



# LPTMS – UMR 8626 Rapport d'activités 2008-2013

Electronic vortex nucleated at a junction (Brazovskii et al, 2013)



## 1. Presentation of the laboratory

*Scientific position.* The LPTMS (Laboratoire de Physique Théorique et Modèles Statistiques) is a joint Paris-Sud University and CNRS unit (Unité Mixte de Recherche 8626). It is located on Orsay campus (presently in the valley), but will change premises in 2018. It is a young theoretical physics laboratory, markedly oriented towards statistical physics. The research pertains to condensed matter in the broad sense, with a strong pluri-disciplinary orientation in the directions of mathematics, computer science, information theory -be it quantum or classical-- with implications in signal processing, chemistry and biology.

Organization. The LPTMS is currently structured into three groups (see the organigram in appendix 3)

- Condensed matter and quantum fluids (headed by G. Shlyapnikov)
- Statistical physics and dynamical systems (headed by S. Majumdar)
- Disordered systems, soft matter and interface with biology (headed by S. Franz)

### 2. Achievements of LPTMS

#### 2.1 Attractiveness and visibility

The lab visibility starts with the quality of the staff, collaborators and corresponding scientific production (e.g. more than 80 papers in the Physical Review Letters in the last 5 years, about 10 in PNAS, and several contributions in Nature Physics, Nature Communications, and Cell), but can also be assessed from other perspectives. During the 2008-2013 time-span, several lab members

• were invited as plenary speakers in main conferences, named lectures, summer schools (ECCS 2008, QFEXT 2009, ICAP 2010, Frontiers in Quantum Gases 2011, International Conference on Mathematical and Theoretical Biology 2012, MECO 2012, StatPhys 2013...), and have been active in the African Institute for Mathematical Sciences.

• received distinctions (Senior BEC award and 2013 advanced ERC grant for G. Shlyapnikov, Prix Langevin de l'Académie des Sciences for E. Bogomolnyi, Gay-Lussac-Humboldt prize for M. Mézard, Tata Excellence Award for S. Majumdar, CNRS bronze medal for G. Schehr, Heisenberg fellowship for T. Emig, Paris-Sud senior chair of excellence for S. Franz, Institut Universitaire de France for E. Trizac, PALM internal junior chair for M. Lenz, Triangle external junior chair for M. Zvonarev who was subsequently appointed CR2, Paris-Sud seed funds for G. Roux, PALM starting grant for P. Vivo, Outstanding APS Referee award for D. Petrov...),

• have participated in various European networks (including ITN), editorial boards (JSTAT, the chief scientific director of which is M. Mézard, Phys. Rev. Lett. where S. Majumdar is a divisional associate editor, J. Stat. Phys., J. Phys. A, EPL, EJP, Synthetic metals...),

• acted in governing bodies at the national or international scale (CNRS where P. Leboeuf was a deputy director for physics during 5 years, Institut Henri Poincaré where A. Comtet was a deputy director and E. Trizac a subsequent member of the board of directors, Laboratoire International Associé Landau-ENS conducted by S. Ouvry who also headed an ANR panel, Les Houches where T. Jolicoeur was the head of the predoctoral program, Comité National de la Recherche Scientifique where among others A. Rosso was active, Conseil National des Universités for C. Texier, ANR for N. Pavloff, European Physical Society...),

• occupied high profile positions (M. Mézard at ENS Paris, P. Leboeuf at CNRS/INP then IDEX Paris-Saclay).

#### 2.2 Report

The following scientific report is for the period between January 2008 and April 2013. It goes with the complete list of publications provided in the appendix, where the references mentioned below can be found. They are ordered by team for convenience, and cited between square brackets (Team 1: condensed matter, quantum fluids; Team 2: Statistical physics, dynamical systems; Team 3: Disordered systems, soft matter and interface with biology).

#### 2.2.1 Condensed matter and quantum fluids

The work of this group pertains to several domains of the physics of quantum gases, strongly correlated systems including mesoscopic physics, fractional quantum Hall effect, low dimensional devices or Kondo phenomenon. Out of equilibrium effects and disorder often play a prominent role in our investigations, a common feature with the two other groups of the laboratory. The physical problems we have in mind may be on the one hand plain condensed matter systems like 1D electronic systems, 2D electron gases, magnetic materials and the like but also on the other hand ultracold quantum gases of atoms or molecules. The group currently consists of 10 permanent researchers, 4 PhD students and 2 postdocs.

#### Quantum disordered systems

The activity on disorder in quantum gases started less than 10 years ago, with the main goal to reveal Anderson localization for neutral atoms and to investigate the subtle question of how the interaction between particles influences localization properties. Our work on quantum gases in disorder is conceivably highlighted by the study of one-dimensional (1D) finite-temperature disordered bosons [T1-58]. It is known that there are no conventional phase transitions in 1D systems at a finite temperature, because long-range correlations are destroyed by thermal fluctuations. We showed that the 1D gas of weakly interacting bosons in disorder can undergo a finite-temperature phase transition between two distinct states: fluid and insulator. None of these states has long-range spatial correlations, but this is a true albeit non-conventional phase transition point. In the fluid phase, mass transport is possible, whereas in the insulator phase it is blocked even at finite temperature. In this sense, we answered the question of how interaction between particles influences the Anderson localization in 1D, where single particles are localized at any energy.

Our studies of a supersonic flow of a Bose-condensed beam through a disordered potential of finite length showed how the interaction effects modify the localization length, thereby providing a critical length above which most of the realizations of the random potential lead to time-dependent flows [T1-45]. We then found the critical velocity at which the superfluidity is destroyed, computed its statistical properties, and showed that they are related to extreme values of the random potential [T1-57]. The other work in this direction includes dipolar oscillations of a trapped Bose-Einstein condensate in the presence of disorder [T1-2] and lattice modulation spectroscopy of strongly interacting bosons in disordered and quasiperiodic lattices [T1-44].

In a series of papers, we discussed the Aubry-Azbel-Harper model, which is a tight binding model for a particle in a primary deep lattice, with a superimposed incommensurate shallow lattice. For a sufficiently large amplitude of the secondary lattice all single-particle states are localized, and the localization is not due to the quantum suppression of classically allowed transport, but rather results from trapping by the potential [T1-56]. We then considered interacting bosons within the Bose-Hubbard approach for this lattice model and determined the phase diagram [P-2], evidencing the competition between Anderson-like localization and Mott localization in a realistic setup. Computations of the dynamical structure factor emphasized that the very nature of the localization mechanism, different from the random disorder



case, could be probed using Bragg spectroscopy [T1-125, see attached graph showing the evolution of Bragg spectrum in the commensurate and incommensurate cases]. In the context of quantum magnetism, we studied the effect of disorder on the paradigmatic model of valence bond solid, the frustrated J1-J2 chain. and found two localization mechanisms at play on the fractional elementary excitations called spinons: a "kinetic" Anderson mechanism and a "interacting"

random confinement mechanism associated to the solitonic nature of the spinons [T1-123]. Lastly, at the boundary between quantum magnetism and cold atoms physics, we computed the weak-disorder predictions on the two-leg hard core bosons model in which the usual interaction-disorder competition is replaced by an coherence-disorder competition that can be addressed within the Luttinger liquid description and numerical techniques in its whole range [T1-86].

