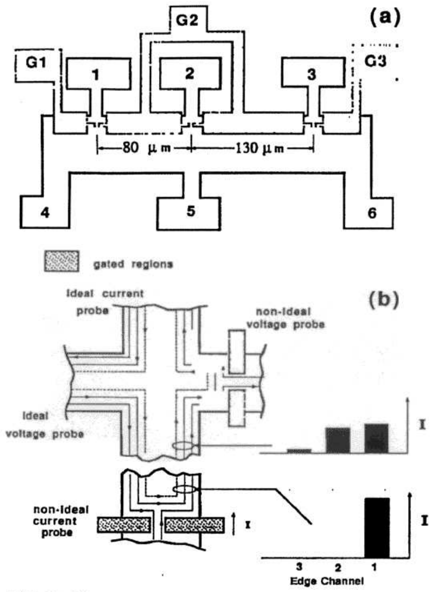
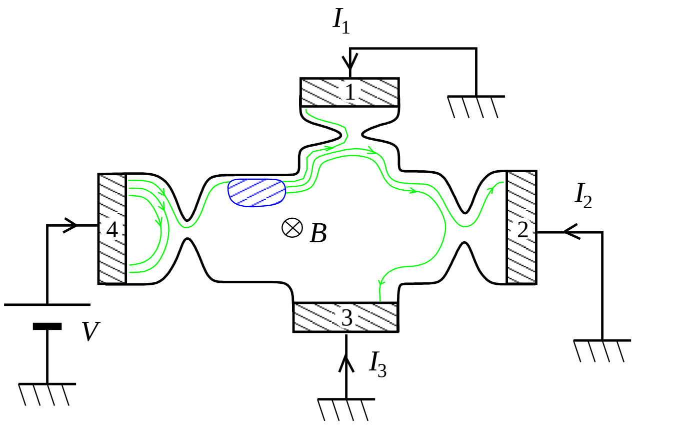


FIG. 2. (a) Schematic representation of the device used in these experiments. The properties of the nonideal probes, [1], [2], and [3], are controlled by voltages on the gates $G1$, $G2$, and $G3$. (b) Hall measurement with nonideal probes for $N=3$ Landau levels. The currents shown correspond to the values determined from the data in Fig. 3.

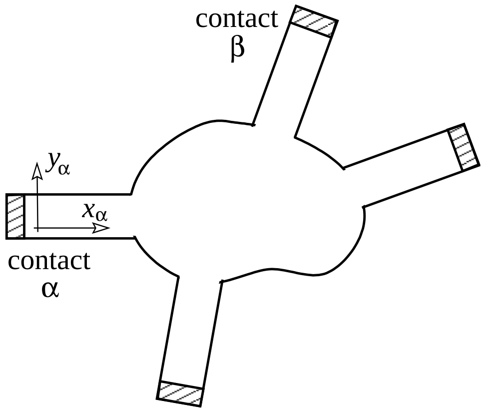
With N available edge states, the current is equilibrated over the $N-1$ lowest channels over $\ell \sim 80\mu$m.



What is the effect of inter-edge state scattering on noise correlations in a HBT experiment ?

2. The scattering approach

$\rightarrow$ apply to phase-coherent conductors



$\rightarrow$ consider the stationary scattering states $|\psi_\alpha^E\rangle$
(incoming state from contact α).

> $s-$matrix : $s_{\beta\alpha}$ is the current amplitude in contact β if the carrier is injected in contact α (*i.e.* in $|\psi_\alpha^E\rangle$)

$\rightarrow$ Current operator :

$$\begin{aligned} \hat{I}_\alpha(\omega) &= \int \mathrm{d}t \int \mathrm{d}y_\alpha \hat{\mathcal{J}}(\vec{r}_\alpha, t)\mathrm{e}^{\mathrm{i}\omega t} \\ &= e \int \mathrm{d}E \sum_{\gamma,\lambda} \hat{a}_\gamma^\dagger(E) A_{\gamma\lambda}^\alpha(E, E+\hbar\omega)\hat{a}_\lambda(E+\hbar\omega) \end{aligned} \quad (1)$$

$\hat{a}_\alpha$ and $\hat{a}_\alpha^\dagger$ are annihilation and creation operator : $\hat{a}_\alpha^\dagger(E)|0\rangle = |\psi_\alpha^E\rangle$

matrix element of $\hat{I}_\alpha$:

$$A_{\gamma\lambda}^\alpha(E, E') = \delta_{\alpha\gamma}\delta_{\alpha\lambda} - s_{\alpha\gamma}^*(E)s_{\alpha\lambda}(E') \quad (2)$$

$\rightarrow$ for many channels : $s_{\beta\alpha}$ is a matrix.

