

## Critical temperature and Ginzburg-Landau equation for a trapped Fermi gas

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We discuss a superfluid phase transition in a trapped neutral-atom Fermi gas. We consider the case where the critical temperature greatly exceeds the spacing between the trap levels and derive the corresponding Ginzburg-Landau equation. The latter turns out to be analogous to the equation for the condensate wave function in a trapped Bose gas. The analysis of its solution provides us with the value of the critical temperature  $T_c$  and with the spatial and temperature dependence of the order parameter in the vicinity of the phase transition point. [S1050-2947(98)50608-7]

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The recent progress in the studies of ultracold trapped Bose gases and the discovery of Bose-Einstein condensation [1–3] have stimulated an interest in macroscopic quantum phenomena in trapped Fermi gases. The most prominent phenomena should be connected with a superfluid phase transition associated with the appearance of the order-parameter-macroscopic wave function of strongly correlated two-particle states on the Fermi surface (Cooper pairs). The possibilities of finding this phase transition in trapped Fermi gases have been discussed in Refs. [4–6].

A remarkable feature of neutral-atom Fermi gases is that the Cooper pairing and, hence, the superfluid phase transition can occur for both attractive and repulsive interparticle interaction. For an attractive interaction (negative scattering length  $a$ ) the pairing occurs in the  $s$ -wave channel, as described by the standard BCS approach, and one has "singlet" Cooper pairs. In a neutral-atom Fermi gas, according to the Pauli principle, such a pair can only be formed by two atoms that are in different hyperfine states. Therefore, the critical temperature  $T_c$  of the transition is very sensitive to the difference in concentrations of the two hyperfine components, and under the condition  $\Delta n/n \equiv (n_1 - n_2)/(n_1 + n_2) \geq T_c/\varepsilon_F \ll 1$ , where  $\varepsilon_F \sim \hbar^2 n^{2/3}/m$  is the Fermi energy, there will be a complete suppression of the spin-singlet pairing. For  ${}^6\text{Li}$  with  $a \approx -1140 \text{ \AA}$  [7], one has  $T_c \approx 30 \text{ nK}$  for the atom density  $n = 4 \times 10^{12} \text{ cm}^{-3}$  [4], and the existence of the  $s$ -wave pairing requires  $\Delta n/n < 3 \times 10^{-2}$ .

For positive scattering length (repulsive interaction) the  $s$ -wave pairing is impossible, and one has to consider the mechanism of  $p$ -wave "triplet" pairing, which originates from the effective interaction caused by polarization effects [8]. Actually, this pairing mechanism is insensitive to the sign of the scattering length. It works equally well for  $a < 0$  in the situation where the direct  $s$ -wave pairing is suppressed. In these cases the Pauli principle allows us to have Cooper pairs formed by two particles that are in one and the same hyperfine state, whereas the particles in other hyperfine states participate only in the formation of the effective pairing interaction. Therefore, the  $p$ -wave "triplet" pairing does not require any severe restriction on  $\Delta n$ . As found [9], the corresponding critical temperature  $T_c$  depends nonmonotonically on  $n_i$  and becomes zero only in the case where all particles are in the same hyperfine state. Since the effective interaction based on polarization effects is weaker than the direct interparticle interaction, the  $p$ -wave triplet pairing re-

sults in a lower value of  $T_c$  as compared to the critical temperature for the  $s$ -wave pairing. For example, for the  $p$ -wave pairing in a gas of  ${}^6\text{Li}$  the critical temperature  $T_c \approx 30 \text{ nK}$  corresponds to densities  $n_i \approx 10^{13} \text{ cm}^{-3}$  of different hyperfine state components. (To reach this value one has to trap three hyperfine states and adjust their concentrations with an accuracy of  $\Delta n/n \sim 0.1$ ; see Ref. [5].)

In this paper we study the influence of a (harmonic) trapping potential on superfluid pairing. We derive the corresponding Ginzburg-Landau (GL) equation for the order parameter, assuming the critical temperature  $T_c$  to be much higher than the level spacing  $\Omega$  in the trap. The analysis of this equation provides us with the value of  $T_c$  for the trapped gas and gives the coordinate and temperature dependence of the order parameter in the vicinity of the phase transition. As found, the critical temperature is slightly lower than that for a spatially homogeneous Fermi gas with density  $n_0$  (maximum density of the trapped gas), and the behavior of the order parameter resembles the behavior of a trapped Bose condensate.

