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(1)

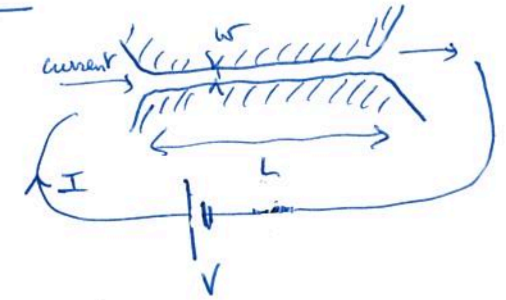
Examen : Ondes en milieux désordonnés et localisation

30 mars 2018

Problème : Transport non linéaire.

→ Consider a quasi 1D wire

$L \gg w \Rightarrow$ equations become effectively 1D.



A. Cooperon and Diffuson

1/ they represent ladder diagram (two particle propagators)

Diffusion : $\sum_n$ 

2/ Quasi 1D approx $\Rightarrow$ eq. for cooperon and diffuson takes the form

$$\underbrace{[\gamma}_{1/L_\varphi^2 + \dots} - \partial_x^2]\, P(x,x') = \delta(x-x')$$

connection to contacts :

$$P(0,x') = P(L,x') = 0$$

a) homogeneous eq : $(\gamma - \partial_x^2)\,\psi = 0$

$\psi_L(x)$ satisfies $\psi_L(0) = 0$

$\psi_R(x)$ —— $\psi_R(L) = 0$

$$\Rightarrow \begin{cases} \psi_L(x) = \operatorname{sh}\sqrt{\gamma}\, x \\ \psi_R(x) = \operatorname{sh}\sqrt{\gamma}\,(L-x) \end{cases}$$

b) matching. $(\gamma - \partial_x^2)\, P = \delta$

Second derivative of P produces a $\delta \Rightarrow P'$ contains a step function and P is continuous.

$$\Downarrow$$

$$\begin{cases} P(x,x')\Big|_{x=x'-}^{x=x'+} = 0 \\ \partial_x P(x,x')\Big|_{x=x'-}^{x=x'+} = -1 \end{cases}$$

c) Ensure continuity :

$$P(x,x') = A \cdot \underbrace{\begin{cases} \psi_L(x)\cdot\psi_R(x') & \text{for } x<x' \\ \psi_R(x)\cdot\psi_L(x') & \text{for } x>x' \end{cases}}_{\text{Satisfies the boundary conditions}}$$

$A = ?$ $\Rightarrow$ one imposes the 2nd condition

$$A\ \underbrace{\left[\psi_R'(x')\,\psi_L(x') - \psi_L'(x')\,\psi_R(x')\right]} = -1$$

we recognize the Wronskian.

$$W[\psi_L, \psi_R] = \psi_L \psi_R' - \psi_L' \psi_R$$

We conclude that

$$\boxed{P(x,x') = -\frac{\psi_L(x_<)\,\psi_R(x_>)}{W[\psi_L,\psi_R]}} \qquad (*)$$

Remarks: (i) this is also valid for a more complex diff. eq. $\quad (\gamma + U(x) - \partial_x^2)\,P(x,x') = \delta(x-x')$
where ψ_L and ψ_R solve the homogeneous eq.

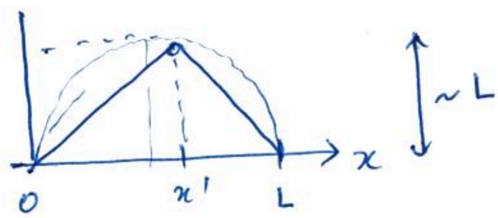
(ii) for such an eq. $W[\psi_L,\psi_R] = \text{cste}$
(easy to check)

Application: $\quad W[\psi_L,\psi_R] = \operatorname{sh}\sqrt{\gamma}x\,(-\sqrt{\gamma})\operatorname{ch}\sqrt{\gamma}(L-x) - \sqrt{\gamma}\operatorname{ch}\sqrt{\gamma}x\operatorname{sh}\sqrt{\gamma}(L-x)$

$$= -\sqrt{\gamma}\ \underbrace{(\operatorname{sh}\operatorname{ch} + \operatorname{ch}\operatorname{sh})}_{\operatorname{sh}[\sqrt{\gamma}(x+L-x)]} = -\sqrt{\gamma}\operatorname{sh}\sqrt{\gamma}L$$

$$\boxed{P(x,x') = \frac{\operatorname{sh}\sqrt{\gamma}x_<\ \operatorname{sh}\sqrt{\gamma}(L-x_>)}{\sqrt{\gamma}\operatorname{sh}\sqrt{\gamma}L}}$$