$\rightarrow$ Applied voltages :
Stationnary scattering states at contact α are occupied up to

$$\mu_\alpha = E_F + eV_\alpha$$

Transport

$\rightarrow$ Average current gives the conductances :

$$\begin{aligned} \langle \hat{I}_\alpha(\omega)\rangle &= e\int \mathrm{d}E \sum_{\gamma,\lambda} A^\alpha_{\gamma\lambda}(E,E+\hbar\omega)\langle \hat{a}^\dagger_\gamma(E)\hat{a}_\lambda(E+\hbar\omega)\rangle \\ &= e\,\delta(\hbar\omega)\int \mathrm{d}E \sum_\beta A^\alpha_{\beta\beta}(E,E) f_\beta(E) \end{aligned}$$

$f_\alpha(E)$ is the Fermi distribution at contact α

In the linear regime $V_\alpha \to 0$:

$$\langle \hat{I}_\alpha(\omega)\rangle = 2\pi\delta(\omega)\sum_\beta G_{\alpha\beta}V_\beta$$

$$G_{\alpha\beta} = \frac{e^2}{h}\int \mathrm{d}E \left(-\frac{\partial f}{\partial E}\right)\left(N_\alpha\delta_{\alpha\beta} - \mathrm{Tr}\left\{s^\dagger_{\alpha\beta}s_{\alpha\beta}\right\}\right) \tag{3}$$

N_α the number of channels at contact α

At $T = 0\,K$:

$$G_{\alpha\beta} = \frac{e^2}{h}\left(N_\alpha\delta_{\alpha\beta} - \mathrm{Tr}\left\{s^\dagger_{\alpha\beta}s_{\alpha\beta}\right\}\right)$$

Nonlinear transport or AC transport :
interaction effects must be included

c.f. Büttiker, J.Phys.C**5** (1993)

Noise

$\rightarrow$ Noise power :

$$S_{\alpha\beta}(\omega)2\pi\delta(\omega+\omega') = \langle \Delta\hat{I}_\alpha(\omega)\Delta\hat{I}_\beta(\omega') + \Delta\hat{I}_\beta(\omega')\Delta\hat{I}_\alpha(\omega)\rangle \qquad (4)$$

$$S_{\alpha\beta} = S_{\alpha\beta}(\omega=0) = \frac{2e^2}{h}\int \mathrm{d}E \sum_{\gamma,\lambda} \mathrm{Tr}\left\{A^{\alpha}_{\gamma\lambda}A^{\beta}_{\lambda\gamma}\right\} f_\gamma(1-f_\lambda)\,, \qquad (5)$$

where f_α is the Fermi distribution at contact α, associated with the chemical potential μ_α .

Two limits :

1) Thermal noise (equilibrium : $\mu_\alpha = \mu$)

$$S_{\alpha\beta} = 2k_BT\,(G_{\alpha\beta}+G_{\beta\alpha})$$

(Fluctuation Dissipation Theorem)

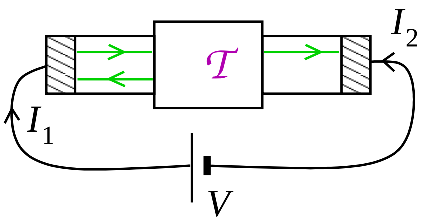
2) Shot noise ($T = 0\,\mathrm{K}$)

$$S_{\alpha\beta} = \frac{e^2}{h}\sum_{\gamma,\lambda} \mathrm{Tr}\left\{s^\dagger_{\alpha\lambda}s_{\alpha\gamma}s^\dagger_{\beta\gamma}s_{\beta\lambda}\right\}|\mu_\gamma-\mu_\lambda|$$

