TD n^{o}9 : Decoherence by electronic interactions

I Influence functional approach

We have seen, within the diagrammatric approach, that the weak localisation (WL) correction to the conductivity $\overline{\Delta\sigma}$ involves a two particle propagator (in the particle-particle channel). In low dimensions ($d \leqslant 2$) and at low temperature ($T \lesssim 1\,\mathrm{K}$), the **dominant decoherence mechanism is the electronic interaction**. As we have seen, the interesting dependence of the WL in the phase coherence length L_φ arises from *large scale (infrared) cutoff.* The treatment of electronic interactions within diagrammatic technics is well formulated in the Fourier space, where small scale-high energy (ultraviolet) cutoff emerges naturally (due to the presence of Fermi distributions), but makes difficult a proper description of large scale cutoff : perturbation theory in Fourier space presents infrared divergences cut off usually by hand (see Fukuyama & Abrahams [1] ; the review by Chakravarty & Schmid [2]).

An alternative approach, pioneered by Al'tshuler, Aronov and Khmel'nitzkiĭ in 1982, was to follow an influence functional approach formulated in real space with path integral.

Let us first recall the expression of the WL seen in the lecture :

$$\overline{\Delta\sigma} = -\frac{2_s e^2}{\pi\hbar} D \int_0^\infty \mathrm{d}t\, \mathrm{e}^{-\Gamma_\varphi t} \overbrace{\int_{\vec{r}(0)=\vec{r}}^{\vec{r}(t)=\vec{r}} \mathcal{D}\vec{r}(\tau)\, \mathrm{e}^{-\int_0^t \mathrm{d}\tau\, \frac{1}{4D}\dot{\vec{r}}(\tau)^2}}^{\mathcal{P}_t(\vec{r}|\vec{r})}\,, \tag{1}$$

where the decoherence (cut off of large length scale $L \gtrsim L_\varphi = \sqrt{D/\Gamma_\varphi}$) was introduced by hand through an exponential damping with the rate Γ_φ. Our purpose is now to provide a microscopic theory justifying such a cutoff arising from electronic interactions.

Altshuler, Aronov and Khmelniskii have proposed to model interaction of a given electron with the surrounding electrons as the interaction with a fluctuating potential $V(\vec{r},t)$ (the vector potential was rather considered in the originial paper). Each contribution in (1) should be thus weighted by $\mathrm{e}^{\mathrm{i}\Phi_V}$, where

$$\Phi_V[\vec{r}(\tau)] = \frac{1}{\hbar}\int_0^t \mathrm{d}\tau\, \left[V(\vec{r}(\tau),\tau) - V(\vec{r}(\tau), t-\tau)\right] \tag{2}$$

is the phase peaked up by the two reversed electronic trajectories in the fluctuating field.

1/ Fluctuation-dissipation theorem.– The electric potential fluctuations are characterised by FDT (written in the classical regime $\hbar\omega \ll k_B T$)

$$\left\langle V(\vec{r},t)V(\vec{r}\,',t)\right\rangle_V \simeq \frac{2e^2 k_B T}{\sigma_0}\,\delta(t-t')\,P_d(\vec{r},\vec{r}\,') \tag{3}$$

where the Diffuson solves $-\Delta P_d(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')$. The Drude conductivity is $\sigma_0 = 2_s e^2 \rho_0 D$ where ρ_0 is the DoS per spin channel. Using the Gaussian nature of the fluctuations, perform averaging over potential fluctuations in $\left\langle \mathrm{e}^{\mathrm{i}\Phi_V}\right\rangle_V$. Show that the result can be interpreted in terms of a **trajectory dependent decoherent rate**

$$\overline{\Delta\sigma} = -\frac{2_s e^2}{\pi\hbar} D \int_0^\infty \mathrm{d}t \int_{\vec{r}(0)=\vec{r}}^{\vec{r}(t)=\vec{r}} \mathcal{D}\vec{r}(\tau)\, \mathrm{e}^{-\int_0^t \mathrm{d}\tau\, \frac{1}{4D}\dot{\vec{r}}(\tau)^2}\, \mathrm{e}^{-\Gamma_{\mathrm{ee}}[\vec{r}(\tau)]\,t}\,. \tag{4}$$

2/ Decoherence in a long wire.– In a long and narrow wire, justify the following expression of the diffuson :

$$P_d(\vec{r},\vec{r}\,') \simeq -\frac{1}{2s}\,|x-x'| \tag{5}$$

where x measures the distance along the wire and s is the cross-section of the wire. Rescaling the variables in the path integral in order to deal with dimensionless coordinate and time, deduce the characteristic time scale and length scale controlling the decoherence in this case. Analyze the temperature dependence. Interpret physically the dependence of the phase coherent time with s and the diffusion constant D (remember FDT). Compare with experimental data.

