Exercice 2 - Exam ICFP - March 30, 2018.

II Decoherence and CBS.

1. $$\alpha_c = \frac{c}{4\pi\ell^2}\int dz_1 dz_2\, e^{-\frac{(z_1+z_2)}{\ell}}\left[P(\vec{k}_\perp, z_1-z_2, \Omega=0) - P(\vec{k}_\perp, z_1+z_2, \Omega=0)\right].$$

where $$P(\vec{k}_\perp, z, \Omega) = \int \frac{dq_\parallel}{2\pi}\, e^{+iq_\parallel z}\, \frac{1}{-i\Omega + Dq_\parallel^2 + D\vec{k}_\perp^2}$$

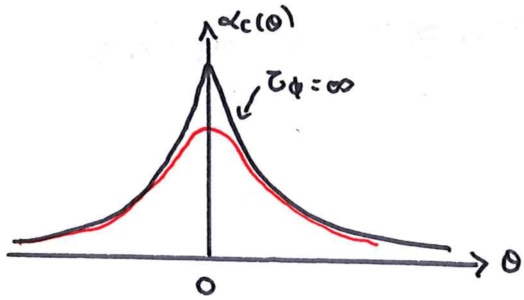
With decoherence, $$P(\vec{k}_\perp, z, \Omega) \longrightarrow \int \frac{dq_\parallel}{2\pi}\, e^{iq_\parallel z}\,\frac{1}{\tau_\phi^{-1} - i\Omega + Dq_\parallel^2 + Dk_\perp^2} = \int \frac{dq_\parallel}{2\pi}\,\frac{e^{iq_\parallel z}}{-i\Omega + Dq_\parallel^2 + D(k_\perp^2 + \frac{1}{\tau_\phi D})}$$

$\Rightarrow$ same as $\tau_\phi = \infty$ with $k_\perp^2 \longrightarrow k_\perp^2 + \underbrace{\frac{1}{D\tau_\phi}}_{\equiv 1/L_\phi^2}$.

CBS lineshape is $$\alpha_c(\theta) = \frac{3}{8\pi}\frac{1}{[1+\sqrt{(\vec{k}_\perp \ell)^2}]^2} \longrightarrow \alpha_c^\phi(\theta) = \frac{3}{8\pi}\frac{1}{\left(1+\sqrt{k_\perp^2\ell^2 + \frac{\ell^2}{L_\phi^2}}\right)^2}$$

i.e $$\boxed{\alpha_c^\phi(\theta) = \frac{3}{8\pi}\frac{1}{\left(1+\sqrt{|k\ell\theta|^2 + \frac{\ell^2}{L_\phi^2}}\right)^2}}$$

$L_\phi = \frac{1}{\sqrt{D\tau_\phi}}$ typical length over which a decoherence process occurs.

2. 

Near $\theta \simeq 0$, $\alpha_c^\phi(\theta) \simeq \frac{3}{8\pi}\frac{1}{(1+\frac{\ell}{L_\phi})^2} < \alpha_c(\theta)$

$\longrightarrow$ CBS peak is reduced

$\longrightarrow$ triangular shape disappears (rounding).

3. At large angles, $\alpha_c^\phi(\theta) \simeq \frac{3}{8\pi}\frac{1}{(k\ell|\theta|)^2} = \alpha_c(\theta)$. This is because only short scattering paths contribute to the wings, and they are less affected by decoherence.

III Decoherence and AL

1. $$\frac{D}{D_B} + \frac{3}{\pi(k\ell)^2} - \frac{3}{2(k\ell)^2}\sqrt{\frac{D_B}{D}}\sqrt{-3i\Omega\tau} = 1. \quad (1)$$

when $k\ell = \sqrt{\frac{3}{\pi}}$, $\frac{D}{D_B} = \frac{\pi}{2}\sqrt{\frac{D_B}{D}}\sqrt{-3i\Omega\tau} \rightarrow D \sim (-i\Omega\tau)^{1/3} \xrightarrow[\Omega\to 0]{} 0$

$k\ell = \sqrt{\frac{3}{\pi}}$ corresponds to the critical point of the Anderson transition.