Ex : 2-terminal conductor with one-channel :

$$S_{11} = \frac{2e^2}{h}\mathrm{Tr}\left\{s_{11}s^\dagger_{11}s_{12}s^\dagger_{12}\right\}|eV|$$

$$S_{11} = S_{22} = -S_{12} = \frac{2e^2}{h}|eV|\,\mathcal{R}\mathcal{T} = 2e\,\langle I_1\rangle\,(1-\mathcal{T})$$

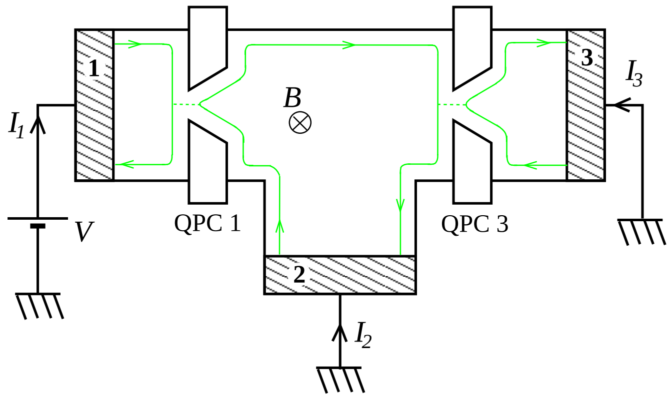


$\Rightarrow S_{11} = 0$ for total reflection ($\mathcal{R}=1$) and perfect transmission ($\mathcal{T}=1$)

Rq : $\boldsymbol{S_{\alpha\beta} < 0}$ for $\boldsymbol{\alpha \neq \beta}$

3. Coherent scattering only

⊛ One-channel case



$\rightarrow$ construct the s-matrix :

$$s = \begin{pmatrix} s_{11} & s_{12} & 0 \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{pmatrix} \qquad \text{with} \quad \begin{array}{lll} |s_{11}|^2 = R_1, & |s_{12}|^2 = T_1, & s_{13} = 0, \\ |s_{21}|^2 = R_3 T_1, & |s_{22}|^2 = R_3 R_1, & |s_{23}|^2 = T_3, \\ |s_{31}|^2 = T_3 T_1, & |s_{32}|^2 = T_3 R_1, & |s_{33}|^2 = R_3. \end{array}$$

Here, information on phases is given by unitarity :

$$s^\dagger s = 1 \Rightarrow \left\{ \begin{array}{l} s_{11}^* s_{12} + s_{21}^* s_{22} + s_{31}^* s_{32} = 0 \\ s_{21}^* s_{23} + s_{31}^* s_{33} = 0 \\ s_{22}^* s_{23} + s_{32}^* s_{33} = 0 \end{array} \right.$$

$$s s^\dagger = 1 \ \Rightarrow \cdots$$

We find for the correlations :

$$S_{23} = -\frac{2e^2}{h} |eV| \, R_3 T_3 T_1^2 \qquad (6)$$

and

$$S_{33} = \frac{2e^2}{h} |eV| \, T_3 T_1 (1 - T_3 T_1) \qquad (7)$$

$\rightarrow$ Experimentally :
Oberholzer *et al.* Physica E (2000)

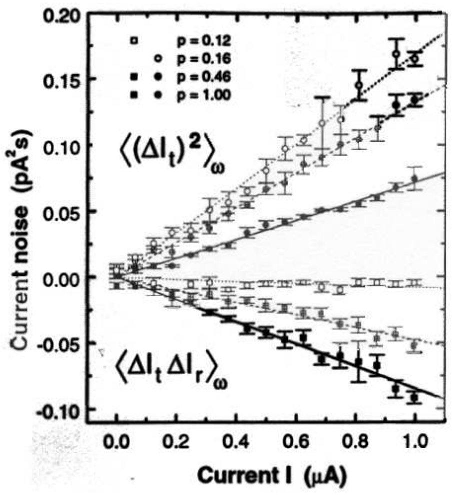
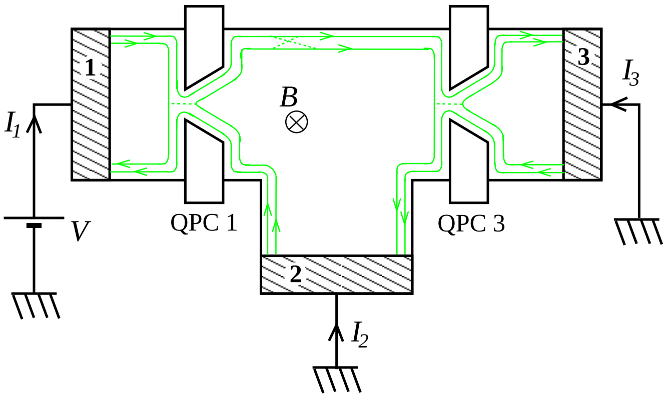


