Master iCFP February 4, 2021
Wave dynamics in random media

TD n°5 : Magneto-conductance of 2D metals

The fit of the anomalous magneto-conductance of 2D electron gas (or metallic films) and wires is a powerful tool which has been extensively used in order to extract the phase coherence length L_φ of metallic devices at low T ($\lesssim$ few K). The fit of $\overline{\Delta\sigma}(\mathcal{B}, L_\varphi)$ is performed at several temperatures what allows to extract the temperature dependence $L_\varphi(T)$ and identify the microscopic mechanisms responsible for dephasing and/or decoherence.

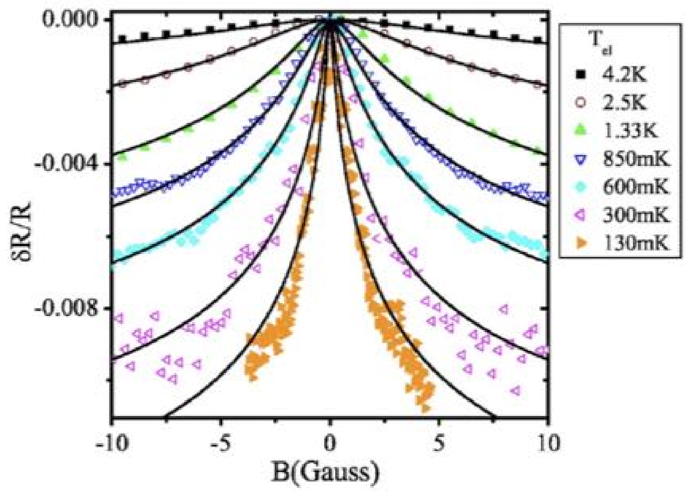

Figure 1: *Magnetoresistance curves for a 2DEG as a function of the magnetic field in Gauss (1 Gauss= 10^{-4} Tesla). From Ref. [1].*

We consider a two dimensional electron gas (2DEG) submitted to a perpendicular magnetic field $\mathcal{B}$. In this case it will be convenient to write the Cooperon as an integral of the propagator in time

$$\overline{\Delta\sigma} = -\frac{2_s e^2 D}{\pi\hbar}\int_0^\infty \mathrm{d}t\,\mathcal{P}_t(\vec{r}|\vec{r})\,\left(\mathrm{e}^{-t/\tau_\varphi} - \mathrm{e}^{-t/\tilde{\tau}_e}\right) \qquad (1)$$

where the second exponential cut off the contribution of small times, that are not described by the diffusion approximation : $\tau_\varphi = L_\varphi^2/D$ and $\tilde{\tau}_e = \ell_e^2/D$. The factor 2_s is the spin degeneracy. The time propagator of the diffusion

$$\mathcal{P}_t(\vec{r}|\vec{r}\,') = \theta_{\mathrm{H}}(t)\,\langle\,\vec{r}\,|\mathrm{e}^{Dt\left(\vec{\nabla}-\frac{2\mathrm{i}e}{\hbar}\vec{A}\right)^2}|\,\vec{r}\,'\,\rangle \qquad (2)$$

solves the diffusion-like equation

$$\left[\partial_t - D\left(\vec{\nabla} - \mathrm{i}\frac{2e}{\hbar}\,\vec{A}\right)^2\right]\mathcal{P}_t(\vec{r}|\vec{r}\,') = \delta(t)\delta(\vec{r}-\vec{r}\,') \qquad (3)$$

1/ Using the mapping onto the Landau problem, compute $\mathcal{P}_t(\vec{r}|\vec{r})$ in the plane.