We consider a two-component neutral gas of fermionic atoms, with a short-range interatomic interaction, trapped in a spherically symmetric harmonic potential. The two (hyperfine) components are labeled by indices  $\alpha = \pm$  and are assumed to have equal concentrations. The Hamiltonian of the system has the form ( $\hbar = 1$ )

$$H = \int_{\mathbf{r}} \sum_{\alpha} \psi_{\alpha}^{\dagger}(\mathbf{r}) \left( -\frac{1}{2m} \partial^2 - \mu + \frac{m\Omega^2 \mathbf{r}^2}{2} \right) \psi_{\alpha}(\mathbf{r}) + \frac{g}{2} \sum_{\alpha, \beta} \int_{\mathbf{r}} \psi_{\alpha}^{\dagger}(\mathbf{r}) \psi_{\alpha}(\mathbf{r}) \psi_{\beta}^{\dagger}(\mathbf{r}) \psi_{\beta}(\mathbf{r}), \quad (1)$$

where  $\mu$  is the chemical potential (Fermi energy),  $\Omega$  the trap frequency,  $g = 4\pi a/m$  the interaction strength, and  $m$  the atom mass.

The interaction effects can be expressed in terms of a small gaseous parameter  $\lambda = 2|a|p_F/\pi < 1$ , where  $p_F = mv_F = (3\pi^2 n_0)^{1/3}$  is the Fermi momentum. In the spatially homogeneous case the system of fermions described by the Hamiltonian (1), with  $\Omega = 0$ , undergoes a superfluid phase transition. The transition temperature  $T_c^{(0)}$  and the type of pairing depend on the sign of  $a$ . For negative  $a$  (attractive interaction) there will be the  $s$ -wave singlet pairing, whereas

for positive  $a$  the  $p$ -wave triplet pairing should take place (see Ref. [8] for details). In both cases the critical temperature  $T_c^{(0)} = C\varepsilon_F \exp\{-1/\Gamma\}$ , where  $\varepsilon_F = p_F^2/2m$  is the Fermi energy,  $\Gamma$  the pairing interaction, and  $C$  a numerical coefficient of order unity. The value of  $C$  and the expression for  $\Gamma$  depend on the type of pairing. For the singlet pairing one has  $\Gamma = \lambda$ , and for the triplet pairing  $\Gamma \approx \lambda^2/13$ .

In the Thomas-Fermi approach ( $\varepsilon_F \gg \Omega$ ) the density profile of the trapped Fermi gas is  $n(r) = n_0(1 - (r/R_{TF})^2)^{3/2}$ , and the Thomas-Fermi radius  $R_{TF} = v_F/\Omega$  turns out to be the natural length scale in the system. One can also introduce the local Fermi momentum  $p_F(r) = p_F(1 - (r/R_{TF})^2)^{1/2}$  and the density of states on the local Fermi surface,  $\nu_0(r) = mp_F(r)/2\pi^2$ . It should be noted that all these quantities are only slightly influenced by the superfluid pairing, because the latter involves only a small fraction of particles ( $\sim T_c/\varepsilon_F \ll 1$ ), with energies close to the Fermi energy.

For describing the phase transition one has to introduce the order parameter, which in the case of singlet pairing is a complex function  $\Delta(\mathbf{r}) \sim \langle \psi_\alpha(\mathbf{r}) \psi_{\alpha'}(\mathbf{r}) \rangle_{\varepsilon_{\sigma\sigma'}}$ , with  $\varepsilon_{\sigma\sigma'}$  being the antisymmetric tensor. For the triplet pairing the order parameter is a  $3 \times 3$  complex matrix  $\Delta_{ij}(\mathbf{r}) \sim (\sigma_2 \sigma_i)_{\alpha\beta} \times \langle \psi_\alpha(\mathbf{r}) \partial_j \psi_\beta(\mathbf{r}) \rangle$ , where  $\sigma_i$  are the Pauli matrices. The time-independent Ginzburg-Landau (GL) equation describes the equilibrium behavior of the order parameter  $\Delta(\mathbf{r})$  below  $T_c$ , assuming  $T_c - T \ll T_c$ . The critical temperature  $T_c$  can be found as the temperature below which this equation has a nontrivial solution. We present the derivation of the GL equation for the trapped Fermi gas in the case of singlet pairing ( $g < 0$ ), relying on the assumption that

$$T_c, T_c^{(0)} \gg \Omega.$$

The derivation of the GL equation for the triplet pairing can be performed along the same lines, and should be based on the results of Ref. [10].

