Correction de l'examen du 29 mars 2017

1/ The oscillations of the resistance with $\mathcal{B}$ at high field are the Shubnikov-de-Haas oscillations : a strong enough magnetic field induces density of states $\rho(\varepsilon_F)$ oscillations (precursor of the Landau spectrum), which induce oscillations of the conductivity $\sigma = 2_s e^2 \rho(\varepsilon_F)\, D$.

2/ We solve (3). The solution is continuous at $x = x_0$:

$$\mathcal{C}(x, x_0; \gamma) = A \begin{cases} \mathrm{e}^{+\sqrt{\gamma}(x-x_0)} & \text{for } x < x_0 \\ \mathrm{e}^{-\sqrt{\gamma}(x-x_0)} & \text{for } x > x_0 \end{cases} \tag{8}$$

Imposing the second boundary condition, we find $-2\sqrt{\gamma}A = -1$, hence

$$\mathcal{C}(x, x_0; \gamma) = \frac{1}{2\sqrt{\gamma}}\, \mathrm{e}^{-\sqrt{\gamma}|x-x_0|} \,. \tag{9}$$

3/ We sandwich (2) between $\chi_n(y)$ and $\chi_m(y)$ and integrate. We use

$$\int_0^w \mathrm{d}y\mathrm{d}y'\, \chi_n(y)\, \partial_y^2 P_c(\vec{r}, \vec{r}\,')\, \chi_m(y') = \int_0^w \mathrm{d}y\mathrm{d}y'\, \underbrace{\partial_y^2 \chi_n(y)}_{-\varepsilon_n \chi_n(y)}\, P_c(\vec{r}, \vec{r}\,')\, \chi_m(y')$$

and

$$\int_0^w \mathrm{d}y\mathrm{d}y'\, \chi_n(y)\, \delta(\vec{r} - \vec{r}\,')\, \chi_m(y') = \delta(x - x') \int_0^w \mathrm{d}y\, \chi_n(y)\, \chi_m(y) = \delta(x - x')\, \delta_{n,m}$$

Finally

$$\left[\gamma + \varepsilon_n - \partial_x^2\right] P_{n,m}(x, x') = \delta(x - x')\, \delta_{n,m} \tag{10}$$

as a consequence

$$P_{n,m}(x, x') = \delta_{n,m}\, \mathcal{C}(x, x'; \gamma + \varepsilon_n) = \delta_{n,m}\, \frac{1}{2\sqrt{\gamma + \varepsilon_n}}\, \mathrm{e}^{-\sqrt{\gamma+\varepsilon_n}|x-x'|} \,. \tag{11}$$

4/ "*Quasi-1D approximation*" : We consider length scales much larger than the width, i.e. $L_\varphi \gg w$ (recall that L_φ set the typical size of trajectories contributing to the weak localisation). The Cooperon is

$$P_c(\vec{r}, \vec{r}\,') = \sum_{n=0}^{\infty} \chi_n(y)\, P_{n,n}(x, x')\, \chi_n(y') \tag{12}$$

The first term $P_{0,0}(x, x') = \frac{L_\varphi}{2} \mathrm{e}^{-|x-x'|/L_\varphi}$ is a function of height L_φ and width L_φ. Using that $\sqrt{\gamma + \varepsilon_n} \simeq n\pi/w$, we see that the next terms $P_{n,n}(x, x') = \frac{w}{2n\pi} \mathrm{e}^{-n\pi|x-x'|/w}$ (height w/n and width w/n) are negligible. Finally we deduce that the sum over traverse modes is dominated by the first term : $P_c(\vec{r}, \vec{r}\,') \simeq \frac{L_\varphi}{2w} \mathrm{e}^{-|x-x'|/L_\varphi}$.

5/ We only need the Cooperon at coinciding points, thus $\Delta\sigma = -\frac{2_s e^2}{\pi\hbar}\frac{L_\varphi}{2w}$. Using $\Delta g = (w/L)\times(h/2_s e^2)\times\Delta\sigma$, we recover the result

$$\boxed{\Delta g = -\frac{L_\varphi}{L}} \tag{13}$$

6/ Strong localisation ?

a) In the strongly localised regime we expect $g \sim \exp[-2L/\xi]$ for $L \gg \xi$, instead of $g \simeq g_{\mathrm{Drude}} = \pi\xi/(2L)$ where $\xi = N_c\ell_e$. Using the data : the number of channels is $N_c = 630/(10\pi) \simeq 20$, thus $\xi = 20\times 340 \simeq 7.2\ \mu$m.

