Supplemental Material: Stable p-wave resonant two-dimensional Fermi-Bose dimers

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(Dated: October 30, 2018)

I. FBB CORRELATOR

In the main text we argue that when we confine one fermion and two bosons to a quasi-two-dimensional slab $l_\perp \times a_{\rm FB} \times a_{\rm FB}$ and use the noninteracting three-body wave function to estimate the three-body correlator, the relaxation rate scales as

$$\nu \sim \frac{K_3}{{l_\perp}^2 a_{\rm FB}^4} \sim \left[\frac{a_{\rm FB}^{\rm (3D)}}{l_\perp}\right]^4 |E_{\rm FB}| \left(\frac{l_\perp}{a_{\rm FB}}\right)^2. \qquad (1)$$

However, the attractive FB interactions increase and the repulsive BB interaction decreases the local three-body correlator. We assume that the effect of these interactions at distances smaller than $l_\perp$ (where the kinematics is essentially three-dimensional) has already been captured by the dependence of K_3 on the three-dimensional scattering lengths. Thus, we are mostly interested in the behavior of the three-body wave function at distances larger than $l_\perp$ where the problem is two dimensional. Here we will use the purely two-dimensional model and treat interactions in the zero-range approximation.

The technical derivation in the rest of this section leads to a rather obvious result, which we state right away. At distances much larger than $a_{\rm BB}$ and much smaller than $a_{\rm FB}$ interactions are weak and the three-body wave function can be written in the Jastrow product form

$$\Psi \propto \ln(r_{\rm FB1}/a_{\rm FB})\ln(r_{\rm FB2}/a_{\rm FB})\ln(r_{BB}/a_{\rm BB}), \qquad (2)$$

where $r_{{\rm FB}i}$ is the (longitudinal) distance from the fermion to the i-th boson and $r_{\rm BB}$ is the distance between the bosons. Equation (2) immediately gives the result claimed in the paper. Namely, interactions modify the rate (1) by powers of logarithmic terms $\ln(l_\perp/a_{\rm BB})$ and $\ln(a_{\rm FB}/l_\perp)$.

Let us denote the two bosons by indices 1 and 2, the fermion by 3, and introduce the mass-scaled two-dimensional Jacobi coordinates

$$\mathbf{x} = \sqrt{\frac{4m_{\rm F}m_{\rm B}}{m_{\rm F}+2m_{\rm B}}}\left(\mathbf{r}_3 - \frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right), \qquad (3)$$

$$\mathbf{y} = \sqrt{m_{\rm B}}(\mathbf{r}_1 - \mathbf{r}_2). \qquad (4)$$

For zero total angular momentum the three-body wave functions depends only on the hyperradius $\rho = \sqrt{x^2+y^2}$, hyperangle $\theta = 2\arctan(|y|/|x|)$, and angle ϕ between vectors $\mathbf{x}$ and $\mathbf{y}$. In these coordinates the Schrödinger equation reads

$$\left[-\frac{\partial^2}{\partial\rho^2} - \frac{3}{\rho}\frac{\partial}{\partial\rho} + \frac{4}{\rho^2}\left(-\frac{\partial^2}{\partial\theta^2} - \cot\theta\frac{\partial}{\partial\theta} - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right) - E\right]\Psi = 0. \qquad (5)$$

The interaction between bosons corresponds to $\theta = 0$ and sets the boundary condition at vanishing y (or θ)

$$\Psi \propto \ln\frac{|y|}{\sqrt{m_{\rm B}}a_{\rm BB}} \approx \ln\frac{\rho\theta}{2\sqrt{m_{\rm B}}a_{\rm BB}}. \qquad (6)$$

The interactions between bosons and fermions correspond to the points $\{\theta=\theta_0, \phi=0\}$ and $\{\theta=\theta_0,\phi=\pi\}$, where $\theta_0 = 2\arctan\sqrt{1+2m_{\rm B}/m_{\rm F}}$. The boundary conditions for the wave function at these points are identical up to the interchange $\phi \leftrightarrow \pi-\phi$. For $\phi = 0$ we have

$$\Psi \propto \ln\frac{|\mathbf{r}_1-\mathbf{r}_3|}{a_{\rm FB}} \approx \ln\frac{\rho(\theta-\theta_0)}{2\sqrt{2\mu}a_{\rm FB}}, \qquad (7)$$

where $\mu = m_{\rm F}m_{\rm B}/(m_{\rm F}+m_{\rm B})$.

