Master iCFP March 21, 2017
Wave dynamics in random media

# TD n$^o$6 : Spin-orbit scattering and weak *anti*-localisation in metallic films

The magneto-resistance of films made of usual metals is not well described by the simple theory presented in the previous exercice session, i.e. by equation

$$\overline{\Delta\sigma}(\mathcal{B}) = \frac{2_s e^2}{h}\frac{1}{2\pi}\left[\psi\left(\frac{1}{2}+\frac{L_{\mathcal{B}}^2}{L_\varphi^2}\right)-\psi\left(\frac{1}{2}+\frac{L_{\mathcal{B}}^2}{\ell_e^2}\right)\right] \tag{1}$$

The reason for this is that electronic spin degree of freedom cannot be ignored.

• First, the presence of spin-orbit coupling

$$H_{\mathrm{SO}} \sim -\frac{1}{m^2c^2}(\vec{p}\times\vec{\nabla}V)\cdot\vec{S} \tag{2}$$

is responsible for an additional phase originating from the rotation of the electronic spin, while the electron is scattered on the disordered potential $V$. Spin-orbit coupling is weak in light metals, like Lithium of Magnesium, but strong in heavy metals like Silver or Gold.

• Second, the presence of residual magnetic impurities is another source of electronic spin rotation :

$$H_{\mathrm{mag}} = -\vec{S}\cdot\sum_i J_i\,\vec{s}_i\,\delta(\vec{r}-\vec{r}_i) \tag{3}$$

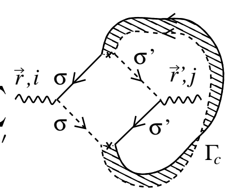
where $\vec{r}_i$ are the positions of the magnetic impurities and $\vec{s}_i$ their spins (considered frozen).

The starting point of the study of quantum transport is the Kubo-Greenwood formula

$$\overline{\widetilde{\sigma}(\omega)} = \frac{e^2}{2\pi m^2}\frac{1}{\mathrm{Vol}}\sum_{k,\,k',\,\sigma,\,\sigma'} k_x\,k'_x\,\overline{G^{\mathrm{R}}_{\sigma,\sigma'}(k,k';\varepsilon_F+\omega)\,G^{\mathrm{A}}_{\sigma',\sigma}(k',k;\varepsilon_F)}\,, \tag{4}$$

where the Green's function carry spin indices. Using that the average Green's function is diagonal in spin indices, we finally obtain the expression of the weak localisation correction

$$\overline{\Delta\sigma} = \sum_{\sigma,\,\sigma'}$$




$$= -\frac{e^2}{\pi}\sum_{\sigma,\,\sigma'} P^{(c)}_{\sigma\sigma',\sigma'\sigma}(\vec{r},\vec{r}) \tag{5}$$

where the Cooperon now propagates a pair of spins (Fig. 1).

$$\Gamma^{(c)}_{\alpha\beta,\gamma\delta}(\vec{r},\vec{r}\,') =$$

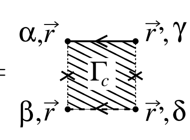



Figure 1: *In the presence of spin flip and spin-orbit scattering, the Cooperon depends on four spin indices.*

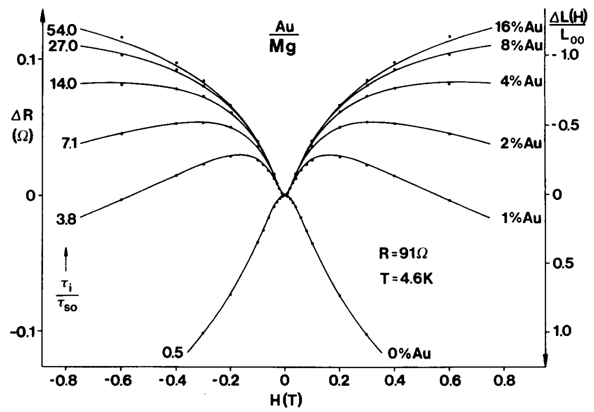



Figure 2: *Magneto-resistance curves for metallic films made of an alloy of Magnesium and Gold for difference concentration of gold. From Ref. [1].*

**1/** Argue that the Cooperon $\Gamma^{(c)}_{\alpha\beta,\gamma\delta}(\vec{r},\vec{r}\,') = \frac{w}{D\tau_e} P^{(c)}_{\alpha\beta,\gamma\delta}(\vec{r},\vec{r}\,')$ can be written as the sum of two separate contributions associated with singlet and triplet channels.

**2/** The efficiency of spin-orbit coupling and scattering by magnetic impurities are controlled by two lengths $L_{\rm so}$ and $L_{\rm m}$. Introducing the projector in the singlet space, $(\Pi_0)_{\alpha\beta,\gamma\delta} = \frac{1}{2}(\delta_{\alpha\gamma}\,\delta_{\beta\delta} - \delta_{\alpha\delta}\,\delta_{\beta\gamma})$, we can write the Cooperon (in the space of the two spins) under the form

$$P^{(c)}(\vec{r},\vec{r}\,') = P_c(\vec{r},\vec{r}\,';1/L_S^2)\,\Pi_0 + P_c(\vec{r},\vec{r}\,';1/L_T^2)\,(1-\Pi_0)\,, \tag{6}$$

where $P_c(\vec{r},\vec{r}\,';\gamma) = \langle\vec{r}\,|\frac{1}{\gamma-\Delta}|\,\vec{r}\,'\rangle$ is the Cooperon in the absence of spin scattering. The two lengths $L_S$ and $L_T$ combine the effect of spin-orbit and magnetic impurities :

$$\frac{1}{L_S^2} = \frac{2}{L_m^2} \qquad \text{and} \qquad \frac{1}{L_T^2} = \frac{2}{3L_m^2} + \frac{4}{3L_{\rm so}^2}\,. \tag{7}$$

Explain why $L_S > L_T$.

**3/** Discuss the effect of time reversal in the singlet and triplet channel. Write $\overline{\Delta\sigma}$ in terms of $P_c(\vec{r},\vec{r}\,';1/L^2_{S,T})$.

Hint : examine the spin structure in the diagram in Eq. (5).

**4/** Using the formula (1) for the magneto-conductance of a thin film, deduce the expression of the weak localisation in the presence of spin-orbit and spin flip scattering. Explains (at least qualitatively) the experimental data of Fig. 2.

**Further reading :** A review article on weak localisation in thin metallic films is the famous article by Bergmann (1984) (Ref. [1]). A more intuitive description can be found in the review article of Chakravarty and Schmid (1986) (Ref. [2]).

## References

[1] G. Bergmann, Weak localization in thin films, Phys. Rep. **107**(1), 1–58 (1984).

[2] S. Chakravarty and A. Schmid, Weak localization: the quasiclassical theory of electrons in a random potential, Phys. Rep. **140**(4), 193–236 (1986).