b) The largest phase coherence length is $L_\varphi(10\ \mathrm{mK}) \simeq 8.2\ \mu$m, i.e. it is comparable, but not much larger. We cannot observe the strong localisation under such conditions (note that in narrow wires, $w \simeq 50$ nm, made of 3D metal deposited on a substrate, the elastic mean free path is shorter $\ell_e \simeq 20$ nm but the number of channels is much larger, $N_c \gtrsim 50\,000$, so that the localisation length is larger $\xi \sim 1$ mm and strong localisation irrealistic).

c) In 2D metal, the Drude dimensionless conductance is $g_{\mathrm{Drude}} = \pi N_c\ell_e/(2L) = \pi\xi/(2L)$. The relative correction is thus

$$\frac{\Delta g}{g_{\mathrm{Drude}}} = -\frac{2L_\varphi}{\pi\xi} \tag{14}$$

The weak localisation is "weak" (validity of the diagrammatic approach) for $L_\varphi \ll \xi$ (no strong localisation). The experiment is slightly at the border of this condition, $L_\varphi(36\ \mathrm{mK}) \simeq 5.3\ \mu$m (and shorter for higher temperatures).

7/ The heuristic argument is based on

$$\Delta\sigma(\mathcal{B}) \sim -\sum_{\mathcal{C}_t \text{ with } t<\tau_\varphi} \mathrm{e}^{4\pi\mathrm{i}\Phi[\mathcal{C}_t]/\phi_0}$$

where the sum runs over closed diffusive trajectories (loops) smaller than L_φ. The magnetic flux encircled by the trajectory is $\Phi[\mathcal{C}_t]$. The magnetic field introduces a second cutoff as it eliminates all trajectories which intercept a flux $\Phi[\mathcal{C}_t] > \phi_0$. This makes the sum decaying with $\mathcal{B}$, so that $\Delta\sigma(\mathcal{B})$ grows with $\mathcal{B}$. This is the anomalous magnetoconductance.

The new cutoff $L_\mathcal{B}$ corresponds to trajectories carrying one quantum flux. The typical surface of a trajectory in the wire is $L_t w$ therefore we write $L_\mathcal{B} w\mathcal{B} \sim \phi_0$ hence $L_\mathcal{B} \sim \hbar/(e\mathcal{B}w)$. At strong magnetic field ($L_\mathcal{B} < L_\varphi$) the magnetic field provides the dominant cutoff so that

$$\Delta g = -\frac{L_\varphi}{L} \longrightarrow \Delta g(\mathcal{B}) \simeq -\frac{L_\mathcal{B}}{L} \sim -1/|\mathcal{B}|$$

8/ We still assume the structure $P_c(\vec r,\vec r\,') \simeq \frac{1}{w}P_{0,0}(x,x')$: this allows to "project" equation (2) on the first transverse mode. Assuming $P_c(\vec r,\vec r\,')$ depends only on x and x' :

$$\int_0^w \frac{\mathrm{d}y}{w}\left[\frac{1}{L_\varphi^2} - \partial_x^2 + \frac{4\mathrm{i}e}{\hbar}A_x(y)\,\partial_x + \frac{4e^2}{\hbar^2}A_x(y)^2\right]P_c(\vec r,\vec r\,') = \frac{1}{w}\delta(x-x')$$

The choice of the gauge leads to $\int_0^w \mathrm{d}y\,A_x(y) = 0$ and $\int_0^w \mathrm{d}y\,A_x(y)^2 = \mathcal{B}^2w^2/12$. Finally we obtain

$$\left[\frac{1}{L_\varphi^2} - \partial_x^2 + \frac{e^2\mathcal{B}^2w^2}{3\hbar^2}\right]P_c(\vec r,\vec r\,') = \frac{1}{w}\delta(x-x')$$

that is the same equation as for $\mathcal{B}=0$, provided

$$\gamma = \frac{1}{L_\varphi^2} \longrightarrow \frac{1}{L_\varphi^{\mathrm{eff}}(\mathcal{B})^2} = \frac{1}{L_\varphi^2} + \frac{1}{L_\mathcal{B}^2} \quad \text{with } L_\mathcal{B} = \frac{\sqrt{3}}{|e\mathcal{B}|w}\,. \tag{15}$$

We have now obtained the precise expression of $L_\mathcal{B}$.