For the  $s$ -wave singlet pairing the equilibrium GL free energy ( $\Delta$ -dependent part of the free energy) can be written in the form

$$F_{GL}[\Delta] = \int_{\mathbf{r}} \frac{|\Delta(\mathbf{r})|^2}{|g|} - T \ln \left\langle T_\tau \exp \left\{ - \int_0^{1/T} d\tau \int_{\mathbf{r}} [\psi_+(\mathbf{r}, \tau) \times \psi_-(\mathbf{r}, \tau) \Delta^*(\mathbf{r}) + \text{H.c.}] \right\} \right\rangle_0, \quad (2)$$

where we use the Matsubara representation, and the symbol  $\langle \dots \rangle_0$  stands for the average over the states of the free-particle Hamiltonian [first term in Eq. (1)]. In the vicinity of the phase transition the quantity  $\Delta$  is small, and the second term in Eq. (2) can be expanded in powers of  $\Delta$ . As usual, we perform this expansion up to the fourth power. The coefficients of the expansions (kernels) can be expressed in terms of the Green function of the normal state (without pairing),  $G_\omega^{(0)}(\mathbf{r}_1, \mathbf{r}_2)$ . As will be justified below, the order parameter varies on a distance scale  $l_\Delta$  that is much larger than the

characteristic distance scale  $\xi_K \sim v_F/T_c$  of these kernels. Accordingly,  $F_{GL}[\Delta]$  can be represented as (hereinafter we use  $R_{TF}$  as a unit of length)

$$F_{GL}[\Delta] = R_{TF}^3 \int_{\mathbf{R}} \left\{ \frac{|\Delta|^2}{|g|} - K_0^{(2)}(\mathbf{R}) |\Delta|^2 - K_1^{(2)}(\mathbf{R}) (\Delta \partial^2 \Delta^* + \Delta^* \partial^2 \Delta - 2 \partial_i \Delta^* \partial_i \Delta) + K^{(4)} |\Delta|^4 \right\}, \quad (3)$$

where  $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ ,  $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ ,  $\Delta \equiv \Delta(\mathbf{R})$ , and

$$K^{(4)}(\mathbf{R}) = \nu_0(\mathbf{R}) \frac{7\zeta(3)}{16\pi^2 T^2}, \quad (4)$$

$$K_0^{(2)}(\mathbf{R}) = R_{TF}^3 T \sum_{\omega} \int_{\mathbf{r}} G_{-\omega}^{(0)}(\mathbf{R}, \mathbf{r}) G_{\omega}^{(0)}(\mathbf{R}, \mathbf{r}), \quad (5)$$

$$K_1^{(2)}(\mathbf{R}) \delta_{ij} = R_{TF}^5 \frac{T}{8} \sum_{\omega} \int_{\mathbf{r}} r_i r_j G_{-\omega}^{(0)}(\mathbf{R}, \mathbf{r}) G_{\omega}^{(0)}(\mathbf{R}, \mathbf{r}). \quad (6)$$

Here  $\zeta(x)$  is the Riemann zeta function, and the summation is performed over the Matsubara frequencies  $\omega = \pi T(2n + 1)$ ,  $n = 0, \pm 1, \dots$ . The condition  $T_c^{(0)} \gg \Omega$  allows us to use the quasiclassical expression for the product of two Green functions:

$$G_{-\omega}^{(0)}(\mathbf{R}, \mathbf{r}) G_{\omega}^{(0)}(\mathbf{R}, \mathbf{r}) = \left( \frac{m}{2\pi r R_{TF}} \right)^2 \times \exp \left\{ -r \frac{|\omega|}{\Omega} \frac{2\sqrt{2}}{[\sqrt{(1-R^2)^2 + (\omega/\varepsilon_F)^2 + 1 - R^2}]^{1/2}} \right\}, \quad (7)$$

