Rare events in turbulence and applications

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My background is wind energy and turbulence

https://www.youtube.com/watch?v=dRht4tkQJIM

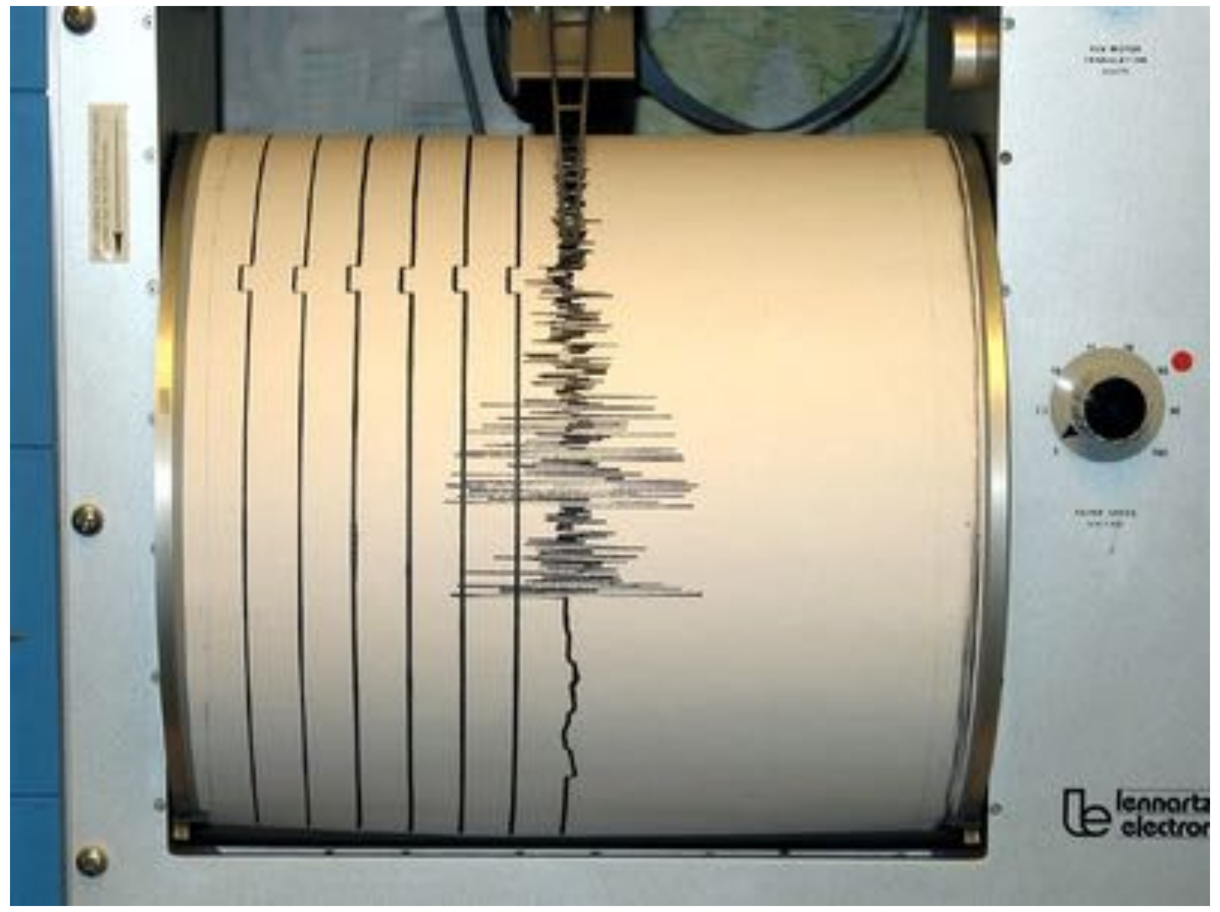
#Tornado #Crowell #Texas

CRAZY TORNADO BENDS WIND TURBINE BLADES!

what is an extreme event?

Snow avalanches

earthquakes



Zerstörte Infrastruktur, Erdbeben 1995, Kobe

06.04.2009 - Erdbeben in Italien: Helfer transportieren einen Verletzten nach dem schweren Erdbeben in der italienischen

epilepsy

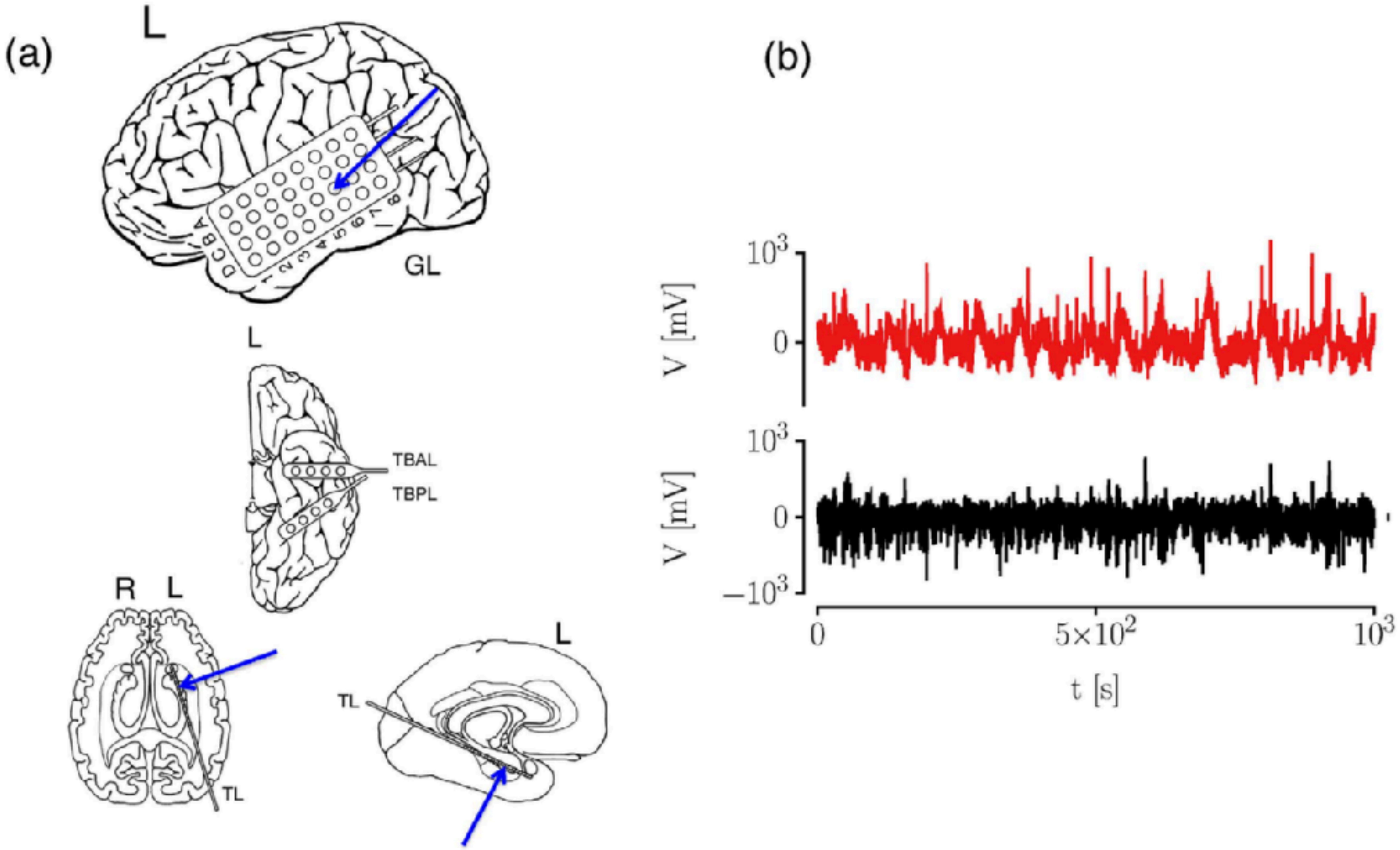


Figure 2. Exemplary recording scheme and intracranial electroencephalographic (iEEG) time series.

market crash

the economist March 1st 2007
stock decline on Febr. 27th

grey tuesday

- extreme event
- „According to Goldman and Sachs, the latest jump in VIx (a measure of stock-market volatility) took it eight standard deviations from is average. If conventional models are correct, such an event should not have happened in the history of the known universe.“

“extreme events are generally easy to recognize but difficult to define.”

Stephenson, D. B. (2008). In H. F. Diaz & R. J. Murnane (Eds.), Climate Extremes and Society. Cambridge,

rare or extreme events categories:

rare or extreme events categories:

1) Extreme events :

- based on statistics
- based on effect - outcome
    - above threshold leads to damage
    - coupled to a system - same event may be extrem or not