Phase coherence and complex geometries: A fundamental quantity in mesoscopic physics is the phase coherence length, which characterizes over which scale quantum interferences are possible and thus sets a frontier between the quantum and classical regimes. At low temperature (T < 1K) and low dimension, the mechanism dominating decoherence is interaction between electrons. Electronic interaction is particular in the sense that the decoherence process is sensitive to the geometrical properties of the system, so that it cannot be characterized by a unique length scale [T1-50]. These ideas have been verified in experiments performed in LPS-Orsay and Institut Néel-Grenoble, on large networks [T1-12] and on a single ring [T1-121]. On the theoretical side, a recent progress was the derivation of a decoherence rate formula for electronic interactions, within the influence functional approach [T1-99]. This advance improved upon previous results, by allowing for a description of both classical and quantum noise regimes. On the technical side, the work on phase coherence in metallic network has relied upon several developments on functional determinants for the Schrödinger operator on metric graphs [T1-28,T1-79,T1-105]. The study of discrete symmetries in disordered systems, for example in the case of Dirac equation, has attracted sustained attention in recent years, in particular motivated by condensed matter physics (disordered spin chains, disordered superconductivity, graphene or more recently topological insulators). In several papers, we studied the breaking of supersymmetry in disordered supersymmetric 1D Hamiltonian, by another disordered potential or by boundary conditions [T1-15,T1-80,T1-M10]. This question has found an application for the analysis of absorption in the Sinai problem (classical diffusion in a random force field) [T1-51]. Other unconventional properties of 1D disordered quantum mechanics involving Lévy noises have been studied in several papers [T1-4,CSP-3,CSP-8,CSP-12].

[Permanent members involved: Leboeuf, Pavloff, Roux, Shlyapnikov, Texier]

#### Molecular gases and light

The domain of dipolar quantum gases is presently one of the "hot topics" in quantum gases, mostly due to a breakthrough in creating ultracold clouds of ground-state polar molecules and successful experiments with magnetic atoms. Our activity on dipolar quantum gases is highlighted by the prediction [T1-34] and description [T1-92] of the topological  $p_x+ip_y$  superfluid phase for microwave-dressed polar molecules in 2D. Due to this dressing, the molecules may acquire an attractive  $1/r^3$  dipole-dipole tail in the interaction potential which leads to efficient superfluid p-wave pairing even in the weakly interacting Bardeen-Cooper-Schrieffer (BCS) regime, with a critical temperature up to 30 nK at common 2D densities of  $10^8-10^9$  cm<sup>-2</sup>. The emerging topological  $p_x+ip_y$  state is collisionally stable and is a promising candidate for topologically protected quantum information processing. In addition, the "old" subject of atom-atom and atom-light interactions was marked by the prediction of novel Feshbach resonances [T1-74]. They are induced by a microwave field and designed for ultralow magnetic fields where ordinary Feshbach resonances do not work. A joint work with the SYRTE group of S.Bize/A.Clairon calculates Feshbach resonances in cesium at very low static magnetic fields and explains experimental observations [T1-114].

The subject of matter-light interations is perhaps highlighted by the work on superfluid motion of light [T1-69]. We have shown that controlling the speed of a light packet with respect to a defect, one can demonstrate the presence of superfluidity and, above a critical velocity, the breakdown through the onset of a dissipative phase. These results open new perspectives in transport optimization, and the experimental realization can be based on the transverse motion through an array of waveguides.

The direction of strongly interacting gases, after remarkable studies of strongly interacting two-component Fermi gases bringing in analogies with neutron matter, was focused on strongly interacting Bose gases and on mixtures of different fermionic atoms. Our joint work with the experimental group of C. Salomon (LKB ENS) [T1-124] defined the region of parameters where a strongly interacting Bose gas exists as a (meta)stable object and suggested directions for further studies of resonantly interacting gases. At ultralow temperatures, such a gas becomes unstable upon approaching a Feshbach resonance because of a greatly enhanced three-body recombination rate. However, at sufficiently high temperatures, the strongly interacting Bose gas is stable in the sense that the elastic collisional rate is higher than the inelastic decay rate. We developed a theory of three-body recombination in this resonantly interacting system and found a good agreement with experiment.



Rate constant of 3-body recombination,  $L_3$ , as a function of the scattering length a (in units of the Bohr radius) at a temperature  $T=6\mu K$ .

Our studies of Fermi mixtures were initially focused on mixtures of heavy and light fermions. Weakly bound molecules (dimers) of such fermions turned out to be collisionally stable due to an effective dimer-dimer repulsion originating from the exchange of the light atoms. We have studied few-body processes in a fermionic mixture near an interspecies Feshbach resonance of finite width

[T1-40,T1-91] and showed that for K-Li mixture the K-KLi atom-dimer scattering should be dominated by a p-wave resonance in a wide range of parameters. Moreover, by introducing a quasi-2D confinement the p-wave atom-dimer interaction can be tuned from attractive to repulsive, allowing for trimer formation. In a joint work with the experimental group of R. Grimm (Innsbruck) we found a strong negative shift in the radiofrequency spectrum of K atoms mixed with KLi dimers and showed that this shift is quantitatively described by the attractive p-wave atom-dimer interaction. Unexpectedly, trimer states were also discovered in one the the prototypical correlated systems, the 1D attractive Hubbard model, provided one has different hoppings for the two spin species [T1-8,T1-41,T1-95,T1-96]. This is due to a novel incommensurability mechanism special to 1D physics. Detailed DMRG investigations have shown the stability region of these unconventional states.

[Permanent members involved: Jolicoeur, Petrov, Roux, Shlyapnikov]

#### From low-dimensional quantum systems...

Conducting polymers [T1-M2,T1-M4] provide low dimensional electronic systems of interest where a combination of two microscopic symmetry breaking may lead to electronic ferro-electricity. The theory predicts the existence of microscopic solitons carrying non-integer electric charges, with spin 0 or 1/2. In a related investigation, we have identified and modeled microscopic topological defects –the amplitude solitons-- which appear [T1-6] via selftrapping of one single electron and can be observed by tunneling microscopy. It carries a spin but no charge, and epitomizes the spinon, a long-searched anomalous particle of correlated states of attracting electrons.

Two-dimensional and one-dimensional quantum gases were created more than 10 years ago by tightly confining the motion of particles in one or two directions, and they attract a great deal of interest due to completely different many-body properties compared to three-dimensional samples. Our work in this direction is perhaps highlighted by the investigation of the dynamics of a mobile impurity in a 1D quantum gas [TT1-112]. Using the Bethe Ansatz, it was shown that the impurity injected with a finite velocity into a 1D gas of fermions never comes to a complete stop. In contrast to the conventional relaxation dynamics, the initially supersonic velocity develops long-lived oscillations and saturates to a value which is almost independent on the initial conditions.

Our studies of the dynamics of 1D systems were also dedicated to reveal the difference in the behavior of integrable and non-integrable systems. We studied the response of a highly excited 1D gas with pointlike interactions to a periodic modulation of the coupling constant [T1-85]. We calculated the corresponding dynamical structure factors and showed that their low-frequency behavior differs dramatically for integrable and non-integrable models. Non-integrable systems are sensitive to excitations with frequencies as low as the mean level spacing, whereas much higher frequencies are required to excite an integrable system. The statistics of levels, localization of eigenstates, decay of excitations, thermalization, response to external perturbations, transport, and other dynamical properties are sensitive to deviations from integrability.

We also developed an effective field theory for describing critical properties of 1D spin gapped fermions at the onset of magnetization, showing that the interaction between spin and charge sectors changes the well-known square-root behavior of magnetization to linear [T1-53].

