TD n°7 : Conductance fluctuations and correlations in narrow wires

The purpose of the exercice is to analyse precisely the correlation function $\overline{\delta g(\mathcal{B})\delta g(\mathcal{B}')}$ for the dimensionless conductance of a narrow wire of length L and width W.

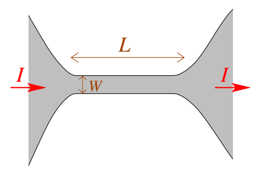

Figure 1: *Transport through a metallic wire of length L and width W.*

A. Preliminary : weak localisation correction and role of boundaries

We recall that the weak localisation correction to the dimensionless conductance of a narrow wire ($W \ll L$) is given by

$$\overline{\Delta g} = -\frac{2}{L^2}\int_0^L \mathrm{d}x\, P_c(x,x) \tag{1}$$

where the Cooperon solves $\left[\gamma - \partial_x^2\right] P_c(x,x') = \delta(x-x')$ where $\gamma = 1/L_\varphi^2$ encodes dephasing. We account for the connections at the two boundaries by imposing some Dirichlet boundary conditions $P_c(x,x')\big|_{x=0,\,L} = 0$.

1/ Show that the weak localisation correction can be expressed in terms of the spectrum of eigenvalues $\{\lambda_n\}$ of the Laplace operator in the wire (i.e. $-\partial_x^2\phi_n(x) = \lambda_n\phi_n(x)$ for $\phi_n(0) = \phi_n(L) = 0$).

2/ Write the weak localisation as $\overline{\Delta g} = -(2/L)\,\mathcal{G}(\gamma)$, where $\mathcal{G}(\gamma)$ is the spatial averaged Cooperon. Give $\mathcal{G}(\gamma)$ and analyse the limiting behaviours $L \gg L_\varphi$ and $L \ll L_\varphi$.

B. Fluctuations and correlations

The correlation function for the narrow wire can be expressed as

$$\overline{\delta g(\mathcal{B})\delta g(\mathcal{B}')} = \frac{4}{L^2}\int \mathrm{d}\omega\,\delta_T(\omega)\int_0^L \frac{\mathrm{d}x\mathrm{d}x'}{L^2}\left[\left|P_d(x,x';\omega)\right|^2 + \frac{1}{2}\,\mathrm{Re}\left\{P_d(x,x';\omega)^2\right\} + (\ P_d \longrightarrow P_c\ )\right] \tag{2}$$

where $\delta_T(\omega)$ is a normalised function of width T such that $\delta_T(0) = 1/(6T)$. The diffuson and cooperon solve

$$\left[-\mathrm{i}\omega/D + \gamma_{d,c} - \partial_x^2\right] P_{d,c}(x,x';\omega) = \delta(x-x')\,, \tag{3}$$

where the expressions of the dephasing rates differ for diffuson and cooperon in the presence of a magnetic field :

$$\gamma_d = \frac{1}{L_\varphi^2} + \frac{1}{L^2_{\frac{\mathcal{B}-\mathcal{B}'}{2}}} \quad\text{and}\quad \gamma_c = \frac{1}{L_\varphi^2} + \frac{1}{L^2_{\frac{\mathcal{B}+\mathcal{B}'}{2}}} \quad\text{where}\quad L_{\mathcal{B}} = \frac{\sqrt{3}\,\hbar}{|e\mathcal{B}|W}\,. \tag{4}$$

Note that we can also write $L_{\mathcal{B}} \equiv \sqrt{3}\frac{\ell_m^2}{W}$ where $\ell_m = \sqrt{\hbar/|e\mathcal{B}|}$ is the 2D magnetic length.

1/ Compare the fluctuations at zero field $\mathcal{B} = \mathcal{B}' = 0$ to the one at large field (no calculation).

2/ Show that the two diffuson contributions can be written in terms of the spectrum $\{\lambda_n\}$ under the form

$$\overline{\delta g(\mathcal{B})\delta g(\mathcal{B}')}^{(\mathrm{diffuson})} = \frac{4}{L^3}\int \mathrm{d}\omega\, \delta_T(\omega)\, \mathcal{F}(\gamma_\omega) \tag{5}$$

where $\gamma_\omega = \gamma_d - \mathrm{i}\omega/D$. Express the function $\mathcal{F}(\gamma_\omega)$ as a series.