Hint : We recall that the spectrum of eigenvalues of the 2D Hamiltonian $H_{\text{Landau}} = -\frac{\hbar^2}{2m}(\vec{\nabla} - \frac{\mathrm{i}}{\hbar}e\vec{A})^2$ for a homogeneous magnetic field is the Landau spectrum $\varepsilon_n = \hbar\omega_c(n + 1/2)$ for $n \in \mathbb{N}$, where $\omega_c = eB/m$ and where each Landau level has a degeneracy proportional to the surface of the plane $d_{\rm LL} = \frac{e\mathcal{B}\mathrm{Surf}}{h}$. The partition function of the Landau problem $Z_{\rm Landau} = \int \mathrm{d}\vec{r}\,\langle\vec{r}|\mathrm{e}^{-\frac{t}{\hbar}H_{\rm Landau}}|\vec{r}\rangle$ can be easily calculated.

2/ *a*) Using the integral given in the appendix, deduce that

$$\boxed{\overline{\Delta\sigma}(\mathcal{B}) = \frac{2_s e^2}{h}\,\frac{1}{2\pi}\left[\psi\left(\frac{1}{2}+\frac{L_{\mathcal{B}}^2}{L_\varphi^2}\right)-\psi\left(\frac{1}{2}+\frac{L_{\mathcal{B}}^2}{\ell_e^2}\right)\right]} \tag{4}$$

where $L_{\mathcal{B}}$ will be related to the magnetic field.

b) What is the magnetic field corresponding to $L_{\mathcal{B}} = 1\,\mu$m ? And $L_{\mathcal{B}} = 20$ nm ? Looking at the range of magnetic field on the experimental curve, argue that it is justified to simplify the result as

$$\overline{\Delta\sigma}(\mathcal{B}) = \frac{2_s e^2}{h}\,\frac{1}{2\pi}\left[\psi\left(\frac{1}{2}+\frac{L_{\mathcal{B}}^2}{L_\varphi^2}\right)-\ln\left(\frac{L_{\mathcal{B}}^2}{\ell_e^2}\right)\right] \tag{5}$$

c) Analyse the zero field value $\overline{\Delta\sigma}(0)$. Discuss the limiting behaviours of $\overline{\Delta\sigma}(\mathcal{B})-\overline{\Delta\sigma}(0)$.

3/ Discuss the experimental data of Fig. 1.

Appendix :

We give the integral (formula 3.541 of Gradshteyn & Ryzhik, Ref. [2])

$$\int_0^\infty \mathrm{d}x\,\frac{\mathrm{e}^{-ax}-\mathrm{e}^{-bx}}{\sinh\lambda x} = \frac{1}{\lambda}\left[\psi\left(\frac{1}{2}+\frac{b}{2\lambda}\right)-\psi\left(\frac{1}{2}+\frac{a}{2\lambda}\right)\right]\ , \tag{6}$$

where $\psi(z)=\frac{\mathrm{d}}{\mathrm{d}z}\ln\Gamma(z)$ is the digamma function. We deduce the functional relation $\psi(z+1)=\psi(z)+\frac{1}{z}$. We give two values $\psi(1)=-\mathbf{C}\simeq-0.577215$ (Euler-Mascheroni constant) and $\psi(1/2)=-\mathbf{C}-2\ln 2$, and the limiting behaviour

$$\psi(x+1/2) \underset{x\to\infty}{=} \ln x+\frac{1}{24x^2}+\mathcal{O}(x^{-3}) \tag{7}$$

5.2 Magneto-conductance in narrow wires

The aim of the exercice is to analyse the magneto-conductance of a long wire of section W submitted to a perpendicular *homogeneous* magnetic field. For simplicity we consider the two-dimensional situation of a wire etched in a two-dimensional electron gas (2DEG). We recall that the weak localisation correction to the conductivity is given by

$$\overline{\Delta\sigma} = -\frac{2_s e^2}{\pi\hbar}\, P_c(\vec{r},\vec{r}) \quad \text{with} \quad \left[\gamma - \left(\vec{\nabla} - \mathrm{i}\frac{2e}{\hbar}\,\vec{A}\right)^2\right] P_c(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')\,, \tag{8}$$

where $\gamma = 1/L_\varphi^2$.