[Permanent members involved: Brazovskii, Petrov, Shlyapnikov, Zvonarev]

#### ... to Hawking radiation

The studies of 1D Bose gases are closely related to the search for Hawking radiation in quantum fluids. This corresponds to a setting where a stationary flow is subsonic in a certain region of space and supersonic in another region. As argued by Unruh in 1981, the boundary between these regions should behave itself as a horizon emitting spontaneous sonic (Hawking-like) radiation. In the context of 1D Bose superfluids, this idea has been verified numerically by the Trento theory group. We provided an analytical description of this



[Permanent member involved: Pavloff]

#### **Out of equilibrium aspects**

showed that one can dynamically reach a realistic lumb hole configuration by sending a Bosecondensed beam onto a localized obstacle [T1-106]. This configuration is promising for the study of the onic Hawking radiation and is interesting by itself, is a configuration of non-linear transport. The obstacle slows down the incoming beam and there levelops a plateau ahead of the obstacle. The plateau region has enhanced density together with ower velocity, and the discontinuity between this egion and the incoming beam typically results in a lispersive shock wave which can be accurately lescribed within the Gurevich-Pitaevskii scheme of Whitham theory.

The focus was on out-of-equilibrium physics of cold atomic bosons, on the so called quantum quench – the behavior of the system under a change in the parameters of the Hamiltonian. We analyzed the quenches of interaction in a homogeneous gas, with the emphasis on sudden quenches and on thermalization [T1-49,T1-77,T1-78]. In particular, we explained the role of integrability and finite size effects on the counter-intuitive observations of this model. The deviations from adiabaticity and the effect of confinement, mixing transport and quench phenomena were also investigated [T1-82,T1-101] in connection with recent experiments. We proposed quenches as a tool to generate entangled many-body wave-packets [T1-104]. In passing, the chaotic features of the Bose-Hubbard Hamiltonian were studied using a recently suggested probe [T1-68], the ratio of consecutive level spacings, for which we eventually provided simple, yet accurate, analytical surmises that could be used more generally in random-matrix theory [CSP-13]. Out of equilibrium physics likewise is momentous in organic metals, chain conductors, conjugated polymers, charge/spin density waves, ferroelectricity [T1-61,T1-102,T1-M3,T1-81,T1-119]. In particular, in crystals with electronically driven instabilities caused by a Fermi-surface nesting, the dynamics of the resulting charge density wave order parameter of electrons after a high pumping by a femto-second laser was identified. The modeling allowed to recover the following sequence of events: erasing of the long range order, high amplitude pendulum oscillations, symmetry breaking trapping via a dynamical phase transition and final relaxation of the stratified state via collapses of domain walls.

[Permanent members involved: Brazovskii, Roux from this group + Bogomolny, Giraud]



#### Kondo physics

The Kondo effect describes the consequences of the coupling of an electron gas with an impurity possessing a spin degree of freedom. It is characterized by the all-important role played by renormalization of interaction between the impurity and conduction electrons due to the virtual processes involving all conduction band electrons. There are thus two temperature regimes: a high-temperature regime with small renormalized interaction and a low-temperature regime of strong coupling, where the impurity spin is

perfectly screened by conduction electrons. The crossover between these two regimes is the so-called Kondo temperature. There has been renewed interest in the last few years in the Kondo physics in the realm of quantum dots, that may behave as Kondo impurities in some range of parameters. In this context, it is thus rather natural to consider the coupling of a Kondo impurity to a mesoscopic electronic bath. This introduces two new energy scales in the problem: the Thouless energy associated to the finite time of flight to cross the system, and the mean level spacing. The question is to understand how these new scales affect the Kondo phenomenon. In this general framework, we focused on the mesoscopic fluctuations in the low-temperature strong-interaction regime. The system can there be described as a Fermi liquid with quasiparticles having eigenstates and eigenenergies that are fluctuating if we vary some external parameter. The correlations of these fluctuations with those of the decoupled Fermi gas have been worked out [T1-100,T1-109,T1-110], making use of a slave-boson/fermion approach as well as an adaptation of the Nozieres-type picture of the local Fermi liquid. We thereby obtained a rather complete description of the fluctuations of the quasiparticle properties when the impurity-Fermi gas coupling is chaotic. Among the remarkable features, these fluctuations have a universal character and a peculiar dependence on time-reversal symmetry [T1-109,T1-110].

[Permanent member involved: Ullmo]

#### Hall effect

In the realm of the fractional quantum Hall effect (FQHE)), the nature of the ground state of 2D electron gas at Landau level filling factor 5/2 has received renewed interest recently due to the fact that this may be a concrete physical system featuring non-Abelian anyonic quasiparticles. Indeed, one of the proposed ground state wavefunctions at filling 5/2 is the so-called Moore-Read Pfaffian state. This state is known to have quasihole excitations with non-trivial braiding properties, suggesting that it is a promising candidate to perform topological quantum computations. Even if such considerations remain speculative, it is nevertheless a pertinent goal to decipher the nature of electron-electron correlations of this somewhat delicate state. At variance with previous expectations, we have shown that increasing the width of the 2D electron gas in real samples may lead to stabilization of the Pfaffian state. Theoretical advances in the understanding of trial wavefunctions [T1-27,T1-71,T1-89] have led to the construction of new candidates to explain some of the still elusive FQHE fractions. Exact diagonalizations have been performed in the cylinder geometry [T1-116, 117] and have led to a new method to extract the Luttinger liquid exponent of the edge theory of principal FQHE liquids.

Rapidly rotating quantum gases likewise attract sustained interest as they bring in analogies with fractional Quantum Hall effect – single-particle Hamiltonian of a neutral atom in the rotating frame is analogous to the Hamiltonian of an electron in a uniform magnetic field. Present experiments have reached the so-called mean field Quantum Hall regime where the ground state is the Abrikosov vortex lattice. In the vortex lattice regime, the novel physics is related to the presence of external confining potentials, which influences the structure of Landau levels, and impinges on interparticle interactions. We have found a mean-field solution for the vortex lattice of harmonically trapped rapidly rotating bosons in the symmetric and narrow channel geometries [CSP-2]. In the latter case, the system exhibits the structure of vortex rows. We showed how an increase in the interaction strength leads to a series of quantum transitions associated with an increase in the number of rows. In the infinite plane geometry, we obtained a mean-field solution for the Tkachenko modes, calculated their damping rate, and studied finite-temperature effects in correlation properties [T1-93]. Deep in the FQHE regime, exact diagonalization investigations have shown that dipolar interactions can stabilize a novel Fermi sea made up of composite fermions instead of the Moore-Read Pfaffian [T1-10].

[Permanent member involved: Shlyapnikov, Jolicoeur from this group + Ouvry]

#### 2.2.2 Statistical physics, dynamical systems

In this group, there are currently 10 permanent researchers, 2 postdocs and 4 PhD students. During the period 2008-2013, the group members have worked on a broad range of topics in statistical physics (both equilibrium and nonequilibrium), extreme value statistics, random matrix theory and its applications, quantum chaos, quantum information theory, conformal field theory, first-passage properties of various stochastic processes, diffusion and localisation in disordered medium, quantum Hall effect, biophysics problems including polymer physics, genetic switches etc. The thematic overlaps with investigations performed in the other groups of the laboratory are consequently quite substantial. Some of the main results are highlighted below.

#### Extreme value statistics and related quantities for strongly correlated time series

Since its first developments in the early 30's, extreme value statistics (EVS) have found an increasing number of applications in a variety of fields including engineering, climatology, finance, biology and also in the physics of disordered systems. In EVS, one is concerned with the distribution of the maximum or minimum of a set of random variables  $\{x_1, x_2, ..., x_N\}$  which may represent the average temperatures or the prices of a stock on *N* successive days, or the energy levels in a disordered system. When the variables  $\{x_i\}$ 's are uncorrelated or weakly correlated, EVS is well understood. Much less is known, however, for *strongly* correlated random variables which happens to be the case in many realistic applications. Significant analytical progresses were recently achieved in our laboratory in characterizing the EVS of strongly correlated variables using various techniques of statistical physics, in particular in two systems: (i) random matrices (where  $x_i$  represents the *i*-th eigenvalue of a random matrix) and (ii) Brownian motion/random walks (where  $x_i$  represents the position of a random walker at the *i*-th step).

