Aging in the Glass Phase of a Two-Dimensional Random Periodic Elastic System

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Using the renormalization group method we investigate the nonequilibrium relaxation of the (Cardy-Ostlund) 2D random sine-Gordon model, which describes pinned arrays of lines. Its statics exhibit a marginal ($\theta = 0$) glass phase for $T < T_g$ described by a line of fixed points. We obtain the universal scaling functions for two-time dynamical response and correlations near T_g for various initial conditions, as well as the autocorrelation exponent. The fluctuation dissipation ratio is found to be nontrivial and continuously dependent on T.

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Tremendous progress has been achieved in recent years in the detailed understanding of the off equilibrium relaxational dynamics (e.g., coarsening) of pure systems. For instance, the scaling forms of response and correlations are well characterized and several autocorrelation and persistence exponents have been computed using powerful methods [1]. By contrast, analytical studies of aging properties of random and complex systems have been mostly achieved within mean field theory [2]. These works have unveiled the existence of nontrivial fluctuation dissipation ratios (FDR), X. These generalize to nonequilibrium situations the fluctuation dissipation relation between (disorder averaged) integrated response and correlation [2] and can be interpreted as an effective temperature $T_{\rm eff} = T/X$ [3]. These were later investigated beyond mean field for pure systems. Although they appear to be trivial for some pure domain growth processes [4], they were later found to be novel universal quantities at pure critical points [5,6].

The description of glassy states beyond mean field needs to take into account a very broad distribution of relaxation times originating from rare sample to sample fluctuations and distribution of barrier heights [7]. In this context it is not obvious how (or whether) the off equilibrium dynamical scaling valid for pure systems extends to this case. Also it is not obvious that various definitions of FDR (using different observables, or different configurational averages) are equivalent and meaningful [8–10]. Some results were obtained for the random mass Ising critical point, via renormalization group (RG). The dynamical (z) and autocorrelation (θ_C) exponent were computed [11]. A definition of the FDR (for the zero mode) was found to be nontrivial [12] and obtained to one loop.

Whether such results hold *within a glass phase,* away from a phase transition, is still an outstanding question. A prototype system which exhibits a glass phase, while analytically more tractable, is the disordered elastic manifold in a random potential [13]. It describes a variety of physical situations, both in or out of equilibrium, ranging from visualization of pinned domain wall relaxation in magnets [14,15], nonequilibrium transport in electronic glasses [16], to mesoscopic fluctuations in vortex physics [17]. Here we investigate the case of a periodic manifold, a case which can be mapped [18,19] onto the so-called Cardy-Ostlund (CO), 2D-random sine-Gordon model [20], defined by the Hamiltonian

$$H[\phi] = \int d^2x \left(\frac{1}{2} [\nabla_x \phi(x)]^2 - \mathrm{Re}[\xi(x) e^{i\phi(x)}] \right), \qquad (1)$$

where $\phi(x) \in]-\infty, +\infty[$ is an XY phase (excluded vortices), $\xi(x)$ is a quenched Gaussian random field, i.e., a complex variable, with random phase, and $\overline{\xi(x)\xi(x')^*} = g\delta^{(2)}(x - x')$. The statics of (1) has been extensively studied analytically and numerically, in the context of planar flux line arrays [with displacement field $u = a\phi/(2\pi)$ and mean spacing a] and solid on solid models with disordered substrates. It is known to exhibit a glass phase [21,22] below T_g ($= 4\pi$) described by a line of fixed points perturbatively controlled by the small parameter $\tau = (T_g - T)/T_g$ (see, however, [23]). By contrast, its (nondriven) nonequilibrium dynamics has been studied only within Gaussian variational approximation [24], known already in statics to yield unreliable results for some observables [25]. There is thus the need for a controlled study via RG.

In this Letter we study the relaxation dynamics for the CO model in the glass phase $T < T_g$ starting from various initial conditions at $t_i = 0$ (e.g., with the same correlations as the pure model at equilibrium at temperature T'). We compute, to lowest order in τ using RG along the fixed line, the two-time ($t' < t$) response $\mathcal{R}$, as well as the connected $\tilde{C}$ and, respectively, off-diagonal C correlations. These are found to be characterized by (T'-dependent) universal scaling functions of $q^z(t - t')$ and t/t', q being the wave vector. We find that an autocorrelation exponent can be consistently defined, i.e., with $\theta_{\tilde{C}} = \theta_R$, only from the connected correlation (while the decay of C is too slow). Similarly, the associated FDR, found to be a scaling function, reaches at large time separation ($t/t' \to \infty$) a nontrivial (and to one loop, q-independent) limit X_∞ which is nonzero only for the connected correlation. This sheds light on relevant definition of the FDR in a glass phase.

