Supplemental Material: Lifetime of the Bose Gas with Resonant Interactions

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I. CALCULATION OF $s_{11}(ka)$

The function $s_{11}(ka)$ has been calculated for $0 < ka \lesssim 10$ in [1]. Here we calculate it for ka of any sign and magnitude including infinity. The three-body Schrödinger equation reads

$$\left[-\nabla^2_{\mathbf{R}} + V_a(\mathbf{R}) - k^2\right]\Psi(\mathbf{R}) = 0, \tag{S1}$$

where V_a is the sum of binary interaction terms parametrized by a and the six-dimensional vector $\mathbf{R}$ is $\{(2\mathbf{r}_3 - \mathbf{r}_1 - \mathbf{r}_2)/\sqrt{3}, \mathbf{r}_1 - \mathbf{r}_2\}$, where $\mathbf{r_1}, \mathbf{r_2}$ and $\mathbf{r_3}$ are atomic coordinates, and we set $\hbar = m = 1$. By definition the first row of matrix s_{ij} gives us the solution of Eq. (S1) with the following asymptotes. For small R we have

$$\Psi(\mathbf{R}) \approx \Phi_1(\hat{R})\left[(kR)^{is_0} + s_{11}(kR)^{-is_0}\right]/\sqrt{2s_0}R^2. \tag{S2}$$

Note that our definition of s_{11} differs from the one of [1] by the factor $-(ka)^{2is_0}e^{-2i\delta_0}$, where $\delta_0 \approx 1.588$ [2]. In the asymptotic region of large R we have

$$\Psi(\mathbf{R}) \approx s_{12}\Psi_2(\mathbf{R}) + \sum_{i=3}^{\infty} s_{1i}\Phi_i(\hat{R})e^{ikR}/\sqrt{2kR}R^2. \tag{S3}$$

In Eqs. (S2-S3) all $\Phi_i(\hat{R})$ are symmetrized and normalized and $\Psi_2(\mathbf{R})$ is the symmetrized wavefunction of the atom-dimer relative outgoing motion normalized to a unit flux. Physically $\Psi(\mathbf{R})$ describes the stationary flow of atoms which are injected at the origin and can either return back with amplitude s_{11} (second term in the right hand side of Eq. (S2)) or travel to infinity by using channels with $i \geq 3$.

Before explaining the numerical method of calculating $s_{11}(ka)$ let us discuss some properties of this function which can be derived analytically. In order to do this it is convenient to use the complex scaling of the Hamiltonian [3] and multiply (rotate in the complex plane) all spatial coordinates by the complex number $ke^{-i\pi/2}$, i.e., we introduce $\tilde{\mathbf{R}} = \mathbf{R}ke^{-i\pi/2}$. Then the problem reduces to calculating properties of the bound trimer state with energy $E = -1$, interaction between the atoms being characterized by the imaginary scattering length $\tilde{a} = ka/i$. Applying the complex scaling to the asymptotes (S2-S3) we see that now the solution is constrained to decay at large distances and to have the short-distance asymptote $\propto \tilde{R}^{is_0} + s_{11}(ka)e^{\pi s_0}\tilde{R}^{-is_0}$. That $\tilde{a}$ is imaginary simplifies the task: if Ψ is a solution for a given real value of a, then Ψ^* is the solution for $a = -a$. This leads to the relation $s_{11}(-ka) = e^{-2\pi s_0}/s^*_{11}(ka)$, and it is thus sufficient to deal, for example, only with $ka < 0$. Since our problem is the inverse to finding the Efimov spectrum versus the three-body parameter and a, the point $ka = \infty$ and its vicinity can be treated analytically: the wavefunction of an Efimov trimer at unitarity is proportional to $\Phi_1(\hat{R})\left[J_{is_0}(i\tilde{R}) - e^{-\pi s_0}J_{-is_0}(i\tilde{R})\right]$ which gives $s_{11}(\infty) = -2^{2is_0}e^{-\pi s_0}\Gamma(1+is_0)/\Gamma(1-is_0)$, the result presented in the main text. Moreover, by using the known analytic formula for the shift of the trimer energy at small $1/a$ with a fixed three-body parameter [4] one obtains $s_{11}(ka \gg 1) \approx s_{11}(\infty)(1 - Cs_0/ka)$, where [4]

$$\begin{aligned} C &= \pi\sinh\left(\frac{s_0\pi}{2}\right)\tanh(s_0\pi)/\left[\cosh\left(\frac{s_0\pi}{2}\right)\right. \\ &+ \left.\frac{s_0\,\pi}{2}\sinh\left(\frac{s_0\pi}{2}\right) - \frac{4\pi}{3\sqrt{3}}\cosh\left(\frac{s_0\pi}{6}\right)\right] \\ &= 2.1126716\ldots \end{aligned} \tag{S4}$$