Rare big events

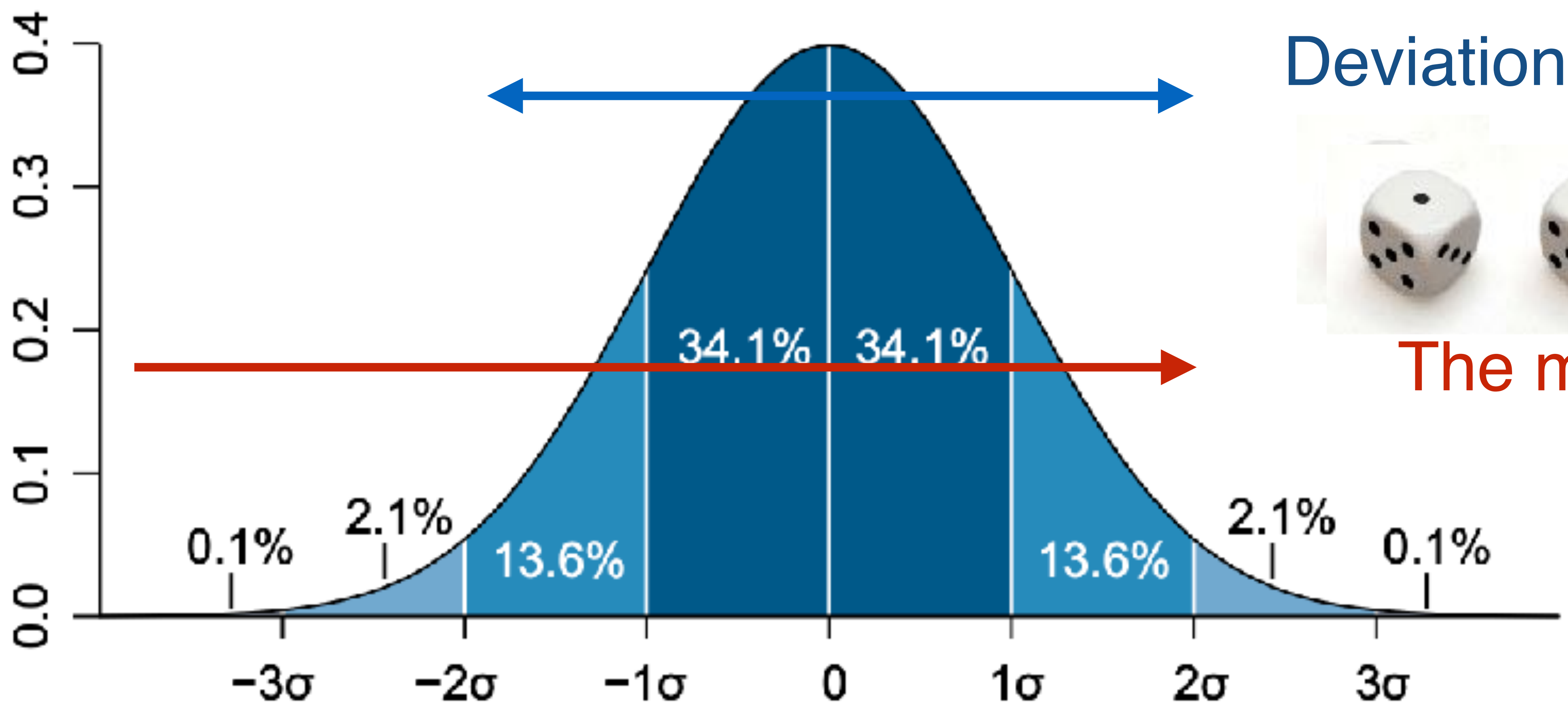
Events x_i, i = 1,…,N

8 σ event

What is the probability of x_i > B,

Gauß-distribution   -> extreme value statistics

$$P(x_i | x_i > B)$$



Deviation from mean value

The magnitude of the events is asked - xx σ

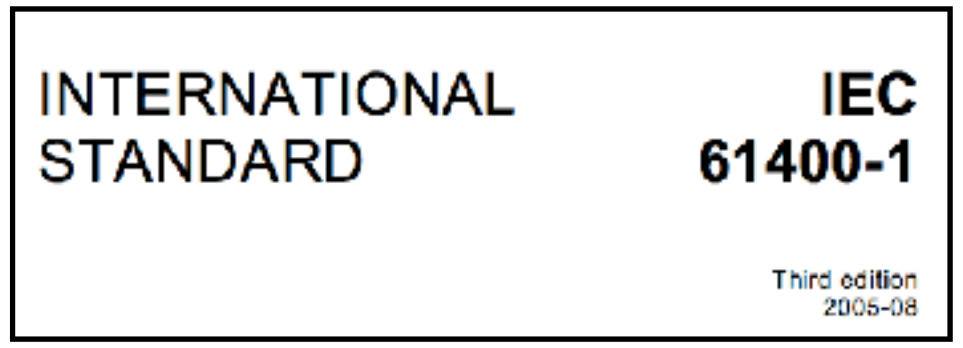
Rare big events

Events x_i, i = 1,…,N

$$P(x_i | x_i > B)$$

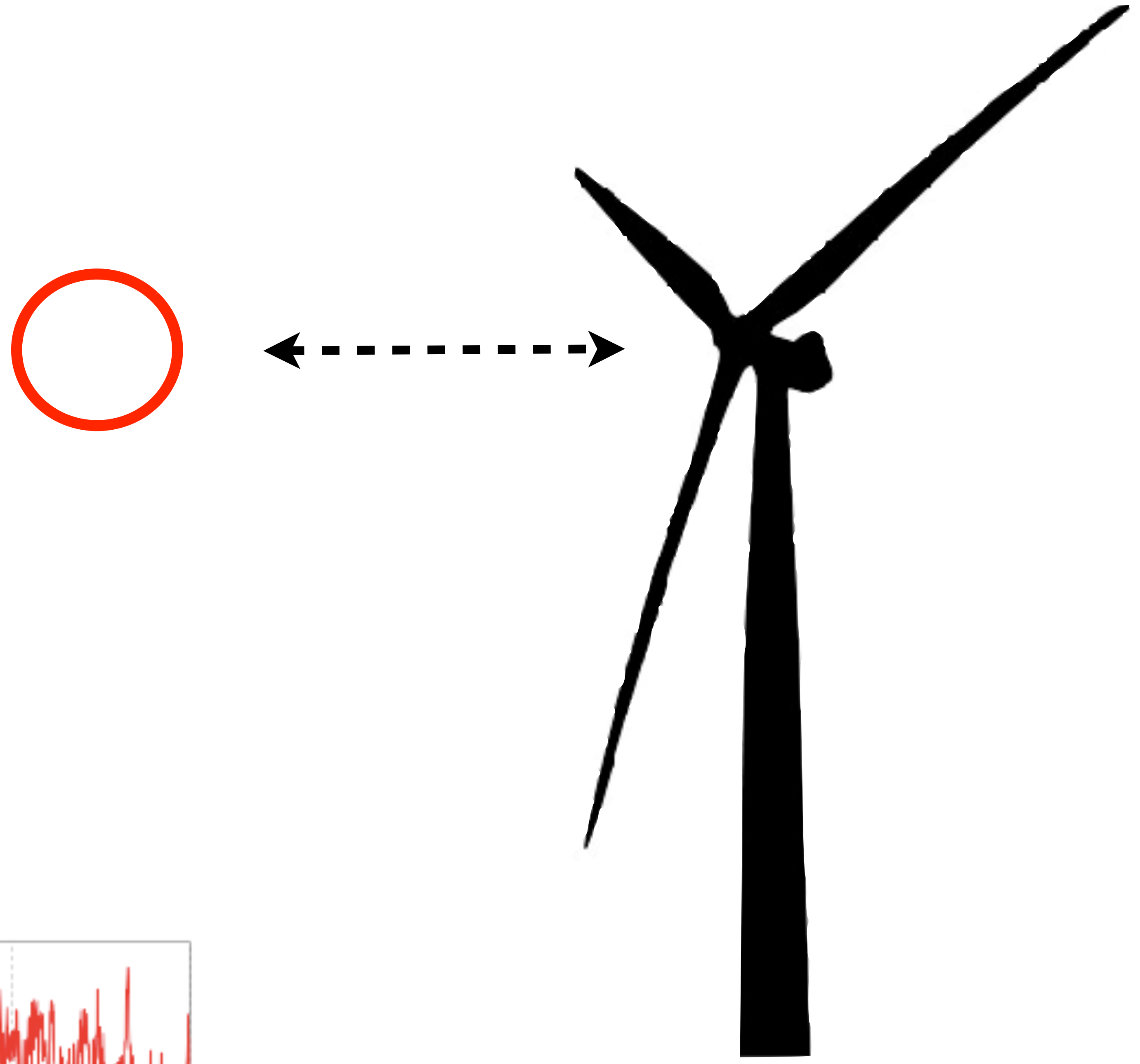
What happen if p is not Gaussian

energy resource: wind gusts

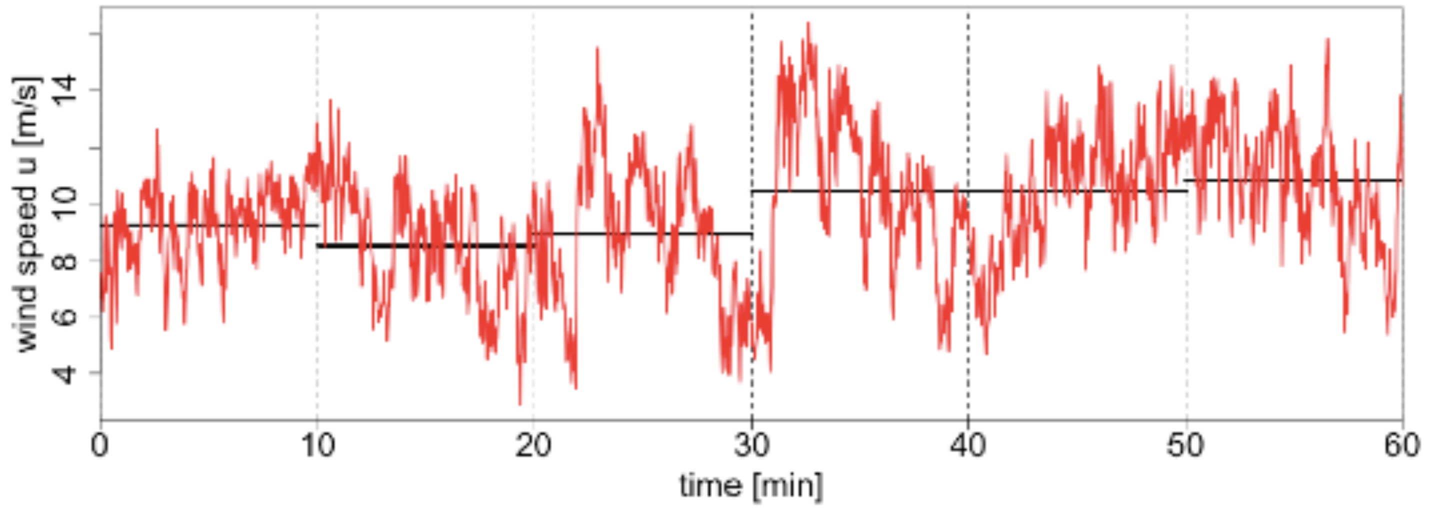

INTERNATIONAL STANDARD

IEC 61400-1

Third edition 2005-08

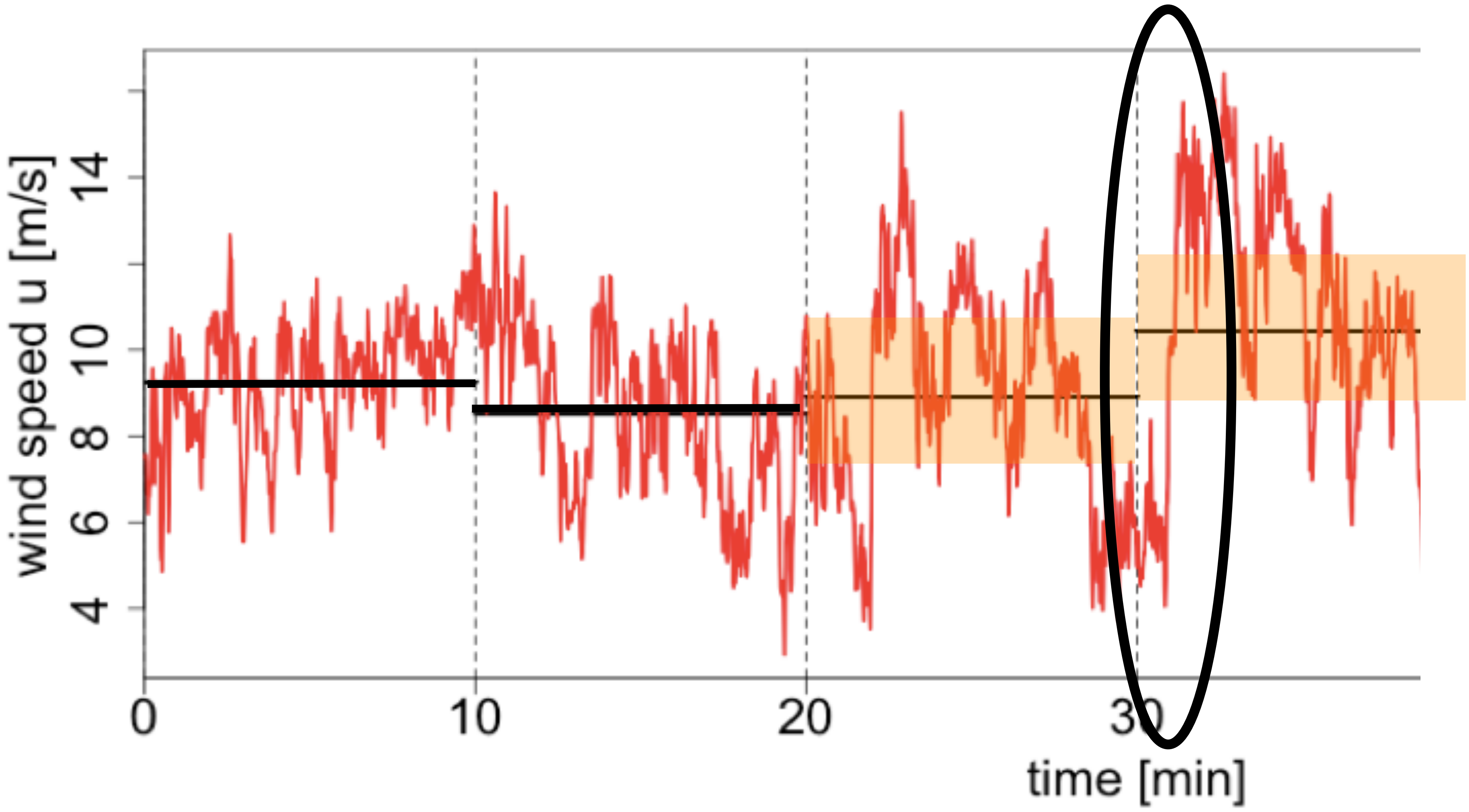


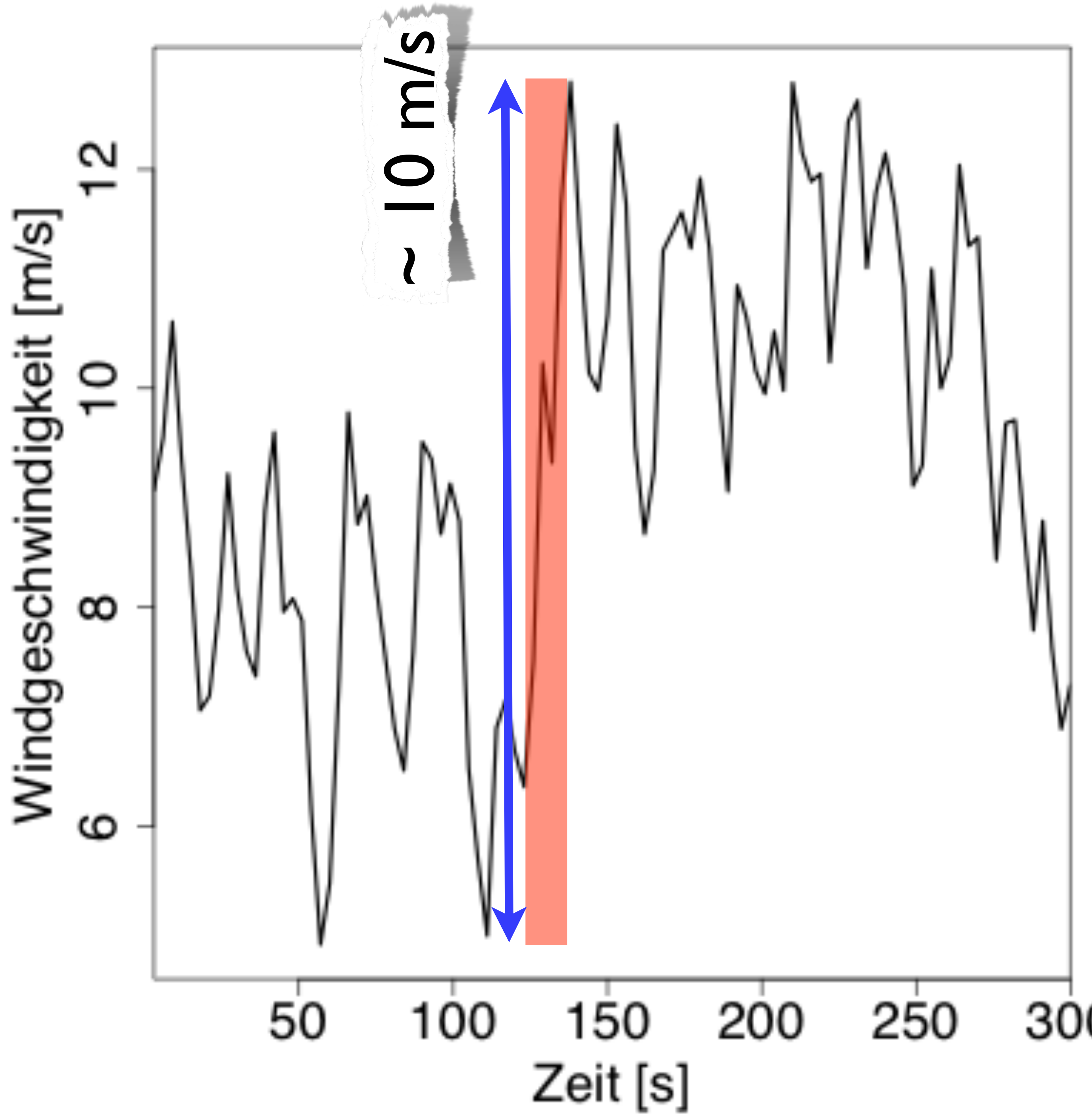
measured time series



wind measurements and data analysis

- characterisation after IEC norm

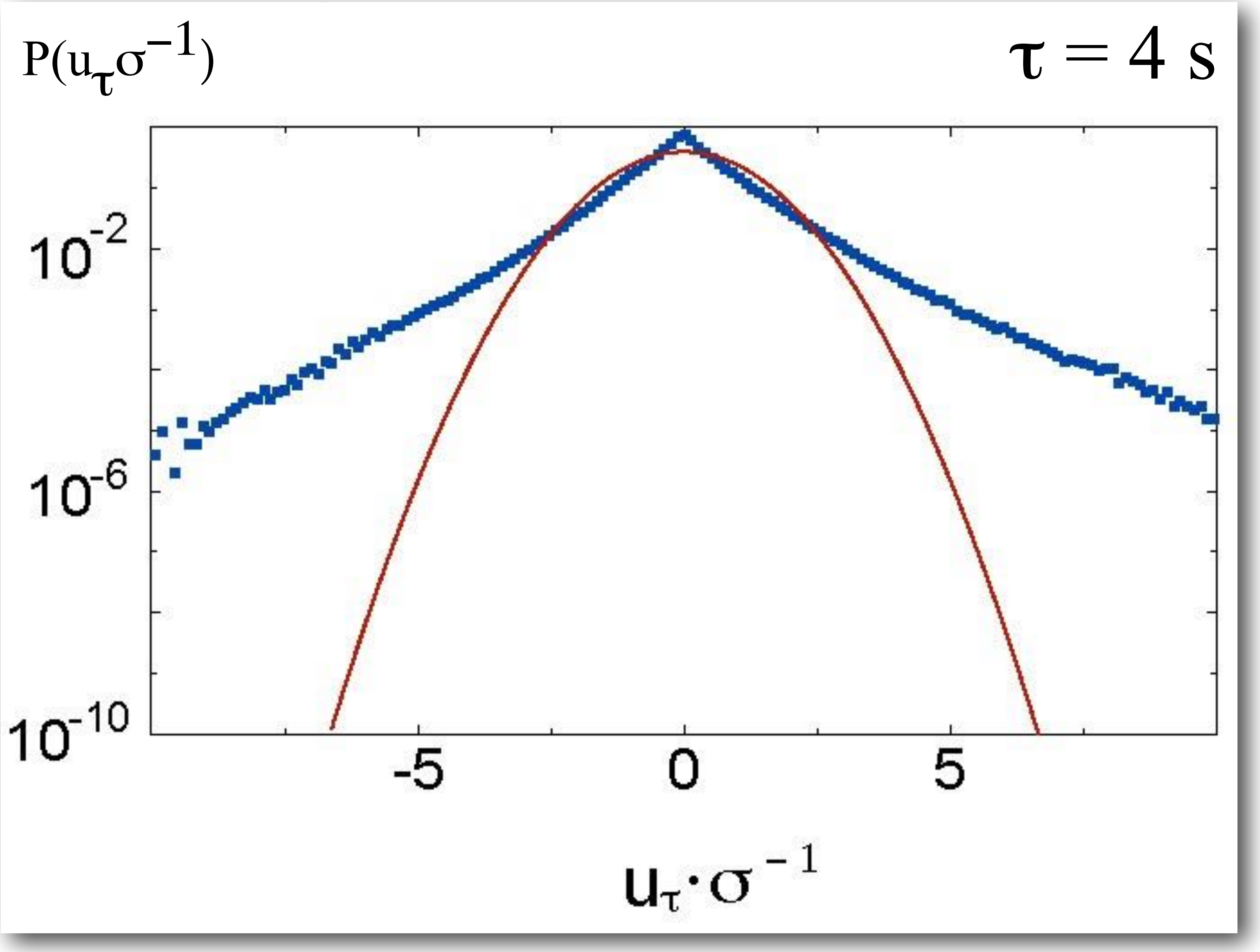


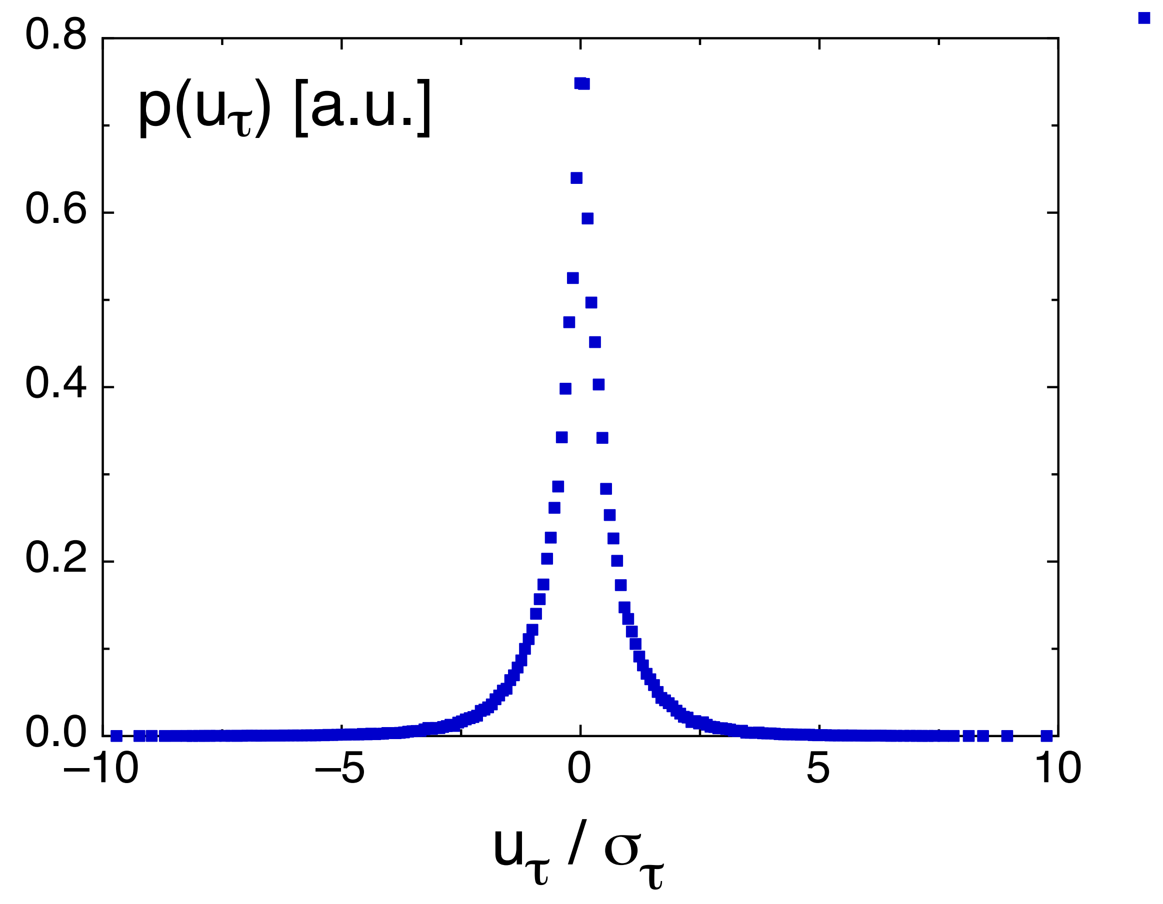


statistics of gusts

- wind fluctuations can be measured by velocity increments

$$u_\tau = u(t+\tau) - u(t)$$

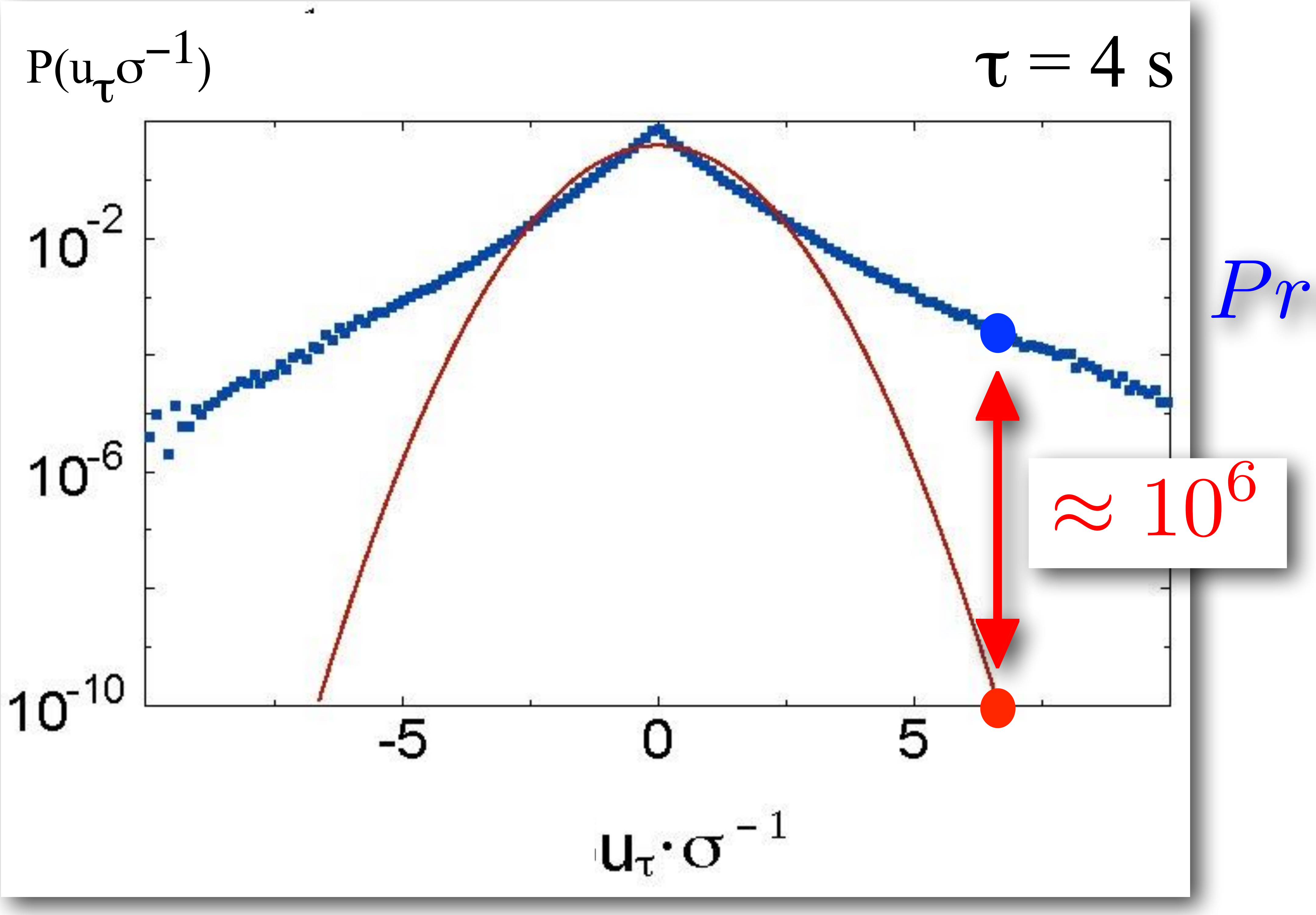




Boundary-Layer Meteorology **108** (2003)

statistics of gusts

non-Gaussian called intermittency



$Prob(u_\tau > 6\sigma) \approx 10^{-4}$ → 1/day

$Prob(u_\tau > 6\sigma) \approx 10^{-10}$ → 1/3000 years

=> Importance of statistics p(x)

Boundary-Layer Meteorology **108** (2003)

finance

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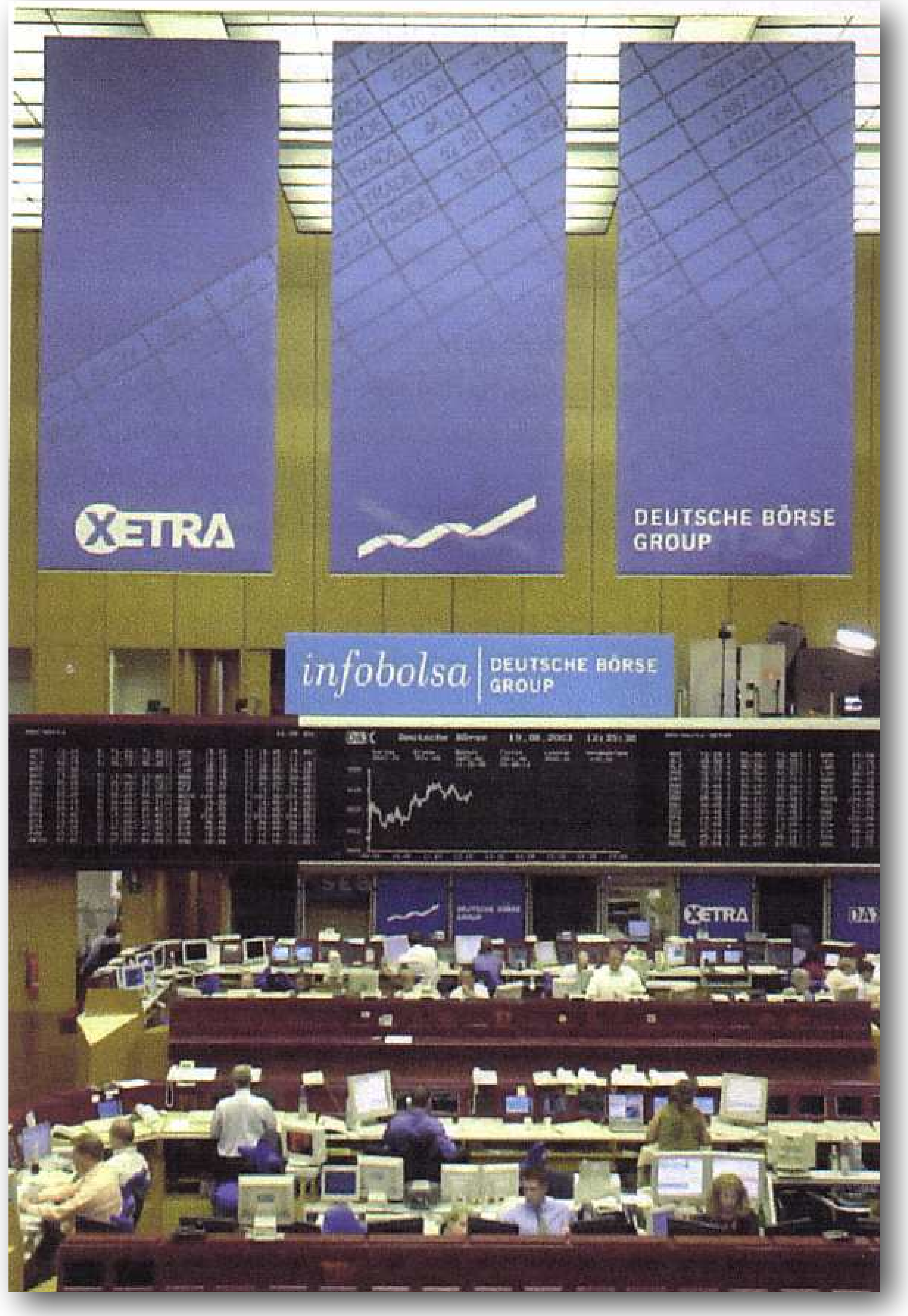
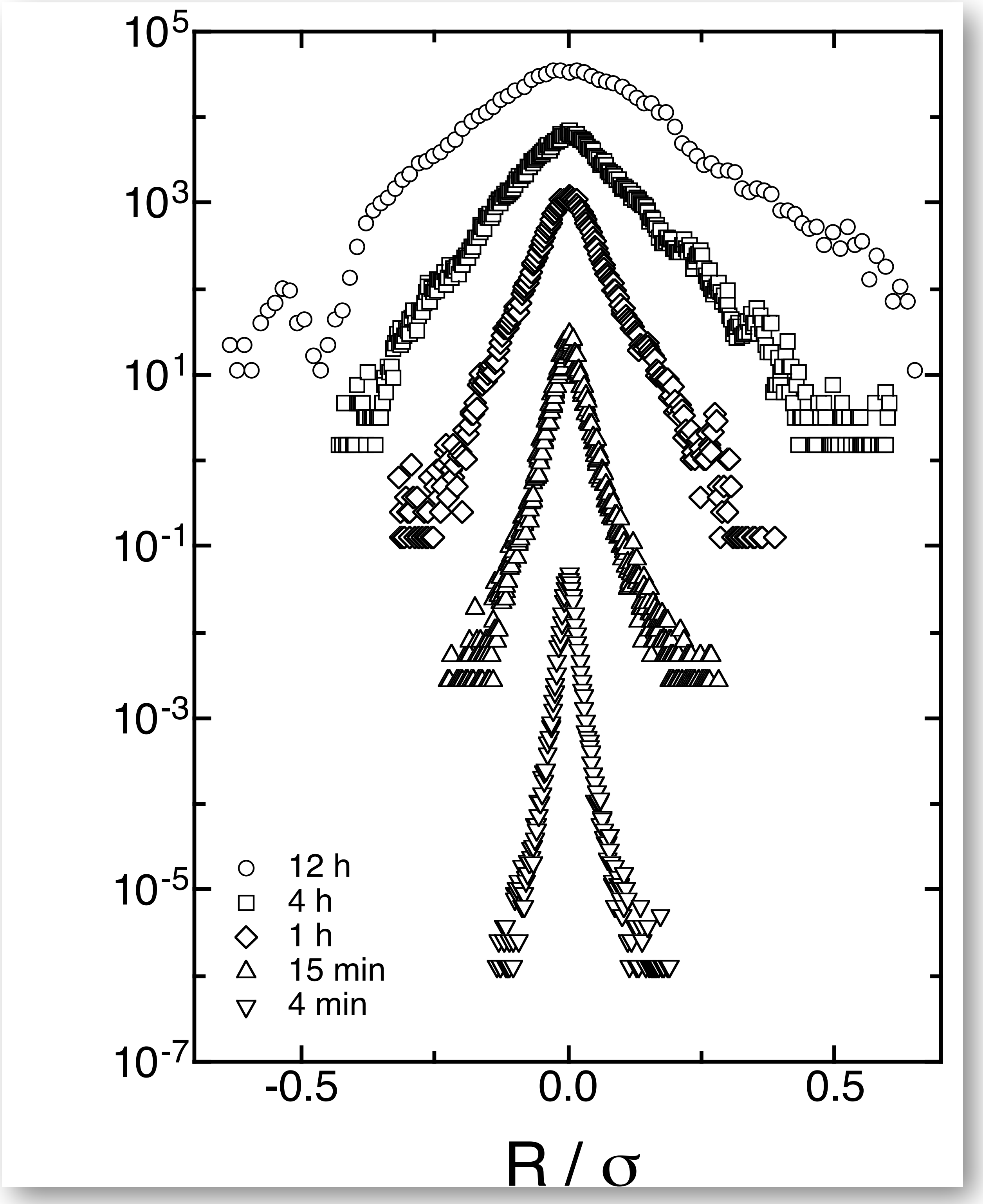
- scale dependent quantity for measuring the disorder

return or log return for different time scales

$$Q(x,r) \Rightarrow r(t,\tau) = \frac{x(t+\tau)}{x(t)} \quad \text{or } R(t,\tau) = \log r(t,\tau)$$





Phys. Rev. Lett. **84**, 5224 (2000)

extreme events

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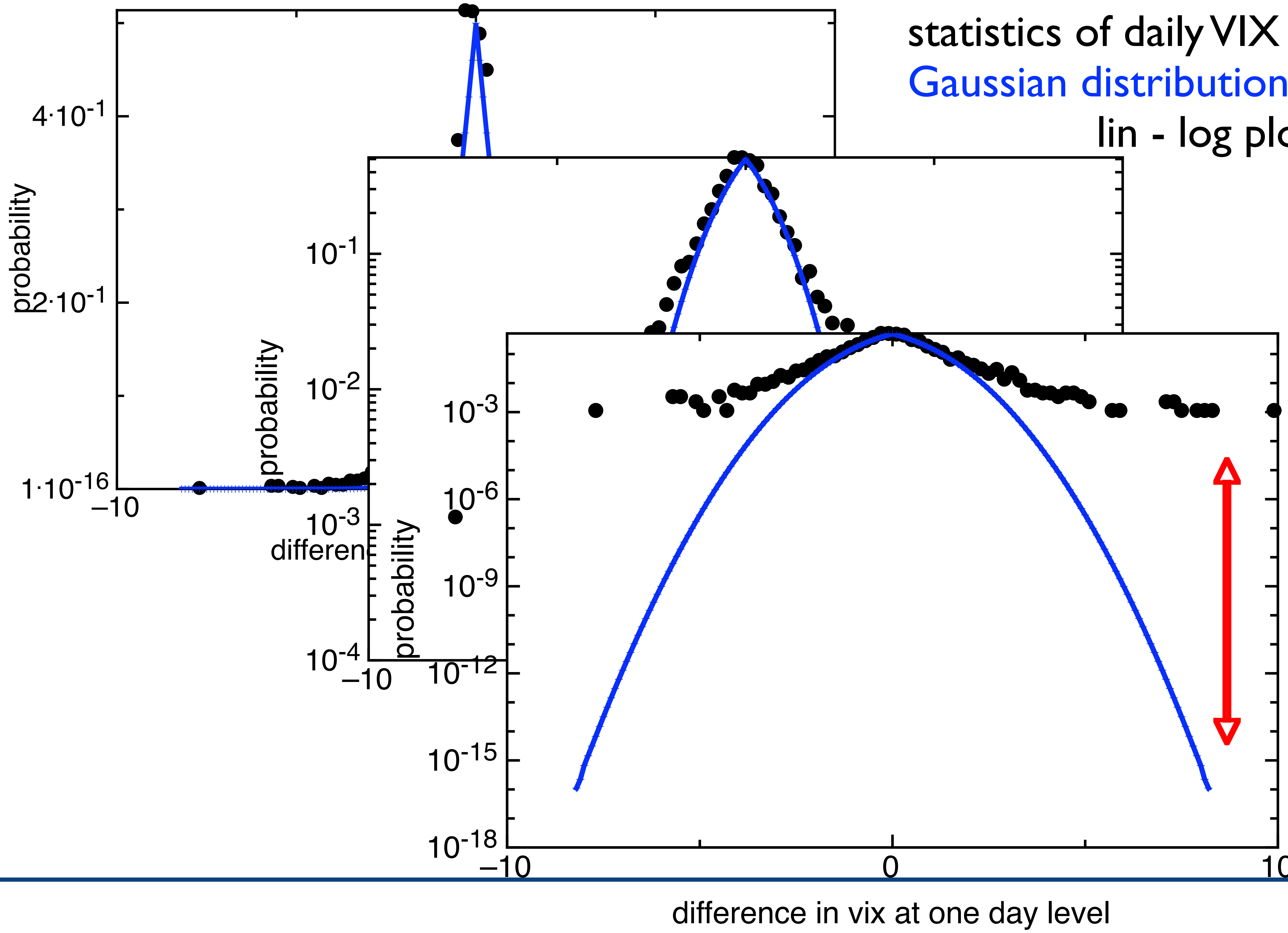