Let us start by a short discussion of the statics. The two-point correlation function exhibits anomalous growth

$$\overline{\langle[\phi(x) - \phi(0)]^2\rangle} = A(\tau)\ln^2(x) + \mathcal{O}(\ln x). \quad (2)$$

The RG to one loop [26] yields the universal [27] $A(\tau) = 2\tau^2 + O(\tau^3)$. Numerical simulations near T_c [28] and at $T = 0$ agree qualitatively and yield $A(\tau = 1) = a_2 = 0.57$ [29] and $A(\tau = 1) = 2(2\pi)^2 B = 0.51$ [30]. A recent work [31] claims an exact solution for the two-point correlation using a conjectured mapping onto disordered free fermion model. The result, translated in the present variables, would yield $A(\tau) = 2\tau^2(1 - \tau^2)$. This, however, is clearly incompatible with numerics, showing that there is more (nonperturbative ?) physics to be understood even in the statics of this model. A more direct signature of the glassy nature of the phase is the sample to sample susceptibility fluctuations [22] which are found (within one loop RG) to be universal and $\sim\mathcal{O}(\tau)$ along the fixed line. The free energy exponent in this glass phase is $\theta = 0$, indicating a free energy landscape with logarithmic roughness. This is consistent with the findings in the equilibrium dynamics: an anomalous diffusion exponent [32], continuously varying along the fixed line has been computed in RG which indicates a logarithmic growth of energy barriers with scale. These properties are characteristic of a marginal glass (by contrast with the case $\theta > 0$ described by a $T = 0$ fixed point). It is reminiscent of a related and simpler case of a vortex in a random gauge XY model where a freezing transition (at $z_c = 4$) was found along a line of fixed points [33].

The relaxational dynamics of the CO model (1) is described by a Langevin equation

$$\eta \frac{\partial}{\partial t}\phi(x,t) = -\frac{\delta H[\phi(x,t)]}{\delta\phi(x,t)} + \zeta(x,t), \quad (3)$$

where the thermal noise $\zeta(x,t)$ is such that $\langle\zeta(x,t)\rangle = 0$, $\langle\zeta(x,t)\zeta(x',t')\rangle = 2\eta T\delta^{(2)}(x - x')\delta(t - t')$, where $T < T_g$ is the temperature and η is the friction ($\eta = 1$ in the following). The system at initial time t_i ($= 0$) is prepared in an equilibrium state of (1) *without* disorder at temperature $T' = \epsilon T$, $[\phi_{q,t=0}\phi_{-q,t=0}]_i = T'q^{-2}$, $\phi_{q,t}$ being the Fourier transform, with respect to (wrt) space coordinates, of the field $\phi(x,t)$. Since the disorder is irrelevant above T_g, this choice of initial condition for $T' > T_g$ describes a quench from a high temperature phase to the glass phase ($T < T_g$). A quench to high temperature ($T > T_g$) is studied in [34].

We focus on the correlation $C^q_{tt'}$ and the connected (wrt the thermal fluctuations) correlation $\tilde{C}^q_{tt'}$:

$$C^q_{tt'} = [\overline{\langle\phi_{qt}\phi_{-qt'}\rangle}]_i,$$
$$\tilde{C}^q_{tt'} = [\overline{\langle\phi_{qt}\phi_{-qt'}\rangle}]_i - [\overline{\langle\phi_{qt}\rangle\langle\phi_{-qt'}\rangle}]_i \quad (4)$$

and the response $\mathcal{R}^q_{tt'}$ to a small external field $f_{-qt'}$,

$$\mathcal{R}^q_{tt'} = \overline{\left[\frac{\delta\langle\phi_{qt}\rangle}{\delta f_{-qt'}}\right]_i}, \qquad t > t', \quad (5)$$

where $\overline{\cdots}$, $\langle\cdots\rangle$, and $[\cdots]_i$ denote, respectively, averages wrt disorder, thermal fluctuations, and initial conditions. We focus on the FDR $\mathcal{X}^q_{tt'}$ associated to the observable ϕ:

$$\frac{1}{\mathcal{X}^q_{tt'}} = \frac{\partial_{t'}\tilde{C}^q_{tt'}}{T\mathcal{R}^q_{tt'}} \quad (6)$$

defined [2] such that $\mathcal{X}^q_{tt'} = 1$ at equilibrium, i.e., when response and correlations depend only on $t - t'$.