(i) Extreme value problems in random matrices: In random matrix theory (RMT), the largest eigenvalue  $\lambda_{max}$ of an  $(N \ge N)$  Gaussian random matrix is known to follow the celebrated Tracy-Widom (TW) distribution (which has now been found to be ubiquitous and occurs in many systems outside random matrices, such as in directed polymers, growth models, sequence alignment problems, quantum dots etc.). TW, however, describes typical fluctuations of  $\lambda_{max}$  around its mean. In recent times, a growing interest has been in the atypically large fluctuations described by large deviation functions. In a series of works [T2-6, T2-51, T2-87, T2-96, T2-136], these large deviation functions have been computed exactly by our group members for various matrix ensembles, making use of Coulomb gas techniques. The major result was the discovery of a 3-rd order phase transition in the underlying Coulomb gas, which gets reflected in an asymmetry and weak singularity of the associated large deviation function of  $\lambda_{max}$ . Similar 3-rd order phase transitions occur in large N gauge theory between strong and weak coupling phases. Our theoretical predictions have subsequently been verified in coupled fiber laser experiments by the Weizmann optics group [M. Fridman et. al. PRE, 85, 020101 (2012)]. Subsequently, similar transitions were found in a number of other systems, such as in the distribution of normal or Andreev conductance through a mesoscopic cavity [T2-32, T2-81, T2-89] and in the distribution of entanglement entropy of a random pure state [T2-76, T2-97]. These problems provide enticing applications of RMT.



Another major result was the discovery of a close connection between RMT and the so called vicious walker problem introduced by De Gennes, where *N* independent Brownian motions propagate in time *without intersection*. Using path integral methods, the joint distribution of the positions of the walkers was shown to be related to the joint distribution of eigenvalues of certain classes of random matrices [T2-28, T2-53]. The exact result [T2-28] for the maximum deviation of the farthest walker from the origin led to the discovery of a deep connection between the vicious walker problem and two-dimensional Yang-Mills gauge theory [T2-28, T2-94, T2-141]. The work in [T2-28] was

reviewed in the 'research highlights' section of Nature Physics (4, 829 (2008)). Figure: Typical space-time trajectories of \$N=7\$ non-intersecting Brownian bridges (vicious bridges), known as the 'watermelon' configuration. Fertile connections with growth models [T2-127, T2-105] or chemical paths on a spanning forest [T2-135] were also established.

(ii) *Extreme value problems in random walks*: For a variety of constrained random walk problems (with 8

applications in finance and queuing theory), a full characterization of the maximum and its time of occurrence was computed exactly using path integral methods [T2-21] and an application to finance was studied in [T2-23]. A related question is the order statistics where one is interested in the statistics of the first maximum, the second one, or more generally the k-th one. Order statistics is a pertinent tool to characterize the phenomenon of crowding near extremes: how many near-extreme events occur in a given time period (not just the global maximum but also events close to the global maximum), such as for instance how many major after or fore-shocks occur around a principal earthquake? For random walks, the distribution of the gap between the k-th and (k+1)-th maximum was computed exactly and was found to be universal, i.e., independent of the jump distribution of the walker [T2-126]. A related problem is the study of records in random walks. The entry  $x_i$  in a time series  $\{x_1, x_2, ..., x_N\}$  is a record if  $x_i$  is bigger than all previous entries. How many such records happen in N steps? In [T2-22], it was shown that for random walks as well as for Lévy flights, the statistics of the number of records and their ages are completely universal, i.e., independent of the jump distribution. In recent works [T2-120, T2-131], these results were generalized to a random walk with a drift and also to the case of n independent random walks. Finally, these results were extended to various non-Markov stochastic processes that arise naturally in non-equilibrium systems, after a quench from the high temperature disordered phase to the low temperature ordered phase [T2-48].

Another interesting application of EVS was found in a 2-d geometric problem. For instance, the mean perimeter and the mean area of the convex hull of *n* independent Brownian motions each of duration *t* in two dimensions was computed exactly, by mapping this nontrivial 2-d problem to a one dimensional extreme value problem using Cauchy's formula [T2-56, T2-72], with potential applications in ecology where the home range of animals is estimated by the convex hull of the union of their trajectories. This mapping from 2-d convex hull to the 1-d EVS problem is very general and has been exploited in several other processes, including random acceleration process arising in polymer physics [CSP-7, CSP-9] and branching Brownian motion (with death) that arises in the context of an epidemics spread in animal population [CSP-14].

Amongst other applications of the 2-d random walks, the fractal properties of the *n*-winding sectors inside the walks was studied numerically [T2-7] with particular emphasis on 0-winding sectors (fjords and lakes). In addition, the probability distribution of the algebraic areas of a random walk (a problem closely related to the quantum Hofstader problem) was studied and an exact result was found for fully directed random walks [T2-52].

Finally, extremes and related problems were also studied analytically in a number of other applications such as in interacting particle systems [T2-58, T2-73, T2-112, T2-114], one-dimensional disordered systems [T1-80], first passage properties of fractional Brownian motion with application to DNA translocation [CSP-10, CSP-15], search problems [T2-92, T2-93, T2-142, CSP-6] and sports [T2-25, T2-90].

[Permanent members involved: Comtet, Desbois, Majumdar, Nechaev, Ouvry, Schehr, Vivo (from this group) + Rosso, Texier]

#### Applications of random matrix theory

Random matrix theory (RMT) is a traditional strong point for LPTMS since the connection between the spectrum of chaotic quantum systems and random matrices was surmised in this laboratory in the early 80's. Several members of this group have been actively involved in various aspects and applications of RMT, beyond quantum chaos. In addition to the question of transport through mesoscopic cavities alluded to above, some of the main results obtained during the period 2008-2013 are summarized below.

(i) Multifractality of quantum states and integrable random matrices: Many physical phenomena can be described in terms of fractals or multifractals (the latter being characterized by the presence of a whole range of fractal dimensions). In the past years, several studies have pointed to multifractality in quantum wave functions in several different physical contexts. This is the case of Anderson localization, where there is a transition separating a localized phase from a delocalized phase where the system is no longer an insulator: at the transition point, the wave functions are neither localized nor ergodic, but are characterized by amplitude distributions which are multifractal. The statistical properties of quantum spectra at the transition is of a type intermediate between the Random Matrix Theory statistics of quantum chaotic system and the Poisson statistics of integrable systems. However, direct observation of multifractality or intermediate spectral statistics is strenuous, both numerically and analytically. Several simple models have been recently identified, in particular quantized maps on a compact phase space [T2-42]; quantum wave functions have been shown numerically to display multifractal properties [P-9]. In [T2-40] a closely related ensemble of critical random matrices was proposed, constructed from Lax matrices of classically integrable onedimensional systems of N interacting particles. The integrable structure has allowed to calculate analytically various spectral properties [T2-84], while multifractal dimensions of eigenvectors have been obtained analytically by a perturbative approach [T2-85, T2-107]. An unexpected relation was found between the

multifractal dimensions and the level compressibility which was checked in various models [T2-82]. The multifractality of quantum wavepackets [T2-115] was studied and various ways to reliably compute multifractal dimensions were proposed [T2-75].