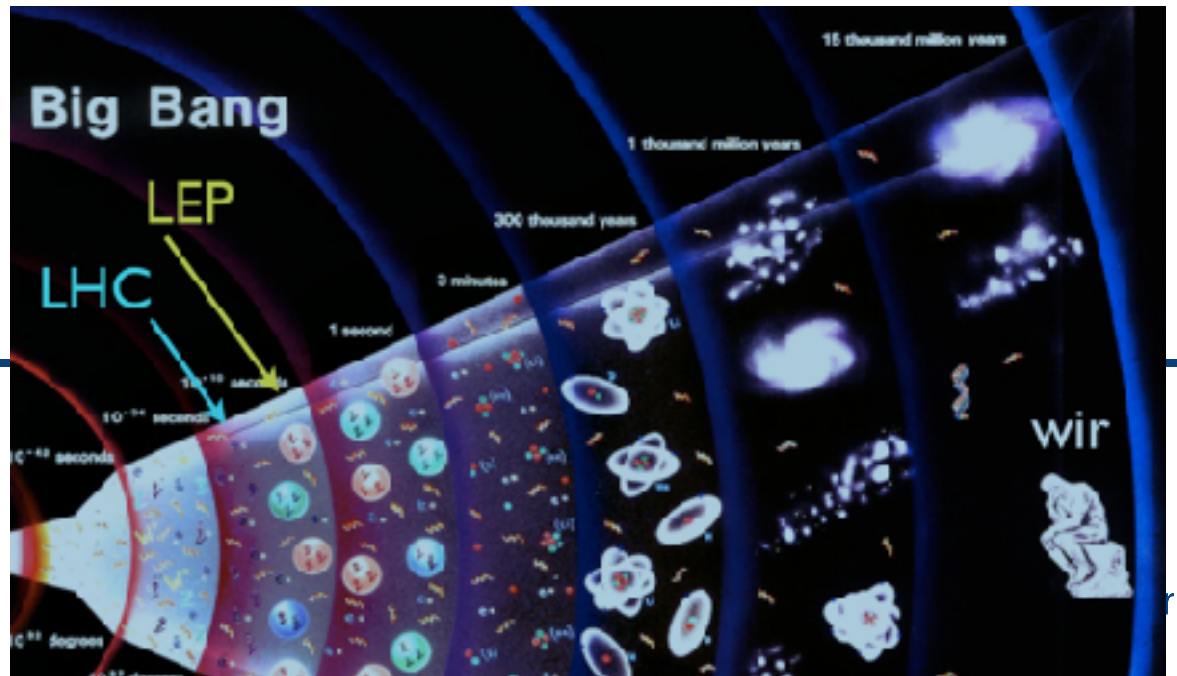
statistics of daily VIX data

Gaussian distribution

lin - log plot

the difference in the probability is > 10^{13}

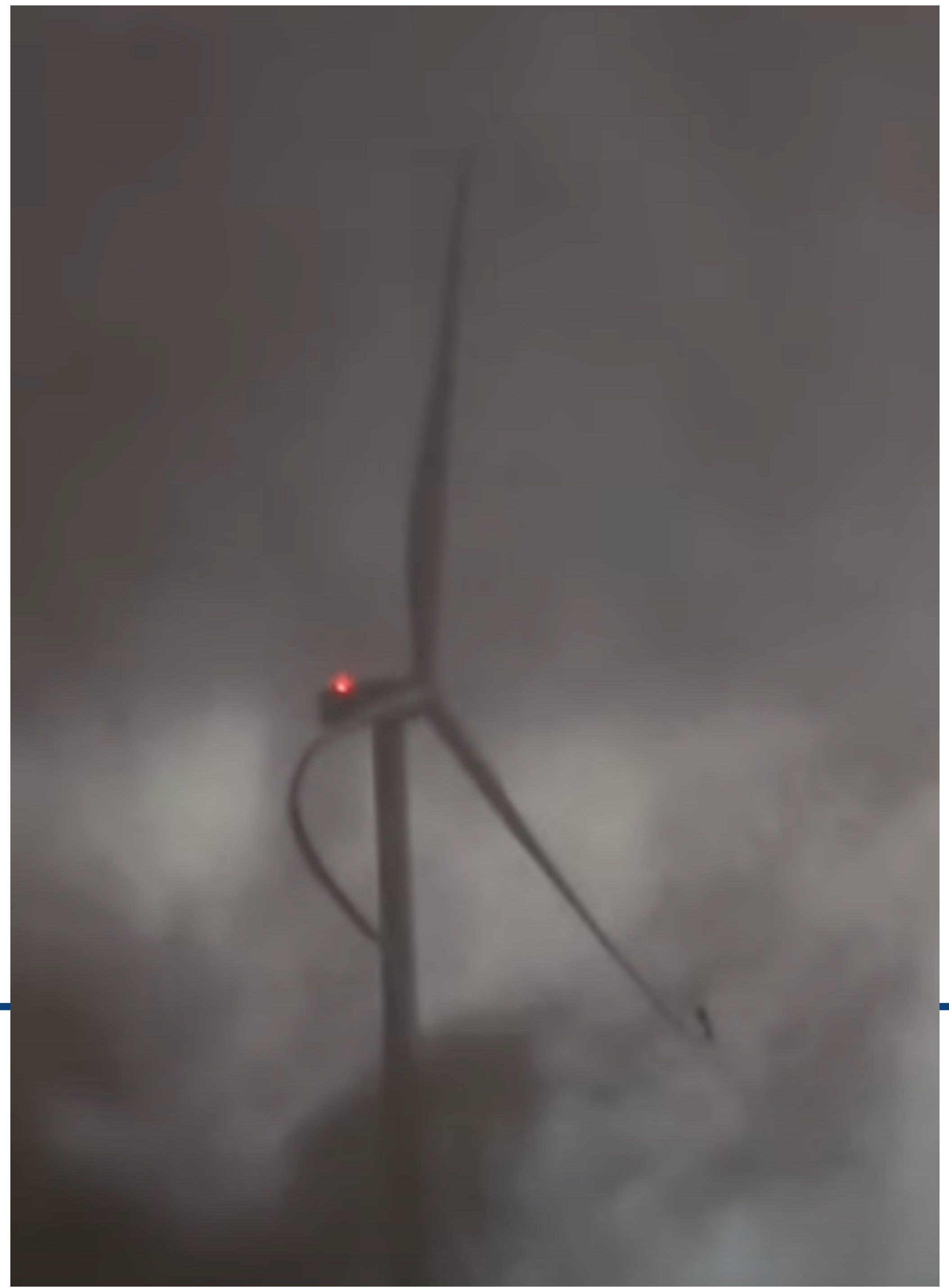
$3*10^{10}$ years are about 10^{13} days



rare or extreme events categories:

1) Extreme events :

- based on statistics
- based on effect - outcome
    - **above threshold** leads to damage
    - coupled to a system - same event may be extrem or not



rare or extreme events categories:

1) Extreme events :

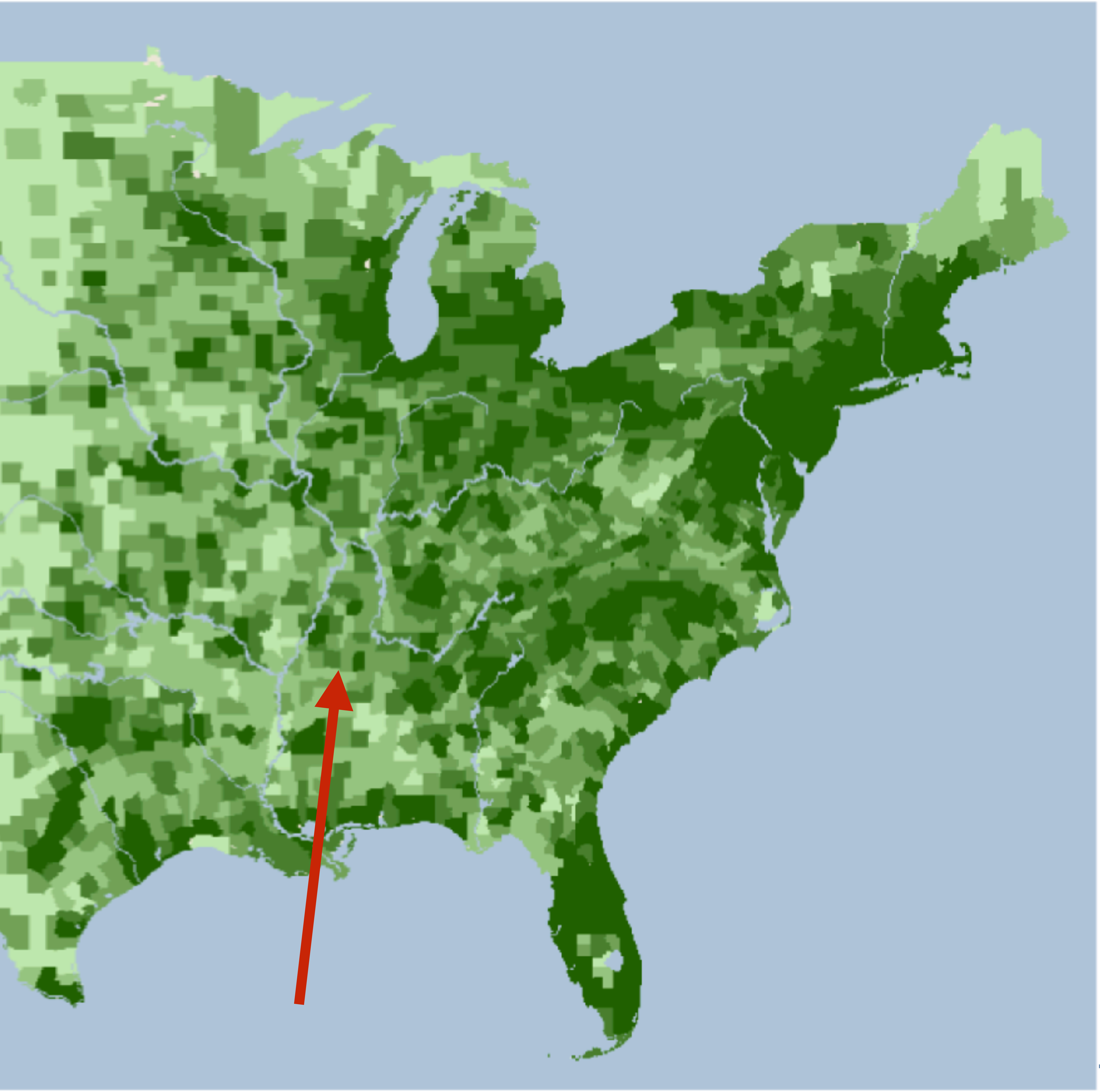
- based on statistics

- based on effect - outcome

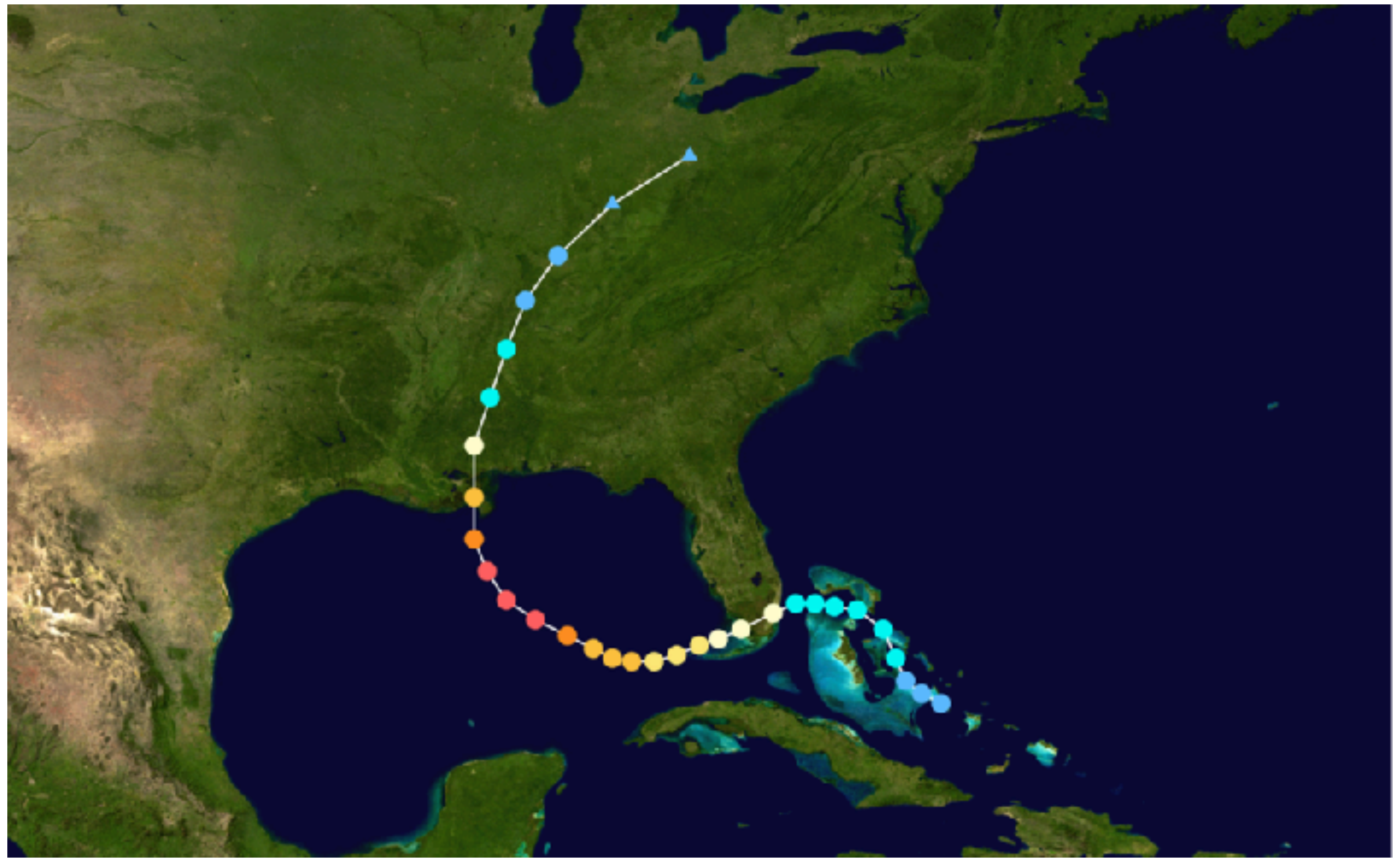
  - **above threshold** leads to damage

  - **coupled to another system** - same event may be extrem or not

Distribution of wealth



The event being extreme depends on the Wealth distribution



rare or extreme events categories:

1) Extreme events :

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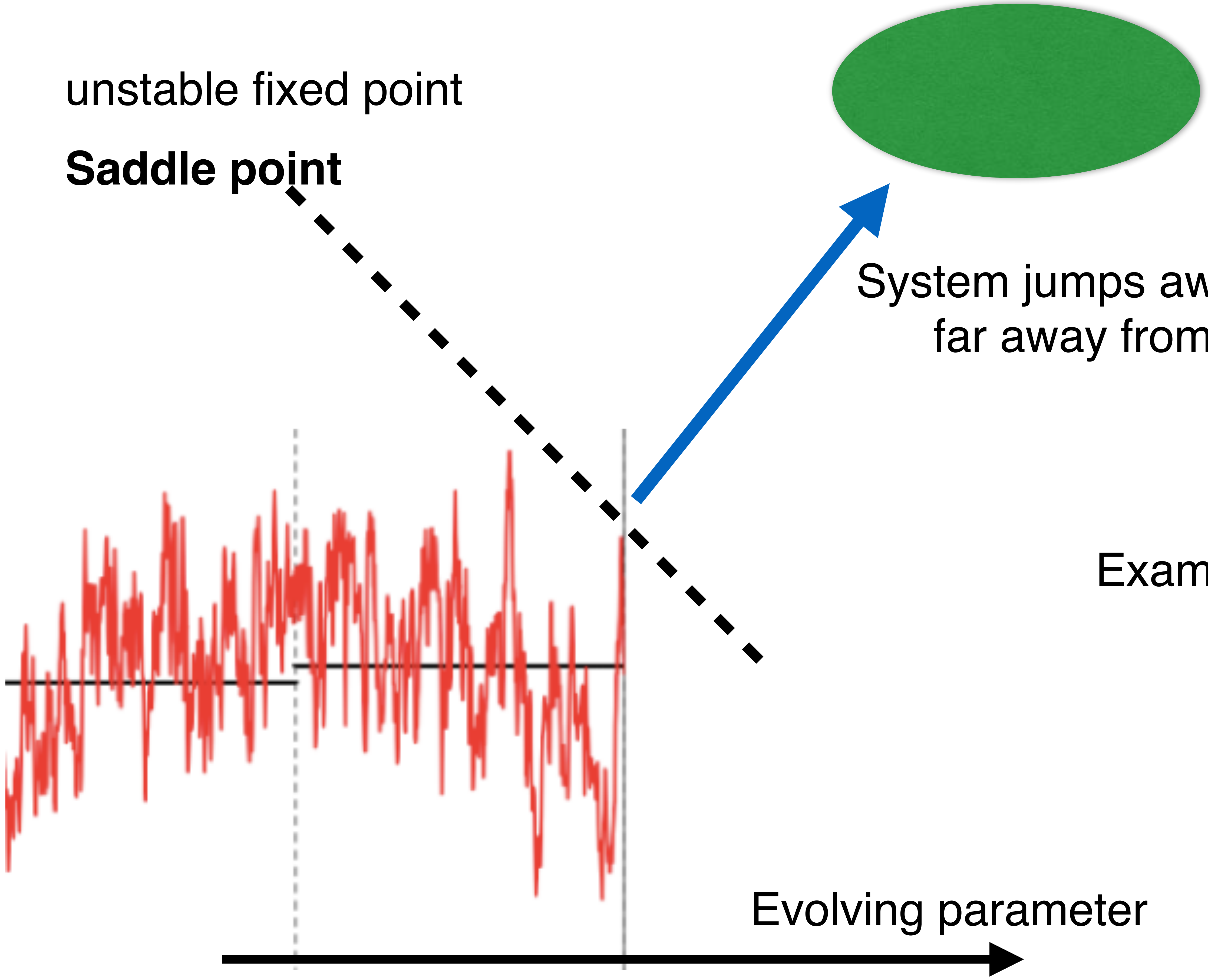
2) event and system

- outside of the system, driven by external parameters
- intrinsic feature of a system

events driven from outside
- Changing parameters

Extrem events :

- **blue sky catastrophe**
  - in phase space a non local effect



Example climate change

rare or extreme events categories:

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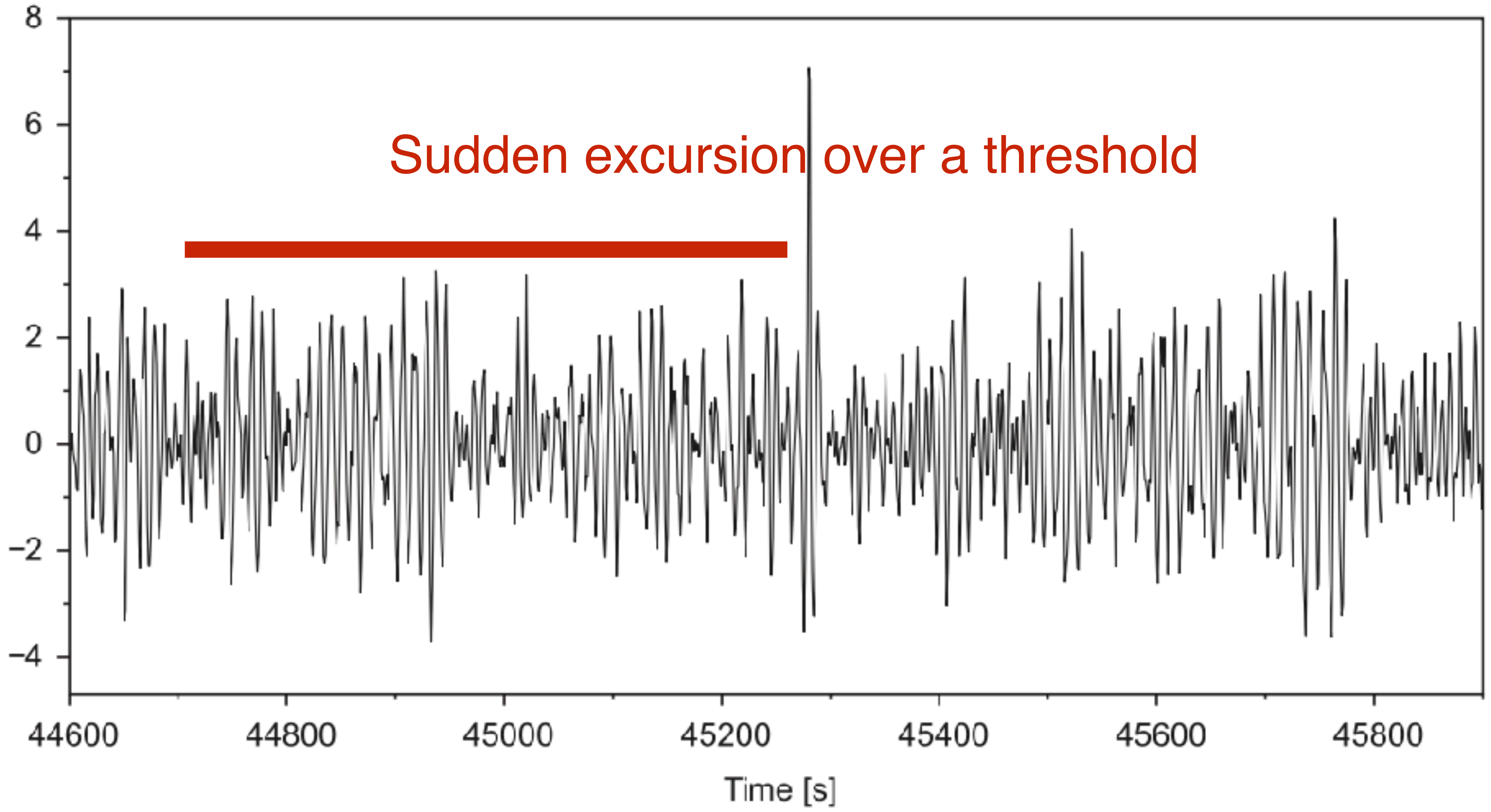
events and the system

Extrem events :

- **blue sky catastrophe** - in phase space a non local effect
- **event within the system**

  Reason may be

  - intrinsic system property



events and the system

Extrem events :

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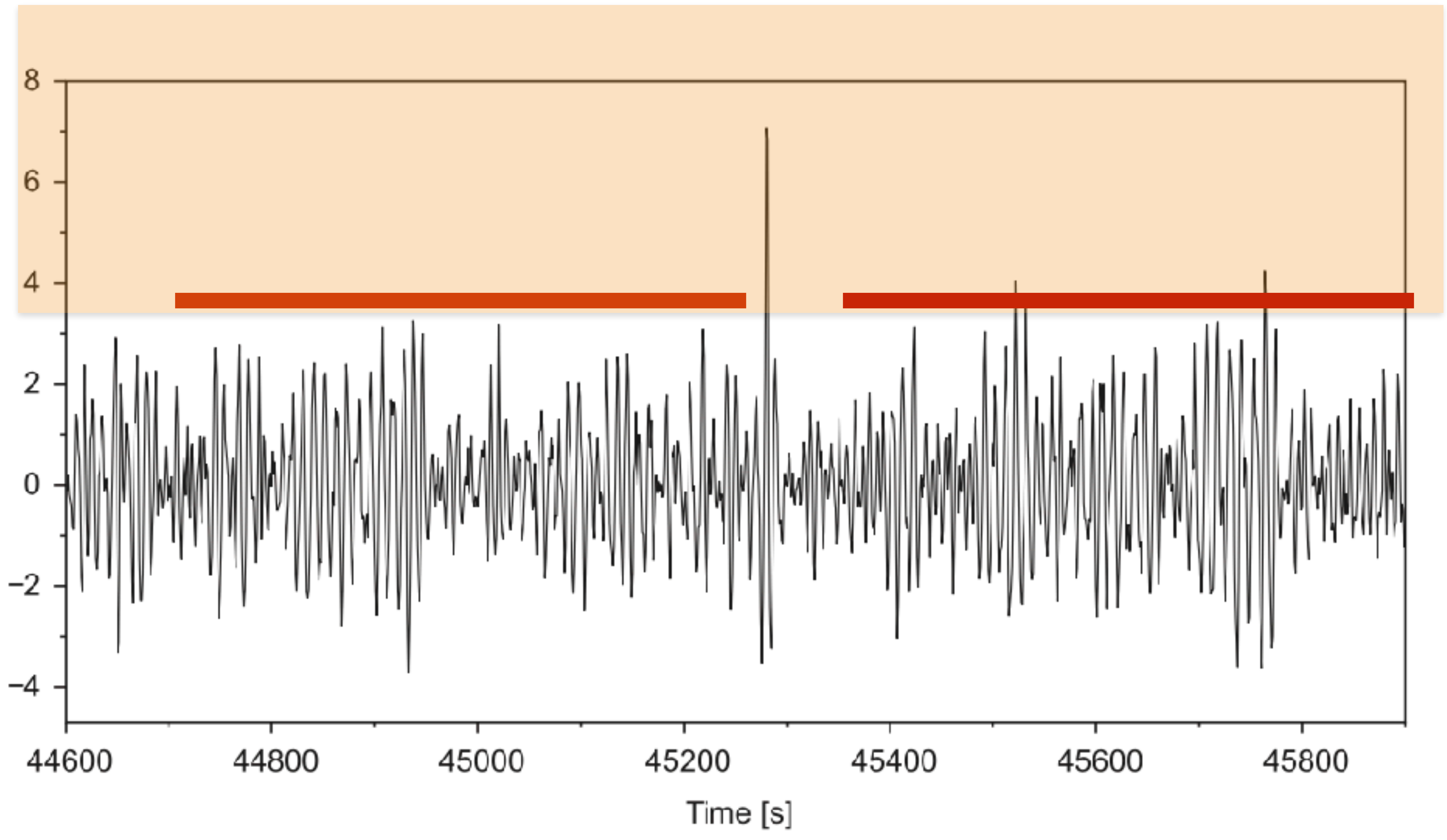
Extreme value statistics - extreme events are rare and independent



events and the system

Extrem events :