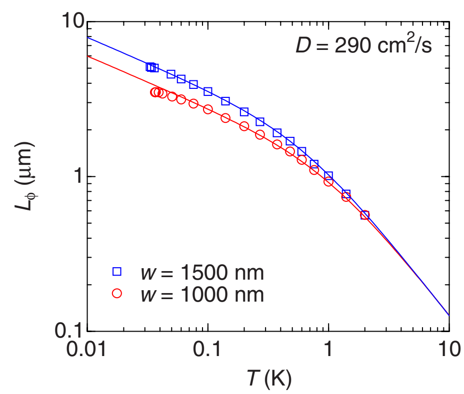
Indication : The change of variable

$$\left\{\begin{array}{l} x=\lambda y \\ t=\eta\tau \end{array}\right. \tag{6}$$

is implemented in the path integral with the help of [1]

$$\int_{x(0)=0}^{x(t=\eta\tau)=\lambda y} \mathcal{D}x\, \mathrm{e}^{-\int_0^t \mathrm{d}t'\,(\dot{x}^2+V(x))} = \frac{1}{\lambda}\int_{y(0)=0}^{y(\tau)=y} \mathcal{D}y\, \mathrm{e}^{-\int_0^\tau \mathrm{d}\tau'\,[\frac{\lambda^2}{\eta}\dot{y}^2+\eta V(\lambda y)]}\,, \tag{8}$$

which can be interpreted as "$\mathcal{D}x=\frac{1}{\lambda}\mathcal{D}y$".



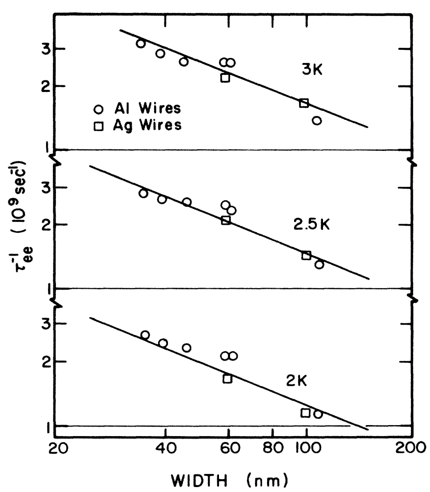


FIG. 3. Electron-electron contribution to $\tau_\phi^{-1}=$ [(total phase-breaking rate) − (electron-phonon rate $= A_{ep}T^3$)] as a function of wire width. The solid lines give the theoretical prediction of Eq. (1). The data are normalized to the $R_\square$ and D of samples Al2, according to Eq. (1).

Figure 1: **Left :** Phase coherence length at low T in wire etched in 2DEG ; from Ref. [3]. **Right :** The phase coherence time in Al and Ag wires as a function of the width ; from [4].

3/ Decoherence in confined geometry.– In a confined geometry, we may simply use the estimate $P_d(\vec{r},\vec{r}\,') \sim \frac{1}{s}L$, where L is the size of the wire. Deduce the new temperature dependence of the phase coherence length.

[1] Proof relies on the fact that

$$K_t(x|0)=\theta_{\mathrm{H}}(t)\int_{x(0)=0}^{x(t)=x}\mathcal{D}x\,\mathrm{e}^{-\int_0^t \mathrm{d}t'\,(\frac14\dot{x}^2+V(x))} \quad \text{solves} \quad \left(\frac{\partial}{\partial t}-\frac{\partial^2}{\partial x^2}+V(x)\right)K_t(x|0)=\delta(x)\,\delta(t)\,. \tag{7}$$

In a recent experiment, the phase coherence lengths obtained in two different devices were compared : from the magneto-conductance of a long wire and the one from the analysis of the Aharonov-Bohm oscillations in a mesoscopic ring of perimeter $L \simeq 14\ \mu$m. Discuss the result (Fig. 2).

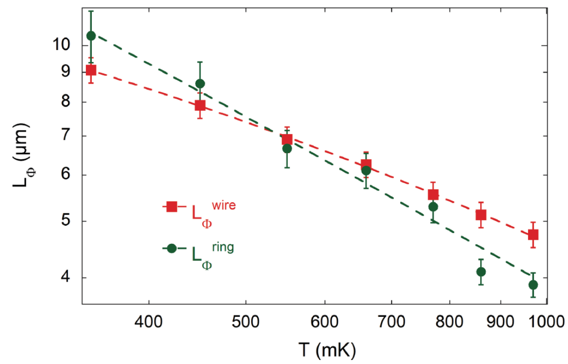


Figure 2: From Ref. [5].

The two-dimensional case : The study of the two-dimensional case along the same lines is more complicated because both large scale cutoff (well acounted for by the influence functional discussed above) and short scale cutoff (more naturally introduced in conventional perturbation theory in Fourier space) matter. The appropriate formulation within influence functional was finally achieved by Jan von Delft and collaborators [6, 7] (see also the review [8]. A more simple discussion was provided in Ref. [9].

Further reading :

- The famous paper : B. L. Altshuler, A. G. Aronov and D. E. Khmelnitsky [10].
- The pedagogical review : S. Chakravarty and A. Schmid [2].
- The present text is inspired by our article [11].

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