2. Ansatz $\Rightarrow$ $$\frac{1}{D} = \frac{1}{D_B}\left[1 + \frac{1}{\pi\ell k}\int \frac{d^3\vec{Q}}{(2\pi)^3}\frac{1}{-i\Omega + D_B Q^2 + \tau_\phi^{-1}}\right]$$

$\longrightarrow$ $\boxed{-i\Omega \rightarrow -i\Omega + \tau_\phi^{-1}}$

$\longrightarrow$ For $\Omega \rightarrow 0$, Eq. (1) becomes $$\boxed{\frac{D}{D_B} + \frac{3}{\pi(k\ell)^2} - \frac{3}{2(k\ell)^2}\sqrt{\frac{D_B}{D}}\sqrt{3\frac{\tau}{\tau_\phi}} = 1} \quad (2)$$

* $k\ell \gg \sqrt{\frac{3}{\pi}}$. D is a constant of the order of D_B.

So (2) $\Rightarrow$ $$\boxed{\frac{D}{D_B} \simeq 1 - \frac{3}{\pi(k\ell)^2} + \frac{3}{2(k\ell)^2}\sqrt{\frac{3\tau}{\tau_\phi}}}$$

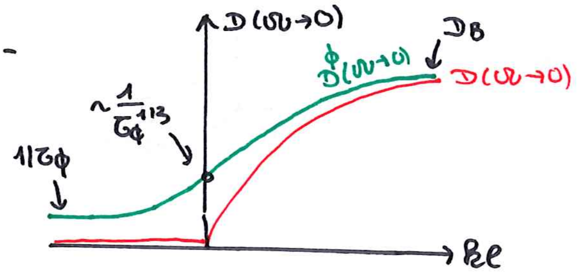
* $k\ell = \sqrt{\frac{3}{\pi}}$ $\Rightarrow$ $$\frac{D}{D_B} = \frac{\pi}{2}\sqrt{\frac{D_B}{D}}\sqrt{3\frac{\tau}{\tau_\phi}} \Rightarrow \left(\frac{D}{D_B}\right)^{3/2} = \frac{\pi}{2}\sqrt{3\frac{\tau}{\tau_\phi}}$$

$$\Rightarrow \boxed{\frac{D}{D_B} \simeq \left(\frac{\pi}{2}\right)^{2/3}\left(3\frac{\tau}{\tau_\phi}\right)^{1/3}}$$

* $k\ell \ll \sqrt{\frac{3}{\pi}}$. D should be close to zero.

So (2) $\Rightarrow$ $$\underbrace{\frac{3}{\pi(k\ell)^2}}_{\gg 1} - 1 - \frac{3}{2(k\ell)^2}\sqrt{\frac{3\tau}{\tau_\phi}}\sqrt{\frac{D_B}{D}} \simeq 0 \Rightarrow \frac{\cancel{3}}{\pi\cancel{(k\ell)^2}} \simeq \frac{\cancel{3}}{2\cancel{(k\ell)^2}}\sqrt{\frac{3\tau}{\tau_\phi}}\sqrt{\frac{D_B}{D}} \Rightarrow \boxed{\frac{D}{D_B} \simeq \frac{3\pi^2}{4}\frac{\tau}{\tau_\phi}}$$

3.



Decoherence kills the Anderson transition, which is transformed into a smooth crossover.

4. Since $D(\omega\to 0)$ is constant, we recover $\langle \vec{r}^2(t)\rangle \propto Dt$ at long times.

$$\langle \vec{r}^2\rangle(t) \propto \frac{t}{\tau_\phi} \Rightarrow \begin{cases} a: & \tau_\phi = 44\tau \\ b: & \tau_\phi = 22\tau \\ c: & \tau_\phi = 9\tau \end{cases}$$