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- **event within the system**

Reason may be

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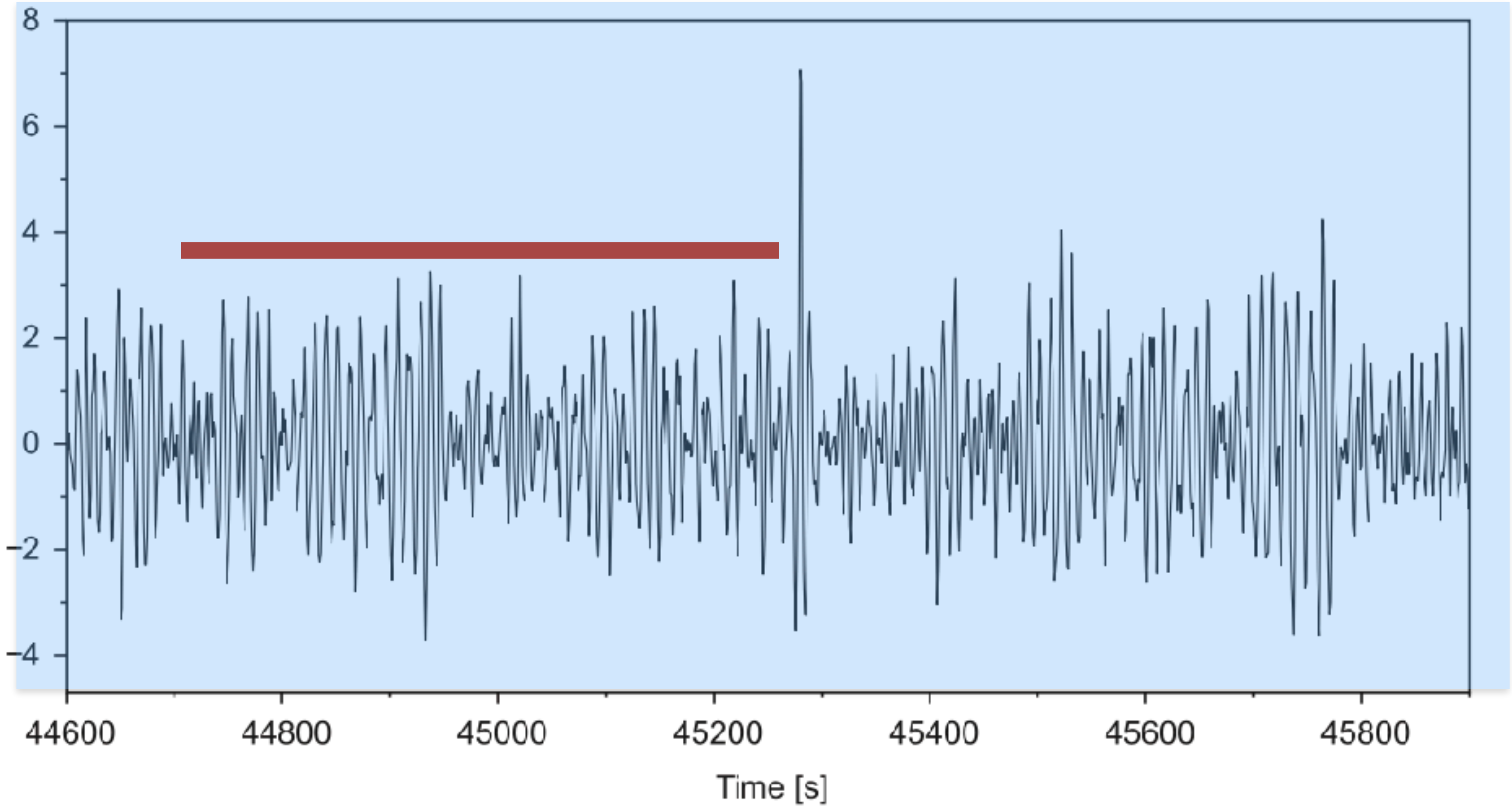
Extreme values are part of the whole system

Of all data



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rare or extreme events categories: — take home message

1) Extreme events :

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Stephenson, D. B. (2008). In H. F. Diaz & R. J. Murnane (Eds.), Climate Extremes and Society. Cambridge,

AGU PUBLICATIONS

Earth’s Future

RESEARCH ARTICLE

10.1002/2017EF000686

Defining Extreme Events: A Cross-Disciplinary Review

Lauren E. McPhillips[1], Heejun Chang[2], Mikhail V. Chester[3], Yaella Depietri[4], Erin Friedman[5], Nancy B. Grimm[6], John S. Kominoski[7], Timon McPhearson[4,8], Pablo Méndez-Lázaro[9], Emma J. Rosi[8], and Javad Shafiei Shiva[10]

rare or extreme events categories:

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- intrinsic feature of a system

3.) what do we want to know?

How likely is an event

Forecasting of events

rare or extreme events categories:

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3.) what do we want to know?

How likely is an event

Forecasting of events

Part B :

Aim of this talk

Rare events :

Statistics and structures

as intrinsic features of a complex system

Turbulence - wind gusts

Water waves - monster waves

Content - Part B

Systems of extreme events

multi-point statistics — main idea

scale-dependent Fokker-Planck equations

consequences

waves and turbulence

Nonlinear Schrödinger equation (NLSE)

$$i\frac{\partial \psi}{\partial x} + \frac{1}{2}\frac{\partial^2 \psi}{\partial t^2} + |\psi|^2 \psi = 0$$

Navier Stokes equation (NSE)

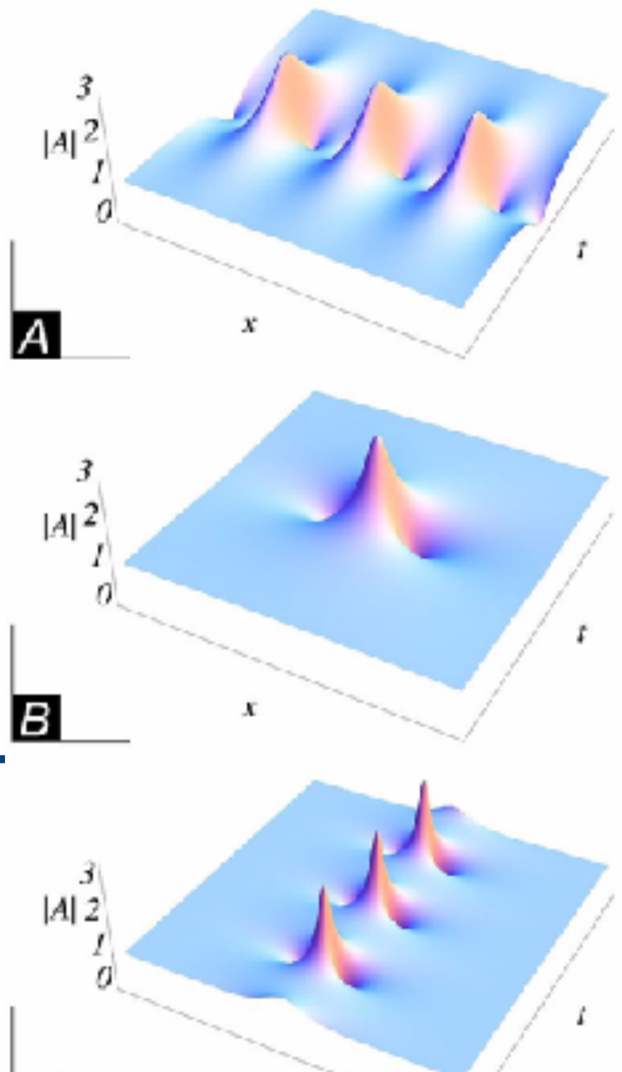
$$\partial_t \vec{u} + (\vec{u}\,\nabla)\,\vec{u} = -\frac{1}{\varrho}\nabla p + \nu \Delta \vec{u}\,, \qquad \nabla \vec{u} = 0$$

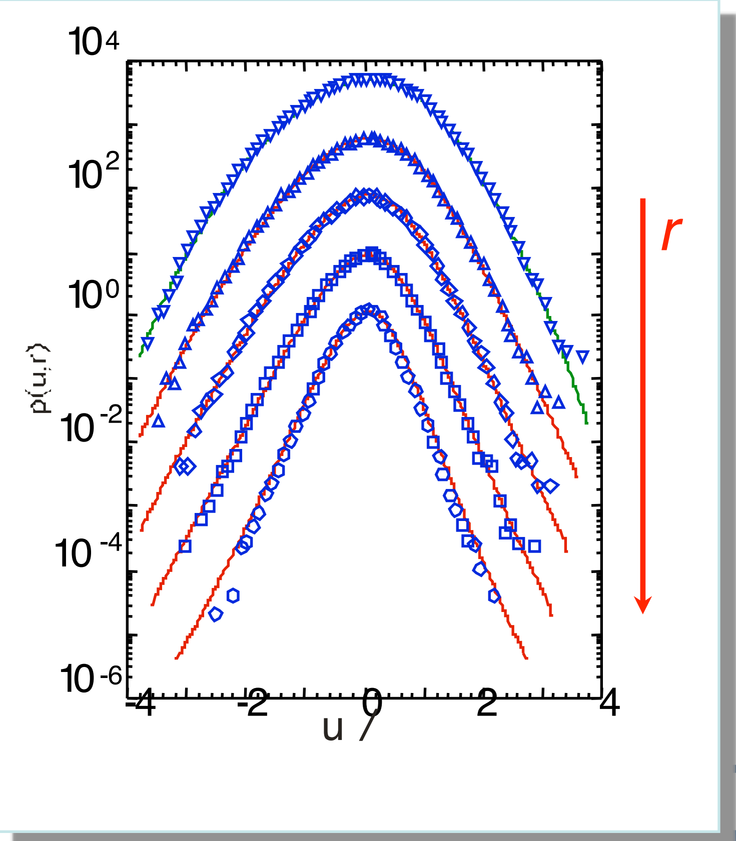
How to approach?

coherent structures (Instabilities)

statistical approach- extreme events

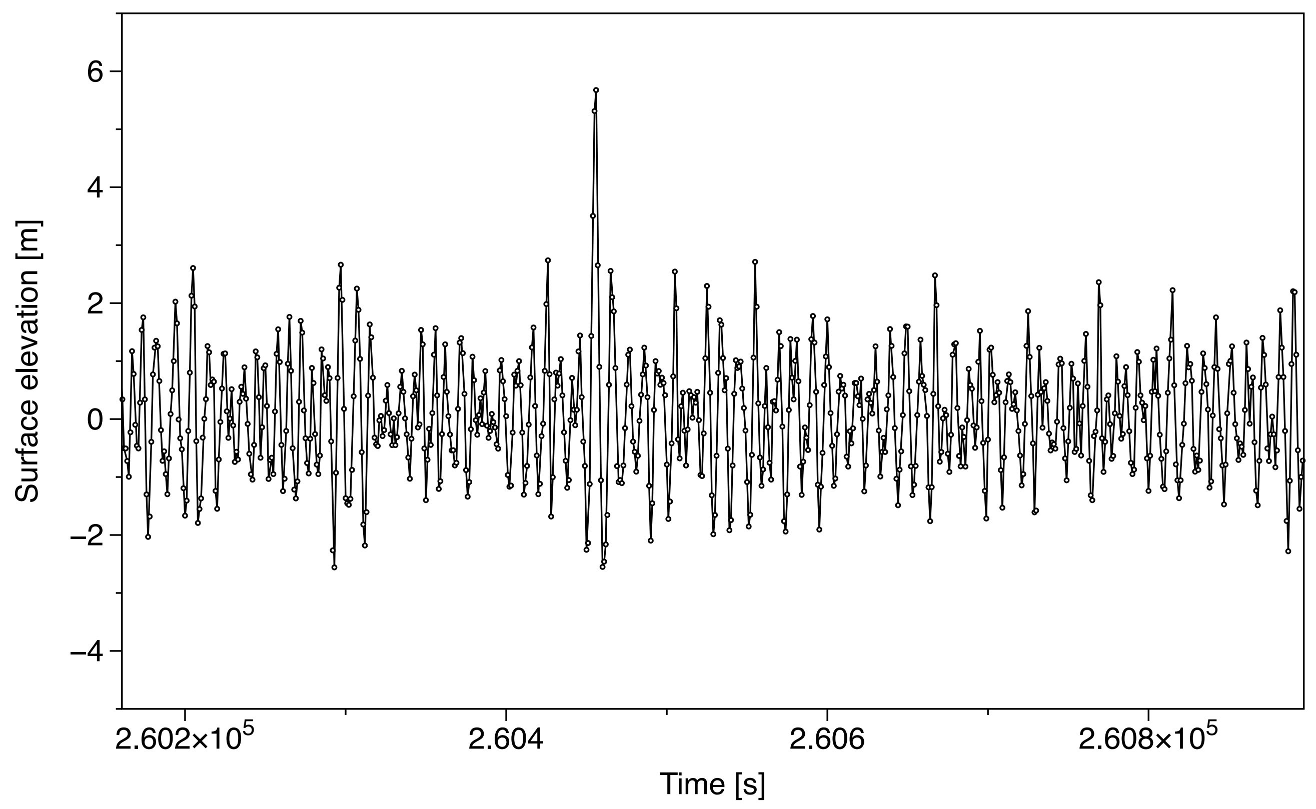
Aim - general joint multipoint statistics combines both sides





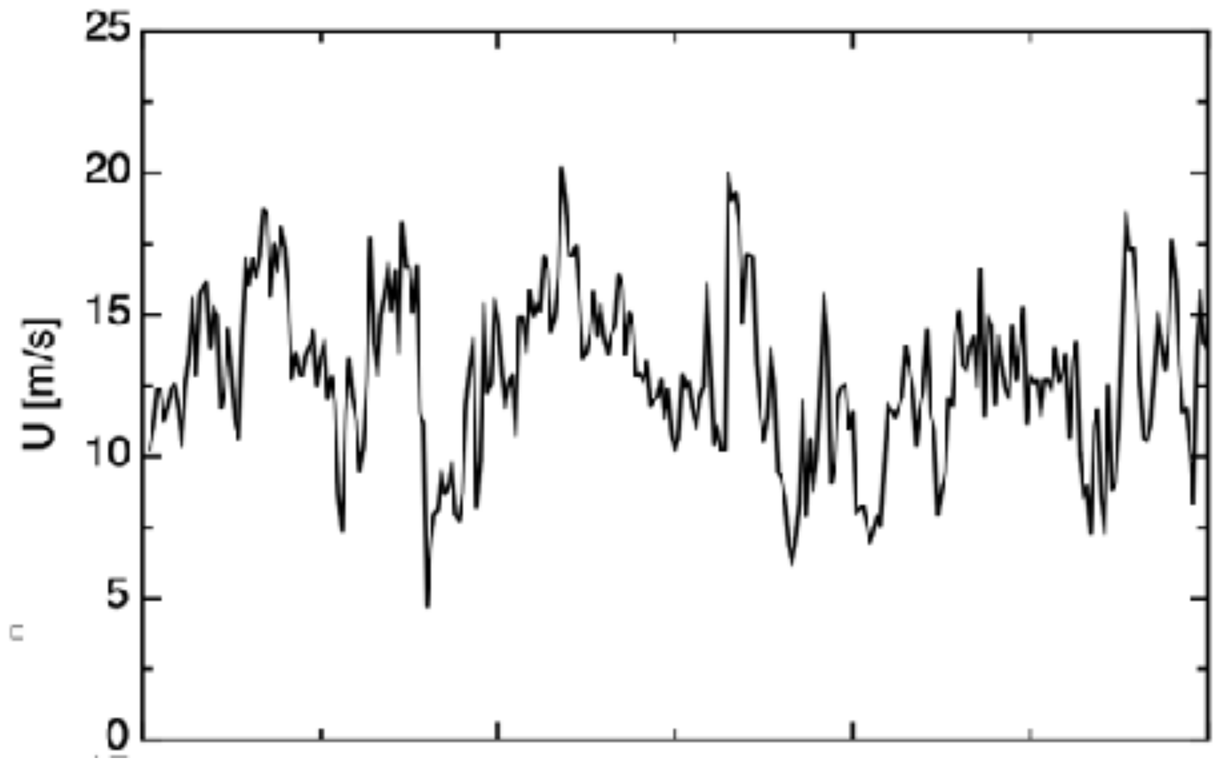
rogue wave - measured data

Sea of Japan data from N. Mori



Wind data

What is an exeme event?



0 100 300
sec

0 100 sec 300

Content - Part B

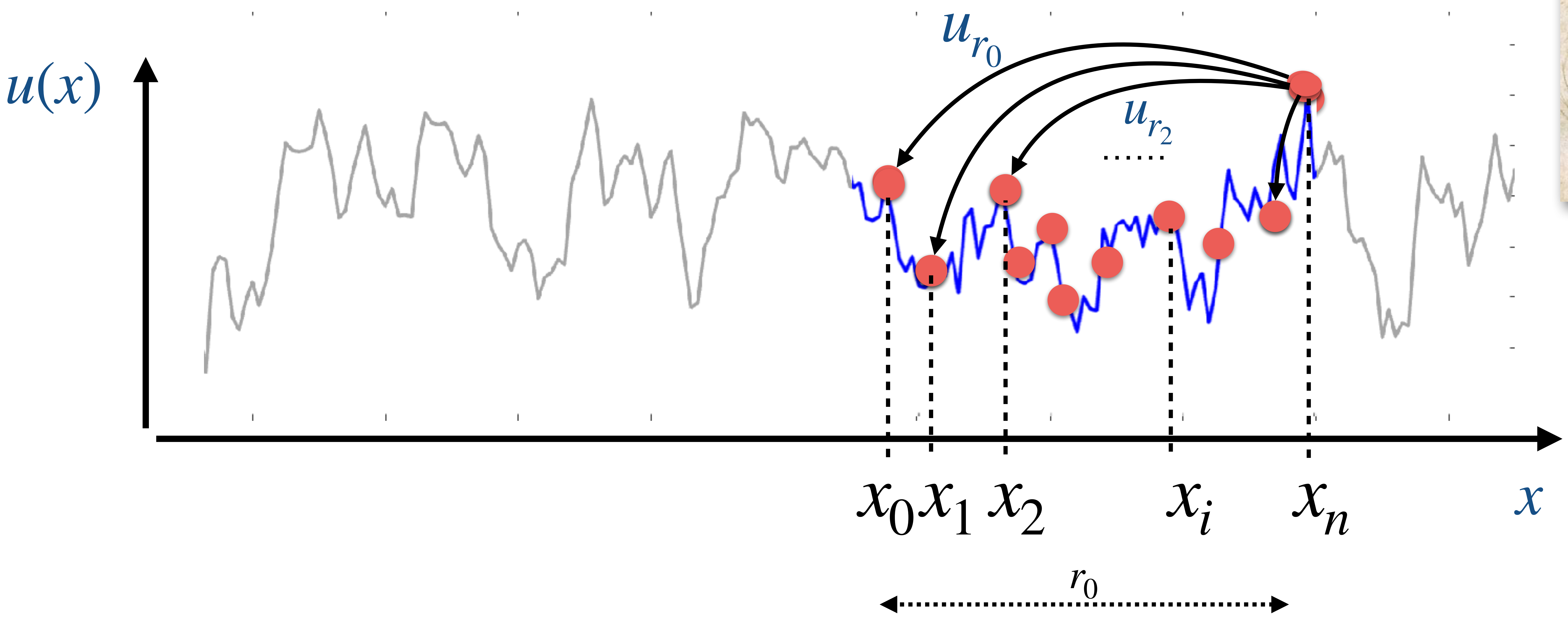
Extreme events

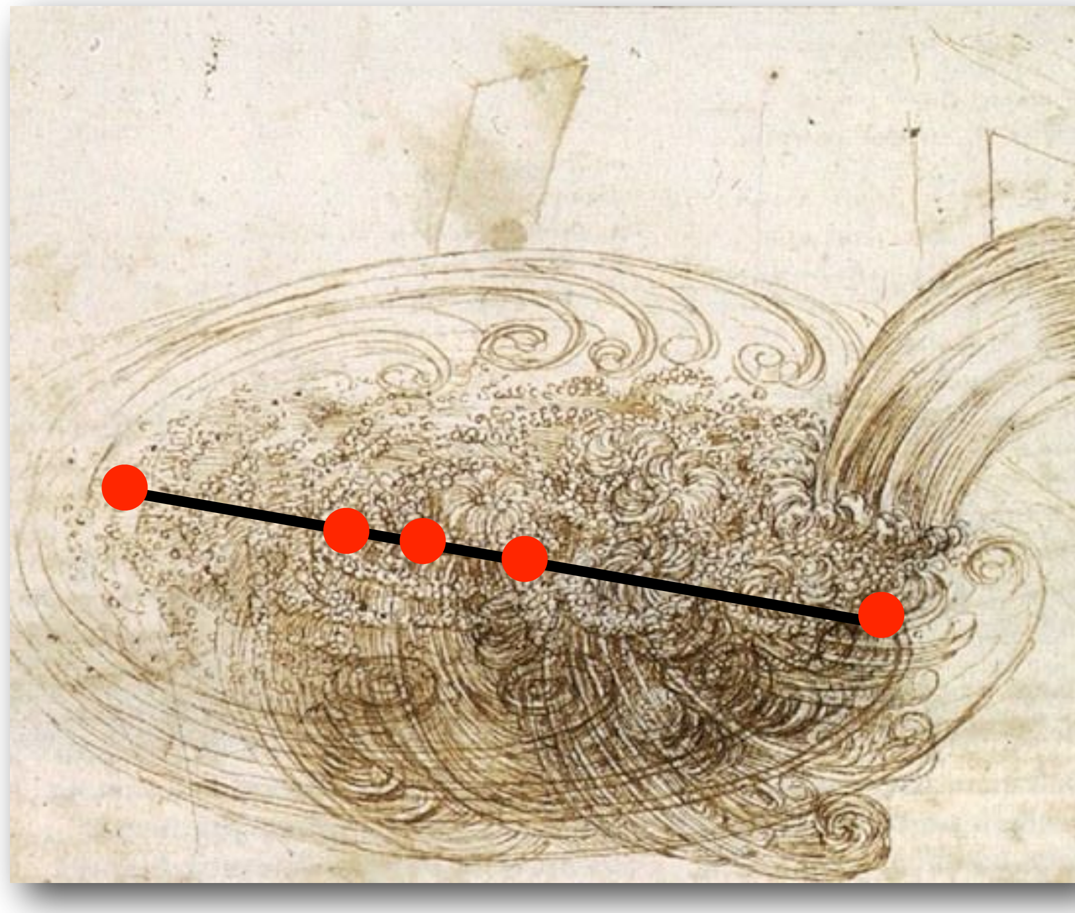
multi-point statistics <— main idea - > we want to get statistics and structures

scale-dependent Fokker-Planck equations

consequences

Data set $u(x)$ / $h(x)$





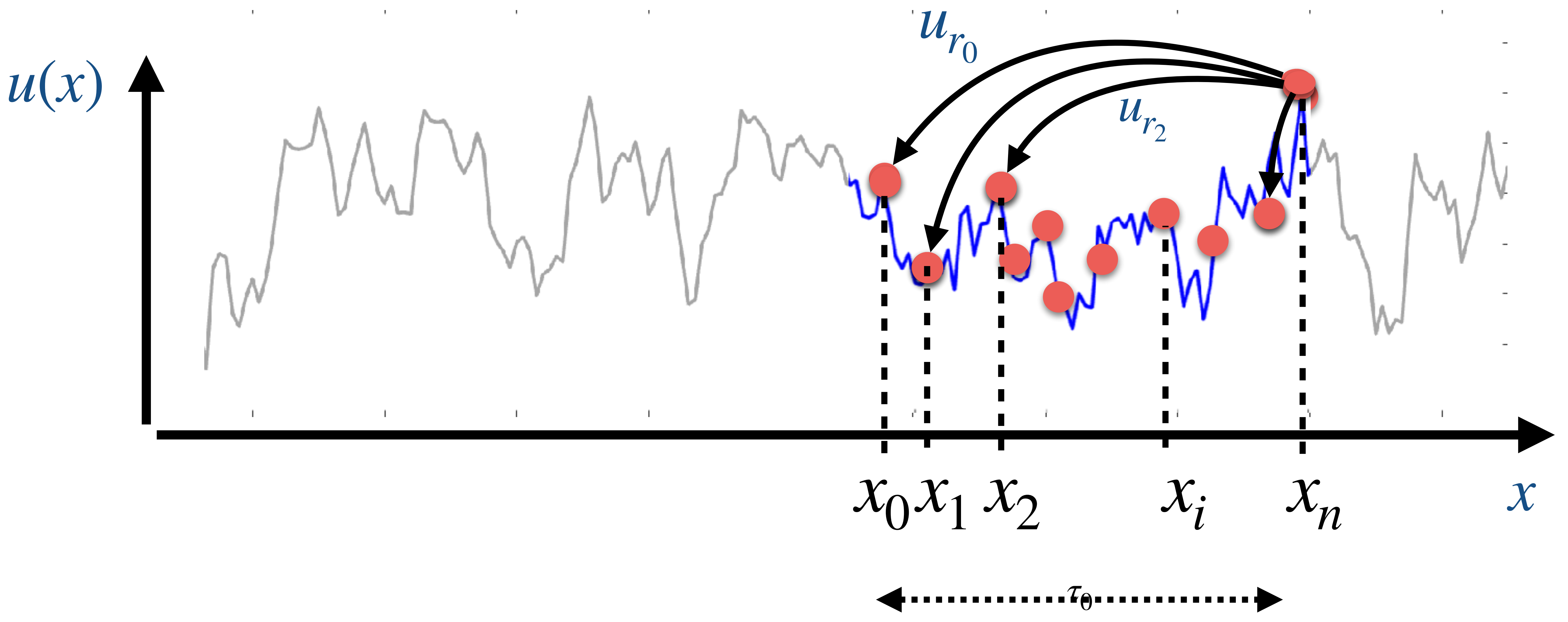
Aim to get the joint multi-time statistics $p(u(x_0), u(x_1), \ldots, u(x_n))$

can be expressed by statistics of **increments** $u_r = u(x + r) - u(x)$

$$p(u(x_0), u(x_1), \ldots, u(x_n)) = p(u_{r_0}, u_{r_1}, \ldots, u_{r_{n-1}}, u(x_n))$$