In the adiabatic hyperspherical formalism we fix ρ and diagonalize the angular kinetic energy operator in Eq. (5)

$$\left[-\frac{\partial^2}{\partial\theta^2} - \cot\theta\frac{\partial}{\partial\theta} - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2} - \lambda(\rho)\right]\chi_{\lambda,\rho}(\theta,\phi) = 0, \qquad (8)$$

with $\chi_{\lambda,\rho}$ satisfying Eqs. (6) and (7). This problem is solvable analytically. For $\phi = 0$ we have

$$\chi_{\lambda,\rho}(\theta,\phi) = C_{\rm BB}\chi^{(0)}(\theta) + C_{\rm FB}\chi^{(0)}(\theta') + C_{\rm FB}\chi^{(0)}(\theta''), \tag{9}$$

where $\theta'(\theta)$ is the polar angle in the coordinate system with z-axis pointing at $\{\theta = \theta_0, \phi = 0\}$, $\theta''(\theta)$ is the polar angle in the coordinate system with z-axis pointing at $\{\theta = \theta_0, \phi = \pi\}$, and

$$\chi^{(0)}(\theta) = \frac{\pi}{2}\tan\frac{\pi\sqrt{1+4\lambda}}{2}{\rm P}_{\sqrt{1/4+\lambda}-1/2}(\cos\theta) + {\rm Q}_{\sqrt{1/4+\lambda}-1/2}(\cos\theta) \tag{10}$$

is the linear combination of the Legendre P and Q functions, that is smooth everywhere except $\theta = 0$ where it behaves as

$$\chi^{(0)}(\theta) = -\ln(\theta/2) - \gamma - [\psi(1/2+\sqrt{1/4+\lambda}) + \psi(1/2-\sqrt{1/4+\lambda})]/2 + O(\theta^2\ln\theta). \tag{11}$$

Here ψ is the digamma function. Substituting Eq. (9) into Eqs. (6) and (7) and using Eq. (11) we obtain two linear homogeneous equations for the coefficients $C_{\rm BB}$ and $C_{\rm FB}$

$$\{\ln(\rho/\sqrt{m_{\rm B}}a_{\rm BB}) - \gamma - [\psi(1/2+\sqrt{1/4+\lambda}) + \psi(1/2-\sqrt{1/4+\lambda})]/2\}C_{\rm BB} + 2\chi^{(0)}(\theta_0)C_{\rm FB} = 0, \tag{12}$$

$$\{\ln(\rho/\sqrt{2\mu}a_{\rm FB}) - \gamma - [\psi(1/2+\sqrt{1/4+\lambda}) + \psi(1/2-\sqrt{1/4+\lambda})]/2 + \chi^{(0)}(2\pi - 2\theta_0)\}C_{\rm FB} + \chi^{(0)}(\theta_0)C_{\rm BB} = 0. \tag{13}$$

We are interested in the case $a_{\rm BB} \ll a_{\rm FB}$. Then, in the region

$$\sqrt{m_{\rm B}}a_{\rm BB} \ll \rho \ll \sqrt{\mu}a_{\rm FB} \tag{14}$$

the logarithms in Eqs. (12) and (13) are large. This physically means that the BB and FB interactions are, respectively, weakly repulsive and weakly attractive. We thus expect $\lambda(\rho)$ to be small in the region (14). Indeed, expanding ψ and $\chi^{(0)}$ at small λ, the consistency condition for the system (12-13) reads

$$\lambda(\rho) \approx 1/\ln(\rho/\sqrt{2\mu}a_{\rm FB}) + 1/2\ln(\rho/\sqrt{m_{\rm B}}a_{\rm BB}), \tag{15}$$

where neglected are higher powers of $1/|\ln(\rho/\sqrt{2\mu}a_{\rm FB})| \ll 1$ and $1/\ln(\rho/\sqrt{m_{\rm B}}a_{\rm BB}) \ll 1$.