2) $\underline{\gamma = 0}$ $\quad P(x,x') = x_<\left(1 - \frac{x_>}{L}\right)$



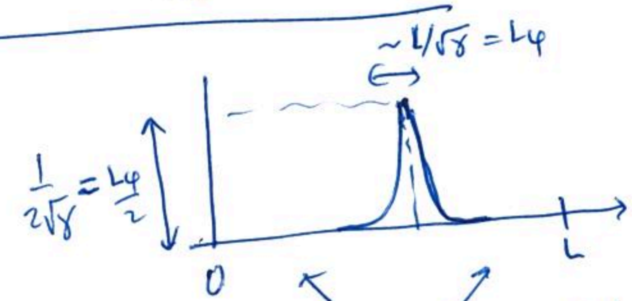
3) $\underset{\Downarrow}{\underline{\text{large } \gamma}}$ $\quad \underbrace{x, x' \text{ "far" from boundaries.}}$

$\sqrt{\gamma}L \gg 1 \qquad \sqrt{\gamma}x \ \&\ \sqrt{\gamma}(L-x) \gg 1$

$$\Downarrow$$

$$P(x,x') \approx \frac{\frac{1}{2}e^{\sqrt{\gamma}x_<}\ \frac{1}{2}e^{\sqrt{\gamma}(L-x_>)}}{\sqrt{\gamma}\,\frac{1}{2}e^{\sqrt{\gamma}L}} = \frac{1}{2\sqrt{\gamma}}\,e^{-\sqrt{\gamma}\overbrace{(x_>-x_<)}^{|x-x'|}}$$

$$P(x,x') \simeq \frac{1}{2\sqrt{\gamma}} e^{-\sqrt{\gamma}|x-x'|}$$ is translation invariant



does not feel boundaries.

Condition $\sqrt{\gamma} L \gg 1$ $\quad \gamma = \frac{1}{L_\varphi^2} = \frac{1}{D\tau_\varphi}$

$1/\gamma \sim D/\text{time}$

$\sqrt{\gamma}L \gg 1 \iff \underbrace{\tau_\varphi}_{\text{time scale probed}} \ll \underbrace{\tau_D}_{\text{Thouless time : time needed to reach the boundaries}} = \frac{L^2}{D} \Rightarrow$ does not feel boundaries

$\sqrt{\gamma}L \lesssim 1 \iff \tau_\varphi \gtrsim \tau_D = \frac{L^2}{D}$

Cooperon / Diffuson involve trajectories for time $\gtrsim L^2/D$ which explore the full sample.

B. Conductance correlator

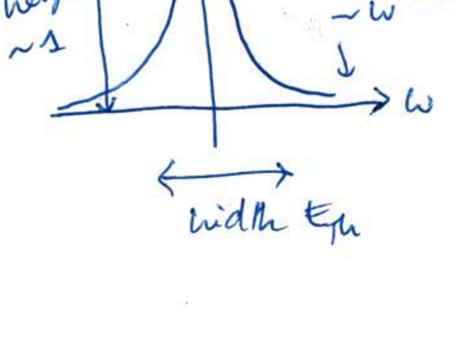
$$\mathcal{C}(\omega) = \overline{\delta g(\omega+\varepsilon_F)\,\delta g(\varepsilon_F)}$$

1/ one diagram: two conductivity bubbles.

two diffusons / cooperons

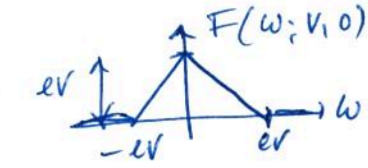
2/ $\mathcal{C}(\omega) \simeq \frac{2}{15} - \frac{2}{1575}\left(\frac{\omega}{E_{Th}}\right)^2$ for $\omega \ll E_{Th}$

$\simeq 3\sqrt{2}\left(\frac{E_{Th}}{\omega}\right)^{3/2}$ for $\omega \gg E_{Th}$



C. I-V characteristic

1/ $\left[\tilde{I}\right] = \left[\frac{h}{e} I\right] = \left[\frac{h}{Q}\frac{Q}{T}\right] = \left[\text{Energy}\right]$

2/ 

$\Rightarrow \int F(\omega; V, 0)\, d\omega = (eV)^2$

$\varphi(\omega) \sim \omega^{-3/2} \Rightarrow \int^{\infty} \varphi(\omega) < \infty$

$\int d\omega\, \underbrace{\varphi(\omega)}_{\simeq f\left(\frac{\omega}{E_{Th}}\right)} = E_{Th} \underbrace{\int du\, f(u)}_{\sim 1} \sim E_{Th}$

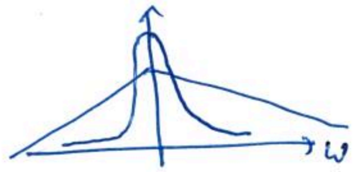
↳ dimensionless fct

3/ $\underline{eV \ll E_{Th}}$ $\overline{\delta \tilde{I}^2} = \int d\omega\, \underbrace{F(\omega; V, 0)}_{\text{narrow}}\, \underbrace{\varphi(\omega)}_{\text{broad}}$

⇓

$\simeq \underbrace{\varphi(0)}_{\frac{2}{15}} \underbrace{\int d\omega\, F(\omega; \cdot)}_{(eV)^2} = \frac{2}{15}(eV)^2$