J.P., et al., Annu. Rev. Condens. Matter Phys., 10 (2019) 107

Data set $u(x)$ / $h(x)$



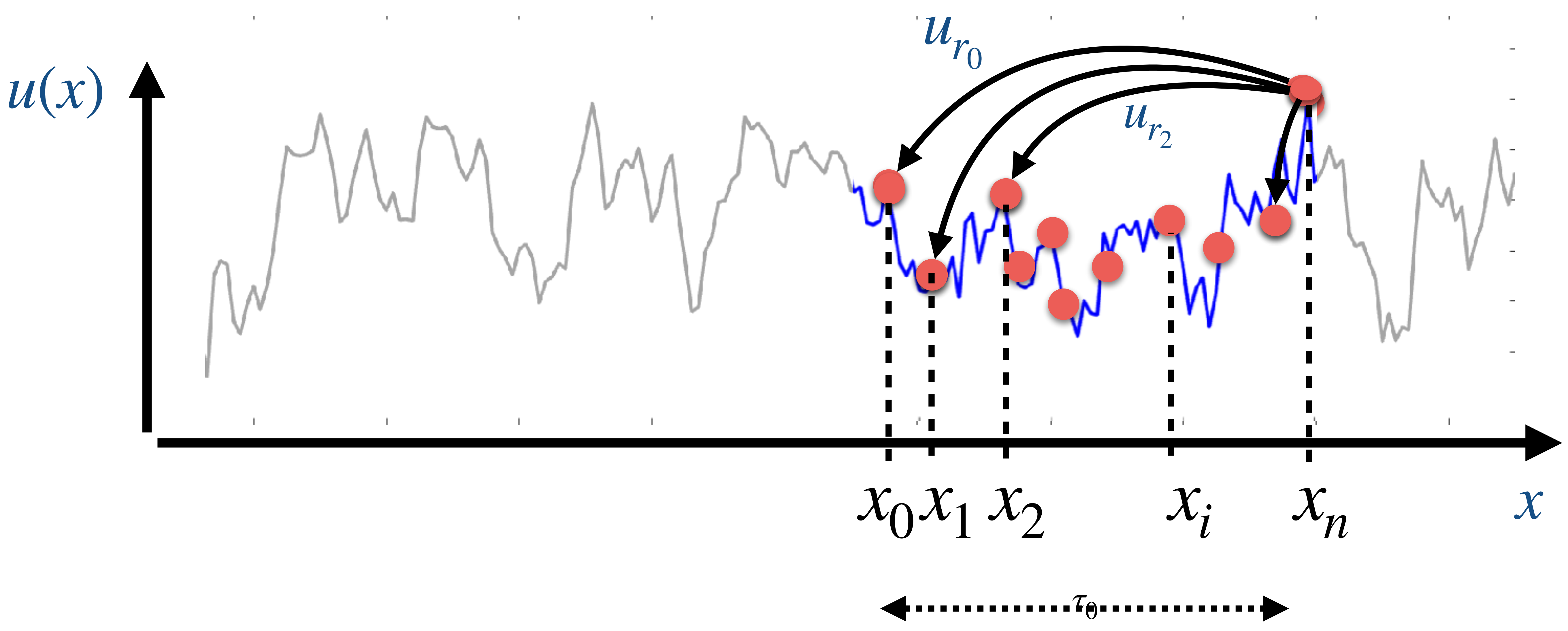
How to simplify

$$p(u(x_0), u(x_1), ..., u(x_n)) = p(u_{r_0}, u_{r_1}, ..., u_{r_{n-1}}, u(x_n))$$

Connection to set of increments - typical turbulent cascade description

J.P., et al., Annu. Rev. Condens. Matter Phys., 10 (2019) 107

Data set $u(x)$ / $h(x)$



How to simplify - use conditional pdfs

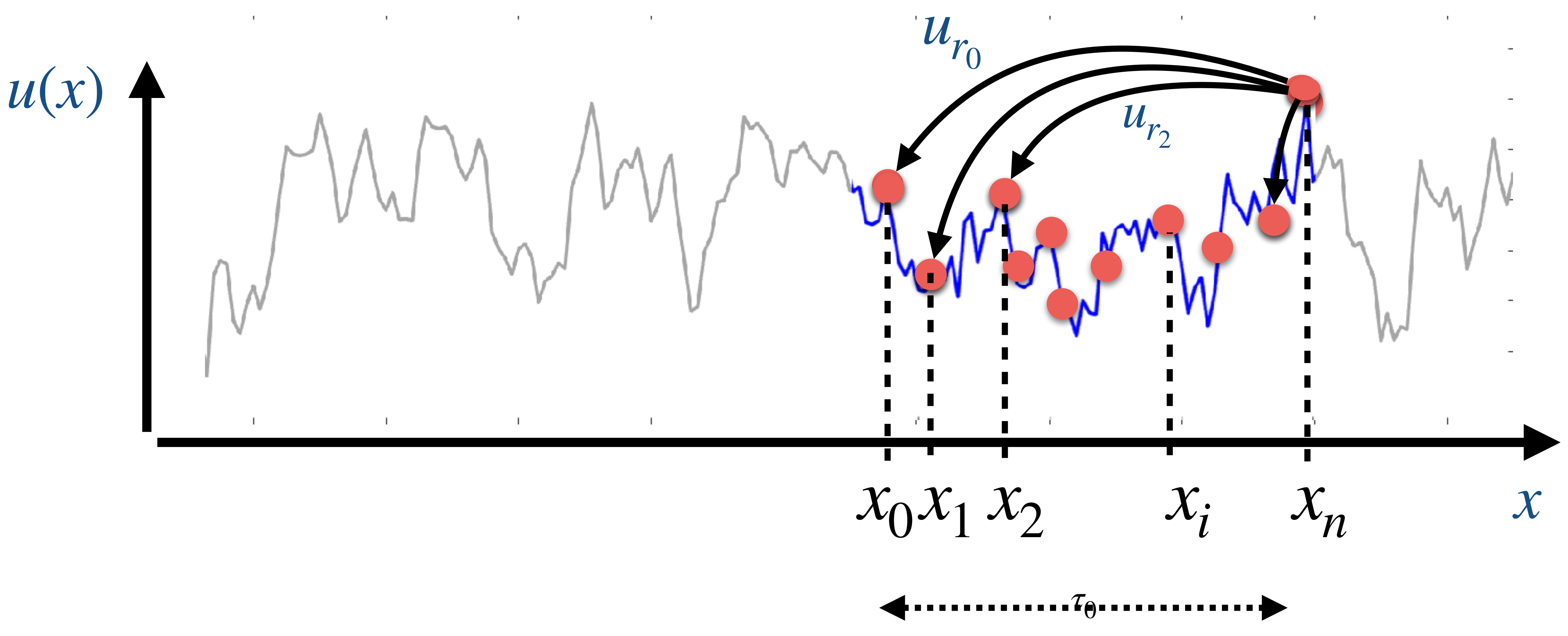
$$p(u(x_0), u(x_1), ..., u(x_n)) = p(u_{r_0}, u_{r_1}, ..., u_{r_{n-1}}, u(x_n))$$
$$= p(u_{r_0}, u_{r_1}, ..., u_{r_{n-1}} | u(x_n)) \; p(u(x_n))$$
$$= ...$$
$$= p(u_{r_{n-1}} | u_{r_{n-2}}, ..., u_{r_0}, u(x_n)) p(u_{r_{n-2}} | ....) .... \; p(u_{r_0} | u(x_n)) \; p(u(x_n))$$

Simplification of multi-conditioned pdfs

$$p(u_{r_i} | u_{r_{i-11}}, ..., u_{r_0}, u(x_n)) = ?$$
$$\stackrel{?}{=} p(u_{r_i} | u_{r_{i-1}}, u(x_n))$$
$$\stackrel{?}{=} p(u_{r_i})$$

J.P., et al., Annu. Rev. Condens. Matter Phys., 10 (2019) 107

Data set $u(x)$ / $h(x)$



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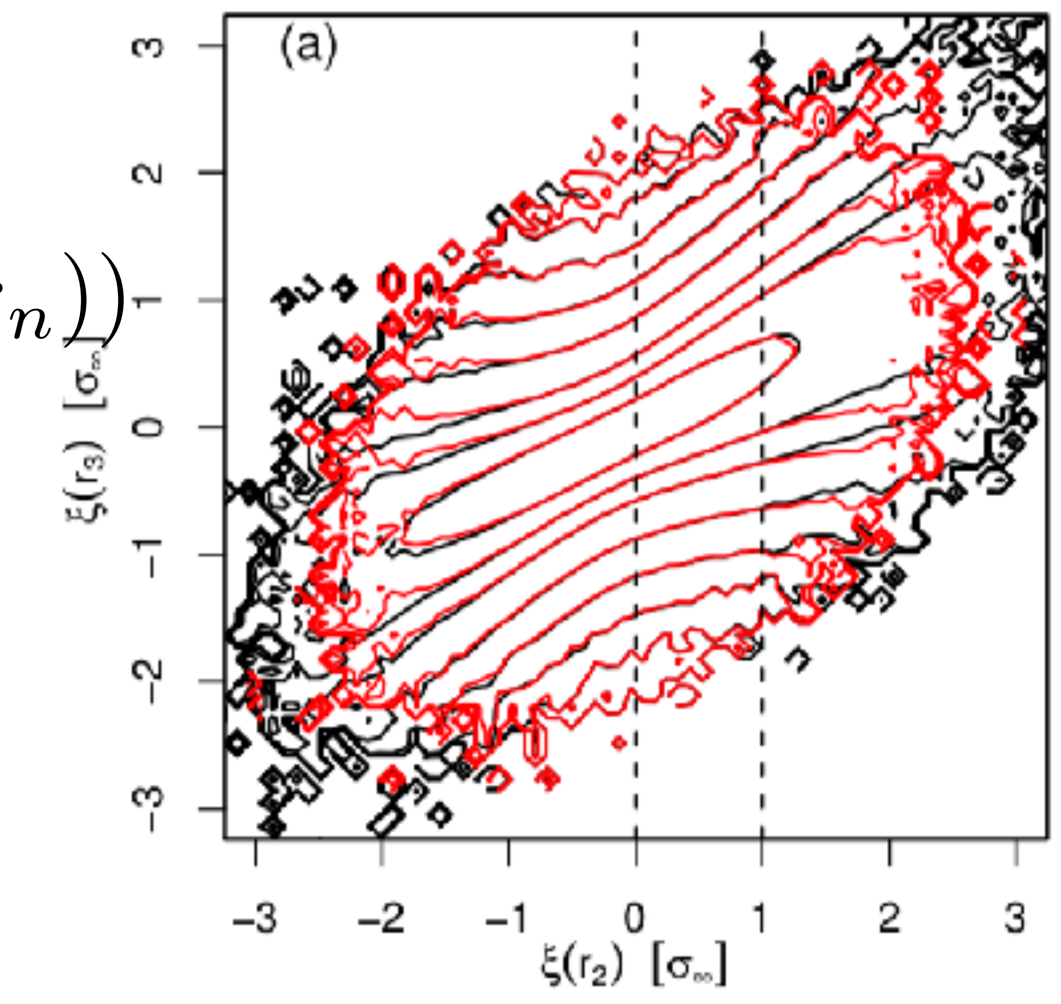
Simplification of multi-conditioned pdfs

$$p(u_{r_i} | u_{r_{i-11}}, ..., u_{r_0}, u(x_n)) = ?$$
$$= p(u_{r_i} | u_{r_{i-1}}, u(x_n))$$

Holds - three point closure

$$\neq p(u_{r_i})$$



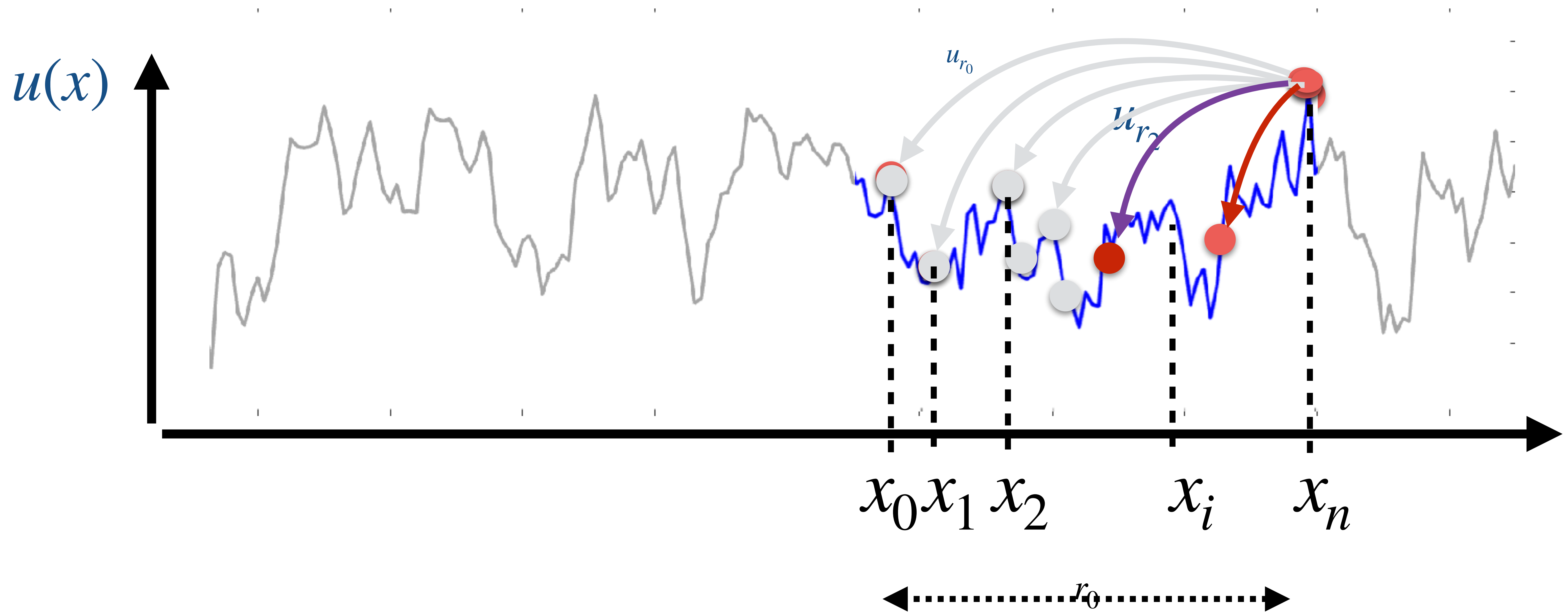
J.P., et al., Annu. Rev. Condens. Matter Phys., 10 (2019) 107

Joint- n-point statistics simplifies:

$$p(u_{r_{n-1}}|u_{r_{n-2}},....,u_{r_0},u(x_n))p(u_{r_{n-2}}|....)....\ p(u_{r_0}|u(x_n))\ p(u(x_n)) = p(u_{r_{n-1}}|u_{r_{n-2}},u(x_n))p(u_{r_{n-2}}|.u_{r_{n-3}},u(x_n))$$

$$p(u_{r_i}|u_{r_{i-1}},....,u_{r_0}) = p(u_{r_i}|u_{r_{i-1}},u(x_n))$$

Simple conditioned pdf = 3 point quantity



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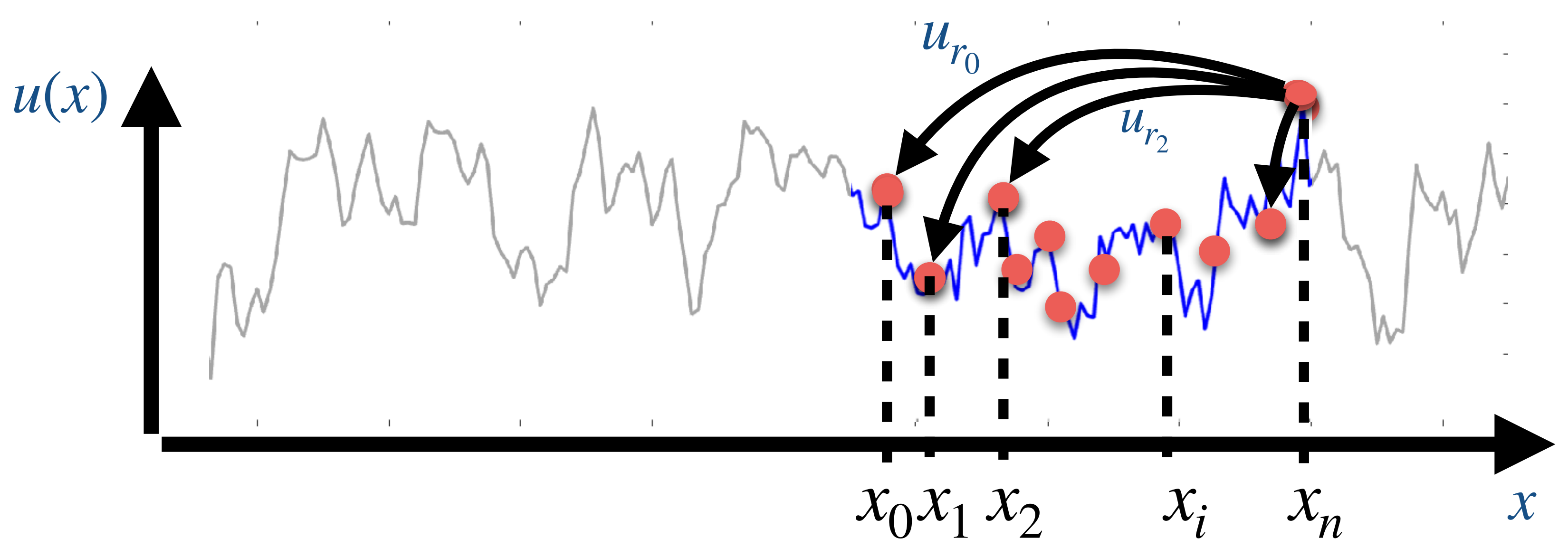

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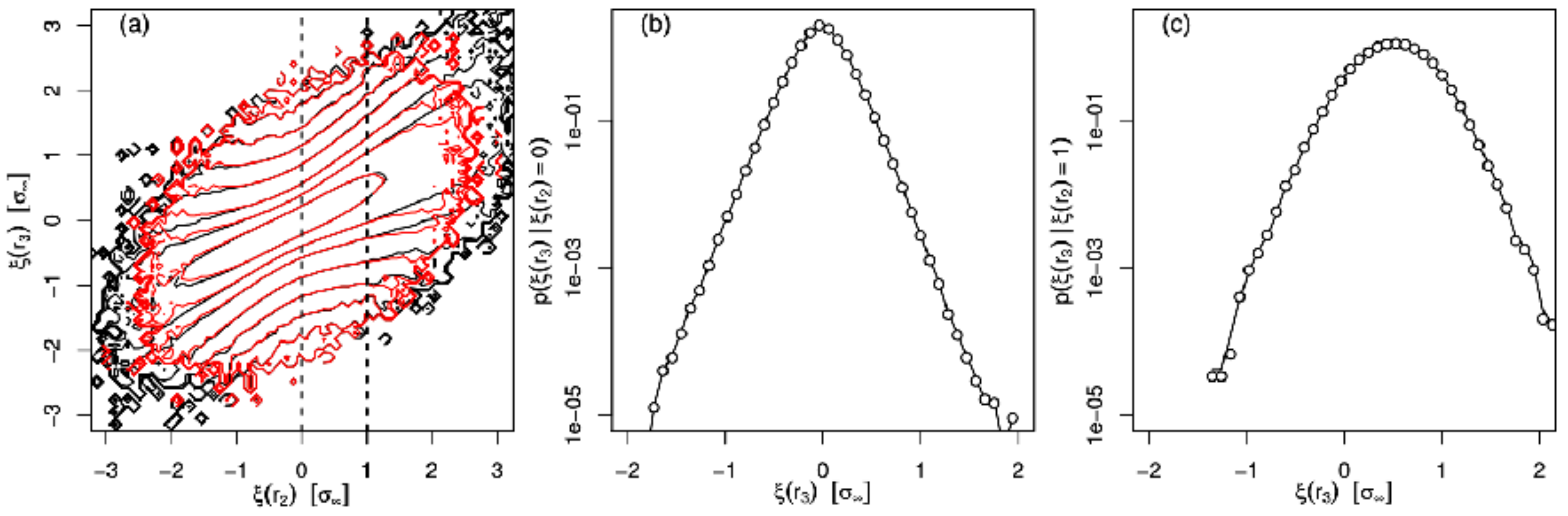
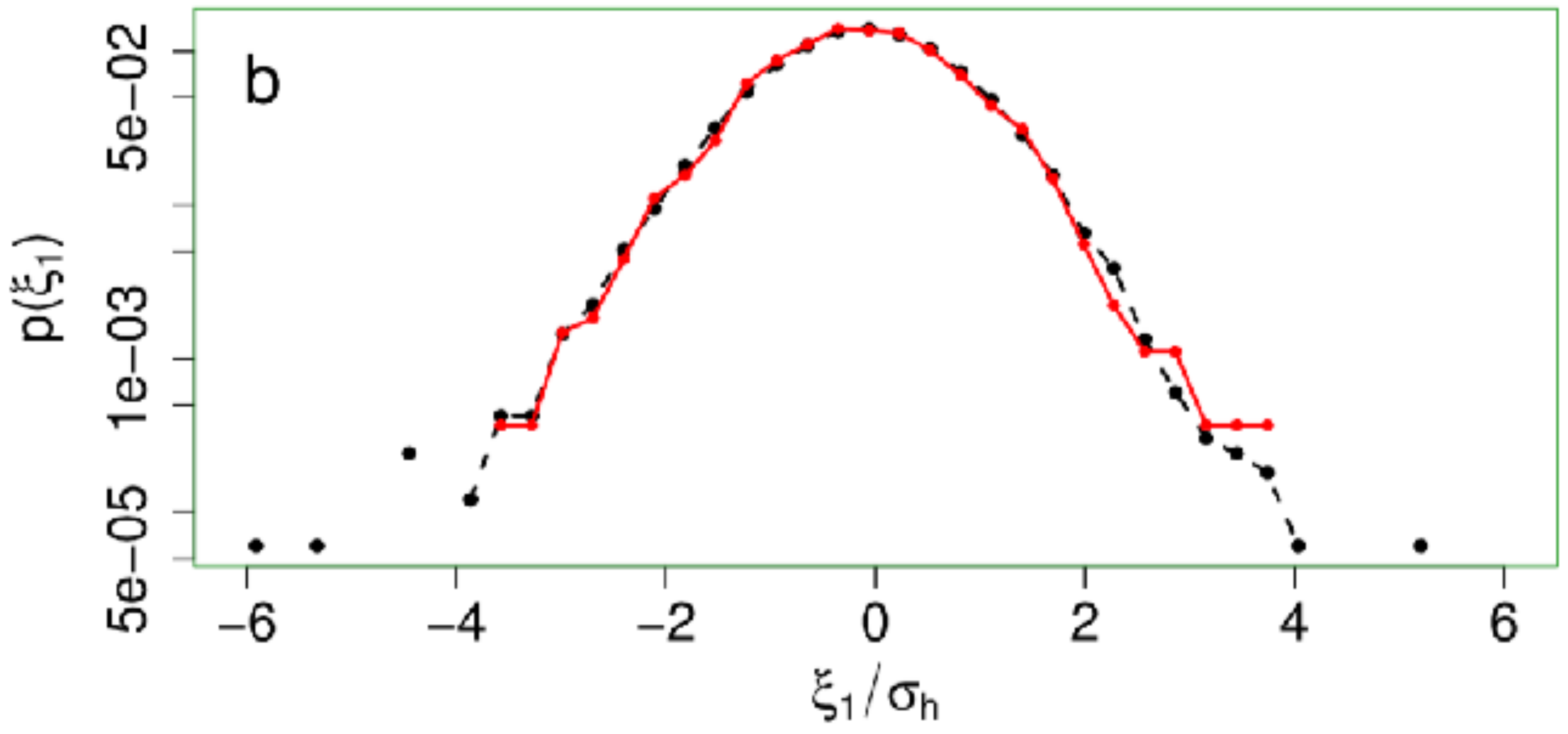
$$p(u_{r_i}|u_{r_{i-1}},....,u_{r_0}) = p(u_{r_i}|u_{r_{i-1}},u(x_n))$$