Finally, the three-body wavefunction in the limit of vanishing total energy has been studied in [2] from which we obtain $s_{11} \approx (k|a|)^{2is_0}e^{-2i\delta_0}$ in the limit $ka \to 0^-$.

In order to calculate s_{12} for arbitrary ka let us introduce the reduced wavefunction $f(\mathbf{r})$ defined by

$$f\left[(2\mathbf{r}_3 - \mathbf{r}_1 - \mathbf{r}_2)/\sqrt{3}\right] = 4\pi\lim_{\mathbf{r}_1\to\mathbf{r}_2}|\mathbf{r}_1 - \mathbf{r}_2|\Psi(\mathbf{R}), \tag{S5}$$

and write down the Skorniakov-Ter-Martirosian (STM) equation for the Fourier transform of $f(\mathbf{r})$ (for more details see [5])

$$(\sqrt{p^2 - k^2} - 1/a)f(p) - \hat{L}_{k^2}f(p) = 0, \tag{S6}$$

where the integral operator $\hat{L}$ is defined by

$$\hat{L}_{k^2}f(p) = \int_0^\infty \ln\left(\frac{p'^2 + p^2 + pp' - 3k^2/4}{p'^2 + p^2 - pp' - 3k^2/4}\right)\frac{4f(p')p'dp'}{\sqrt{3}\pi p}. \tag{S7}$$

Note that in Eq. (S6) we use $f(\mathbf{p}) \equiv f(p)$ since, in the case of three identical bosons, higher spherical harmonics of this function correspond to the non-Efimovian kinematics, do not contribute to the asymptote (S2), and do not lead to (strong) recombination losses.

As usual, the branches of the logarithm and of the square root are chosen as if the momentum k (or energy k^2) is slightly shifted into the upper complex half-plane, or, alternatively, p and p' are slightly shifted to the lower half-plane. In fact, the complex scaling discussed above

means that we rotate p and p' in the clockwise direction all the way to the negative imaginary axis and rescale them by k. Then changing variables in such a way that the integration goes along the positive real axis, we obtain the same Eq. (S6) in which $k \to i$ and $1/a \to i/ka$. The resulting equation does not have singularities on the real axis and is extremely easy to solve numerically. The large-p asymptote of the solution can be written as $C_1 p^{-2-is_0} + C_2 p^{-2+is_0}$, and it is straightforward to show that $s_{11}(ka) = (C_2/C_1)\,[\Gamma(1+is_0)/\Gamma(1-is_0)]\,e^{-\pi s_0}$.

In Fig. S1 we plot $|s_{11}|$ and $\mathrm{Arg} s_{11}$ versus $ka < 0$ (solid line). The dashed and dotted lines correspond, respectively, to the limits $k|a| \gg 1$ and $k|a| \ll 1$ discussed above.

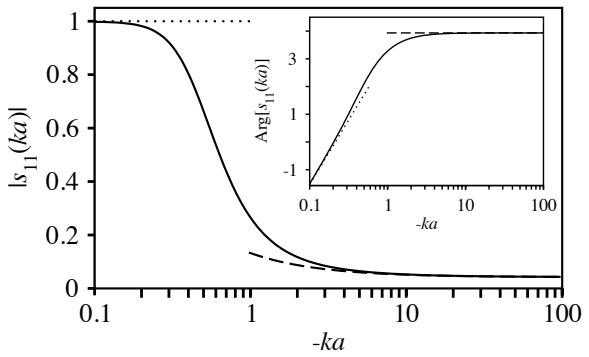

FIG. S1: Modulus and phase of s_{11} versus $ka < 0$. Dashed and dotted lines are analytic limits, see text.