Fig. 2. Measured spectral densities of correlations between current-fluctuations as a function of the current I of the incident beam at $T = 2.5$ K. $\langle \Delta I_t \Delta I_r \rangle_\omega$ denotes the cross-correlation between the transmitted and reflected beams and $\langle(\Delta I_t)^2\rangle_\omega$ the auto-correlation in the transmitted beam. The offset noise arising from thermal fluctuations and residual amplifier noise has been subtracted. The absolute slopes are $(0.23 \pm 0.030) \cdot 2e$ and $(0.26 \pm 0.037) \cdot 2e$ for the auto-correlation and cross-correlation, respectively, in agreement with the expected prefactor $t(1-t) = 1/4$. If the incident beam is diluted by lowering the transmission probability p, the cross-correlation gets smaller and disappears for p close to zero. The auto-correlation itself increases, because the incident beam is noisy.

⊛ Two edge-states are open



→ s-matrix has the form :

$$s = \left(\begin{array}{cc|cc|cc} & & \times & & & \\ & \times & & \times & & \\ \hline & & & & \times & \\ \times & \times & & \times & & \times \\ \hline \times & \times & & \times & & \\ \times & \times & & \times & & \times \end{array} \right) \tag{8}$$

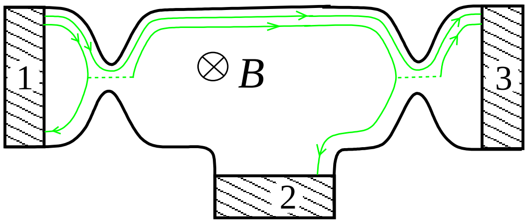
→ Introduce the inter-edge state scattering probability ε.

→ We find for the correlation :

$$\begin{aligned} S_{23}^{\mathrm{coh}} = -\frac{2e^2}{h}|eV|R_3[(1-\varepsilon)^2 T_3 T_1^2 + \varepsilon^2 T_3 \\ + \varepsilon(1-\varepsilon)(1 - 2R_3 T_1 + T_1^2)] \,. \end{aligned} \tag{9}$$

→ Limiting cases

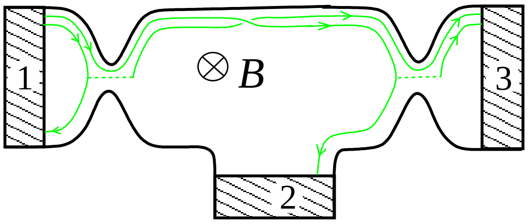
- $\underline{\varepsilon = 0}$



$$S_{23}^{\rm coh} = -\tfrac{2e^2}{h}|eV|\, R_3 T_3 T_1^2$$

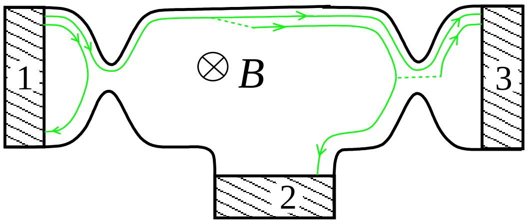
→ same as one-channel case

- $\underline{\varepsilon = 1}$



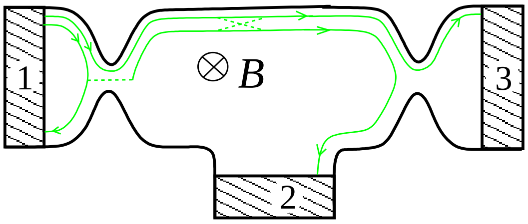
$$S_{23}^{\rm coh} = -\tfrac{2e^2}{h}|eV|\, R_3 T_3$$