(ii) *Ratio of consecutive spacings in random matrix ensembles*: Usually, when predictions of random matrix ensembles are compared with numerical or experimental results, it is necessary to perform 'unfolding' of spectrum reducing the mean spectral density to a constant. For many-body problems though, the Weyl expansion for the mean spectral density is unknown with necessary accuracy, which diminishes the precision of statistical tests. To circumvent this difficulty, it was proposed recently to consider a non-standard correlation function, namely the ratio of two nearest level spacings which is independent on the level density. In [CSP-13], an accurate approximate formula was found for this quantity and new observables with similar statistical properties were proposed.

(iii) *Complex networks and random matrices*: Networks describing complex systems have been widely studied in the past decade, allowing for a better understanding of phenomena from disease spreading to sociological or chemical networks. Properties of complex networks were investigated via the spectral properties of the associated weighted adjacency matrix (the so-called Google matrix), and the localization properties of its eigenvectors. The delocalization transition for matrices describing part of the world wide web [P-16] or lexical networks [T2-69] was studied. In [T2-116] it was shown that the network approach could give some insight into statistical properties of the game of go, with possible applications to the improvement of computer simulators, which are still defeated by amateur players. In another development, a new class of random matrices-the 'randomized Parisi matrices' was proposed in [T2-34, T2-35, T2-36, T2-60, T2-61]. Their spectral properties were investigated with applications in scale-free networks, as well as for kinetic properties of proteins at low temperatures.

(iv) *Index distribution for random matrices*: The number of negative eigenvalues of a matrix is known as its index, and is pivotal in characterizing the stability of a stationary point in a random landscape (such as in glasses and liquids, or in cosmology), where the Hessian matrix associated to the landscape is random. Similarly, for Wishart random matrices (random covariance matrices), the index plays a major role in applications such as in Principal Component Analysis (PCA). In a recent series of works [T2-50, T2-95, T2-119], the full probability distributions of the index of Gaussian and Wishart matrices were computed exactly for large matrices using Coulomb gas methods. The most unexpected feature was a logarithmic singularity in the associated large deviation function of the index. In addition, the variance of the index was found to grow universally with matrix size N as  $\ln N$  for large N.

(v) Quantum entanglement in a bipartite system via RMT: Another interesting application of RMT lies in the study of the distribution of quantum entanglement of a random pure state of a bipartite system composed of two coupled Hilbert spaces of respective dimensions M and N (for instance, qubits and environment). Several exact results were obtained for the distribution of entanglement entropy (von Neumann and Renyi entropy), both for finite M and N [P-29, P-31, P-32], as well as in the asymptotic large M, N limit (with M/N fixed) [T2-76, T2-97]. The entropy distribution was shown to have two singular points, signaling two phase transitions in the underlying Coulomb gas problem. This analysis negated a widespread belief that a randomly prepared pure state has near maximal entropy (which stemmed from the calculation of the mean entropy and not its full distribution).

(vi) Generalised  $\beta$ -Wishart matrices: Traditionally, the Wishart ensemble describes random covariance matrix which is usually characterized by an integer Dyson index  $\beta = 1$  (real Gaussian entries),  $\beta = 2$  (complex Gaussian entries) and  $\beta = 4$  (quaternion entries). In all these cases, the spectral density is given by the Marcenko-Pastur (MP) law which has a finite support over the positive real axis. Recently, new classes of random matrices have been put forward where  $\beta$  can be any positive real number-the so called generalised  $\beta$ -Wishart matrices. For any finite  $\beta$  (~ O(1)), the spectral density is given universally by the MP law and it is hard to deviate from the MP basin of attraction. In [T2-132], it was shown that when  $\beta = c/N$ , the entropy of the Coulomb gas becomes of the same order as the energy, leading to an unbounded spectral density parametrized by c, that continuously interpolates between the MP law (large c) and the Gamma law (low c), thereby providing a new family of spectral densities.

(vii) One dimensional disordered systems and random matrices: The deep relationship between products of random matrices and generalized quantum points scatterers was explored in [CSP-3]. A continuum limit was identified where the Lyapunov exponent can be computed analytically [CSP-12]. This work provided a *general classification* of the possible types of one dimensional disordered models viewed as continuum limits of discrete models described by products of random matrices.

(viii) *Interdisciplinary application of RMT*: A class of correlation matrices with power-law tails with potential applications to financial historical time series was studied [P-23, P-24, P-28]. Studies were undertaken to characterize the financial stability in complex markets and emergent instabilities [P-25] and a

comprehensive game-theoretical analysis of lowest-unique bid auctions [T2-125] in economics, and also the application of RMT to biophysical problems such as the collective behaviour of proteins in the elastic regime [P-26].

[Permanent members involved: Bohigas, Bogomolny, Comtet, Giraud, Majumdar, Nechaev, Schehr, Vivo + Roux, Texier]

#### Applications of conformal field theory to random systems

Conformal field theory (CFT) is a powerful tool to obtain analytical and non-perturbative results for classical and quantum systems, where a large number of degrees of freedom are strongly correlated. Over the past few years, the deep relation between CFT correlators and the 2 + 1 topological phases of matter was explored. This allowed to analyse the integrable structures of many-body wave-functions describing non-Abelian phases. Among the results [T2-43, T2-46, T2-67, T2-68, T 2-78, T2-111], the most important achievements are (i) the proof of a conjecture which relate many families of fractional quantum Hall wave-functions to a particular class of symmetric polynomials, the Jack polynomials; these results also provided efficient ways to construct many-body quantum Hall states and to study their entanglement properties (ii) the discovery of integrable structures, related to the Benjamin-Ono integrable non-linear equations, hidden in the non-Abelian quantum Hall states. As a by-product, a new family of eigenfunctions of the celebrated Calogero-Sutherland Hamiltonians was discovered and connections with four dimensional super-symmetric gauge theories were found. In addition, significant progresses were made in the study of fractal interfaces in disordered and nonminimal critical models (determining the value of their fractal dimension). The results of these works, although partial and sometimes numerical, have suggested the existence of elegant yet elusive CFT structure. New CFT's were developed with predictions for observables in conformally invariant random processes. In particular two dimensional geometrical problems such as self-avoiding walks, percolation or random clusters in spin lattices models were studied [T2-55, T2-77, T2-99, P-5, P-6, P-8], including an extension of SLE approaches to study critical interfaces.

[Permanent member involved: Santachiara]

#### Selection of other problems with diverse applications

(i) *Dielectric micro lasers*: Articles [T2-3, T2-9, T2-62, T2-83, T2-106, T2-108] are devoted to the investigation of semiclassical approximation for dielectric cavities of different forms. The importance of this



thematics is connected, to a large extent, to the possibility to use such cavities as resonators for micro lasers and to observe experimentally light emission from these objects. Trace formulae and perturbation series were worked out [T2-3,T2-8], and successfully compared to experiments in a joint effort with ENS Cachan [T2-9]. [Bogomolny, Giraud]. Figure: Eigenmodes for two isospectral cavities with Dirichlet boundary conditions: fundamental mode

#### (left pair), 15th excited mode (right pair).

(ii) *Multifractality of spin chains*: The success of quantum chaos is, to a large part, related to the development of computer power, which permitted to calculate numerically a large number of eigenvalues for small-dimensional models. Nowadays computers are capable of finding numerically properties of one-dimensional many-body systems like the Hubbard model. The development of semi-classical methods for such models is important, as for the first time it is possible to compare theoretical predictions not only with numerics but also with experiments with cold atoms. The transfer of the methods and ideas of quantum chaos to this type of problems will, without doubt, give new and interesting results. The simplest models like the one-dimensional quantum Ising and Heisenberg models of spin 1/2 and their generalizations were studied and unexpected results were found, namely, the ground state functions of all these models are multifractals with non-trivial fractal dimensions [T2-104].