II. LARGE ka ASYMPTOTE OF $|s_{12}(ka)|$

The quantity s_{12} is the amplitude of the atom-dimer outgoing wave, see Eq. (S3). By construction it is zero for $a < 0$ since there are no shallow dimers and, as we argue in the main text and in Sec. III, for $a > 0$ it becomes important for determination of L_3. The question that we address now is whether $|s_{12}|^2$ vanishes for $a \to +\infty$ or not. Unfortunately, numerical results for $s_{12}(ka)$ are available only for $ka \lesssim 10$ [1] and do not allow us to make any statement on the large ka behavior of this quantity.

In the region $|\mathbf{r}_1 - \mathbf{r}_2| \sim a$ and $R \gg a$ the atom-dimer wavefunction $\Psi_2(\mathbf{R})$ introduced in Eq. (S3) can be written as

$$\Psi_2(\mathbf{R}) = \frac{\exp(-|\mathbf{r}_2 - \mathbf{r}_1|/a)}{\sqrt{2\pi a}|\mathbf{r}_2 - \mathbf{r}_1|}\frac{\exp(ip_0 R)}{\sqrt{24\pi p_0} R}, \qquad \text{(S8)}$$

where $p_0 = \sqrt{k^2 + 1/a^2} > k$ is the atom-dimer relative momentum. The outgoing wave (S8) corresponds to the pole of $f(p)$ at $p = p_0$:

$$f(p) \approx \frac{2\pi s_{12}(ka)}{\sqrt{3a p_0^3}}\frac{1}{p - p_0 - i0}, \; |p - p_0| \ll p_0 - k. \quad \text{(S9)}$$

Therefore, in order to calculate s_{12} one has to solve Eq. (S6) (with the correct boundary condition at large p) and find the residue of this pole. Here we solve this problem perturbatively using $1/ka$ as a small parameter.

The solution of the three-body problem at unitarity is given in terms of the Bessel functions. The corresponding correctly normalized function $f_0(p)$ up to a phase factor can be written as

$$f_0(p) = \frac{2\pi\sqrt{C s_0 \cosh(\pi s_0)}\exp(-\pi s_0)}{\sqrt{3}\sinh(\pi s_0/2) p\sqrt{p^2 - k^2}} \qquad \text{(S10)}$$
$$\times \left[k^{2is_0} e^{\pi s_0}(p + \sqrt{p^2 - k^2})^{-is_0} - (p + \sqrt{p^2 - k^2})^{is_0}\right].$$

In deriving Eq. (S10) we used the small $\hat{R}$ asymptote of the normalized hyperangular Efimov wavefunction $\Phi_1(\hat{R})$ which we took from [4]. The constant C is defineed in Eq. (S4).

Let us now write the solution of Eq. (S6) at small $1/ka$ as $f(p) = f_0(p) + \delta f(p)$, where $\delta f(p)$ tends to zero when $a \to \infty$. Equation (S6) now reduces to

$$(\sqrt{p^2 - k^2} - 1/a)\delta f(p) = f_0(p)/a + \hat{L}_{k^2}\delta f(p). \quad \text{(S11)}$$

Looking at the right hand side of this equation at $p = p_0$ we observe that the first term tends to a finite value as $a \to \infty$ since $f_0(p)$ is singular at $p \to k$. In contrast, the integral operator smooths singularities and makes the second term vanish uniformly for large a. Therefore, the dominant contribution to s_{12} can be obtained by neglecting the second term, and we finally obtain

$$|s_{12}(ka)|^2 \approx 2C s_0\,[1 + \exp(-2\pi s_0)]\,/ka, \; ka \gg 1, \quad \text{(S12)}$$

i.e., we have managed to show that $s_{12} \to 0$, as one approaches the resonance.

III. ATOM-DIMER CHEMICAL EQUILIBRIUM NEAR RESONANCE

On the positive side of the resonance the loss rate in the system is no longer solely due to the recombination to deep molecular states. Three atoms can recombine to a shallow dimer and depending on how its binding energy, $E_D = \hbar^2/ma^2$, compares to the trap depth, U, the products of such a three-body event may or may not leave the trap. Moreover, even if they have enough energy to leave, they can collide with the remaining atoms and redistribute their excess energy into heat. This dynamical problem, in general, goes far beyond calculating the loss rate in three-atom collisions. Obviously, this complication is absent very far from the resonance where, starting from a purely atomic sample, one counts any recombination event as the loss of three atoms. By contrast, if $E_D < U$, the shallow dimers stay in the system and mix with atoms.