- $\underline{T_1 = 0}$



$$S_{23}^{\rm coh} = -\tfrac{2e^2}{h}|eV|\, [\varepsilon(1-\varepsilon)R_3 \\ +\varepsilon^2 R_3 T_3]$$

- $\underline{T_3 = 0}$ (separation of the 2 channels at QPC 3)



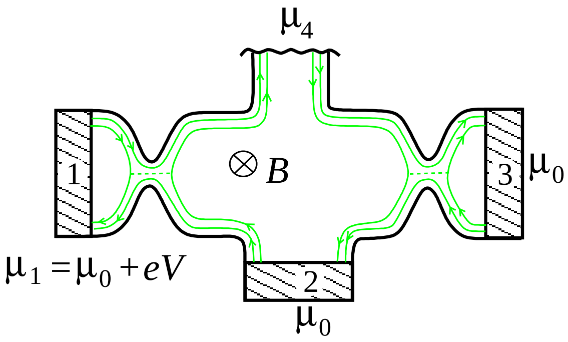
$$S_{23}^{\rm coh} = -\tfrac{2e^2}{h}|eV|\, \varepsilon(1-\varepsilon) R_1^2$$

$$\varepsilon = 1/2: \quad \begin{cases} S_{23}^{\rm coh} = -\dfrac{e^2}{h}|eV|\dfrac{R_1^2}{2} \\[2ex] S_{22}^{\rm coh} = S_{33}^{\rm coh} = \dfrac{e^2}{h}|eV|\dfrac{1+T_1}{2}R_1 \end{cases}$$

4. Incoherent scattering

⊛ Description of the model

→ Add a fictitious contact (4).
⇒ Relate the problem to the study of a coherent one.



$I_4 = 0 \Rightarrow$ Determines the distribution $f_4(E)$
(potential μ_4 fluctuates)

• Model of inelastic scattering

Büttiker, 1986

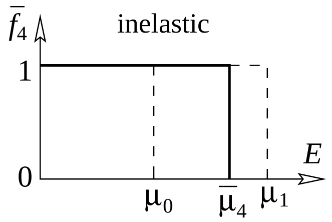
$$\boxed{I_4 = 0 \text{ and } \bar{f}_4 \text{ is a Fermi distribution}}$$

Write

$$I_\alpha = \frac{1}{e} \sum_\beta G_{\alpha\beta} \mu_\beta + \delta I_\alpha = \langle I_\alpha \rangle + \Delta I_\alpha \, . \tag{10}$$

δI_α : intrinsic part of the fluctuations.
$S_{\alpha\beta} = \langle \delta I_\alpha \delta I_\beta \rangle$: correlations of the 4-terminal conductor

$a)$ $\langle I_4 \rangle = 0$
$\Rightarrow$ average potential is $\bar{\mu}_4 = -\frac{G_{41}\mu_1 + G_{42}\mu_2 + G_{43}\mu_3}{G_{44}}$



$\bar{\mu}_4 = \mu_0 + \frac{1+T_1}{2} eV$

$b)$ $\Delta I_4 = 0$
$\Rightarrow$ fluctuations of potential : $\mu_4 - \bar{\mu}_4 = -\frac{e}{G_{44}} \delta I_4$

Then the fluctuating part of the current is :

$$\Delta I_\alpha = \delta I_\alpha - \frac{G_{\alpha 4}}{G_{44}} \delta I_4 \tag{11}$$

$\rightarrow$ Then correlations $S^{\text{in}}_{\alpha\beta} = \langle \Delta I_\alpha \Delta I_\beta \rangle$ are

$$S^{\text{in}}_{\alpha\beta} = S_{\alpha\beta} - \frac{G_{\alpha 4}}{G_{44}} S_{\beta 4} - \frac{G_{\beta 4}}{G_{44}} S_{\alpha 4} + \frac{G_{\alpha 4} G_{\beta 4}}{G_{44}^2} S_{44} \,, \tag{12}$$

where $S_{\alpha\beta}$ are intrinsic correlations for
the 4-terminal conductor with $\mu_4 \rightarrow \bar{\mu}_4$.