We consider the geometry of a infinitly long quasi-1D wire, i.e. $x \in \mathbb{R}$ and $y \in [0, W]$.

1/ Relate the conductivity σ of the wire to the conductance $G = I/V$.

We choose the Landau gauge such that A_x is an **antisymmetric** function of the transverse coordinate. If $y \in [0, W]$ we choose $A_x(W-y) = -A_x(y)$, i.e.

$$A_x(y) = (W/2 - y)\,\mathcal{B} \quad \text{and} \quad A_y = 0\,. \tag{9}$$

We assume that the confinment imposes Neumann boundary conditions

$$\partial_y P_c(\vec{r},\vec{r}\,')\big|_{y=0\ \&\ W} = 0\,. \tag{10}$$

2/ Zero field.– The aim is to construct the spectrum of the Laplace operator $\Delta = \partial_x^2 + \partial_y^2$ in the wire.

a) Use the separability of the problem to find the spectrum of eigenvectors and eigenvalues of the Laplace operator in the infinitly long wire of width W.

b) **Green's function.**– Justify the following representation

$$P_c(\vec{r},\vec{r}\,') = \sum_{n=0}^{\infty} \chi_n(y)\, \underbrace{\langle\, x\,|\frac{1}{\gamma + \varepsilon_n - \partial_x^2}|\, x'\,\rangle}_{P_c(x,x')\ \text{for}\ \gamma\to\gamma+\varepsilon_n}\, \chi_n(y') \tag{11}$$

The functions $\chi_n(y)$ satisfy the differential equation $-\partial_y^2\chi_n(y) = \varepsilon_n\,\chi_n(y)$ on $[0, W]$ with appropriate boundary conditions.

Under what condition on W and L_φ can the Cooperon be approximated by the 1D Cooperon $P_c(x,x') = \langle\, x\,|\big(\gamma - \partial_x^2\big)^{-1}|\, x'\,\rangle$?

3/ Weak magnetic field.– In the diffusion approximation, the Cooperon can be interpreted as the Green's function of the operator $-(\nabla - \frac{\mathrm{i}}{\hbar}2eA)^2$, Eq. (8). We recall that this treatment of the magnetic field in the diffusion approximation supposes that $\ell_e \ll R_c$, where $R_c = v_F/\omega_c$ is the cyclotron radius of electrons with energy ε_F ($\omega_c = e\mathcal{B}/m_*$ is the cyclotron pulsation). Our aim is to compute the Cooperon in the weak magnetic field limit.

a) Projecting the differential equation (8) (i.e. $\int_0^W \frac{\mathrm{d}y}{W}\times\cdots$), show that the effect of the magnetic field can be absorbed by a transformation of the phase coherence length in the one-dimensional cooperon

$$\frac{1}{L_\varphi^2} \longrightarrow \frac{1}{L_\varphi^{\mathrm{eff}}(\mathcal{B})^2} \stackrel{\mathrm{def}}{=} \frac{1}{L_\varphi^2} + \frac{1}{L_\mathcal{B}^2} \qquad \text{where } \frac{1}{L_\mathcal{B}^2} = \frac{4e^2}{\hbar^2}\int_0^W \frac{\mathrm{d}y}{W} A_x(y)^2\,. \tag{12}$$

b) Deduce explicitly $L_\mathcal{B}$ and discuss the range of validity of this approximation, i.e. what is the condition on $\mathcal{B}$, W and L_φ ?

c) We recall the expression of the 1D Cooperon $P_c(x,x) = \langle\, x\,|\frac{1}{1/L_\varphi^2-\partial_x^2}|\,x\,\rangle = L_\varphi/2$. Deduce the expression of the magneto-conductivity $\overline{\Delta\sigma}(\mathcal{B})$ of the infinitly long wire and show that the WL correction to the dimensionless conductance can be written as

$$\boxed{\overline{\Delta g}(\mathcal{B}) = \frac{\overline{\Delta g}(0)}{\sqrt{1+(\mathcal{B}/\mathcal{B}_\varphi)^2}}} \tag{13}$$

Give the expression of the scale $\mathcal{B}_\varphi$ and interpret physically this expression.

d) Discuss the experimental data of Fig. 2 at the light of this calculation. In particular, how can one interpret the evolution of the curve when the sample is cooled down ?