We now focus on the regime $E_D \ll k_B T$, where the situation greatly simplifies. Let us assume chemical equilibrium between atoms and shallow dimers, and validate

this assumption *a posteriori*. The dimer density n_D is then related to the atomic density n by

$$n_D = n^2 \lambda_{\rm th}^3 \, 2\sqrt{2} \, e^{E_D/(k_B T)} \simeq n^2 \lambda_{\rm th}^3 \, 2\sqrt{2}. \qquad \text{(S13)}$$

The two reverse processes of three-atom recombination to a shallow dimer and of atom-dimer breakup (i.e. dissociation of a shallow dimer after collision with an atom) then balance each other and give a vanishing total contribution to dn/dt. Hence $dn/dt = -L_3 \, n^3 - L_2^{AD} \, n \, n_D$, where L_3 is the rate constant for recombination to deep dimers and L_2^{AD} is the rate constant for atom-dimer relaxation (i.e. formation of a deeply bound dimer after collision of a shallow dimer with an atom). The expression of $L_3(T)$ for $a > 0$ was obtained in [1] and differs from Eq. (4) of the main text only by the replacement of the term $1 - |s_{11}|^2$ by $1 - |s_{11}|^2 - |s_{12}|^2$. For the incoming atom-dimer channel $i = 2$, the expression of the loss probability P_2 was given above Eq. (4) in the main text, and leads after thermal averaging to

$$L_2^{AD}(T) = \frac{3\sqrt{3\pi}\hbar^2 [1 - \exp(-4\eta_*)]}{2\,(m k_B T)^{3/2}} e^{-E_D/k_B T} \int_{-E_D}^{\infty} \frac{|s_{12}|^2}{|1 + (|k|R_0)^{-2is_0} e^{-2\eta_*} s_{11}|^2} e^{-E/k_B T} dE. \qquad \text{(S14)}$$

Here the integration variable $E = \hbar^2 k^2/m$ is the total energy of the three-atom system in the center of mass reference frame. Thus, the integration over negative E decribes the atom-dimer relaxation events below the breakup threshold. We should also note that the matrix elements s_{11} and s_{12} are functions of $\sqrt{E}a$, which becomes imaginary for $E < 0$. To show that L_2^{AD} vanishes in the large a limit, we treat separately the contributions from positive and negative E: For $E > 0$, we have seen above that s_{12} vanishes as $1/a$, see Eq. (S12), which leads to a contribution $\propto 1/a$ to L_2^{AD}; for $E < 0$, the integration is limited to the narrow window $[-E_D, 0]$, and the integrand can be bounded from above thanks to $P_2 \leq 1$, leading to a contribution $\propto 1/a^2$ to L_2^{AD}. This allows us to neglect L_2^{AD} in the rate equation for dn/dt, which then reduces to Eq. (1) of the main text [14].

Finally, let us validate our chemical equilibrium assumption. For a given dimer, the event rates for relaxation and breakup (after collision with an atom) are respectively $\gamma_{\rm rel} = n \, L_2^{AD}(T)$ and $\gamma_{\rm break} = n \, \alpha_{\rm shallow}(T) \, \lambda_{\rm th}^{-3} \, \sqrt{2} \, e^{-E_D/(k_B T)}$, where $\alpha_{\rm shallow}(T)$ is the event rate constant for three-atom recombination to a shallow dimer, and we used Eq. (S13). In the regime $a \gg \lambda_{\rm th}$ considered here, we can estimate from [1] that $3\alpha_{\rm shallow}(T)$ saturates to a value $\gtrsim 10 \, L_3^{\rm max}(T)$ [15]. Evaluating the leading-order behavior of L_2^{AD} as explained above then gives $\gamma_{\rm rel}/\gamma_{\rm break} \lesssim 0.03 \, \lambda_{\rm th}/a \ll 1$ [16]. Hence the relaxation events do not destroy chemical equilibrium, as they happen much less frequently than the breakup events (and thus also than the reverse dimer-formation events). The relaxation rates in dimer-dimer and dimer-atom-atom collisions are also smaller than $\gamma_{\rm break}$ by factors $\propto n\lambda_{\rm th}^3 \ll 1$ (with unknown prefactors which depend on the four-body problem). The last condition to check is $\gamma_{\rm break} \gg \gamma_3$, i.e., a given dimer should be likely to break up (and to be replaced by a newly formed dimer) within a time much smaller than the timescale $1/\gamma_3$ over which the cloud decays. Estimating $\gamma_{\rm break}$ as above and using our result for γ_3 gives $\gamma_3/\gamma_{\rm break} \lesssim 0.1 \, n\lambda_{\rm th}^3 \ll 1$.