- Model of quasi-elastic scattering

Beenakker, de Jong '96

$$\boxed{j_4(E) = 0}$$

Write

$$I_\alpha = \int \mathrm{d}E\, j_\alpha(E) = \int \mathrm{d}E \left(\frac{1}{e}\sum_\beta G_{\alpha\beta} f_\beta + \delta j_\alpha(E)\right) \tag{13}$$

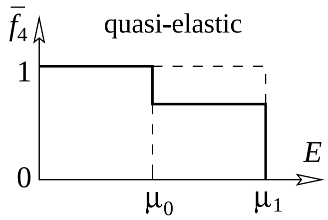
$$= \int \mathrm{d}E\, \left(\langle j_\alpha(E)\rangle + \Delta j_\alpha(E)\right) \tag{14}$$

$\delta j_\alpha(E)$: intrinsic part of the fluctuations.

a) $\langle j_4(E)\rangle = 0$

$\Rightarrow$ average distribution is

$\bar{f}_4(E) = \frac{1+T_1}{2} f_1(E) + \frac{R_1}{2} f_2(E)$



b) $\Delta j_4(E) = 0$

$\Rightarrow$ fluctuations of distribution : $f_4 - \bar{f}_4 = -\frac{e}{G_{44}}\delta j_4(E)$

Then the fluctuating part of the current is :

$$\Delta j_\alpha(E) = \delta j_\alpha(E) - \frac{G_{\alpha 4}}{G_{44}}\delta j_4(E) \tag{15}$$

$\rightarrow$ correlations $S^{\mathrm{qe}}_{\alpha\beta} = \langle \Delta I_\alpha \Delta I_\beta\rangle$ are

$$S^{\mathrm{qe}}_{\alpha\beta} = S_{\alpha\beta} - \frac{G_{\alpha 4}}{G_{44}}S_{\beta 4} - \frac{G_{\beta 4}}{G_{44}}S_{\alpha 4} + \frac{G_{\alpha 4}G_{\beta 4}}{G_{44}^2}S_{44}\,. \tag{16}$$

where $S_{\alpha\beta}$ are intrinsic correlations for the 4-terminal conductor with $f_4 \rightarrow \bar{f}_4$.

⊛ Application to the 3-terminal conductor :

• Quasi-elastic case

$$S_{23}^{\rm qe} = -\frac{e^2}{h}|eV|\frac{R_3}{4}[2T_3(1+T_1)^2 + (1+T_3)R_1^2]\,. \tag{17}$$

<u>$T_3 = 0$:</u>

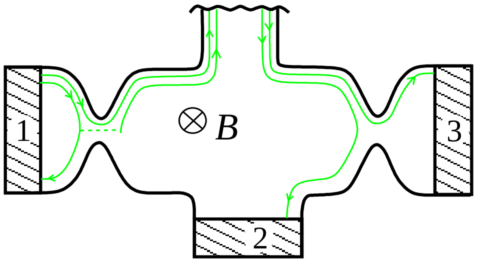
$S_{23}^{\rm qe} = -\frac{e^2}{h}|eV|\frac{R_1^2}{4} = \frac{1}{2}\left(S_{23}^{\rm coh} \text{ for } \varepsilon = 1/2\right)$

$S_{22}^{\rm qe} = S_{33}^{\rm qe} = \frac{e^2}{h}|eV|\frac{R_1}{2}\frac{1+3T_1}{2} \neq \left(S_{33}^{\rm coh} \text{ for } \varepsilon = 1/2\right).$

• Inelastic case

$$S_{23}^{\rm in} = -\frac{e^2}{h}|eV|\frac{R_3}{2}[2T_3(1+T_1) - (1+T_3)R_1T_1] \tag{18}$$