The dynamics (3) is then studied using the standard Martin-Siggia-Rose formalism, using the Ito prescription. The correlations (4) and response (5) are then obtained from a dynamical (disorder averaged) generating functional or, equivalently, as functional derivatives of the corresponding dynamical *effective* action Γ. This functional can be perturbatively computed [35] using the exact RG equation associated to the multilocal operators expansion introduced in [36]. It allows one to handle arbitrary cutoff functions $c(q^2/2\Lambda_0^2)$ and check universality, independence wrt $c(x)$, and the ultraviolet scale Λ_0. It describes the evolution of Γ when an additional infrared cutoff Λ_l is lowered from Λ_0 to its final value $\Lambda_l \to 0$, where a fixed point of order $\mathcal{O}(\tau)$ is reached. In this limit, one obtains $\mathcal{R}^q_{tt'}$ and $\tilde{C}^q_{tt'}$ (for $t > t'$) from

$$\partial_t\mathcal{R}^q_{tt'} + [q^2 + \mu(t)]\mathcal{R}^q_{tt'} + \int_{t_i}^t dt_1\Sigma_{tt_1}\mathcal{R}^q_{t_1t'} = 0, \quad (7)$$

$$\tilde{C}^q_{tt'} = 2T\int_{t_i}^{t'} dt_1\mathcal{R}^q_{tt_1}\mathcal{R}^q_{t't_1} + \int_{t_i}^t dt_1\int_{t_i}^{t'} dt_2\mathcal{R}^q_{tt_1}D^c_{t_1t_2}\mathcal{R}^q_{t't_2}, \quad (8)$$

with $\mu(t) = -\int_{t_i}^t dt_1\Sigma_{tt_1}$ and where the self-energy $\Sigma_{t_1t_2}$ and the noise-disorder kernel $D^c_{t_1t_2}$ are directly obtained from Γ at the fixed point [37]. One finds [35,38]

$$\Sigma_{tt'} = -2^{\epsilon-1}e^{\gamma_E}\tau R^{x=0}_{tt'}e^{-1/2(B^{(0)}_{tt'}+B^{(d)}_{tt'})}, \quad (9)$$

$$D^c_{tt'} = T_g 2^{\epsilon-1}e^{\gamma_E}\tau e^{-1/2(B^{(0)}_{tt'}+B^{(d)}_{tt'})}(1 - e^{-C^{(d)}_{tt'}}), \quad (10)$$

where γ_E is the Euler constant, $B^{(d)}_{tt'} = C^{(d)}_{tt} + C^{(d)}_{t't'} - 2C^{(d)}_{tt'}$, $C^{(d)}_{tt'} = \gamma(t + t') - \gamma(|t - t'|)$ is the bare Dirichlet propagator at coinciding points, $B^{(0)}_{tt'} = \epsilon[2\gamma(t + t') - \gamma(t) - \gamma(t')]$, arising from the average over the initial condition. We have defined $\gamma(t) = \frac{T}{4\pi}\int_a \hat{c}(a)\ln(\Lambda_0^2 t + \frac{a}{2})$, using the parametrization $c(x) = \int_a \hat{c}(a)e^{-ax}$ for the cutoff function, and denote $R^{x=0}_{tt'} = \theta(t - t')\int_a \hat{c}(a)[\Lambda_0^2(t - t') + \frac{a}{2}]^{-1}$ the bare response at coinciding points. Up to a boundary term, the correlation $C^q_{tt'}$ satisfies (8) setting the $e^{-C^{(d)}_{tt'}}$ term to zero in the above expression for $D^c_{tt'}$.

Studying Eq. (7) (setting $t_i = 0$), in the scaling regime $q/\Lambda_0 \ll 1$, $\Lambda_0^2 t$, $\Lambda_0^2 t' \gg 1$ and keeping fixed

$$v \sim q^z(t - t'), \qquad u = t/t', \tag{11}$$

one finds a solution consistent with the scaling form:

$$\mathcal{R}^{\,q}_{tt'} = q^{z-2}(t/t')^{\theta_R} F_R(q^z(t - t'), t/t') \tag{12}$$

similar to the form obtained for the response for critical systems [11,39,40]. Here the two exponents z and θ_R are identified from the logarithmic singularities of the scaling function, respectively, at $u \to 1$ and $u \to \infty$. We find that z identifies with the equilibrium dynamical exponent $z - 2 = 2e^{\gamma_E}\tau + O(\tau^2)$ and is thus, as expected, independent of initial conditions under study. The exponent θ_R in (12), characteristic of long time nonequilibrium relaxation is also independent of ϵ:

$$\theta_R = e^{\gamma_E}\tau + \mathcal{O}(\tau^2). \tag{13}$$

One also finds, with this choice of θ_R, that $\lim_{u\to\infty} F_R(v, u) = F_{R\infty}(v)$. The full scaling function $F_R(v, u)$, however, depends on the initial conditions and is universal (up to a single overall nonuniversal length scale $q \to \lambda q$). Its explicit expression, given in [38], contains both nonequilibrium and equilibrium ($u \to 1$) regimes. We have checked to this order that it naturally splits into $F_R(v, u) = F_R^{\rm eq}(v) + F_R^{\rm noneq}(v, u)$. Here we give the nonequilibrium scaling function only in the large time separation limit $u \to \infty$:

$$\begin{aligned} F_{R\infty}^{\rm noneq}(v) &= e^{-v}\left[\int_0^v dt_2 \int_0^{t_2} dt_1 (e^{t_2 - t_1} - 1)\tilde{\Sigma}_{t_1 t_2} + \sigma\right], \\ \tilde{\Sigma}_{t_1 t_2} &= \tau e^{\gamma_E} \frac{1}{(t_2 - t_1)^2}\left[\left(\frac{t_2 + t_1}{2\sqrt{t_2 t_1}}\right)^{1-\epsilon} - 1\right], \end{aligned} \tag{14}$$

where the constant $\sigma = \int_1^\infty dt_2 \int_0^1 dt_1 \tilde{\Sigma}_{t_1 t_2}$ is a monotonic decreasing function of ϵ. The large v behavior is a power law. The form (14) is convergent due to the subtraction of the pole at $t_2 = t_1$. The logarithmic divergence associated to this pole yields a nontrivial z exponent. The subtracted piece precisely gives the equilibrium scaling function for the response $F_R^{\rm eq}(v)$, up to $\mathcal{O}(\tau^2)$:

$$F_R^{\rm eq}(v) = e^{-v} + \tau e^{\gamma_E}[(v - 1){\rm Ei}(v)e^{-v} + e^{-v} - 1], \tag{15}$$

where ${\rm Ei}(v)$ is the exponential integral. The same result is also obtained if t_i is taken to be $t_i = -\infty$ from the outset (e.g., at large but fixed system size), showing that the nonequilibrium regime of the scaling function merges smoothly with the equilibrium one [41].

We now turn to correlation functions. To obtain the equilibrium correlation one can simply use fluctuation-dissipation theorem which holds in this regime (i.e. $\mathcal{X}^q_{tt'} = 1$) and $\tilde{C}^{q{\rm eq}}_{tt'} = Tq^{-2}F_C^{\rm eq}(v)$, with $\partial_v F_C^{\rm eq}(v) = -F_R^{\rm eq}(v)$. One finds that at equilibrium and to this order in τ, $\tilde{C}$ and C coincide.

The nonequilibrium connected correlation is already nontrivial in the absence of disorder [42]. It also takes a scaling form $\tilde{C}^q_{tt'} = Tq^{-2}F^0_{\tilde{C}}[q^2(t - t'), t/t']$ with $F^0_{\tilde{C}}(v, u) = e^{-|v|} - e^{-v(u+1)/(u-1)}$, $z = 2$. The FDR $X^q_{tt'} = (1 + e^{-2v/(u-1)})^{-1}$ interpolates between 1 and 1/2 as $u = t/t'$ increases from 1 to ∞ (for $q \neq 0$), while $X^{q=0}_{tt'} = \frac{1}{2}$ is the "random walk" value. In presence of disorder one can solve (8) perturbatively in τ using the above solution for the response. One obtains that $\tilde{C}^q_{tt'}$, in the scaling regime, is consistent with the scaling form [43]

$$\tilde{C}^{\,q}_{tt'} = Tq^{-2}(t/t')^{\theta_{\tilde{C}}} F_{\tilde{C}}(q^z(t - t'), t/t'). \tag{16}$$

The calculation [38] shows that $\theta_{\tilde{C}} = \theta_R$, yielding an autocorrelation exponent $\lambda_{\tilde{C}} = -z(\theta_{\tilde{C}} - 1)$:

$$\lambda_{\tilde{C}} = 2 + \mathcal{O}(\tau^2). \tag{17}$$

This value of $\theta_{\tilde{C}}$ is also such that $\lim_{u\to\infty} uF_{\tilde{C}}(v, u) = F_{\tilde{C}\infty}(v)$. We have obtained the complete expression of $F_{\tilde{C}}(v, u)$ [38] from which we can extract the large u behavior. We then discover the relation, valid for *any* ϵ,

$$F_{\tilde{C}\infty}(v) = (2 + 4e^{\gamma_E}\tau)vF_{R\infty}(v) + \mathcal{O}(\tau^2). \tag{18}$$