Turbulence

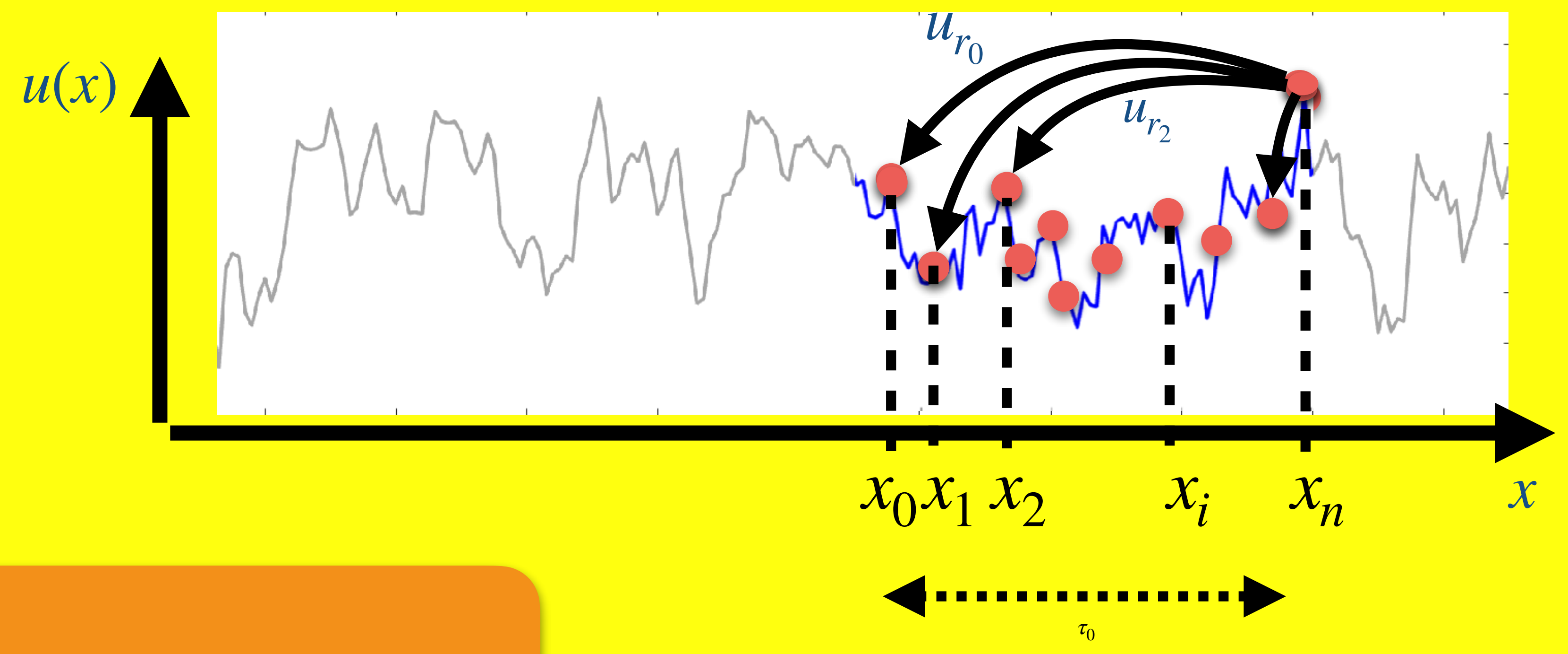


Waves



Joint- n-point statistics simplifies:

Three point closure



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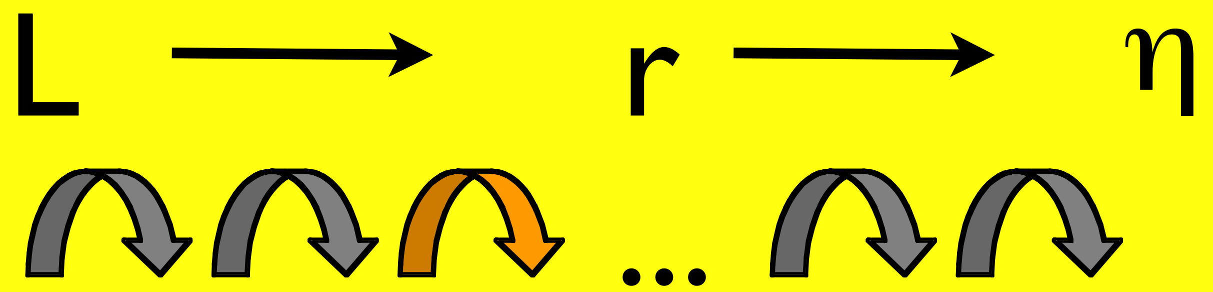
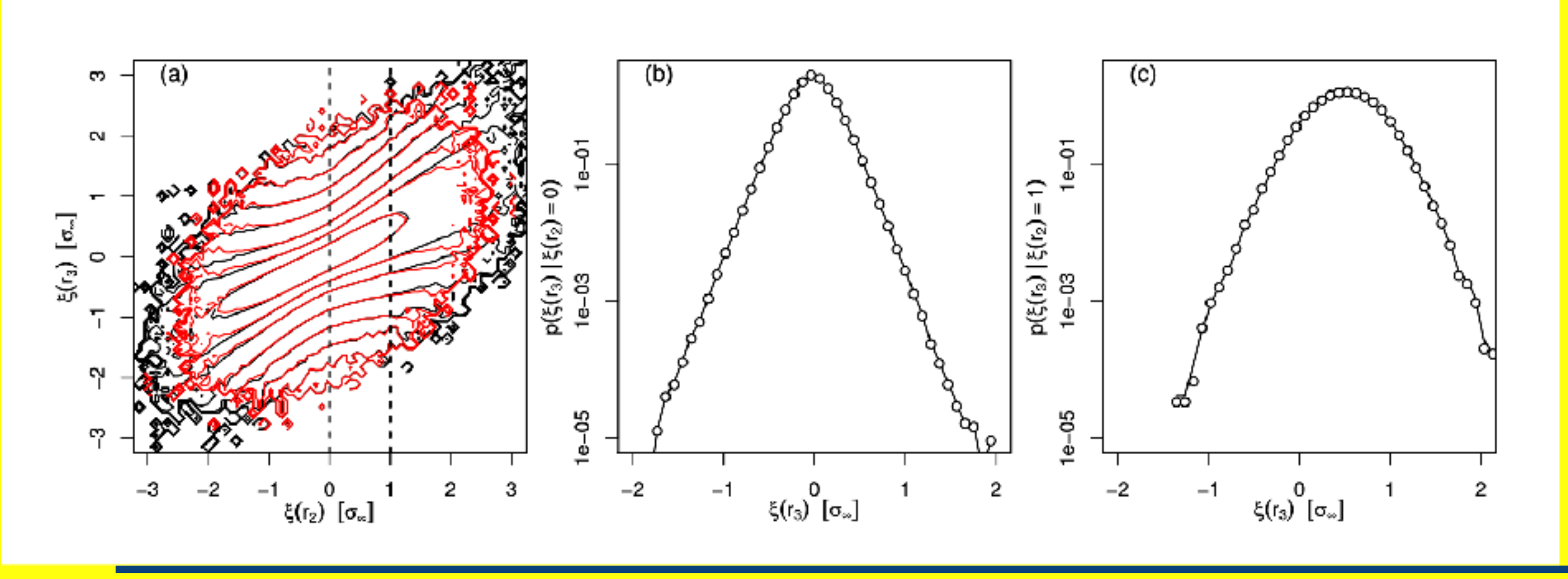
$$p(u_{r_i}|u_{r_{i-1}},....,u_{r_0}) = p(u_{r_i}|u_{r_{i-1}},u(x_n))$$

Idea of cascade - increments from large, L, to small scales η



Turbulence



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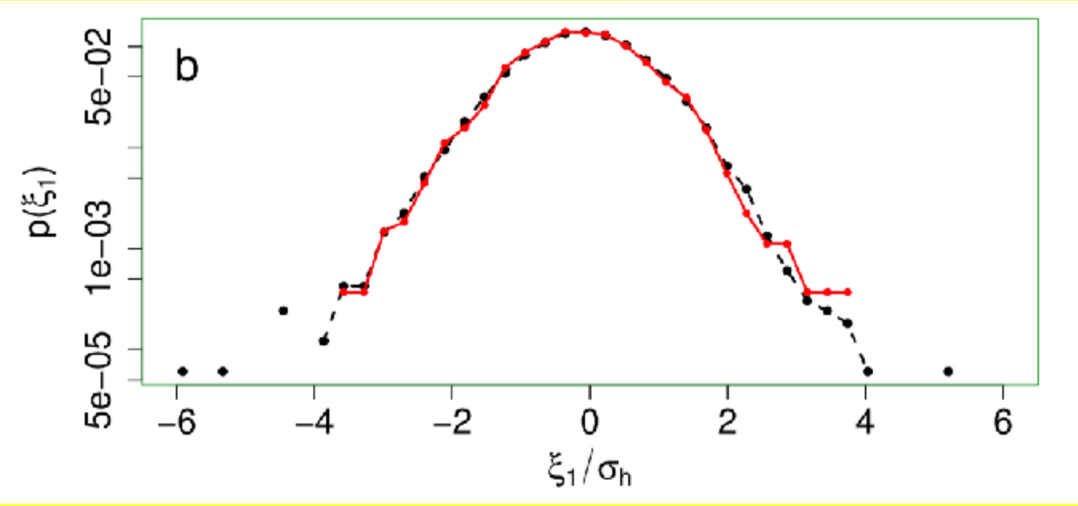


Content

Extreme events

multi-point statistics

-> structures and forecasting

consequences

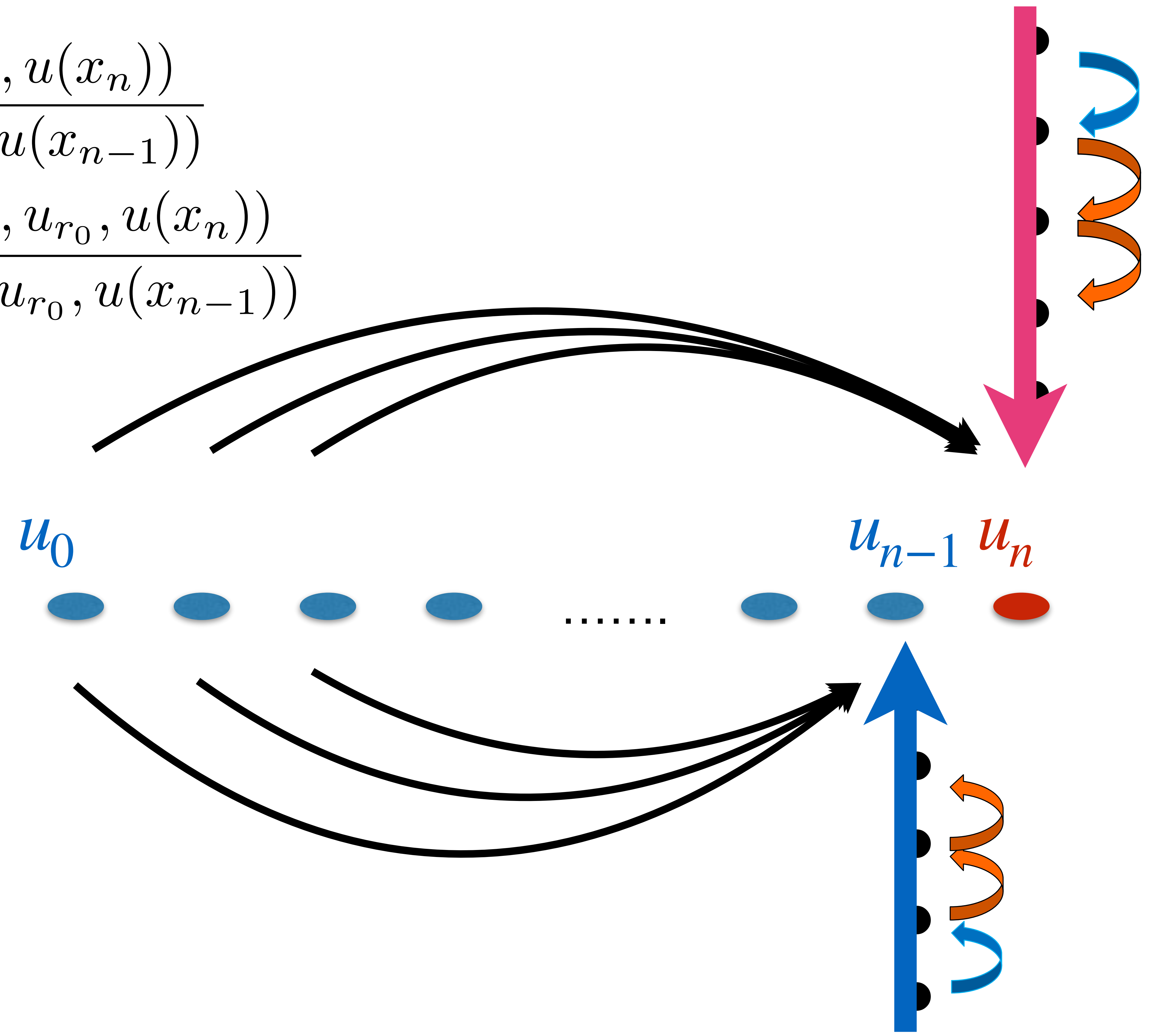
results and claims

cascade process in r and estimation of Fokker-Planck equation,

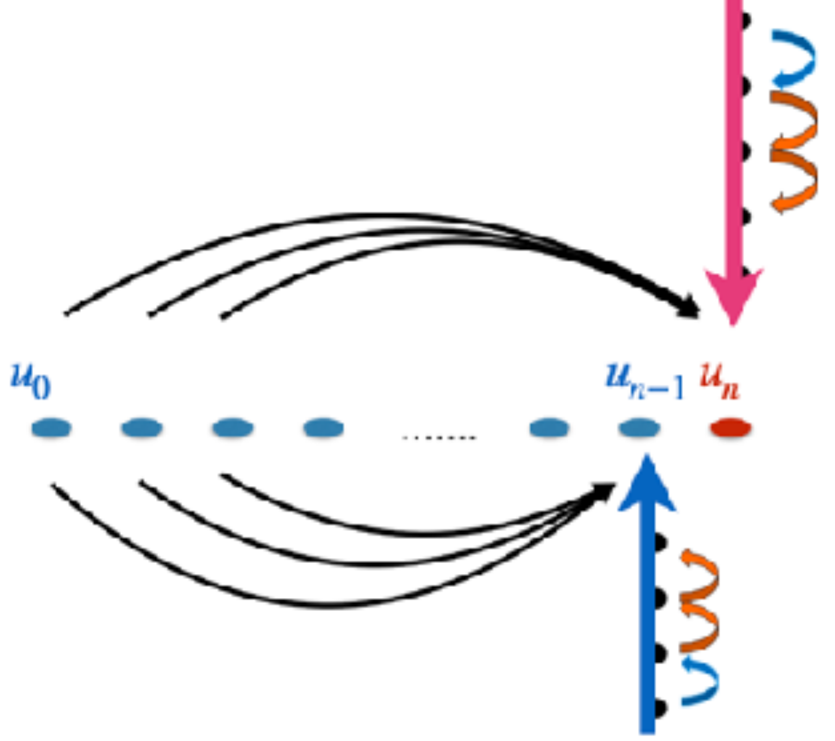
-> new class of stochastic processes for n-point statistics. => forecasting of future (here waves but also wind gusts…)

$$p(u(x_n)|u(x_{n-1}),...,u(x_0)) = \frac{p(u(x_0),u(x_1),...,u(x_n))}{p(u(x_0),u(x_1),...,u(x_{n-1}))}$$

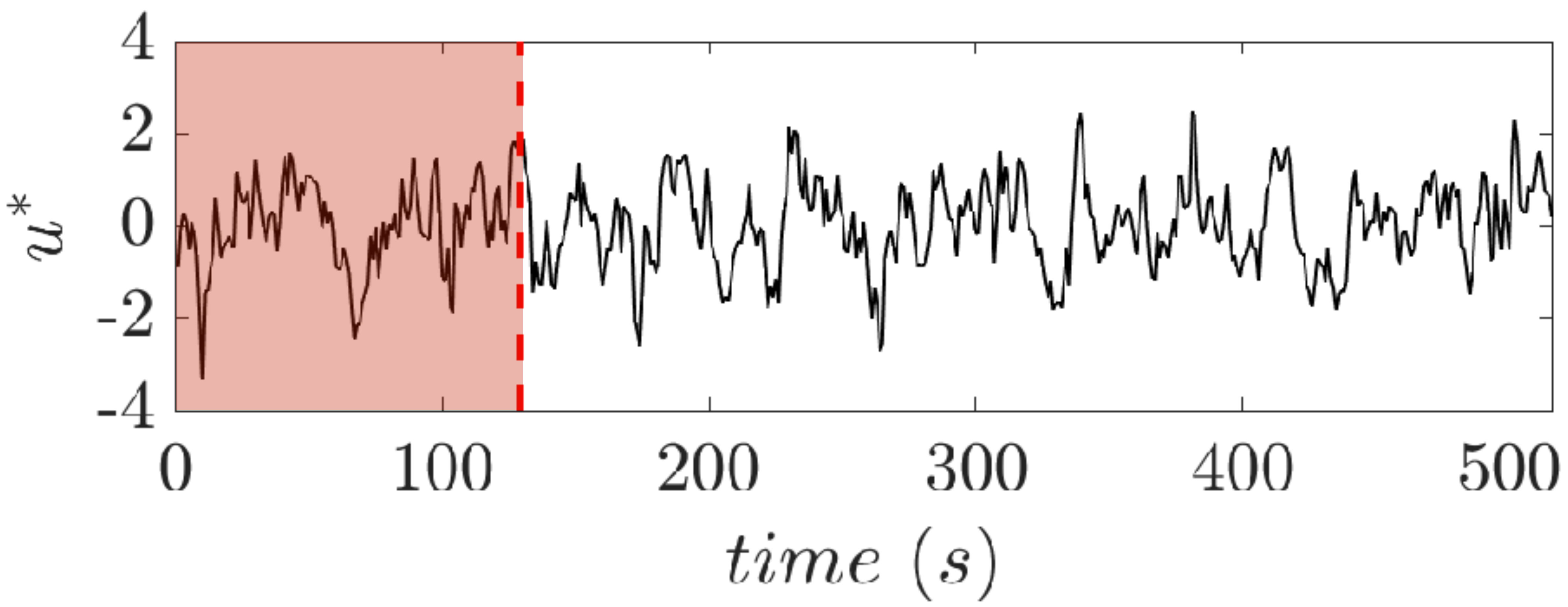
$$= \frac{p(u_{r_{n-1}},u_{r_{n-2}},...,u_{r_0},u(x_n))}{p(u_{r_{n-2}},u_{r_{n-2}},...,u_{r_0},u(x_{n-1}))}$$



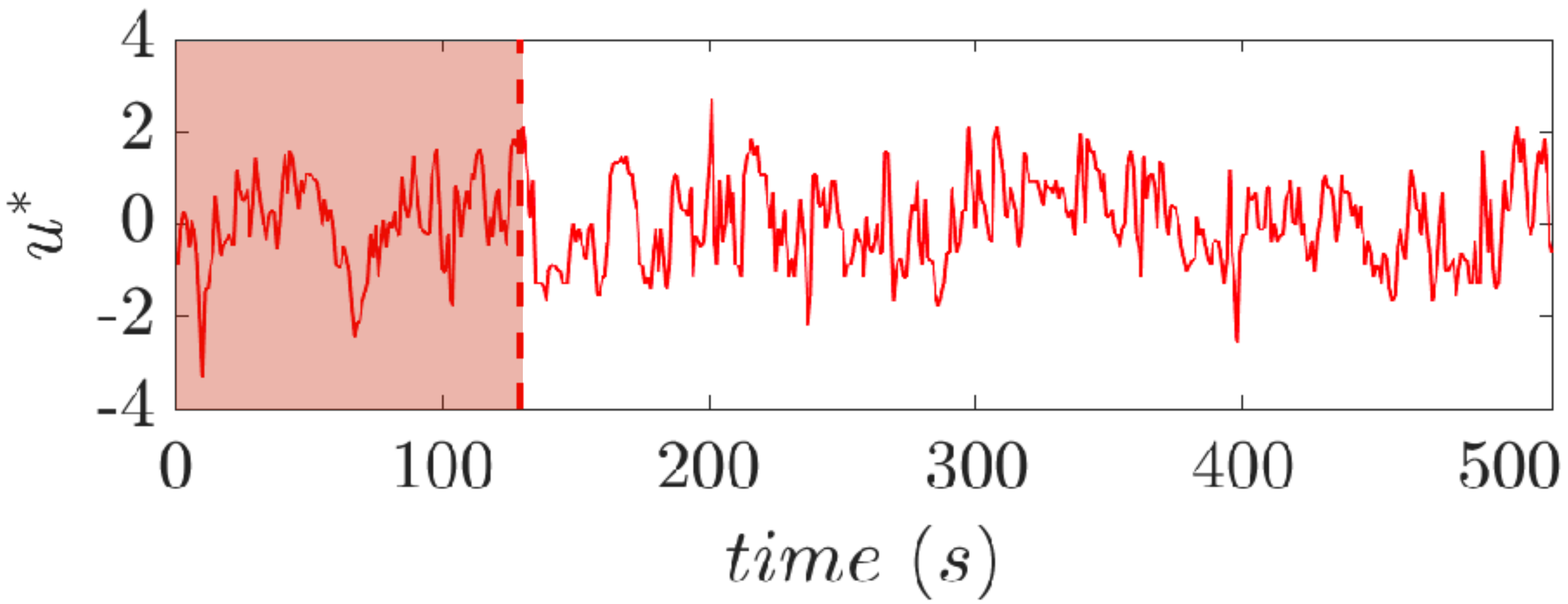
Surrogate wind speed fluctuations from Fokker-Planck Equation