9/ *a*) Performing the substitution (15) in $\Delta g = -\frac{L_\varphi}{L}$, we deduce the magnetoconductance

$$\boxed{\Delta g(\mathcal{B}) = -\frac{L_\varphi}{L}\frac{1}{\sqrt{1+(\mathcal{B}/\mathcal{B}_\varphi)^2}} \qquad \text{with } \mathcal{B}_\varphi \stackrel{\text{def}}{=} \frac{\sqrt{3}\hbar}{ewL_\varphi} = \frac{\sqrt{3}}{2\pi}\frac{\phi_0}{wL_\varphi}} \tag{16}$$

b) The limiting behaviours are $\Delta g(\mathcal{B}) - \Delta g(0) \simeq +\frac{L_\varphi}{2L\mathcal{B}_\varphi^2}\,\mathcal{B}^2$ at $\mathcal{B} \ll \mathcal{B}_\varphi$ and $\Delta g(\mathcal{B}) \simeq -\frac{L_\mathcal{B}}{L} \sim -1/|\mathcal{B}|$ as expected for $\mathcal{B} \gg \mathcal{B}_\varphi$.

c) The magnetoconductance curve $\Delta g(\mathcal{B})$ has height L_φ/L and width $\mathcal{B}_\varphi \propto 1/L_\varphi$. The experimental data show that $L_\varphi(T) \searrow$ as $T \nearrow$. As temperature increases, the extrinsic degrees of freedom are activated and decoherence becomes more efficient.

d) The validity of the quasi-1D treatment is $L_\varphi \gg w$ at $\mathcal{B}=0$. Similarly we expect that the validity at high field is $L_\mathcal{B} \gg w$, i.e.

$$|\mathcal{B}| \ll \frac{\phi_0}{w^2}\,.$$

Using $w = 0.63\,\mu$m this gives $|\mathcal{B}| \ll 100$ Gauss, which is the case for the data.

e) For strong magnetic field such that $|\mathcal{B}| \gg \phi_0/w^2$, the cutoff is shorter than the width. Assuming that the diffusion approximation still holds, this means that one enters into the 2D regime (the diffusion cannot anymore be considered as effectively 1D). In 2D we have $\Delta\sigma \simeq -\frac{2_s e^2}{\pi h}\ln(L_\varphi/\ell)$. At strong field $L_\varphi \to L_\mathcal{B}^{(2D)} = \sqrt{\phi_0/\mathcal{B}}$ hence $\Delta\sigma(\mathcal{B}) \sim -\ln L_\mathcal{B}^{(2D)} \sim +\ln|\mathcal{B}|$.

It is not really what the experimental data show : for strong field, there is probably a problem with the diffusion approximation in these samples with very large ℓ_e...

To know more about it

• This analysis was proposed in a well-known paper :
B. L. Al'tshuler and A. G. Aronov, *Magnetoresistance of thin films and of wires in a longitudinal magnetic field*, JETP Lett. **33**(10), 499 (1981).

• **Semi-ballistic regime.**– Many experiments (like the one studied here) are performed with long wires etched in a 2DEG at the interface of two semiconductors ($GaAs/GaAl_{1-x}As_x$). In this case the elastic mean free path $\ell_e^{(2D)}$ of the original 2DEG is usually larger than the section of the wire (this is not the case here). The effective elastic mean free path in such wires is also larger than the section $\ell_e^{(1D)} > w$, mostly due to reflections on the boundaries. The dephasing by the magnetic field involves the different length scales due the phenomenon of flux cancellation. This has been described by semiclassical methods by Dugaev-Khmelnitski and Beenakker-van Houten :.
V. K. Dugaev and D. E. Khmel'nitzkiĭ, *Magnetoresistance of metal films with low impurity concentrations in parallel magnetic field*, Sov. Phys. JETP **59**(5), 1038 (1984).
C. W. J. Beenakker and H. Van Houten, *Boundary scattering and weak localization of electrons in a magnetic field*, Phys. Rev. B **38**(5), 3232 (1988).

• The experiment analysed here was performed at the Institut Néel (Grenoble), cf. :
Y. Niimi, Y. Baines, T. Capron, D. Mailly, F.-Y. Lo, A. D. Wieck, T. Meunier, L. Saminadayar, and C. Bäuerle, *Quantum coherence at low temperatures in mesoscopic systems : Effect of disorder*, Phys. Rev. B **81**, 245306 (2010).