IV. MATRIX s_{ij}: NEGATIVE a

In the case $a < 0$ the atom-dimer channel is closed and the structure of matrix s_{ij} is as follows. We have a single discrete small-R Efimov channel and a continuum of large-R channels, $\{\Phi_i(\hat{R}),\ i \geq 3\}$ being a complete orthonormal set of hyperangular functions, for example, eigenfunctions of the hyperangular kinetic energy operator in the absence of interactions. Given s_{ij} we change this basis in favor of another orthonormal set $\{\tilde{\Phi}_i(\hat{R})\}$ in which we choose

$$\tilde{s}_{13} \tilde{\Phi}_3 = \sum_{i \geq 3} s_{1i} \Phi_i, \qquad \text{(S15)}$$

The normalization condition which we impose on $\tilde{\Phi}_3$ uniquely defines this function and $\tilde{s}_{13}$ (up to an irrelevant phase factor). The corresponding asymptotic triatomic channel is defined by the incoming, $\psi_3 = \tilde{\Phi}_3(\hat{R}) e^{-ikR}/\sqrt{2kR}R^2$ and outgoing, ψ_3^*, waves. Note that we do not touch the Efimov channel and, therefore, $\tilde{s}_{11} = s_{11}$. From Eq. (S15) and the unitarity of s_{ij} we can deduce that $|\tilde{s}_{11}|^2 + |\tilde{s}_{13}|^2 = 1$. Since the new matrix $\tilde{s}_{ij}$ should also be unitary, we conclude that $\tilde{s}_{11}$ and $\tilde{s}_{13}$ are the only non-zero entries of its first row. Let us now write explicitly the corresponding wavefunction [cf. Eqs. (S2-S3)]

$$\Psi = \begin{cases} \psi_1 + \tilde{s}_{11}\psi_1^*, & R \to 0, \\ \tilde{s}_{13}\psi_3^*, & R \to \infty, \end{cases} \qquad \text{(S16)}$$

where we denote the incoming Efimov wave as $\psi_1 = \Phi_1(\hat{R})(kR)^{is_0}/\sqrt{2s_0}R^2$.

Choosing an appropriate linear combination of Ψ and

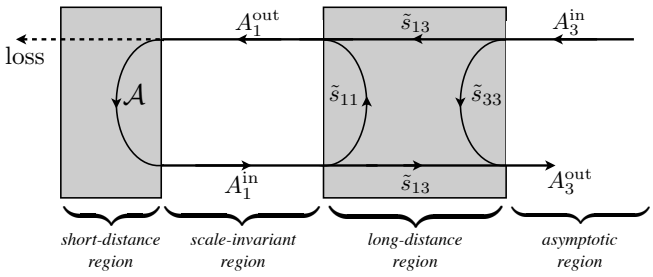

FIG. S2: A three-body wave arriving from large hyperradius R with amplitude $A_3^{\rm in}$ in the triatomic channel $i = 3$ can follow various pathways before it either returns to large R, or gets lost at $R \sim R_e$ by turning into an atom and a deep dimer. One can imagine a Fabry-Perot interferometer, the mirrors of which are formed by the *short-distance* and *long-distance* regions. Multiple reflections by these regions can lead to the resonant denominator in the three-body loss rate formula.