#### [Permanent member involved: Bogomolny].

(iii) A surprising link was discovered between diffusion processes and continued fractions [T2-88]. A diffusion process in a deterministic environment can be analyzed in terms of its excursions which are the paths followed by the process between two successive visits to the starting point. In this way one can associate, to any diffusion process, a Ricatti equation which can be solved in terms of an infinite continued fraction. The probabilistic significance of the coefficients was studied and results were illustrated by considering some examples in deterministic and random environments. Another interesting diffusion problem concerns the motion of a particle in a one dimensional random environment. This can be mapped to

the quantum mechanical problem of a particle in a super-symmetric potential. In the case where the superpotential is a Levy process one can prove that the complex Lyapunov exponent may be expressed in terms of the positive solution of a difference equation [CSP-8]. The problem of computing the density of states thus reduces to a Stieltjes moment problem which, in some cases, can be solved explicitly in terms of special functions.

#### [Permanent member involved: Comtet, Texier].

(iv) *Quantum information*: Quantum information investigates how quantum states can be entangled, manipulated and measured in an optimal way in order to process information efficiently. Although algorithms allowing for efficient solutions (exponentially, or at least polynomially faster than classical algorithms) are known to exist, surprisingly few have been provided so far. We have discussed algorithms implementing efficient calculations for various physical problems: trace formulas from quantum chaos [P-11], wavelet transforms [P-14] or the equilibrium distribution for classical statistical mechanics systems [T2-66]. Experimentalists now manage to manipulate systems with a small number of qubits. Mathematically, these quantum states belong to a finite-dimensional Hilbert space, whose geometry is nontrivial. The problem is slightly simpler for symmetric states under permutation of qubits. We defined 'classical' spin states as the quantum states [T2-117] and identified the least classical (or the most quantum) states [T2-71, T2-74].

#### [Permanent member involved: Giraud].

(v) Morphological transition in random RNA alignment: The fraction f of nucleotides involved in the formation of a cactus-like secondary structure of random heteropolymer RNA-like molecules was studied as a function of the number c of different nucleotide species (alphabets) [T2-98, T2-128, T2-129, T2-130]. It was shown that the secondary structures of RNA undergoes a morphological transition as a function of c at a critical value  $c_{cr}$ , such that for  $c \le c_{cr}$  one has a perfect 'gapless' secondary structure while for  $c > c_{cr}$ , gaps survive. The strict upper and lower bounds  $2 \le c_{cr} \le 4$  were proven.

#### [Permanent member involved: Nechaev].

(vi) *Rotating Bose-Einstein condensates*: A rotating Bose-Einstein condensate in a narrow channel was studied within the Lieb-Liniger model in the infinite coupling constant limit [T2-54]. It was shown that contrary to popular belief, the gas is entirely fermionic and that no one-dimensional experiment can possibly detect a bosonic signature of the gas. In [T2-137], a rapidly rotating Bose gas with interaction was shown to be soluble in a given angular momentum sector.

[Permanent member involved: Ouvry].

#### 2.2.3 Disordered systems, soft matter and physics-biology interface

The team is currently composed of 6 permanent members (including one member on a "détachement" position), 5 PhD students and 4 post-docs. The research lines embrace the statistical physics of disordered and glassy systems, their connections with computer science and information theory, fluctuation induced forces in soft-condensed matter and beyond, together with an activity at the physics-biology interface. The results we deem most interesting are the following:

#### Statistical physics of disordered systems and interdisciplinary applications

(i) Introduction and first studies of spin glasses on Dyson lattices. One of the obstacles in the theoretical understanding of finite dimensional disordered systems is the lack of a reliable approximation scheme of Real Space Renormalization Group (RG). Morever, the presence of disorder induces non trivial structures of interfaces that are not well understood theoretically. In the absence of disorder, Dyson lattices provide models where interfaces are particularly simple and that can be solved exactly through the RG transformation. One of our lines of research has consisted in studying spin glass models on Dyson lattices, with the twofold scope of (1) obtaining hints on the structure of interfaces in glassy systems and (2) studying the real space RG transformation in finite range interaction disordered systems in a simplified setting. The interfaces have been analyzed using the replica method and numerical simulations on a spin glass model with pair interaction [T3-43]. The main result of the analysis is the evidence of a lower critical dimension above which large scale order parameter heterogeneities imply a divergent free-energy cost. The study of the RG transformation, which turns out to be very involved in general, has been addressed in the context of a simplified model (a hierarchical Random Energy Model) where on the one hand an exact algorithmic solution for finite sizes can be achieved in a polynomial time on the other hand one can study the problem by exact high temperature expansions that can be pushed to high order [T3-63]. Both methods give rise to a coherent picture of the model with an ideal glassy phase at low temperature. While this is the first time that an ideal glass transition has been evidenced in a non-mean field model, the analysis shows the non mean-field nature of the transition.

(ii) Heterogeneities and fluctuations of glassy relaxation. Dynamical fluctuations in supercooled liquids and their role in the route to glassyness have been a longly debated issue in the last years. Fluctuations in the socalled "beta relaxation regimes" of relatively short times can be understood as quasi-equilibrium fluctuations in presence of a constraint [T3-107]. This observation allows to use static techniques to study dynamic heterogeneities, at the price of expressing parametrically the time dependence of correlation functions. This gives rise to a detailed description of dynamical heterogeneities both in schematic models, which are exactly solvable, and in realistic systems treated through liquid theory approximations [T3-143]. A critical field theory emerges which, to all orders in perturbation theory, coincides with the one describing the spinodal point of a random field Ising model, where the random initial condition in dynamics are at the origin of the random field. The predictions of this universal theory are currently tested in numerical simulations by several groups. The quasi-equilibrium analysis has been extended to the slow (alpha) relaxation [T3-179]. It is shown that guite surprisingly, the resulting effective dynamics coincides with -and thus explains the- slow dynamics (equilibrium and aging) in mean field models, and can be fruitfully employed in first principle liquid state theories. A unified description of static and dynamic properties ensues, beyond mode-coupling. The study of the formal and physical properties of this dynamics have just started and will be developed in the next years.

(iii) The Quantum Cavity Method. The cavity method, developed in the last decades in large part at LPTMS, is by now a well established and powerful technique in statistical physics for solving classical spin models on finite connectivity graphs without short loops. Besides providing the most sophisticated form of mean-field approximations for physical systems, its development has been instrumental in providing analytic and algorithmic solutions to random satisfiability problems in computer science, error correcting codes, and more generally inference problems in information theory. The development of a quantum versions of the cavity method to various degrees of approximation allows to tackle problems where analytic solutions are rare and provides a complementary view to the approaches based on low dimensional models. We have put forward two quantum extensions of the cavity method. The first consisted in using the Trotter-Suzuki formula to reduce the problem to a classical dynamical counterpart [T3-23]. A non-trivial aspect has been to find a parametrization of the trajectories that avoid an exponential computational cost as the temperature is decreased. The resulting procedure, which is powerful when the system is glassy or highly frustrated, has been used to obtain the phase diagram of the Ising model in a transverse field on a random graph. The comparison with the results of quantum Monte Carlo shows the adequacy of the method to get approximations for finite dimensional systems. The second method does not use path integration and

operates directly at Hilbert space level [T3-66], producing a flow of Hamiltonians that can be projected onto simple sub-spaces to obtain approximate solutions. The resulting accuracy increases with the connectivity of the underlying graph. The basic idea is to use the cavity Hamiltonian in absence of a site in the graph, as the basic « message » to be iterated in the graph. The method has been applied to the study of the superconductor-insulator transition in strongly disordered superconductors [T3-73]. This system, which can be viewed as an Ising model in a random transverse field, turns out to have a glassy phase at low temperature, similar to the one of directed polymers on trees [T3-141]. One important consequence is that superconductivity can persist for finite levels of disorder, while more naive mean-field theories would predict superconductivity only in absence of disorder.