Having determined both response and correlation we now obtain, in the scaling regime, the FDR (6). It is also characterized by the universal scaling function

$$(\mathcal{X}^q_{tt'})^{-1} = F_X(q^z(t - t'), t/t'), \tag{19}$$

which also depends on ϵ and has a complicated form [38]. In the limit of large time separation, i.e., $q^z(t - t')$ fixed, $t/t' \to \infty$, using (14) and (18) it simplifies into

$$\lim_{u\to\infty} (\mathcal{X}^q_{tt'})^{-1} = 2 + 2e^{\gamma_E}\tau + \mathcal{O}(\tau^2) = (X_\infty)^{-1} \tag{20}$$

independently of v and ϵ, i.e., of a (small) wave vector and the initial condition. There is thus a nontrivial asymptotic FDR in the CO glass phase, which similarly to the case of pure critical points is in the interval [1/2, 0[. Here, however, it continuously depends on temperature T.

As in previous works [6,10,12] one can also examine the "diffusive" mode $q = 0$. It is possible to obtain a simple analytical form for any $u = t/t'$ in the case $\epsilon = 0$ [flat initial conditions $\phi(x, t = 0) = 0$], up to order $\mathcal{O}(\tau^2)$,

$$(\mathcal{X}^{q=0}_{tt'})^{-1} = F_X^{\rm diff}(u) = 2 + 2\tau e^{\gamma_E}[1 - {\rm arccoth}(\sqrt{u})]. \tag{21}$$

Although this quantity depends on ϵ, in general it reaches an ϵ-independent limit for $u \to \infty$. It is also interesting to compute $\mathcal{X}^{x=0}_{tt'}$ in real space. In previous works [6], it was argued that

$$\lim_{t'\to\infty}\lim_{t\to\infty} \mathcal{X}^{x=0}_{tt'} = \lim_{t'\to\infty}\lim_{t\to\infty} \mathcal{X}^{q=0}_{tt'} = X_\infty. \tag{22}$$

The v independence found above (20) puts in the present case these heuristic arguments on firmer ground [6].

Following the discussion in [2], this X_∞ leads to an effective temperature $T_{\rm eff} = T/X_\infty$, which can be measured by a thermometer coupled to the field $\phi(x, t)$. Indeed, Fourier transforming Eqs. (12) and (16), one checks that the *local* $\mathcal{R}_{tt'}^{x=0}$ and $\tilde{C}_{tt'}^{x=0}$ are precisely of the form given in [45].

It turns out that it is crucial to consider the connected correlation to obtain a nontrivial FDR. We have also performed the calculation [35] for the correlation function $C_{tt'}^{q}$ (4). It exhibits a scaling form similar to (16) which decreases more slowly than $\tilde{C}_{tt'}^{q}$ for large u. If we impose $\lim_{u\to\infty} u F_C(v, u) = F_{C\infty}(v)$ one finds, for $\epsilon = 0$, that $\theta_C = \theta_{\tilde{C}} + \frac{1}{2}$, leading to $\lambda_C = 1 - e^{\gamma_E}\tau + \mathcal{O}(\tau^2)$. The FDR is found to be $\mathcal{X}_{tt'}^{q=0} = (2 + \tau e^{\gamma_E}\sqrt{u})^{-1}$ and thus does not seem to approach (uniformly in τ) a nontrivial limit at large u. Thus, although both correlations give the same equilibrium result to order $\mathcal{O}(\tau)$, only the connected one, as defined here, yields a nontrivial asymptotic FDR in the nonequilibrium regime. For $\epsilon \neq 0$, the large u behavior is dominated by the initial condition and $F_C(v, u) \sim F_{C\infty}(v)$ [40,46]. The present considerations are also of interest for pure systems when initial conditions are nonzero, e.g., in a quench from an ordered phase (or correlated initial conditions $\epsilon \neq 0$) [34,40,46]. Indeed, in that case one can distinguish connected and nonconnected correlations (4) which can also lead to different behaviors of the FDR.

We have found a physically relevant glass phase in which one can show analytically the existence of a nontrivial FDR. It is a robust quantity independent of the initial condition under study and, for some observables, it appears to be related to an effective temperature T_{eff}. Its continuous dependence on T reflects the marginal character of this glass phase. Our analytical predictions can be tested in numerical simulations [47]. They are in good agreement near T_g, and it would be interesting to investigate the aging behavior at lower temperature, where hints of some new physics already appear in the statics.

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