which can be obtained from the corresponding expression for the spatially homogeneous case, with the replacement  $p_F \rightarrow p_F(R)$ . The validity of Eq. (7) requires the condition  $(1 - R^2) \gg (\Omega/T)^2$ . The use of Eq. (7) for calculating the kernels  $K_0^{(2)}$  and  $K_1^{(2)}$  is justified by the fact that the pairing takes place only in the central region of the gas cloud, and the characteristic size of this region  $l_\Delta \ll 1$ . The main contribution to  $K_1^{(2)}$  comes from frequencies  $|\omega| \ll \varepsilon_F$ , and a straightforward calculation yields

$$K_1^{(2)}(\mathbf{R}) = \frac{1}{4} \nu_0(R) \kappa^2, \quad (8)$$

where  $\kappa = \sqrt{7\zeta(3)/48\pi^2} (\Omega/T) = 0.13(\Omega/T) \ll 1$ .

The calculation of  $K_0^{(2)}$  is more subtle, because the frequency sum in Eq. (5) diverges. The divergency can be eliminated in a standard way by renormalization of the bare interaction  $g$ , and finally one has

$$\frac{1}{|g|} - K_0^{(2)}(\mathbf{R}) = \frac{m}{4\pi|a|} - \nu_0(R) \ln \frac{C\varepsilon_F(R)}{T}.$$

Then, the final expression for the GL free energy can be written in the form

$$F_{GL}[\Delta] = R_{TF}^3 \int_{\mathbf{R}} \nu_0(R) \left\{ -\frac{\kappa^2}{4} (\dots) + \left[ \frac{1}{\lambda} \left( \frac{\nu_0}{\nu_0(R)} - 1 \right) - \ln \left( \frac{T_c^{(0)}}{T} \frac{\varepsilon_F(R)}{\varepsilon_F} \right) \right] |\Delta|^2 + \frac{7\zeta(3)}{16\pi^2} \frac{1}{T^2} |\Delta|^4 \right\}, \quad (9)$$

where  $\nu_0(R) = \nu_0 \sqrt{1-R^2}$ ,  $\varepsilon_F(R) = \varepsilon_F(1-R^2)$ , and the symbol  $(\dots)$  stands for the same combination of derivatives of  $\Delta$  as in Eq. (3). The analogous expression for the ‘‘triplet’’ pairing can be obtained from Eq. (9), with the replacements  $\lambda \rightarrow \Gamma \approx \lambda^2/13$ ,  $|\Delta|^2 \rightarrow \text{tr}(\Delta^\dagger \Delta)$ , and  $|\Delta|^4 \rightarrow \text{tr}(\Delta^\dagger \Delta)^2$ .

Since only small distances  $R \sim \sqrt{\kappa} \ll 1$  in Eq. (9) are important for pairing, we will make an expansion in powers of  $R$  and retain only the largest (quadratic) terms. Then the minimization of Eq. (9) with respect to  $\Delta^*$  gives the GL equation

$$\left[ -\kappa^2 \partial^2 + \frac{1+2\lambda}{2\lambda} R^2 - \ln \frac{T_c^{(0)}}{T} \right] \Delta + \frac{7\zeta(3)}{8\pi^2} \frac{|\Delta|^2}{T^2} \Delta = 0. \quad (10)$$

We stress once more that Eq. (10) is valid under the condition  $\Delta/T \ll 1$ , which, in turn, implies that  $T_c - T \ll T_c$ .

It is interesting to emphasize that Eq. (10) for  $\Delta$  is formally equivalent to the nonlinear Schrödinger equation for the condensate wave function  $\Psi_0$  in a Bose gas of neutral particles of ‘‘mass’’  $1/2\kappa^2$  in a harmonic confining potential with ‘‘frequency’’ (‘‘level spacing’’)  $2\tilde{\kappa} = 2\kappa(1+1/2\lambda)^{1/2}$ . The last (nonlinear) term on the left-hand side plays the role of repulsive interparticle interaction, and the third term  $\ln(T_c^{(0)}/T)$  the role of the chemical potential. Accordingly, the calculation of the shape of  $\Delta$  is similar to the calculation of the shape of  $\Psi_0$  in a trapped Bose gas, performed in, e.g., [11,12]. But there is an important difference. In the Bose gas the amplitude of  $\Psi_0$  is determined by the normalization condition. Together with the Schrödinger equation this condition gives the chemical potential as a function of the particle number. Hence, for a small interparticle interaction the nonlinear term is not important at all. In the Fermi gas the amplitude of  $\Delta$  is always determined by the nonlinear term.