Ψ^* we obtain the relation

$$\frac{\Psi^* - \tilde{s}_{11}^*\Psi}{\tilde{s}_{13}^*} = \begin{cases} \tilde{s}_{13}\psi_1^*, & R \to 0, \\ \psi_3 - (\tilde{s}_{13}\tilde{s}_{11}^*/\tilde{s}_{13}^*)\psi_3^*, & R \to \infty, \end{cases} \quad \text{(S17)}$$

the right hand side of which defines the second ($i = 3$) row of the matrix $\tilde{s}_{ij}$. Namely, $\tilde{s}_{31} = \tilde{s}_{13}$, $\tilde{s}_{33} = -(\tilde{s}_{13}\tilde{s}_{11}^*/\tilde{s}_{13}^*)$, and $\tilde{s}_{3j} = 0$ for $j > 3$.

Because $\tilde{s}$ is unitary one sees that $\tilde{s}_{i1} = \tilde{s}_{i3} = 0$ for $i > 3$. Therefore, the upper left 2×2 block completely separates from the rest of the matrix. The problem of calculating the three-body loss rate then reduces to the problem of finding the four amplitudes $A_1^{\rm in/out}$ and $A_3^{\rm in/out}$, which are the coefficients in front of the corresponding incoming and outgoing waves in the three-body wavefunction, see Fig. S2. The coefficient $A_3^{\rm in}$ is found by projecting the initial correctly normalized six-dimensional plane wave into the state ψ_3. The amplitudes $A_1^{\rm in/out}$ are related by the three-body contact condition $A_1^{\rm in} = \mathcal{A}\, A_1^{\rm out}$ where $\mathcal{A} = -(kR_0)^{-2is_0}e^{-2\eta_*}$. Finally, the relation between the incoming and outgoing amplitudes given by the matrix $\tilde{s}_{ij}$ provides the last two linear equations necessary to solve the problem: $A_1^{\rm out} = \tilde{s}_{11}A_1^{\rm in} + \tilde{s}_{13}A_3^{\rm in}$ and $A_3^{\rm out} = \tilde{s}_{31}A_1^{\rm in} + \tilde{s}_{33}A_3^{\rm in}$. The loss rate is then obtained by calculating the difference between incoming and outgoing fluxes either for $R \to \infty$ or $R \to 0$. Averaging over the thermal distribution one recovers the formula for L_3 presented in Eq. (4) of the main text, where we now see that the non-trivial k-dependence of the integrand comes from the interference between the various pathways represented in Fig. S2. The simplified approximate formula given in Eq. (5) of the main text corresponds to neglecting any reflection from the long-distance region, hence no more interferences and no log-periodic modulation of L_3T^2 with $\lambda_{\rm th}/R_t$.

V. MATRIX s_{ij} AT UNITARITY

At unitarity, the procedure of transforming the matrix s_{ij} into block diagonal form is very simple. Having an infinite a does not introduce a lengthscale into the problem and, as a consequence, the adiabatic hyperangular eigenfunctions do not depend on the hyperradius, leading to the complete separability [7] between the hyperangular and hyperradial problems. Namely, the three-body wavefunction can be written as

$$\Psi(\mathbf{R}) = \sum_s \phi_s(\hat{R})F_s(R)R^{-2}, \quad \text{(S18)}$$

where $\phi_s(\hat{R})$ and s^2 are, respectively, the (normalized) eigenfunctions and eigenvalues of the hyperangular kinetic energy operator supplemented with the unitary two-body contact conditions. The hyperradial wavefunctions satisfy

$$\left(-\frac{d^2}{dR^2} - \frac{1}{R}\frac{d}{dR} + \frac{s^2}{R^2}\right) F_s(R) = k^2\, F_s(R). \quad \text{(S19)}$$

In the case of three identical bosons considered here, the set $\{s\}$ contains a single imaginary number $s = i\,s_0 \simeq i1.00624$ (Efimovian sector) and an infinite number of real numbers (non-Efimovian sectors). In the Efimovian sector the attractive $-s_0^2/R^2$ potential gives rise to the following asymptotic behavior of F_{is_0}: for $R \ll 1/k$ we have $F_{is_0}(R) \propto R^{\pm is_0}$ and in the opposite limit $F_{is_0}(R) \propto \exp(\pm ikR)/\sqrt{R}$. These two asymptotes of the same function actually define the two channels $i = 1$ and $i = 3$ discussed in Sec. I, the hyperangular wavefunctions being $\Phi_1 = \tilde{\Phi}_3 = \phi_{is_0}$. The rest of ϕ_s, appropriately relabelled, form the rest of the set $\tilde{\Phi}_i$. The corresponding matrix $\tilde{s}_{ij}$ has a 2×2 block in its upper left corner, which describes the transmission and reflection of the wavefunction $F_{is_0}(R)$ by the *long-distance* region $R \sim 1/k$. The rest of $\tilde{s}_{ij}$ is simply diagonal because (i) these channels are decoupled from each other and (ii) the repulsive s^2/R^2 potentials do not allow (in the zero-range approximation) for a transmission of the corresponding waves to the *short-distance* region $R \sim R_e$. Solutions of Eq. (S19) can be written in terms of Bessel functions. In particular, the wave that has properties of Eq. (S16) can be written by setting