(iv) Directed polymers and interface dynamics in a random medium. A directed polymer in a random medium (DPRM) is one incarnation of Kardar-Parisi-Zhang (KPZ) equation, the paradigm of growth dynamics out of equilibrium. This problem has received renewed attention in the last years thanks to an exact solution obtained combining Bethe Ansatz and replica method. We have focused on the high temperature limit of the DPRM, where two regimes appear: (i) at short times, a crossover from the simple Edwards-Wilkinson universality class to the KPZ class takes place, and (ii) at large times, KPZ scaling holds [T3-61]. Moreover, the model is integrable via Bethe Ansatz and the solution we provided [T3-62] is already well received in the mathematical physics community. A related question is that of the dynamics of elastic interfaces in a disordered medium under a driving force. It exhibits typical glassy phenomena such as metastability, pinning, creep and avalanches. Despite intensive study in the last decades, our understanding of these phenomena is still incomplete. A powerful quantitative method for such strongly out of equilibrium systems is provided by the Functional Renormalization Group (FRG). We developed and employed efficient numerical methods to study the avalanche statistics of one and two-dimensional interfaces at the depinning transition [T3-54]. Our results show an impressive agreement with the FRG, in particular with respect to the most spectacular "cusp" singularity in the renormalized disorder distribution. More recently, we have used the same numerical scheme to study the toughness of brittle materials. Our study reveals the existence of a collective pinning regime where the material resistance can be expressed as a function of a few parameters only, that can be extracted from the material micro-structural features.

(v) Statistical physics approaches to information theory: The problem of compressed sensing. The last years have witnessed a spectacular convergence of scopes and methods between statistical physics and information theory. In particular, many inference problems in information theory can be formulated as spin models with disordered interactions. Among these problems, compressed sensing is a hot topic in information theory, since the seminal work of Candès, Tao and Donoho in 2006. For almost all natural signals (e.g. images), there exists a base in which the representation is parsimonious. This is at the basis of compression used for



data storage. Compressed sensing aims at acquiring data directly in a compressed format. Using a widespread linear model of data acquisition, we proposed a new message passage algorithm, capable to reach the ultimate theoretical limit of compression. It has been possible to achieve this goal coding the information in a mean-field disordered model with spatial extension [T3-157, T3-158]. Data reconstruction appears as a phase of the system that has to compete with glassyness. In the glassy metastability region. the stable reconstruction phase may be induced by a seed

in a limited region of space, which gives rise to a wave front leading to the solution of the original inference problem. This work has been considered as a break-through in the information theory community. Several applications and extensions are currently under scrutiny (see the perspectives in the remainder). Figure: Comparison of the "Seeded Belief Propagation" algorithm (bottom) with previously used state-of-the-art algorithms (l1 reconstruction: top) in image reconstruction:  $\alpha$  the fraction of pixels acquired during sensing [T3-157].

[Members involved: Franz, Mézard, Rosso; part of the activity pertaining to anomalous diffusion has been addressed in the report of the group "Statistical physics and dynamical systems"].

#### Soft condensed and granular matter

In the realm of charged colloids, a particular effort has been invested in developing an analytically exact strong coupling approach, after having spent several years --and continuing-- investigating the opposite limit

where mean-field prevails, and where a particularly powerful mapping to Painlevé III equations can be brought to bear. The hitherto dominant approach was the so-called Virial Strong Coupling route, in principle a systematic expansion around infinite coupling. We have shown that it fails to account for collective effects, that turn prevalent. We have also realized that geometrical constraints somehow cure these deficiencies, in the one-dimensional geometries that have been investigated previously, but not in more general conditions (under dielectric mismatch, or with curved macromolecules). These shortcomings have been circumvented within a new scheme, dressing the underlying Wigner crystal that forms under sufficient coupling by the relevant low T excitations. The method was thus dubbed 'Wigner Strong-Coupling' expansion [T3-127], and provides the first exact treatment of the problem. It is free of infra-red divergences, in excellent agreement with Monte Carlo simulation data, and relevant for describing deionized charged colloids at room temperature with multi-valent counter-ions (say tri- or tetra-valent). Similar ideas have led to the study of a generalized Thomson plum-pudding model, under strong Coulombic couplings [T3-100]. Likewise, exact results could be obtained on the polarizability of a model macromolecule, considering a two-dimensional one component plasma at the free fermion point [T3-173]. On the other hand, a large gamut of colloidal systems are amenable to a mean-field Poisson-Boltzmann in essence treatment, which was performed in three directions: 1/ study of colloidal complexes on two-dimensional periodic substrates generated by optical



trapping techniques where the possibility of pattern switching under external field was analyzed [T3-104], 2/ description of self assembly of spherical interpolyelectrolyte complexes from oppositely charged polymers [T3-135], 3/ development of a renormalized jellium model as an alternative to the antique although quite robust Wigner-Seitz cell model [T3-38]. *Figure: illustration of colloidal behavior on a periodic optical trap.* 

Coarse-grained approaches for dissipative systems and granular gases. We have uncovered and studied a non-

equilibrium phase transition in a granular gas, revealed by an impurity in a sheared system [T3-109]. The impurity may carry a finite fraction of the total kinetic energy of the system, with an ensuing diverging temperature ratio. This extreme violation of equipartition is monitored from an exact solution to the dissipative Boltzmann equation, which in turn shows that the intuitive picture of an impurity enslaved to the host fluid breaks down. A related endeavor allowed to derive the hydrodynamic description for a driven granular gas, and to explain puzzling simulation data, that showed a non vanishing rescaled variance of energy fluctuations upon approaching the elastic limit [T3-45]. These are fingerprints of subtle 1/N corrections to two-body correlation functions. A key aspect, hitherto overlooked, lies in the structure of the velocity scaling function, that is not of the common one parameter form, but requires the inclusion of a second parameter that can be viewed as measuring the distance to the steady state [T3-181]. In addition, a connection between granular and mass transport models has been established, with the aim to understand the real-space condensation transition in polydisperse hard rods [CSP-5]. The stochastic process results, where a factorized steady state emerges, can be recovered by a density functional route. This incidentally raises a general question pertaining to equilibrium and dealing with the entropy of continuous mixtures, which is plagued by a 'measure' problem resulting from improper weighting of phase space. Shannon himself identified the difficulty, but concluded that the entropy has an absolute status for discrete probabilities, but a relative one for continuous cases, depending on the coordinate system. Such a viewpoint is not acceptable, and we could provide the 'absolute' entropy formulation for a class of models whereby a large number of agents or molecules randomly and repeatedly interact in pairs with prescribed conservation law(s) [T3-119]. [Member involved: Trizac from this group + Majumdar]