The critical temperature  $T_c$  for the trapped Fermi gas is the maximum temperature  $T$  at which Eq. (10) has a non-trivial solution. As usual,  $\Delta \rightarrow 0$  for  $T \rightarrow T_c$ , and for finding  $T_c$  the nonlinear term in Eq. (10) can be omitted. Then the GL equation becomes similar to the Schrödinger equation for spherically symmetrical oscillator, and we obtain

$$\frac{T_c^{(0)} - T_c}{T_c^{(0)}} \approx \ln \frac{T_c^{(0)}}{T_c} = 3\tilde{\kappa} \ll 1. \quad (11)$$

One can see from Eq. (11) that the critical temperature  $T_c$  for the trapped gas is only slightly lower than  $T_c^{(0)}$  for the homogeneous gas with density  $n_0$ .

As well as in the case of a trapped Bose condensate, the shape of the order parameter in Eq. (10) is predetermined by the ratio  $z$  of the nonlinear term  $|\Delta|^2/T^2$  to the level spacing  $2\tilde{\kappa}$ . Since the nonlinear term is on the order of the difference between the ‘‘chemical potential’’  $\ln(T_c^{(0)}/T)$  and its minimum value (11), we have

$$z = \frac{1}{2\tilde{\kappa}} \ln \frac{T_c}{T} \approx \frac{\delta T}{\Omega} (1+1/2\lambda)^{-1/2}, \quad (12)$$

where  $\delta T = T_c - T$ .

For  $z \ll 1$  and, hence,  $T$  very close to  $T_c$ , the nonlinear term in Eq. (10) does not influence the shape of the order parameter, and the latter takes the form of a Gaussian:

$$\Delta_c(\mathbf{R}) \sim \varphi_0(R) \equiv (\pi l_\Delta^2)^{-3/4} \exp(-R^2/2l_\Delta^2). \quad (13)$$

The linear size of the spatial region where the pairing takes place,  $l_\Delta = \kappa/\sqrt{\tilde{\kappa}} \ll 1$ , is finite for  $T \rightarrow T_c$ . Moreover,  $l_\Delta \gg \xi_K \sim \Omega/T_c$ , which justifies the gradient expansion in Eq. (3). For finding the amplitude of the order parameter,  $\Delta(R=0)$ , in the limiting case  $z \ll 1$  we write  $\Delta$  in the form  $\Delta(\mathbf{R}, T) = \alpha(T) [\varphi_0(R) + \delta\varphi(\mathbf{R}, T)]$ , where  $\varphi = \varphi_0 + \delta\varphi$  obeys the normalization condition  $\int_{\mathbf{R}} |\varphi(\mathbf{R}, T)|^2 = 1$ , and  $\delta\varphi \rightarrow 0$  for  $T \rightarrow T_c$ . Then Eq. (10) is transformed to

$$\begin{aligned} & \left[ \kappa^2 \partial^2 - \frac{1+2\lambda}{2\lambda} R^2 + 3\tilde{\kappa} \right] \delta\varphi + \ln(T_c/T) (\varphi_0 + \delta\varphi) \\ & = \alpha^2 \frac{7\zeta(3)}{8\pi^2} \frac{(\varphi_0 + \delta\varphi)^3}{T_c^2}. \end{aligned} \quad (14)$$

Equation (14) gives  $\alpha$  and  $\delta\varphi$  as series of rational powers of  $\ln(T_c/T) \approx \delta T/T_c$ , the small parameter of expansion being  $z$ . Multiplying both sides of this equation by  $\varphi_0(\mathbf{R})$ , integrating over  $\mathbf{R}$ , and omitting the terms containing  $\delta\varphi$ , to the leading order we obtain

$$\alpha = T_c l_\Delta^{3/2} \left( \frac{16\pi^3 \sqrt{2\pi}}{7\zeta(3)} \ln \frac{T_c}{T} \right)^{1/2},$$