$$F_{is_0}(R) = \frac{2^{is_0}\Gamma(1+is_0)}{\sqrt{2s_0}}[J_{is_0}(kR) - e^{-\pi s_0}J_{-is_0}(kR)] \quad \text{(S20)}$$

in Eq. (S18). Expanding Eq. (S20) at small R we obtain the result for $s_{11}(\infty)$, which has already been mentioned. For completeness, from the large-R asymptotes of $J_{\pm is_0}$ we get $\tilde{s}_{13} = 2^{is_0}\sqrt{2/\pi s_0}\,\Gamma(1 + is_0)\sinh(\pi s_0)\exp(-\pi s_0/2 - i\pi/4)$.

VI. MATRIX s_{ij}: POSITIVE a

In the case $a > 0$ we have to take into account another discrete channel: the large-R atom-dimer one denoted by $i = 2$. By using a similar construction as in the case of negative a, one can show that the matrix s_{ij} can be reduced to a block-diagonal form with a 4×4 block in the upper left corner, i.e., there are actually two triatomic channels, $i = 3$ and $i = 4$, coupled to the atom-dimer and Efimov ones, and decoupled from the rest of the triatomic continuum, $i > 4$. As we have shown in Sec. II, $s_{12}(ka) \to 0$ as $a \to \infty$. It is then straightforward to show that in this limit channels 1 and 3 approximately decouple from channels 2 and 4. This means that dimers existing in the system are more likely to break-up or scatter elastically [17] than to relax to deeply bound states. This is consistent with our earlier conclusion on the atom-dimer chemical quasi-equilibrium close to the resonance.

VII. EVAPORATION AND ANTI-EVAPORATION

For $a<0$, the three-body recombination to deeply bound states gives the contribution $\dot{N}_{3body} = -\int L_3 n^3(\mathbf{r}) d^3r$ to the atomic decay, and the corresponding energy loss rate equals

$$\dot{E}_{3body} = -\int d^3r \left\{ \frac{L_3 n^3(\mathbf{r})}{3} \left[3U(\mathbf{r}) + \frac{3k_B T}{2} \right] + \frac{n^3(\mathbf{r})}{3} \frac{72\sqrt{3}\pi^2\hbar(1-e^{-4\eta_*})}{m k_T^6} \int_0^\infty \frac{\hbar^2 k^2}{m} \frac{(1-|s_{11}|^2)e^{-k^2/k_T^2} k dk}{|1+(kR_0)^{-2is_0} e^{-2\eta_*} s_{11}|^2} \right\} \tag{S21}$$

where $L_3 n^3(\mathbf{r})/3$ is the frequency of three-body events per unit volume, $3U(\mathbf{r})$ and $3k_BT/2$ are the loss of trapping potential energy and of center-of-mass kinetic energy by each recombining triple, and the last term is the loss of relative-motion kinetic energy [13]. Let us write the lost energy per lost atom as $\dot{E}_{3body}/\dot{N}_{3body} = (3-\delta)k_BT$ where $\delta\, k_BT$ is the excess energy as compared to the average energy per atom $3k_BT$.