#### Geometry and fluctuations

Motivated by the lack of predictions for the Casimir interaction between objects of arbitrary shape and material composition, we have developed a universal and exact approach for computing the Casimir forces between such objects [T3-7,T3-52]. The corresponding energy is obtained as an interaction between multipoles that are generated by quantum fluctuations of currents. This approach leads to a general expression for the interaction between any given number of objects in terms of the individual scattering matrices, describing their shape and material. The formulation is universal, i.e., body-independent. We could successfully apply this method to solve a number of previously open problems [T3-29,T3-41,T3-53,T3-71]. More recently, we generalized the scattering approach to dielectric objects immersed in media other than vacuum, at nonzero temperatures, and with spatial arrangements in which one object is enclosed in the other [T3-86,T3-87]. We examined whether fluctuation-induced forces can lead to stable levitation. We analyzed a collection of classical objects at finite temperature with fixed and mobile charges, and showed that any arrangement is unstable to small perturbations in position [T3-79]. This extends Earnshaw's theorem for electrostatics, by including thermal fluctuations of internal charges. Quantum fluctuations of the electromagnetic field are responsible for Casimir/van der Waals interactions. Neglecting permeabilities, we found that any equilibrium position of items subject to such forces is also unstable if the permittivities of all objects are higher or lower than that of the enveloping medium; the former being the generic case for ordinary materials in vacuum. Existing approximations could not deal with objects that have sharp edges and tips. Those objects naturally appear though in the design of micro-electromechanical (MEM) devices. We developed a novel approach to electromagnetic scattering, suitable for perfect conductors with sharp edges and tips, specifically wedges and cones [T3-117]. The Casimir interaction of these objects with a metal plate (and among themselves) was then computed systematically by a multiple-scattering series. For the wedge, we obtained analytical expressions for the interaction with a plate, which should provide a particularly useful



tool for the design of MEMs. Our result for the Casimir interactions between conducting cones and plates applies directly to the force on the tip of a scanning tunneling probe. We found an unexpectedly large temperature dependence of the force in these configurations, that should attract experimental interest. Recently, we have extended the scattering approach for Casimir interactions to heat transfer and Casimir forces in nonequilibrium cases, where each body and the environment is at a different temperature [T3-114,T3-115]. We derived general trace

formulas [T3-156] that yield non-equilibrium forces, heat radiation and transfer in terms of the bodies scattering amplitudes. The formalism takes into account evanescent tunneling modes that are usually ignored in heat transfer. As a first application, we computed the radiation from a cylinder, emphasizing its polarized nature, and obtained the heat transfer between a sphere and a plate. Combining the scattering approach and a conformal map in three dimensions (bi- spherical coordinates) we could solve exactly the experimentally highly relevant case of a metallic plate and sphere described by a Drude model [T3-138]. Our analytical results provide exact expressions for the force at any distance, displaying an intricate structure of deviations from the commonly employed proximity force approximation. *[Member involved: Emig]* 

### **Physics-biology interface**

*Biophysics of proteins.* In an effort to understand the intertwining between biomolecular folding and function, we worked out a simple 'fly-casting' model for proteins, based on a capillarity picture and polymer chain statistics [T3-84]. Related ideas allowed to decipher the effects of confinement on protein dimerization [T3-56], and the influence of topology on the kinetics of repeat (sausage-like) proteins, thereby differing from the more spread globular objects usually encountered. We explored how interfaces between repeated units affect cooperativity and folding characteristics [T3-151].

Protein-membrane interactions. Eukaryotic cells are enclosed by a thin, impermeable lipid bilayer. To allow

vital exchanges with the outside world, specialized proteins must constantly tubulate, cut, deform, and fuse compartments made of this membrane. One of these proteins, dynamin, severs cell membrane tubules by forming a polymeric helix around them, and then constricting through GTP hydrolysis. To disentangle the multiple mechanical effects regulating this process, we first developed a general hydrodynamic theory of dynamin's constriction [P-51]. This model showed good agreement with experiments from our collaborator Aurélien Roux (U. Genève), demonstrating the importance of dynamin's friction with the membrane and its sustained mechanical integrity during the conformational change [P-59]. This prompted us to propose several hypotheses for the fission mechanism [P-54], one of which was recently validated through a joint experimental and theoretical approach [T3-161]. In parallel, we demonstrated the influence of membrane elasticity on the polymerization of the dynamin helix [P-55]. We also investigated other membraneassociated proteins, showing that the deformation of the membrane by the protein complex ESCRT-III could be akin to a physical process of buckling [P-52]. In Ref. [P-53], we characterized the cytoskeletoninfluenced active fluctuations of the red blood cell membrane in collaboration with Timo Betz (Institut Curie), then related cytoskeleton-membrane interactions in the framework of tumorigenesis in Ref. [P-56]. Finally, we considered stereocilia, membrane-coated vibrating structures of the animal inner ear, and proposed that their physiologically important shape is robustly determined by the stochastic detachment of cross-linking proteins within its cross-linked actin core [P-57].



Figure: Progressive deformation of the cell membrane upon binding of ESCRT-III filaments.

*Cytoskeleton contractility.* Cellular movement heavily relies on contractile structures within the cytoskeleton made of polar filamentous actin (F-actin) and the molecular motor myosin. While it is often assumed that the mechanisms generating this contractility are well understood, we showed that this phenomenon actually poses a conceptual challenge in the case of disordered structures [P-61] such as biomimetic actomyosin bundles studied in collaboration with Margaret Gardel [P-60]. We further showed through a combination of theory and experiments that this symmetry breaking occurs through the buckling of the actin filaments [P-62, T-187]. This work opens exciting perspectives –to be outlined below-- where new modes of actomyosin contractility can be envisioned for the myriad of cytoskeletal structures that contract in the absence of a clear polarity organization.

Biological networks. Biological networks represent a change of paradigm over the past decade whereby the functioning of a (biological) system requires not only a list of parts but also knowledge of how these parts interact and work together. As such, networks are ubiquitous in systems biology and have become mainstrean in our representation of intra-cellular processes. We worked mainly on metabolic and genetic intra-cellular networks [T3-131]. Genome scale metabolic modeling took off with the pioneering work of the Palsson group that showed that a framework based on flux balance analysis could provide good agreement with experimental results. The corresponding map between genotypes (presence/absence of reaction enzymes) and phenotypes (growth on various media, be they minimal or rich) provides a fertile ground for investigating evolutionary questions such as robustness, evolvability or exaptation as well as a path towards synthetic biology. Using a Monte Carlo Markov Chain approach that allowed us to sample the space of all genome scale metabolic genotypes having a given phenotype, we investigated how the metabolic phenotype influences the network architecture of the underlying biochemical reactions. Similarly, in the context of genetic regulation, we asked to what extent regulatory functions constrain network structure. Such structures include the presence of specific pairwise interactions and motifs as defined by other groups (Alon). Considering the phenotypes to be either generic functional features or actual biological patterns of gene expression, we uncovered the ubiquity of the « network function shapes network structure » scenario [T3-94] and a new kind of motif we coined "loopless feed forward cascade".

*Crossover formation*. Crossover formation is at the heart of meiosis --the key phase of sexual reproduction. The "purpose" of meiosis is to go from a diploid cell to a haploid one. Specifically, for each pair of

homologous chromosomes in the diploid cell, it is essential that exactly one of the two homologs be transmitted to the haploid one. Crossovers lead to the shuffling of alleles; they are thus major actors of evolutionary dynamics and also of direct interest for practical applications in breeding programs. We have been studying the two putative pathways of crossover formation in plants, comparing state of the art data sets with mathematical statistical modeling. One major result was to show that the two pathways are indeed present in maize [T3-42]. We have also been refining our models thanks to high quality *Arabidopsis* data, to the point where we now have reached the conclusion that the standard model assumes characteristics for each of the two pathways that simply are not valid.

[Members involved: Lenz, Martin, Trizac]