and, hence,

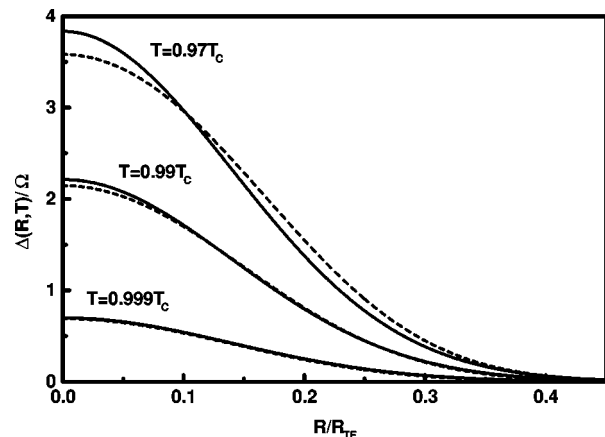


FIG. 1. The order parameter versus  $R$  for various temperatures. The solid lines correspond to  $\Delta_0(R, T)$  [Eq. (15)], and the dashed lines to numerical solutions of Eq. (10).

$$\begin{aligned}\Delta(\mathbf{R}, T) &\approx \Delta_0(\mathbf{R}, T) = T_c \sqrt{\frac{16\pi^2\sqrt{2}}{7\zeta(3)} \ln \frac{T_c}{T}} \exp\left(-\frac{R^2}{2l_\Delta^2}\right) \\ &\approx 5.15T_c \sqrt{\frac{T_c - T}{T_c}} \exp\left(-\frac{R^2}{2l_\Delta^2}\right).\end{aligned}\quad (15)$$

As in the spatially homogeneous case, we have  $\Delta \sim \sqrt{T_c - T}$  for  $T \rightarrow T_c$ . For the triplet pairing one will have  $\Delta_{ij}(\mathbf{R}, T) = \delta_{ij}\Delta_0(\mathbf{R}, T)$ .

It is important to mention, however, that the corrections to Eq. (15), which can be obtained from Eq. (14), are physically meaningless. They have the same order of magnitude [ $\propto (\delta T/T_c)^{3/2}$ ] as the corrections originating from terms with higher powers of  $\Delta$  (for example,  $\Delta^5/T_c^4$ ) or higher derivatives, neglected in deriving Eq. (10). For the same reason one should not go beyond the first term in expanding  $\ln(T_c/T)$  in powers of  $\delta T/T_c$ . In Fig. 1 we present the approximate solution  $\Delta_0$  (solid lines) and the corresponding numerical solutions of Eq. (10) (dashed lines) for  $T_c^{(0)}/\Omega = 5$ ,  $\lambda = 0.3$ , and  $\delta T/T_c = 0.001, 0.01, 0.03$ . For these values of  $T_c^{(0)}/\Omega$  and  $\lambda$  we have  $\tilde{\kappa} = 4.4 \times 10^{-2}$ , and Eq. (11) gives the critical temperature  $T_c = 0.87T_c^{(0)}$ , which is only 1% higher than  $T_c$ , following from the exact numerical solution of Eq. (10).

For lower temperature, where  $z \gg 1$  (but still much smaller than  $\tilde{\kappa}^{-1}$ , as required by the condition  $\delta T \ll T_c$ ), one can neglect the Laplacian term in Eq. (10), and write the approximate solution for the order parameter in the form

$$\begin{aligned}\Delta(\mathbf{R}, T) &= T_c \sqrt{\frac{8\pi^2}{7\zeta(3)} \ln \frac{T_c}{T}} (1 - R^2/R_c^2)^{1/2} \\ &\approx 3.06T_c \sqrt{\frac{T_c - T}{T_c}} (1 - R^2/R_c^2)^{1/2}\end{aligned}\quad (16)$$

for  $R \leq R_c = \sqrt{(\delta T/T_c)(1 + 1/2\lambda)^{-1}} = l_\Delta \sqrt{2z} \ll 1$ , and zero otherwise. The solution (16) is completely analogous to that for the Bose condensate wave function in the quasiclassical (Thomas-Fermi) regime [13,14].

Equations (15) and (16) show that in the vicinity of the phase transition the superfluid pairing takes place only in a small central region of the gas sample. This, together with the fact that the superfluid pairing involves only a small fraction ( $\sim T_c/\varepsilon_F \ll 1$ ) of atoms, makes it very difficult to detect the presence of pairing through the measurement of the gas density profile. On the other hand, as well as in the spatially homogeneous case, the pairing should influence the spectrum of elementary excitations. In this respect we believe that the measurement of eigenfrequencies of oscillations of the gas cloud can be one of the most promising ways of identifying the phase transition in trapped Fermi gases.

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