4/ $\underline{eV \gg E_{Th}}$ $\overline{\delta \tilde{I}^2} = \int d\omega\, \underbrace{F(\omega; V, 0)}_{\text{broad}}\, \underbrace{\varphi(\omega)}_{\text{narrow}}$

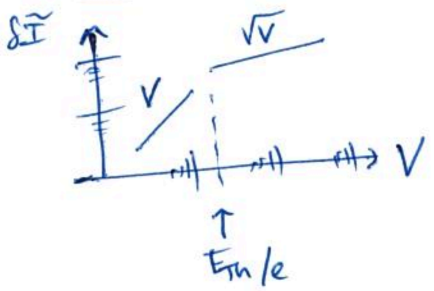


$\simeq \underbrace{F(0; V, 0)}_{eV} \underbrace{\int d\omega\, \varphi(\omega)}_{\sim E_{Th}}$

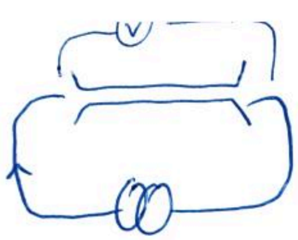
$\overline{\delta \tilde{I}^2} \sim eV \cdot E_{Th}$

Conclusion:

$$\boxed{\delta \tilde{I} \sim \begin{cases} eV & \text{for } eV \ll E_{Th} \\ \sqrt{eV \cdot E_{Th}} & \text{for } eV \gg E_{Th} \end{cases}}$$



5/ <u>Experiment.</u>



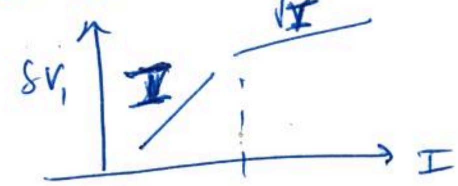
voltage
mesoscopic fluctuations

$$\begin{cases} V \to \dfrac{\tilde{I}}{\bar{g}} \\ \delta\tilde{I} \to \delta V.\bar{g} \end{cases} \Rightarrow \quad \delta V \sim \begin{cases} \dfrac{e\tilde{I}}{\bar{g}^2} & \text{for } \tilde{I} \lesssim \dfrac{E_{th}\,\bar{g}}{e} \\ \dfrac{\sqrt{e\tilde{I}E_{th}}}{\bar{g}^{5/2}} \end{cases}$$

for practical reasons (we need to detect a very small voltage)
we impose an oscillating current $I(t) = I_0 \cos\omega t$
<u>small ω</u>

linear resp. $\Rightarrow \delta V(t) = (\dots)\cos\omega t$
$\delta V \propto I$
non-linear resp. $\Rightarrow \delta V(t) = \underset{\substack{\downarrow \\ \text{non linear} \\ \text{fct}}}{h(I)} = \underbrace{\sum_n (\dots) I^n}_{\substack{\Downarrow \\ \text{contains} \\ \text{all} \\ \text{harmonics}}} = \sum_n \delta V_n \cos n\omega t$

the experimental data show
the expected behaviour

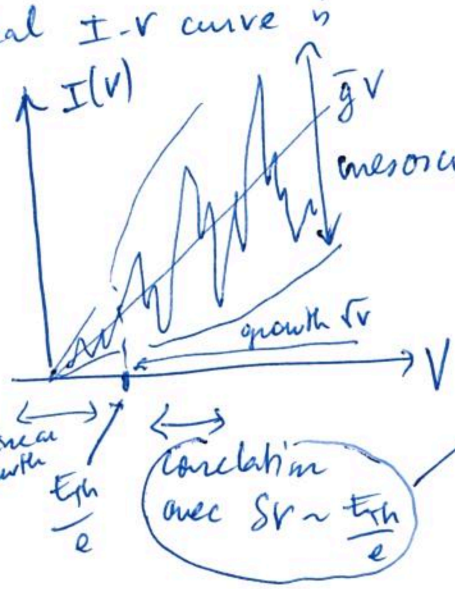


linear at low current $\quad \propto\sqrt{I}$ at high current

Remark: I don't understand why $\delta V_n \neq 0$ (for $n>1$) at low I !!

6/ $\delta\tilde{I}$ presents mesoscopic fluctuations, around $\bar{g}.V$

the typical I-V curve is



mesoscopic fluct. grow as $\sim\sqrt{V}$

$$\delta\tilde{I} \sim E_{th} \times \sqrt{\frac{eV}{E_{th}}} \quad \text{for } eV \gtrsim E_{th}$$

correlations over $\sim E_{th}$

$$\Rightarrow g_d(V) = \frac{d\tilde{I}}{e\,dV} \sim \frac{\delta\tilde{I}}{E_{th}} + \bar{g}$$

$$\underline{\delta g_d(V) \sim \sqrt{\frac{eV}{E_{th}}} \gtrsim 1} \quad \text{for } eV \gtrsim E_{th}$$

change in sign and <u>grows</u>
(fluctuations)

$\Rightarrow$ <u>$g_d(V)$ can become negative</u>