<u>$T_3 = 0$:</u>



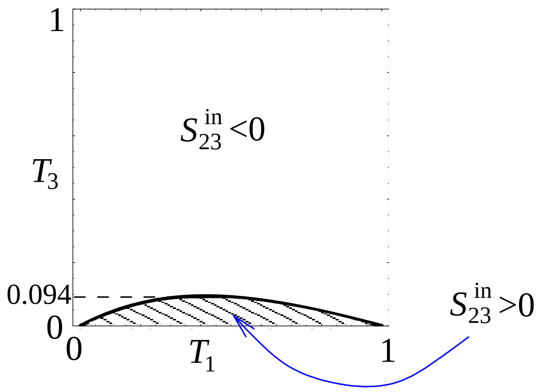
$S_{23}^{\rm in} = +\frac{e^2}{h}|eV|\frac{R_1T_1}{2}$

$S_{22}^{\rm in} = S_{33}^{\rm in} = \frac{e^2}{h}|eV|\frac{R_1T_1}{2}$

⇒ Possibility of injection of current in the two channels in a correlated way due to fluctuations of potential μ_4

⊛ How to destroy the positive correlation $S_{23}^{\rm in}$?

- Finite transmission T_3



- Finite temperature

$$S_{23}^{\rm in} = \frac{e^2}{h}\Big[- k_B T\,(2 + R_1 + R_1 T_1) \\ + \frac{R_1 T_1}{2} eV \coth\frac{eV}{2k_B T}\Big] \,. \tag{19}$$

$\rightarrow eV = 0 \Rightarrow$ FDT :

$$S_{23}^{\rm in} = -\tfrac{2e^2}{h} k_B T \left(1 + \tfrac{R_1}{2}\right) = 2k_B T (G_{23}^{\rm in} + G_{32}^{\rm in}) < 0$$

where $G_{\alpha\beta}^{\rm in} = G_{\alpha\beta} - \frac{G_{\alpha 4} G_{4\beta}}{G_{44}}$

$\rightarrow$ Critical temperature :

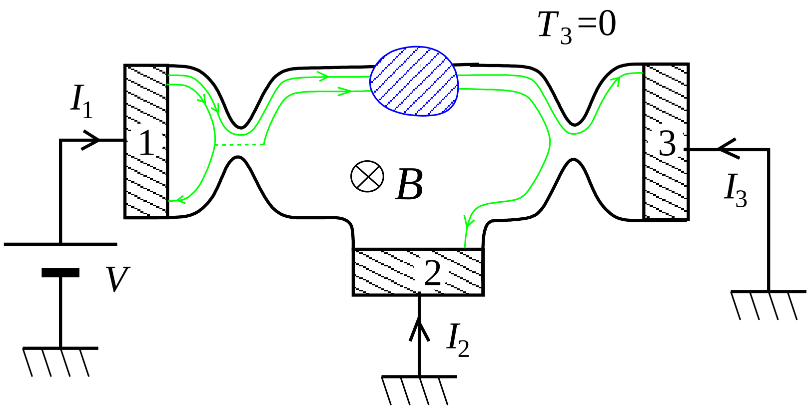
$S_{23}^{\rm in} > 0$ below T_c.

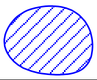
$*$ small transmission $T_1 \ll 1$: $k_B T_c \simeq |eV| \frac{T_1}{6}$

$*$ large transmission $R_1 \ll 1$: $k_B T_c \simeq |eV| \frac{R_1}{4}$.

$*$ optimum transmission $T_1 = 3 - \sqrt{6} \simeq 0.55$: $k_B T_c^{\rm max} \simeq \frac{|eV|}{21.8}$.

5. Conclusion



	S_{23}	S_{22}	S_{33}
no scattering	0	$2R_1T_1$	0
coherent (with $\varepsilon = 1/2$)	$-\frac{R_1^2}{2}$	$\frac{1+T_1}{2}R_1$	$\frac{1+T_1}{2}R_1$
quasi-elastic	$-\frac{R_1^2}{4}$	$\frac{1+3T_1}{2}\frac{R_1}{2}$	$\frac{1+3T_1}{2}\frac{R_1}{2}$
inelastic	$+\frac{R_1T_1}{2}$	$\frac{R_1T_1}{2}$	$\frac{R_1T_1}{2}$

$\Rightarrow$ Noise measurements provide information on the nature of inter-edge state scattering.