3/ With the help of the representation obtained in the previous question, show that the expression of the correlator involves a new cutoff at the length given by the **thermal length** $L_T \stackrel{\mathrm{def}}{=} \sqrt{D/T}$. Identify a "low temperature" regime and a "high temperature" regime by comparing the three lengths L_T, L_φ and L.

a) "Low" temperature regime ($L_T = \infty$)

In this case we simplify the calculation by performing the substitution $\delta_T(\omega) \to \delta(\omega)$ which allows for a straightforward integration over frequency.

1/ Show that the correlator $\overline{\delta g(\mathcal{B})\delta g(\mathcal{B}')}^{(\mathrm{diffuson})}$ can be *formally* related to the weak localisation (i.e. that the two functions $\mathcal{F}(\gamma)$ and $\mathcal{G}(\gamma)$ are related).

2/ Analyse the limiting behaviours for $L \gg L_\varphi$ and $L \ll L_\varphi$. Recall the physical origin of the decay of $\overline{\delta g^2}$ with L/L_φ when $L \gg L_\varphi$.

3/ Adding the cooperon contribution, analyse the structure of the correlator as a function of the magnetic fields in the limit $L_\varphi \ll L$.

b) "High" temperature regime (small L_T)

1/ In the "high temperature" regime, justify the substitution $\delta_T(\omega) \to \delta_T(0)$ (the value $\delta_T(0)$ was given above). Deduce that the correlator $\overline{\delta g(\mathcal{B})\delta g(\mathcal{B}')}^{(\mathrm{diffuson})}$ is simply related to the WL, i.e. expressed in terms of $\mathcal{G}(\gamma)$.

2/ Analyse the limiting behaviours for $L \gg L_\varphi$ and $L \ll L_\varphi$.

3/ Analyse the magnetic field dependence of the full correlator (diffuson and cooperon) when $L_\varphi \ll L$.

c) Experiment

The weak localisation correction $\overline{\Delta g(\mathcal{B})}$ and the conductance fluctuations $\overline{\delta g(\mathcal{B})^2}$ have been measured in a short wire etched in a 2DEG with a direct averaging procedure (Fig. 2). Discuss the experimental data at the light of your calculations.

Appendix

$$\sum_{n=1}^{\infty} \frac{1}{(n\pi)^2 + y^2} = \frac{1}{2y}\left(\coth y - \frac{1}{y}\right) .$$

$$\coth x = \frac{1}{x} + \frac{1}{3}x - \frac{1}{45}x^3 + \frac{2}{945}x^5 + \mathcal{O}(x^7) .$$

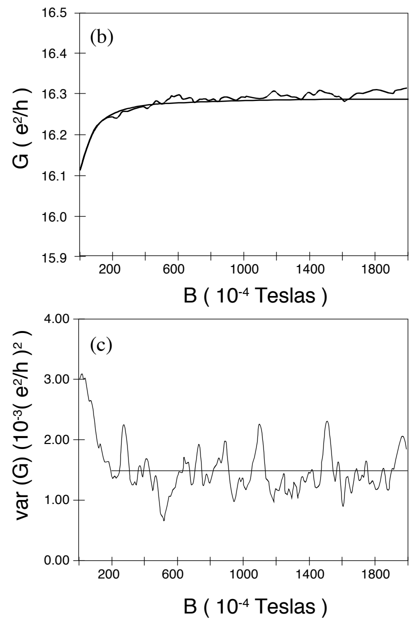

Figure 2: *WL and CF of a short wire (*$L \simeq 10\,\mu\mathrm{m}$*) etched in a 2DEG at* $T = 45\,\mathrm{mK}$*. Curves from Ref. [*?*].*

Further reading :

• The effect of boundary conditions in wires has been studied in the paper of Al'tshuler, Aronov and Zyuzin B. L. Al'tshuler, A. G. Aronov and A. Yu. Zyuzin, *Size effects in disordered conductors*, Sov. Phys. JETP **59**, 415 (1984).

• A measurement of conductance fluctuations with direct averaging over disorder configurations has been performed by Dominique Mailly and Marc Sanquer, *Sensitivity of quantum conductance fluctuations and* $1/f$ *noise to time reversal symmetry*, J. Phys. I France **2**, 357 (1992).

• For a recent theoretical paper (with a short review) : Christophe Texier & Gilles Montambaux, *Four-terminal resistances in mesoscopic networks of metallic wires: Weak localisation and correlations*, Physica E **75**, 33–46 (2016), special issue "*Frontiers in quantum electronic transport – in memory of Markus Büttiker*", available as preprint cond-mat arXiv:1506.08224.