We can now use (15) as the potential energy surface to find the hyperradial part of the three-body wave function, which satisfies

$$\left[-\frac{\partial^2}{\partial\rho^2} - \frac{3}{\rho}\frac{\partial}{\partial\rho} + \frac{4\lambda(\rho)}{\rho^2} - E\right]\Psi(\rho) = 0. \tag{16}$$

We are interested in energies $E \sim 1/\mu a_{\rm FB}^2$ and, therefore, can neglect E in Eq. (16) for ρ belonging to the region (14). Note that although the perturbation $\lambda(\rho)$ is weak, it acts in a wide region of $\ln\rho$. One can see this more clearly by introducing the new variable $z = \ln\rho$ and substituting $\Psi(\rho) = e^{-z}W(z)$ into Eq. (16) which then transforms into

$$\left[-\frac{\partial^2}{\partial z^2} + \frac{4}{z - \ln(\sqrt{2\mu}a_{\rm FB})} + \frac{2}{z - \ln(\sqrt{m_{\rm B}}a_{\rm BB})} + 1\right]W(z) = 0 \tag{17}$$

and which one can solve in the region (14) analytically by using the semiclassical WKB approximation.

Alternatively, we can just note that the ansatz

$$\Psi(\rho) \propto [\ln(\sqrt{2\mu}a_{\rm FB}/\rho)]^\alpha[\ln(\rho/\sqrt{m_{\rm B}}a_{\rm BB})]^\beta \tag{18}$$

with $\alpha = 2$ and $\beta = 1$ solves Eq. (16) to the leading order in $1/\ln$ [i.e., $1/\ln(\rho/\sqrt{2\mu}a_{\rm FB})$ and $1/\ln(\rho/\sqrt{m_{\rm B}}a_{\rm BB})$].

Equation (18) is the main result of this derivation. One can easily check that it is nothing else than the hyperradial part of the Jastrow product (2) (to the leading order in $1/\ln$).

In the paper we are interested in the case $m_{\rm F} \sim m_{\rm B}$. For completeness we mention that for $m_{\rm F} \ll m_{\rm B}$ (two heavy bosons interacting with a light fermion) the right inequality of (14) can be rewritten as the constraint on the distance between the heavy bosons $|\mathbf{r}_1 - \mathbf{r}_2| \ll \sqrt{m_{\rm F}/m_{\rm B}}a_{\rm FB}$. At distances $\sqrt{m_{\rm F}/m_{\rm B}}a_{\rm FB} \ll |\mathbf{r}_1 - \mathbf{r}_2| \ll a_{\rm FB}$ the dependence of λ on ρ becomes linear, so that the effective potential $4\lambda(\rho)/\rho^2$ is hydrogen-like (cf. Ref. [62] in the main text).

Finally, let us comment on the three-dimensional case. Consider a gas of weakly-bound three-dimensional FB molecules of size $a_{\rm FB}^{(3D)}$ (much larger than the van der Waals range $R_{\rm vdW}$) and density $\sim [a_{\rm FB}^{(3D)}]^{-3}$. As in the two-dimensional case, the relaxation rate of such molecules is on the order of the relaxation rate in the three-body FBB system confined to the volume $[a_{\rm FB}^{(3D)}]^3$. In the case $a_{\rm BB} \ll a_{\rm FB}$ the three-body wave function at hyperradii $\sqrt{m_{\rm B}}a_{\rm BB}^{(3D)} \ll \rho \ll \sqrt{\mu}a_{\rm FB}^{(3D)}$ is proportional to ρ^{-2} (see Ref. [66] in the main text; we neglect the Efimov log-periodic oscillations here) to be compared to the scaling ρ^0 in the noninteracting case. This means that interactions enhance the probability of finding three atoms in the recombination region $\rho \sim R_{\rm vdW}$ by $\sim [a_{\rm FB}^{(3D)}/R_{\rm vdW}]^4$ leading to relaxation rates comparable to the molecule binding energy. Whether the lifetime of this system can be increased by introducing a BB repulsion is an open question (see Ref. [69] in the main text).