Measured



Surrogates

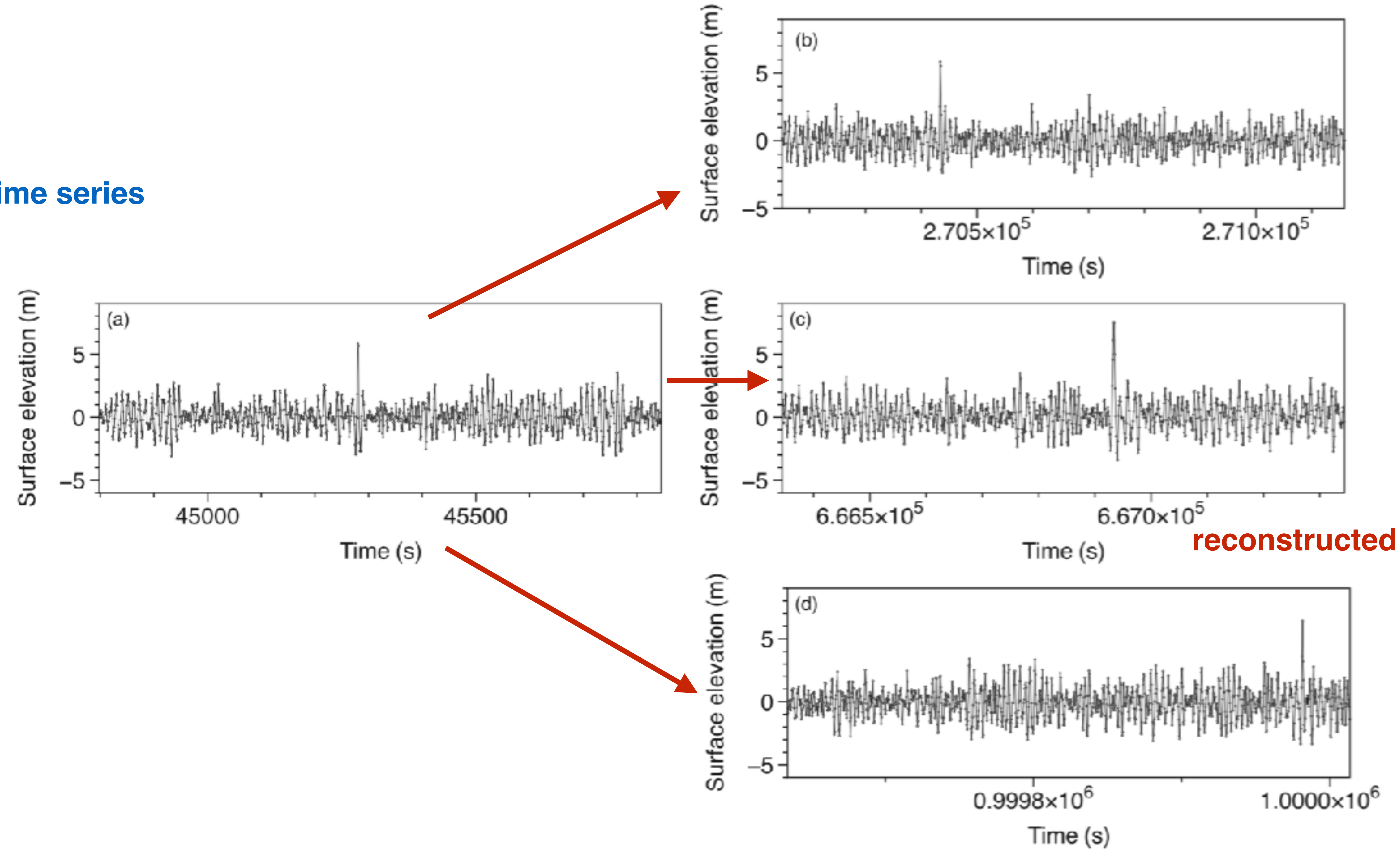


See also Poster F. Köhne

new data sets

-> new class of stochastic processes for n-point statistics

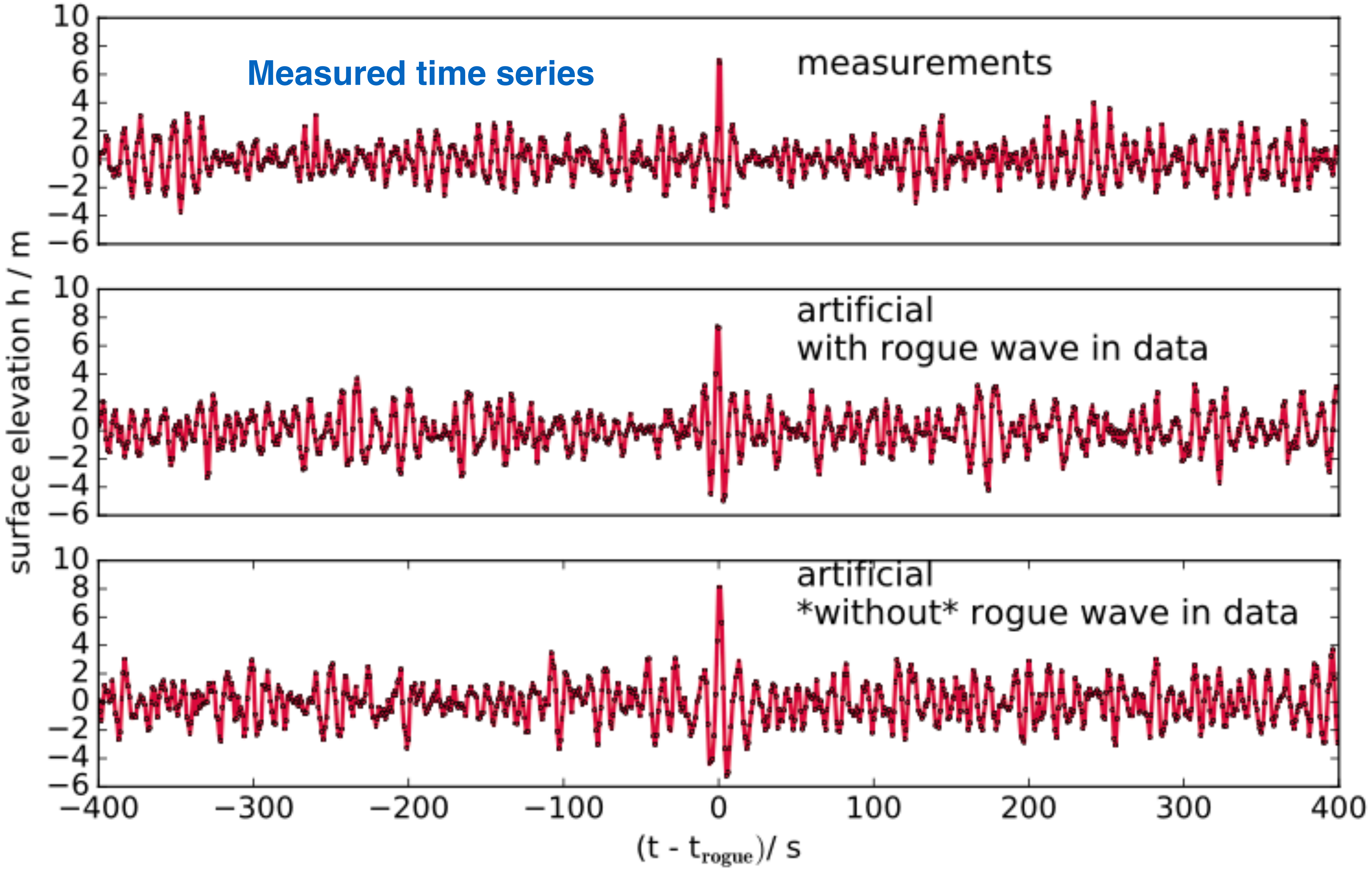
Measured time series



reconstructed time series

new data sets

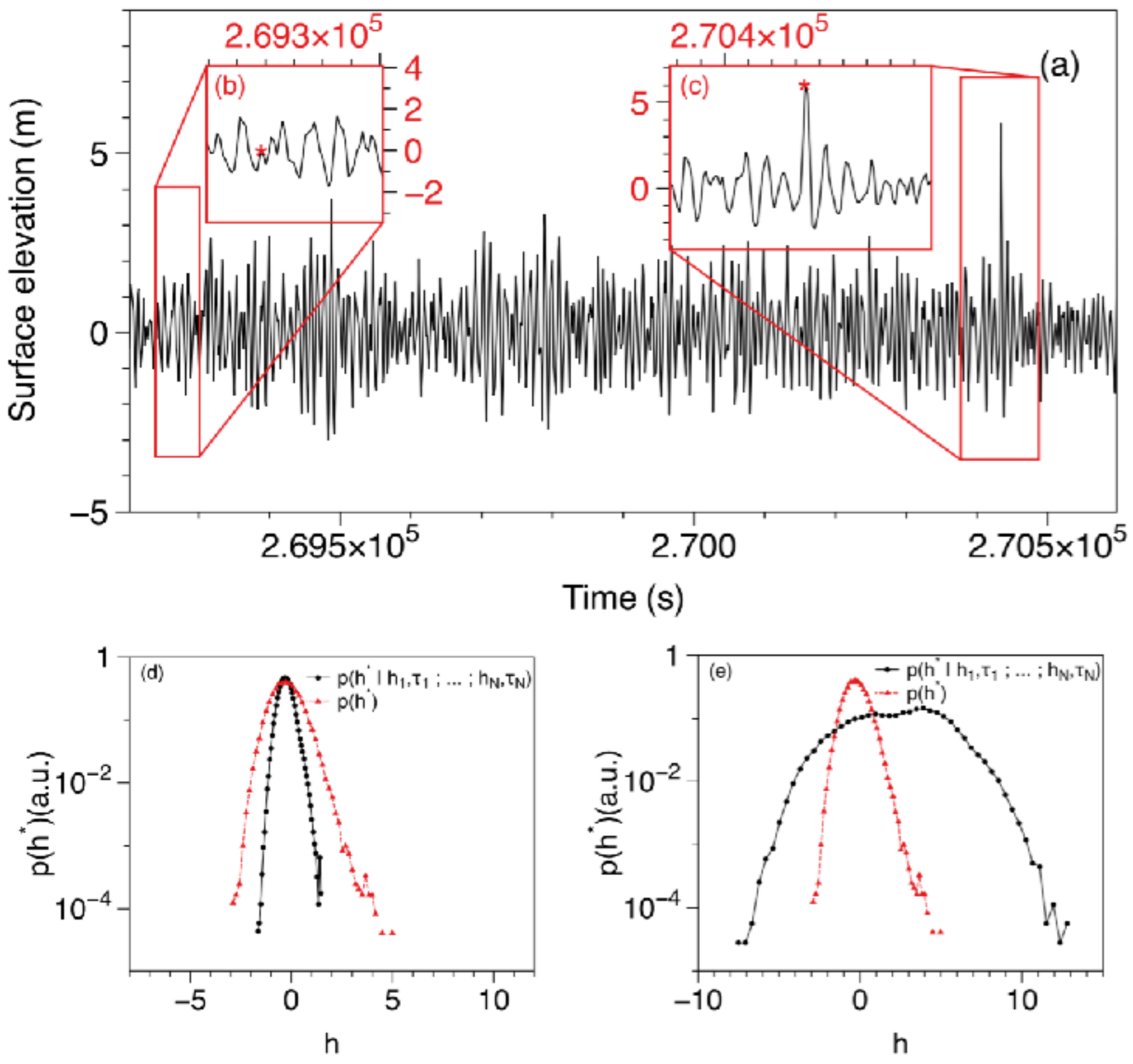
-> new class of stochastic processes for n-point statistics

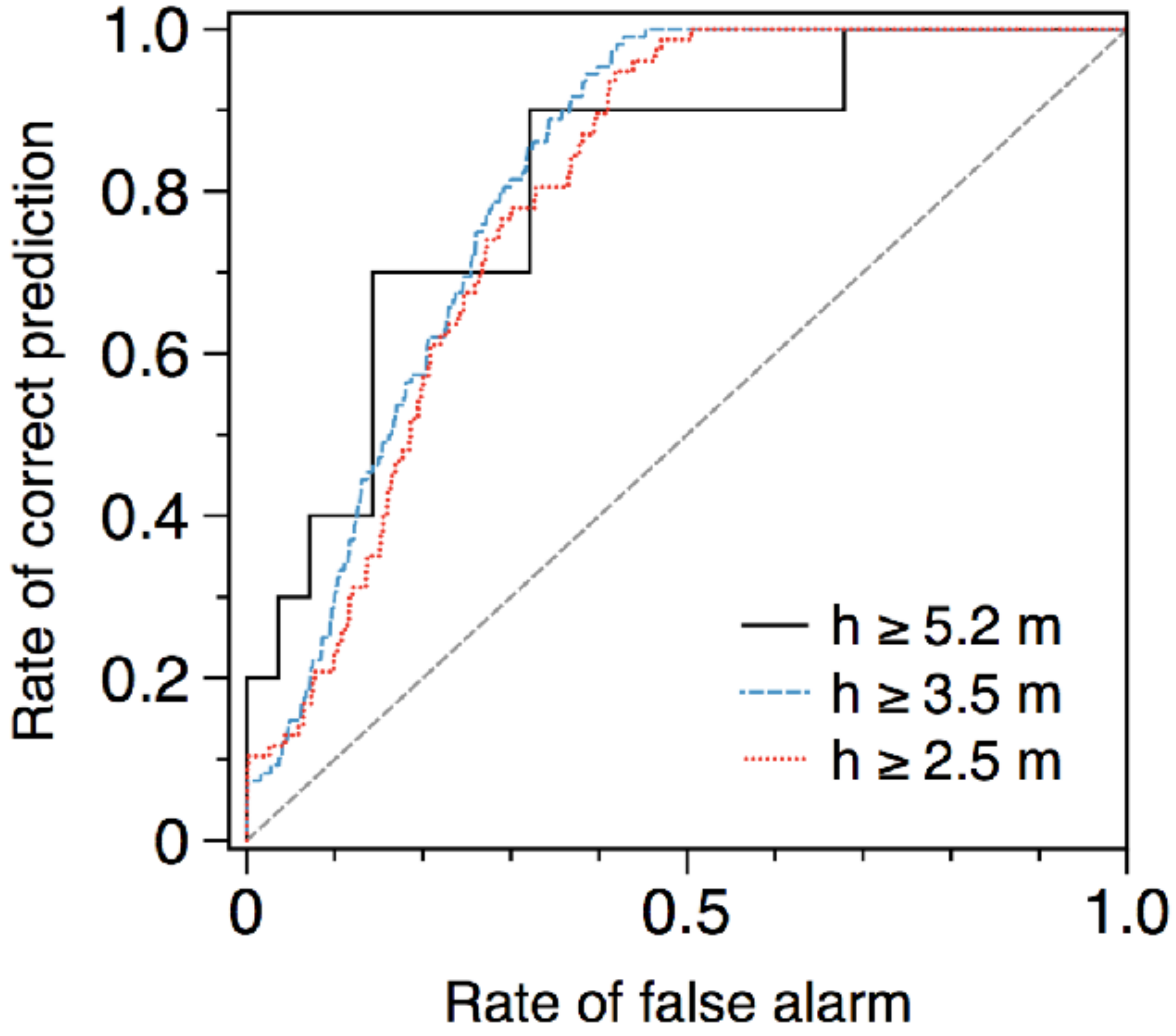


reconstructed time series

Forecasting of rogue waves - ROC analysis

$$P_{extreme} = \int_{h_r}^{\infty} p(h^* | h_1, \tau_1; \ldots; h_N, \tau_N) dh^*$$

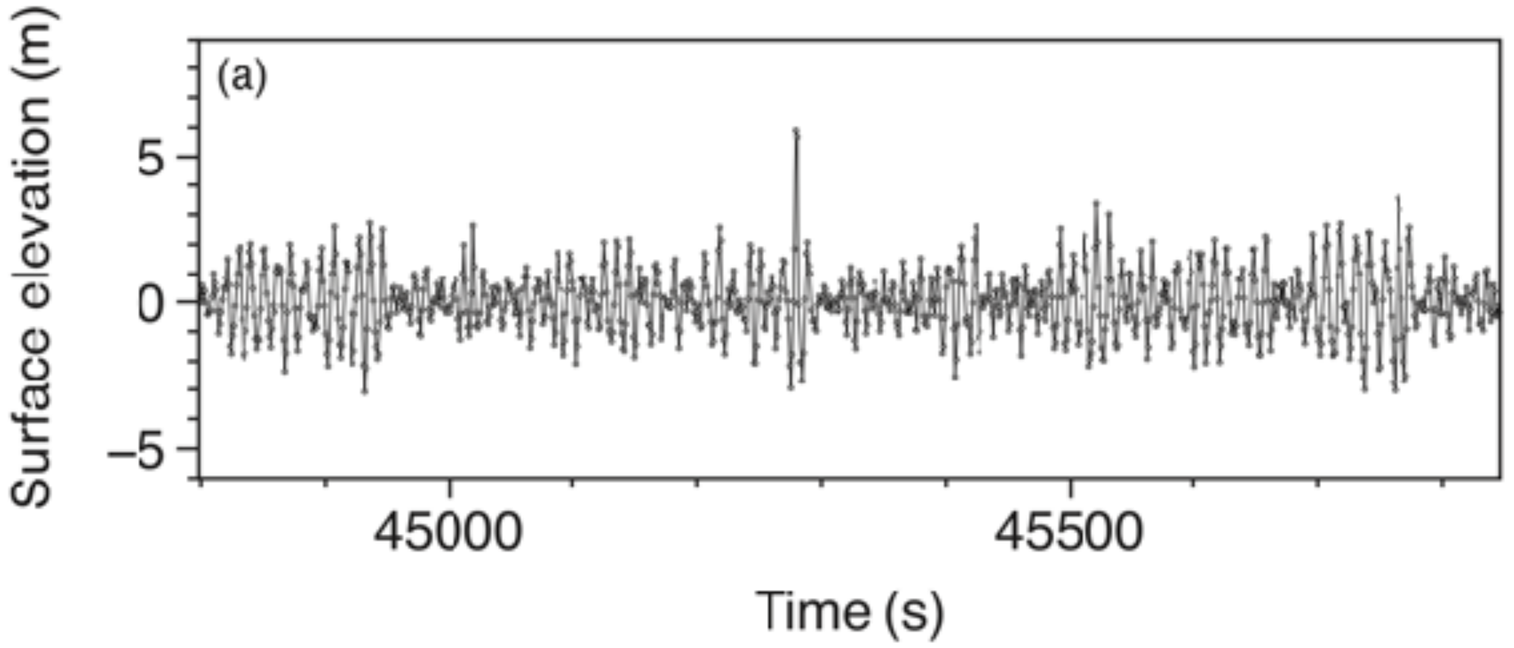




rogue wave - large deviation

Data from sea of Japan

=> F. P. Equation

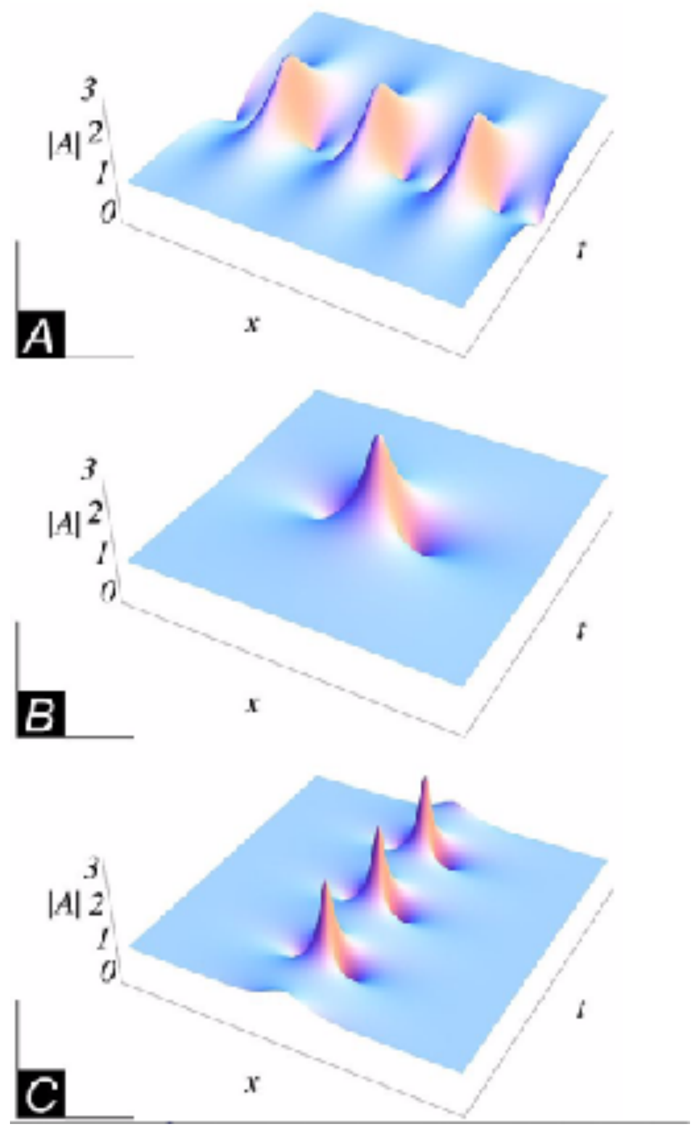


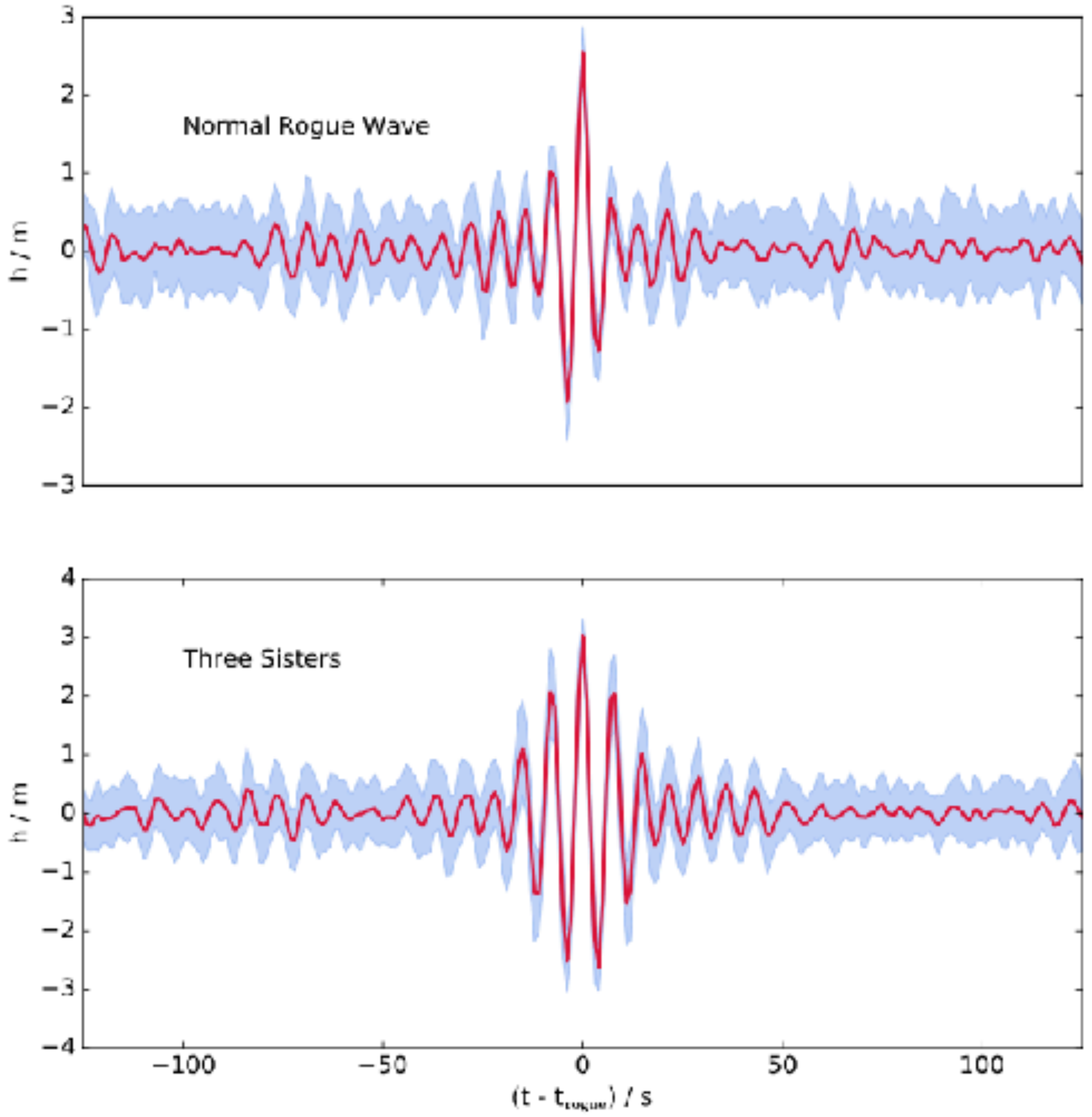
stochastic simulation of very rare events -

“Trajectory-Adaptive Multilevel Sampling” (TAMS)

Lestang, T. ; Ragone, F. ; Bréhier, C.-E. ; Herbert, C. ; Bouchet, F. :

Journal of Statistical Mechanics: Theory and Experiment 2018 (2018), Nr. 4, S. 043213





non-linear Schrödinger equation

Predicts structures of rogue waves

Akhmediev Peregrine

Carl von Ossietzky universität Oldenburg

ForWind Zentrum für Windenergieforschung Bremen Hannover Oldenburg

Multipoint statistics by 3-point closure leads to realistic data
- including extreme events and forecasting
- predicts also unseen events

We get correct
- probabilities,
- Spectra,
- higher order structure function

.....

Multipoint statistics by 3-point closure. - literature

ANNUAL REVIEWS

Annual Review of Condensed Matter Physics

The Fokker–Planck Approach to Complex Spatiotemporal Disordered Systems

J. Peinke,[1,2] M.R.R. Tabar,[3] and M. Wächter[1]

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[2] Fraunhofer Institute for Wind Energy Systems, D-26129 Oldenburg, Germany

[3] Department of Physics, Sharif University of Technology, Tehran 11155-9161, Iran

Annu. Rev. Condens. Matter Phys. 2019. 10:107–32

Physics of Fluids

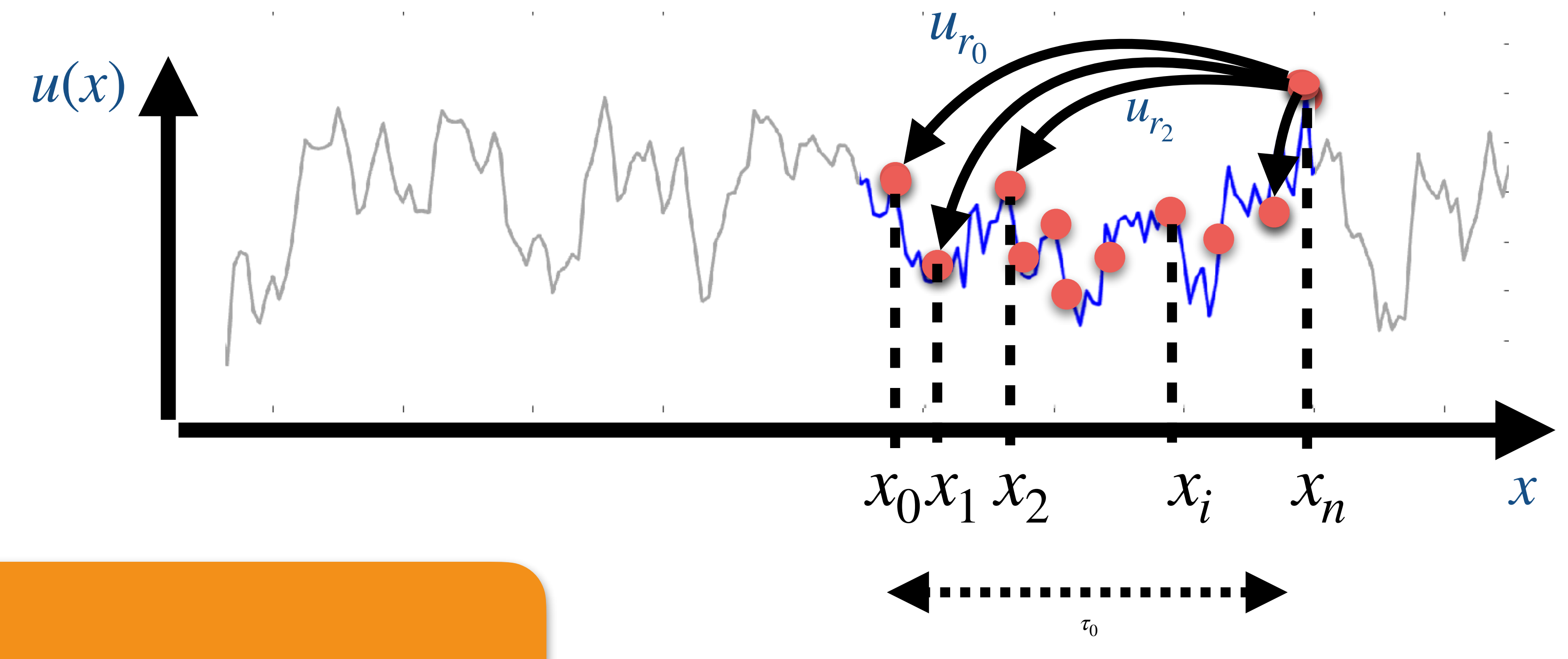
An open source package to perform basic and advanced statistical analysis of turbulence data and other complex systems

Cite as: Phys. Fluids **34**, 101801 (2022); https://doi.org/10.1063/5.0107974
Submitted: 07 July 2022 • Accepted: 16 September 2022 • Accepted Manuscript Online: 18 September 2022 • Published Online: 21 October 2022

http://github.com/andre-fuchs-uni-oldenburg/open_fpe_ift

2nd lecture ???