For evaporation, $\dot{E}_{evap}/\dot{N}_{evap} \approx (\eta+\kappa)k_BT$, where we can take the expression of κ in terms of η for a harmonic trap and given in terms of incomplete gamma functions in [8]. Indeed, as realized in [9], two-body collisions leading to an evaporative loss occur mainly in the cloud center where the trap is harmonic, and the relative momentum for such a collision is approximately fixed by the trap depth so that the result derived in [8] for an energy-independent two-body cross-section is applicable. Typically, we have $\kappa \simeq 0.68$ for $\eta = 6$ and $\kappa \simeq 0.78$ for $\eta = 8$. The condition of constant temperature means that $\dot{E}_{evap} + \dot{E}_{3body} = 3k_B T\,(\dot{N}_{evap} + \dot{N}_{3body})$, which yields $\dot{N}_{evap}/\dot{N}_{3body} = \delta/(\eta+\kappa-3)$. At unitarity, we can neglect s_{11} in (S21), which gives $\delta \approx 5/3$. For $-a \ll \lambda_{\rm th}$, we recover $\delta = 1$ as in [11].

For $a > 0$, we use $\delta = 5/3$ when $a > \lambda_{\rm th}$. In the opposite limit $a \ll \lambda_{\rm th}$ we use $\delta = 1$ from [11].

VIII. DISCUSSION OF UNCERTAINTIES

We make use of the grand-canonical equation of state for a degenerate Bose gas in the mean-field limit to calibrate our measurement of the value of λ_3 [12]. We produce a condensate at $a = 200\,a_0$ and measure the normalized pressure h versus the gas parameter $\nu = \frac{\mu}{g} a^3$, where $g = 4\pi\hbar^2 a/m$. Next, we find that in order to match $h(\nu)$ to the mean-field prediction, we must multiply the pressure by a constant $\xi = 2.45$. ξ corrects for errors in the calibrations of our experimental system, *e.g.* our absolute atom counting, through the product ${\omega_r}^2/({\omega_z}^4 (px)^3 \sigma_{\rm A})$, where px is the size of a camera pixel magnified through the imaging system to the gas location and $\sigma_{\rm A}$ is the atomic absorption cross-section for imaging light of finite linewidth. When fitting Eq. (3) to our data, we extract two fit parameters: $\gamma_3 = A(T) L_3(T) N^2(0)$ and $N(0)$. Consequently, our result for $\lambda_3 = L_3 T^2 \propto \gamma_3 T^5/(N^2(0)\omega_r{}^4 \omega_z{}^2)$ scales as ${\omega_z}^8 (px)^6 {\sigma_{\rm A}}^2/{\omega_\rho}^4$. This factor is exactly ξ^{-2}. Ultimately, we estimate our uncertainty in ξ^{-2} to be 25 %, dominated by the uncertainty in our trap frequency and pixel size. For the data in Fig. 3, we have an additional 20 % uncertainty arising from the T^2 scaling with $(px)^4$.

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[14] We should note that our imaging does not count the atom number N, but rather N plus twice the number N_D of shallow dimers. Fortunately, this effect is negligible in the considered near-resonant chemical-equilibrium regime, where $N_D/N = n_0\,\lambda_{\rm th}^3 \lesssim 1\%$.
[15] For $a \to \infty$, using our result $s_{12} \to 0$, the expression of [1] simplifies to $3\alpha_{\rm shallow}(T) = c \cdot L_3^{\rm max}(T)$ with $c = 1 - |s_{22}(\infty)|^2 + \sum_{J=1}^{\infty}(2J{+}1) f^{(J)}(\infty)$. The functions $f^{(J)}(ka)$, which correspond to each angular momentum sector J of the three-body problem, are plotted in [1] for $J \le 6$ and $ka < 10$, indicating that $f^{(J)}(\infty) \gtrsim f^{(J)}(ka = 10)$ and thus $c \gtrsim \sum_{J=1}^{6}(2J{+}1) f^{(J)}(ka = 10) \gtrsim 10$.
[16] $\gamma_{\rm rel}/\gamma_{\rm break} \approx C\, s_0\, (1{-}e^{-2\pi s_0})(\sqrt{2}\,c)^{-1}\, (1{-}e^{-4\eta_*})\, \lambda_{\rm th}/a$.
[17] The function $|s_{22}(ka)|$ has been calculated in [1] for $ka \lesssim 10$. For large ka it shows a tendency for saturation at a finite value around 0.4. Assuming that this is true in the limit $ka \to \infty$ we have $|s_{24}|^2 = 1 - |s_{22}|^2 \approx 0.84$ and we can conclude that the break-up is significantly more probable that the elastic atom-dimer collision.