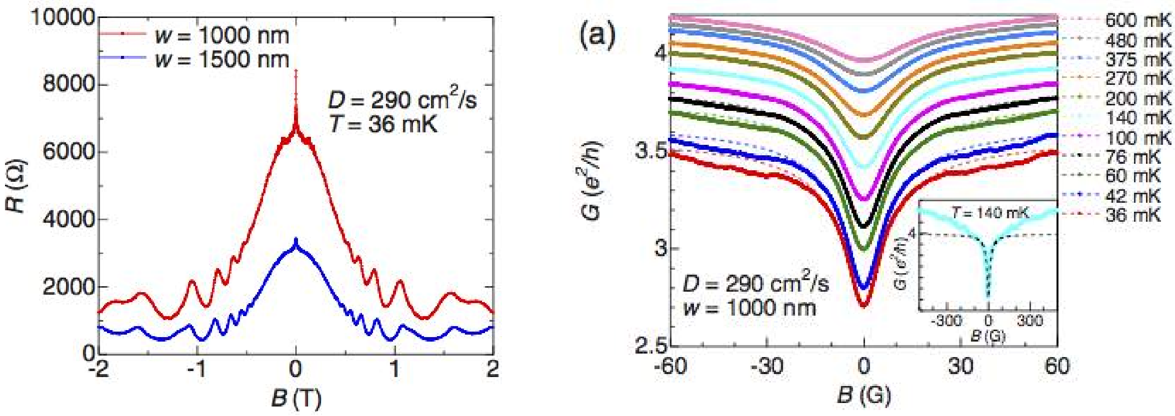

Figure 2: *Magnetoconductance curves for a long wire etched in a 2DEG as a function of the magnetic field in Gauss (*1 *Gauss=* 10^{-4} *Tesla). Length of the wire is* $L = 150\,\mu$m*, lithographic width* $W_{\text{litho}} = 1\,\mu$m *and effective width* $W = 630\,$nm*. Electronic density is* $n_e = 1.5\times10^{15}\,\text{m}^{-2}$*. Left : Resistance over a large window in* $\mathcal{B}$ *field,* $[-2\,\text{T}, +2\,\text{T}]$*. Right : Conductance over small window around zero field,* $[-6\text{ mT}, +6\text{ mT}]$*. From Niimi et al. Phys. Rev. B* ***81****, 245306 (2010) [****?****].*

e) In the "high field" regime, $L_\mathcal{B} < W$, what expression do you expect for the MC ?

Remarks :

• This analysis was performed in a well-known paper : by Altshuler and Aronov, Sov. Phys. JETP (1981) (Ref. [**?**]).

• **Semi-ballistic regime.**– Many experiments are performed on long wires etched in a two-dimensional electron gas (2DEG) at the interface of two semiconductors ($\text{GaAs}/\text{GaAl}_{1-x}\text{As}_x$). In this case the elastic mean free path $\ell_e^{(2\text{D})}$ of the original 2DEG is usually larger than the section of the wire. The effective elastic mean free path in the wire is also larger than the section $\ell_e^{(1\text{D})} > W$. The dephasing by the magnetic field involves different length scale due the phenomenon of flux cancellation. This has been described by semiclassical methods by Dugaev and Khmelnitskii [**?**] and Beenakker and van Houten, Phys. Rev. B (1988) (Ref. [**?**]).

References

[1] M. Eshkol, E. Eisenberg, M. Karpovski, and A. Palevski, Dephasing time in a two-dimensional electron Fermi liquid, Phys. Rev. B **73**(11), 115318 (2006).

[2] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series and products*, Academic Press, fifth edition, 1994.