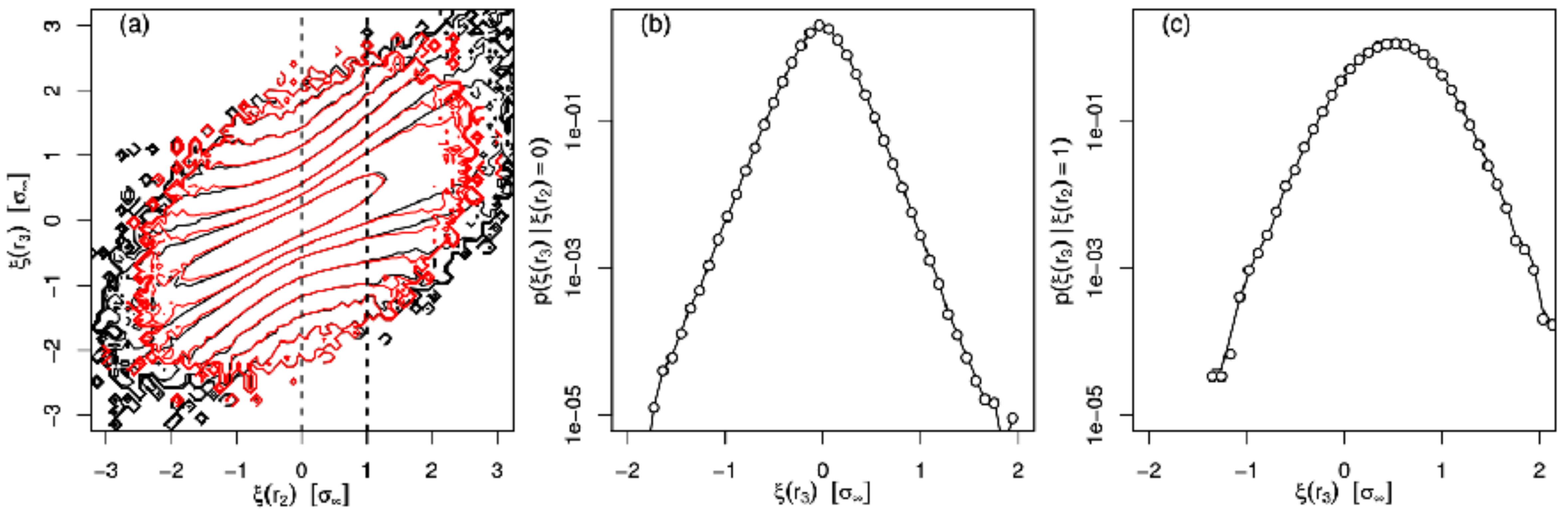
Joint- n-point statistics simplifies:



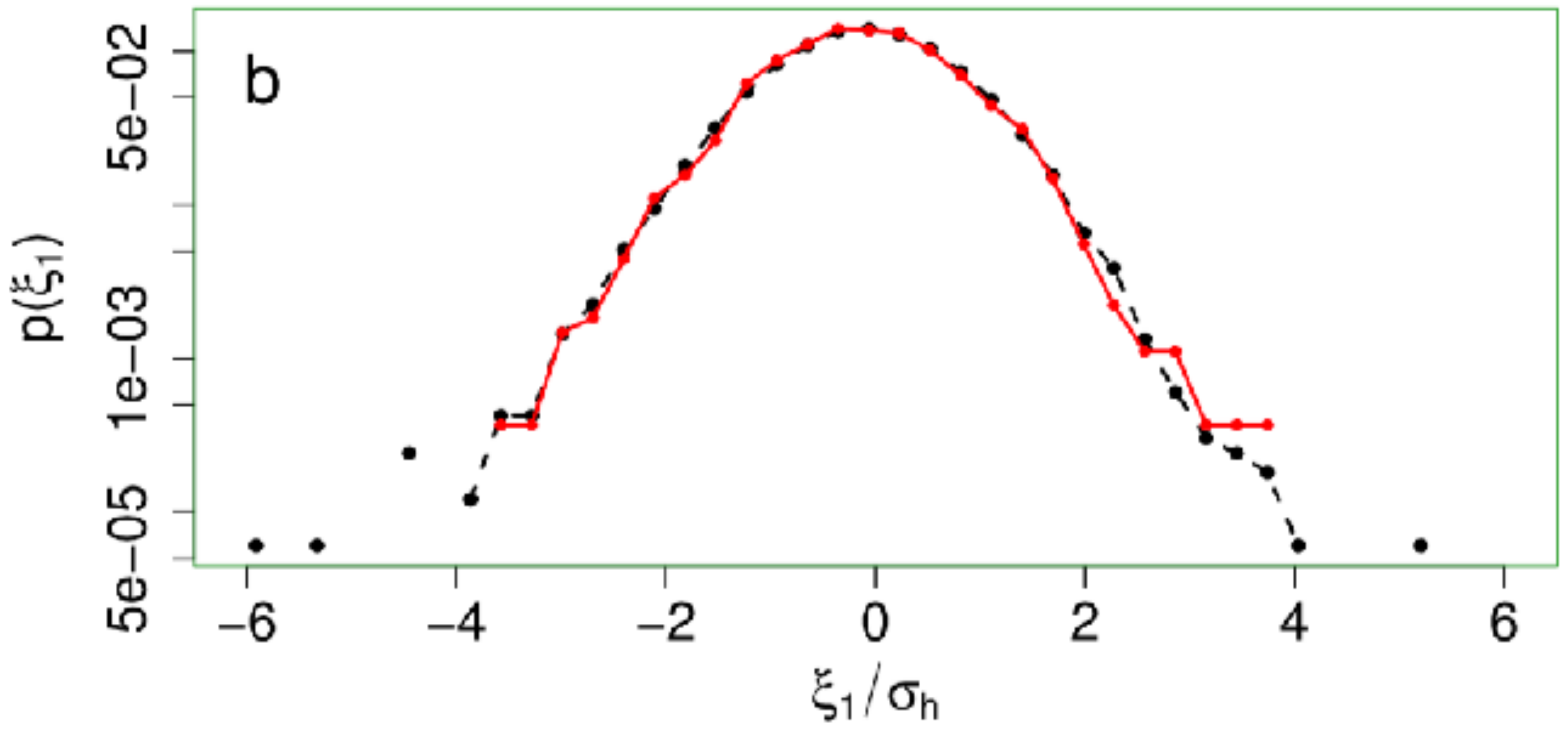
$$p(u_{r_{n-1}}|u_{r_{n-2}},....,u_{r_0},u(x_n))p(u_{r_{n-2}}|....)....\ p(u_{r_0}|u(x_n))\ p(u(x_n)) = p(u_{r_{n-1}}|u_{r_{n-2}},u(x_n))p(u_{r_{n-2}}|.u_{r_{n-3}},u(x_n)$$

$$p(u_{r_i}|u_{r_{i-1}},....,u_{r_0}) = p(u_{r_i}|u_{r_{i-1}},u(x_n))$$

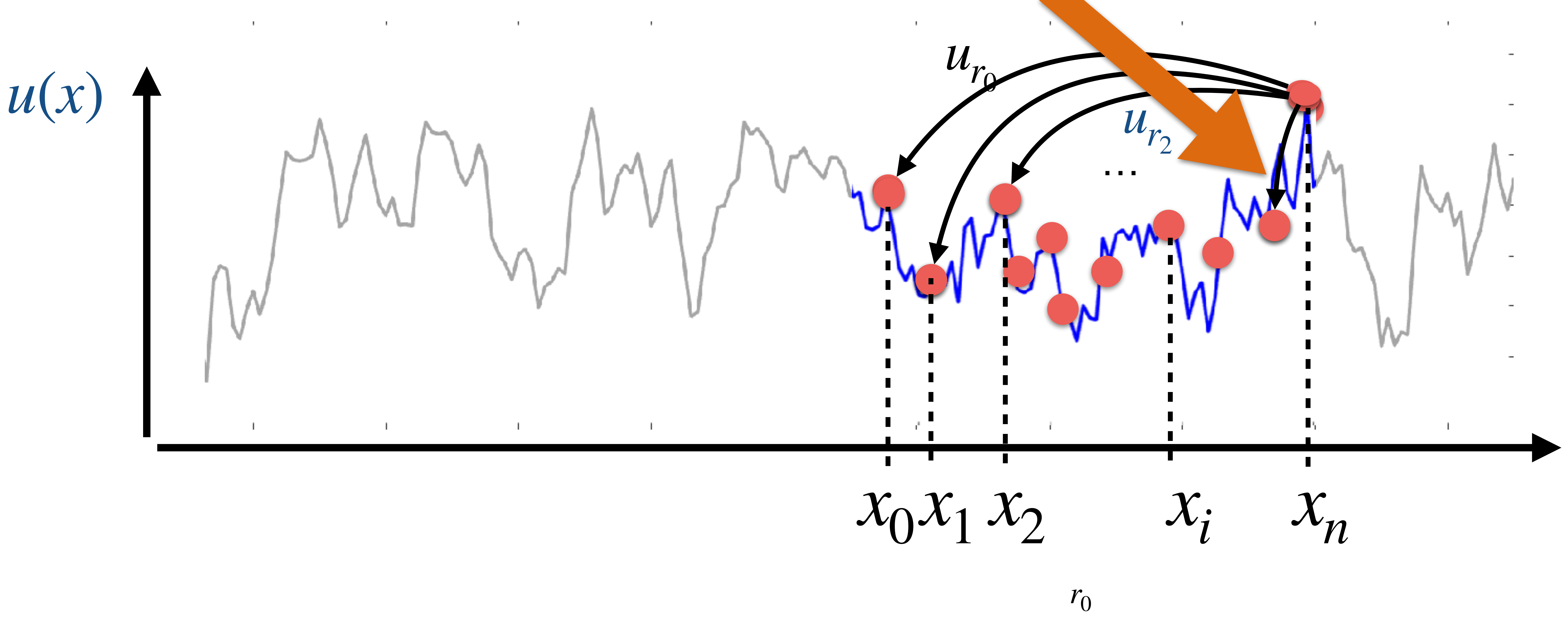
Turbulence

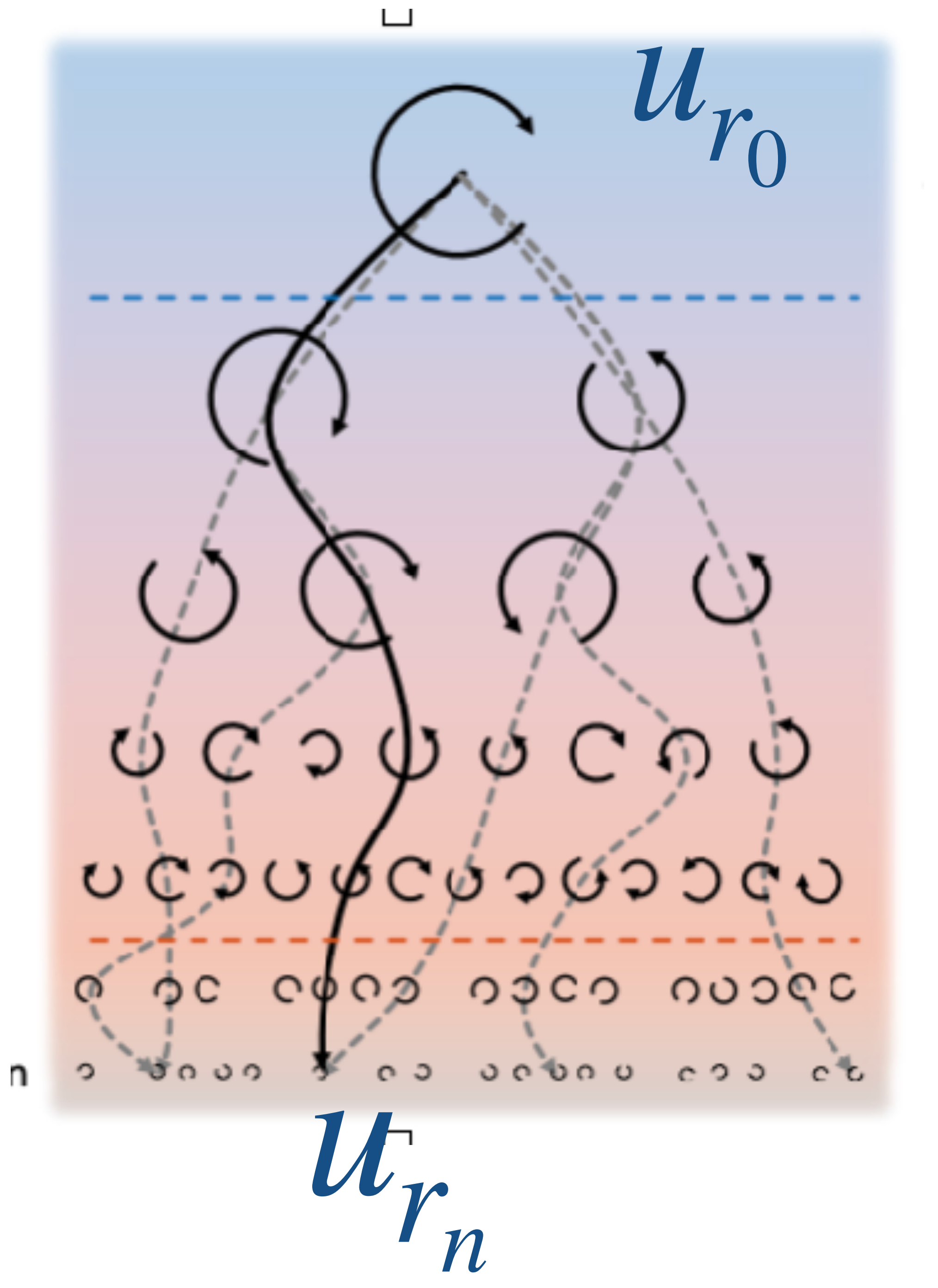


Waves



Joint- n-point statistics and three point closure => Markow process in scale





increments $u_r = u(x+r) - u(x)$

Cascade path $u(\cdot)$ goes from u_{r_0} to u_{r_n} and $r_0 > r_1 > \dots r > \dots r_n$

Markov- process in scale -> Fokker Planck equation (Kolmogorov equ.)

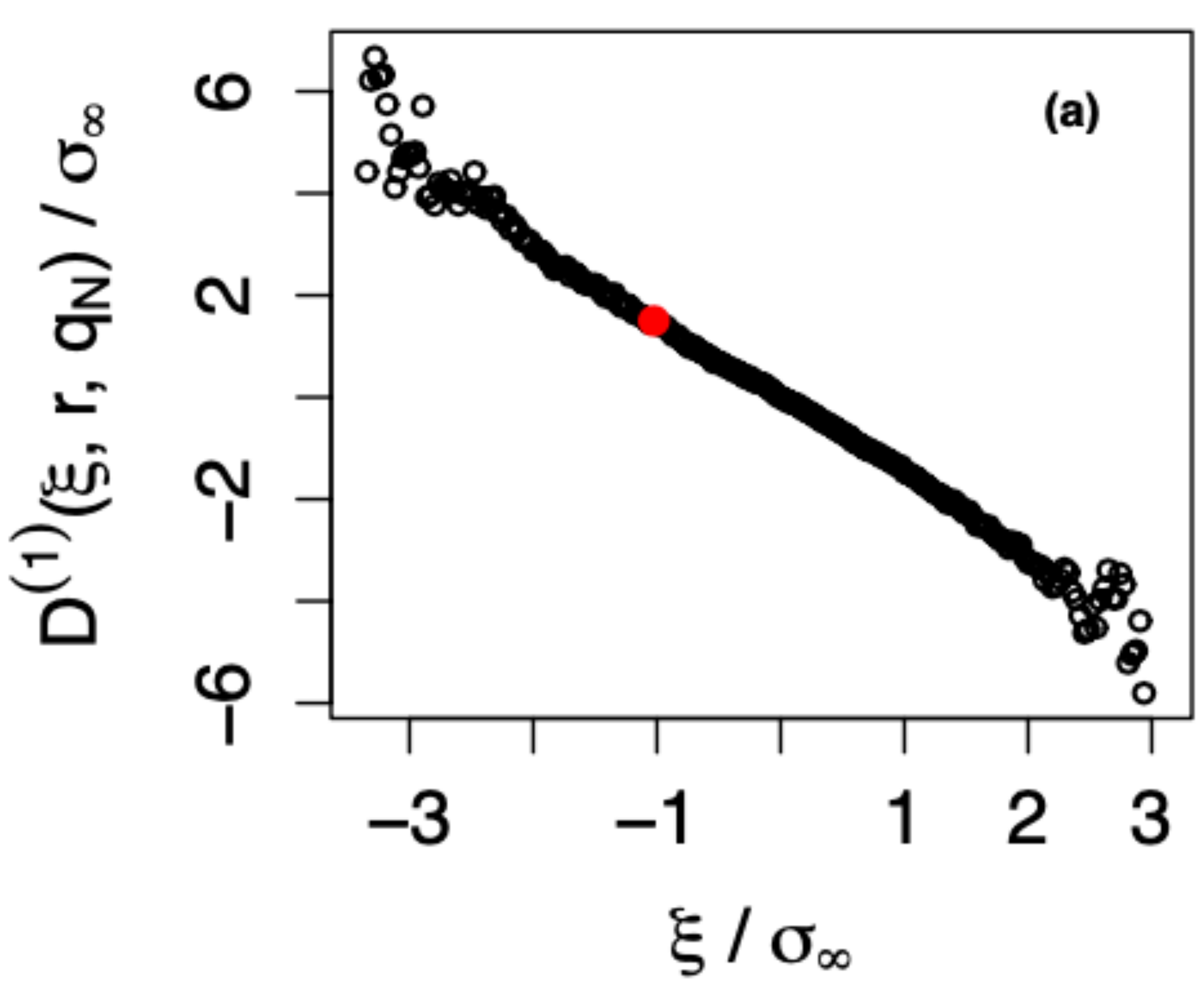
$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j}|u_{r_k}, u(x_1)) = \{-\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1))\} \, p(u_{r_j}|u_{r_k}, u(x_1))$$

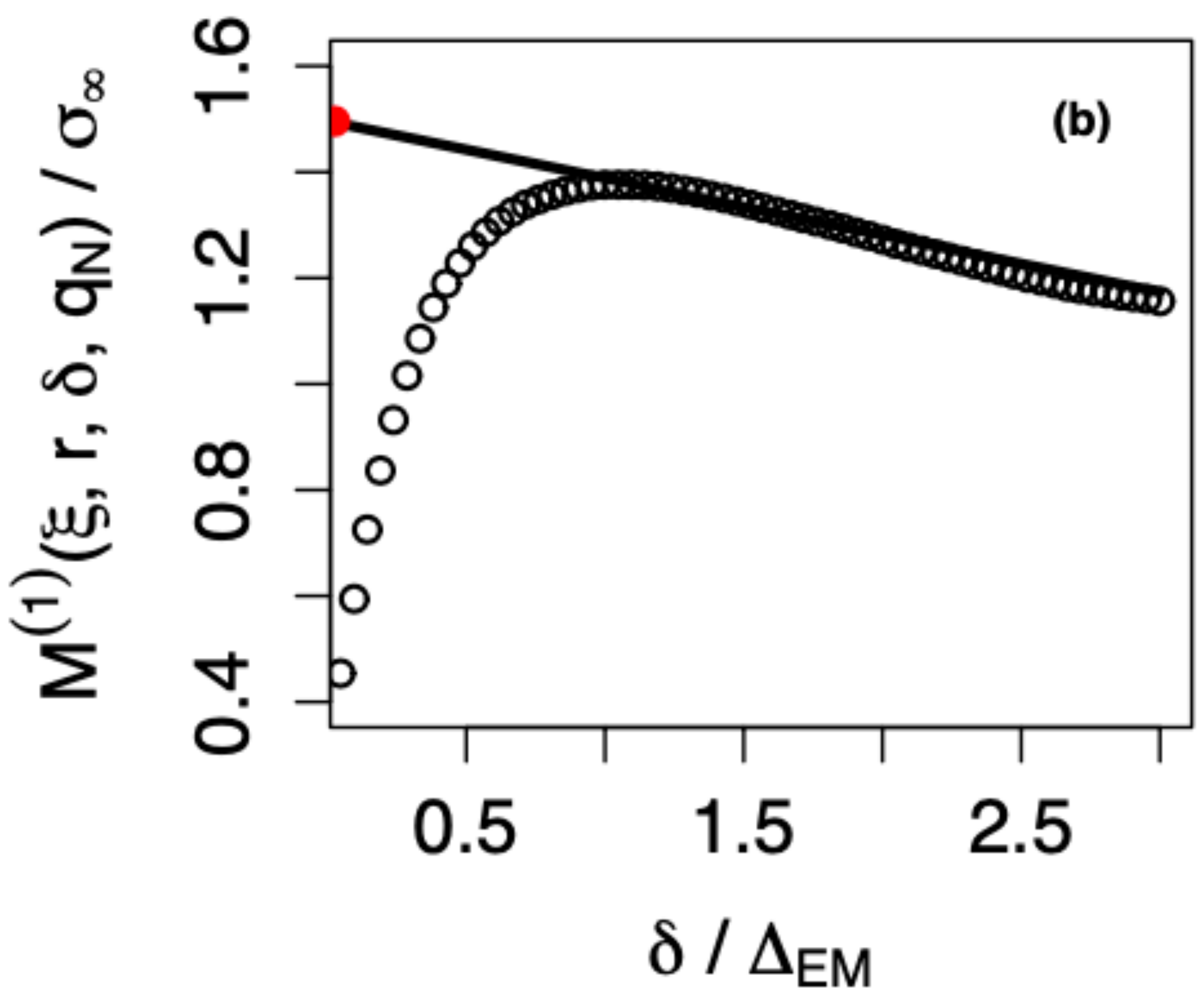
Drift and Diffusion coefficient can be **"measured"**

Kramer_Moyal coeff.

$$D^{(k)}(u,r) = \lim_{\Delta r \to 0} \frac{r}{k!\Delta r} M^{(k)}(u,r,\Delta r),$$

$$M^{(k)}(u,r,\Delta r) = \int_{-\infty}^{+\infty} (\tilde{u}-u)^k p(\tilde{u}, r-\Delta r | u, r) d\tilde{u}$$



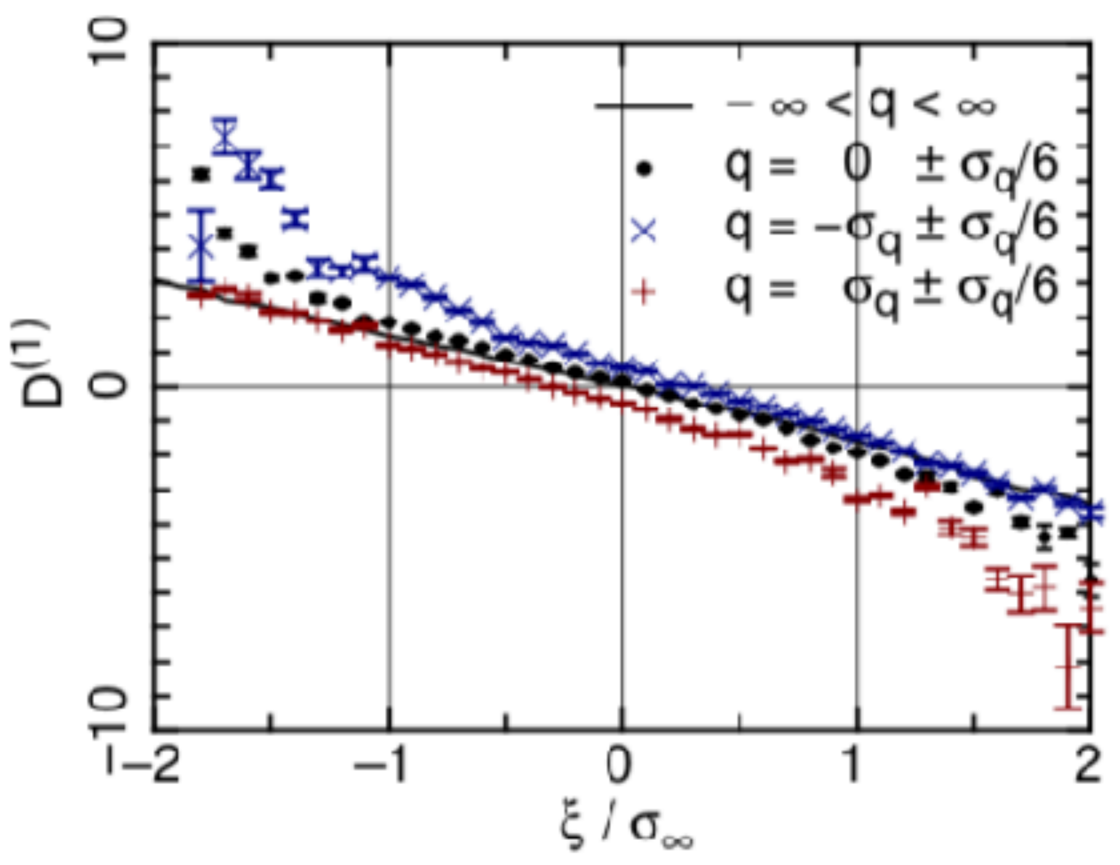


multi-point characterisation (FP for turbulence)

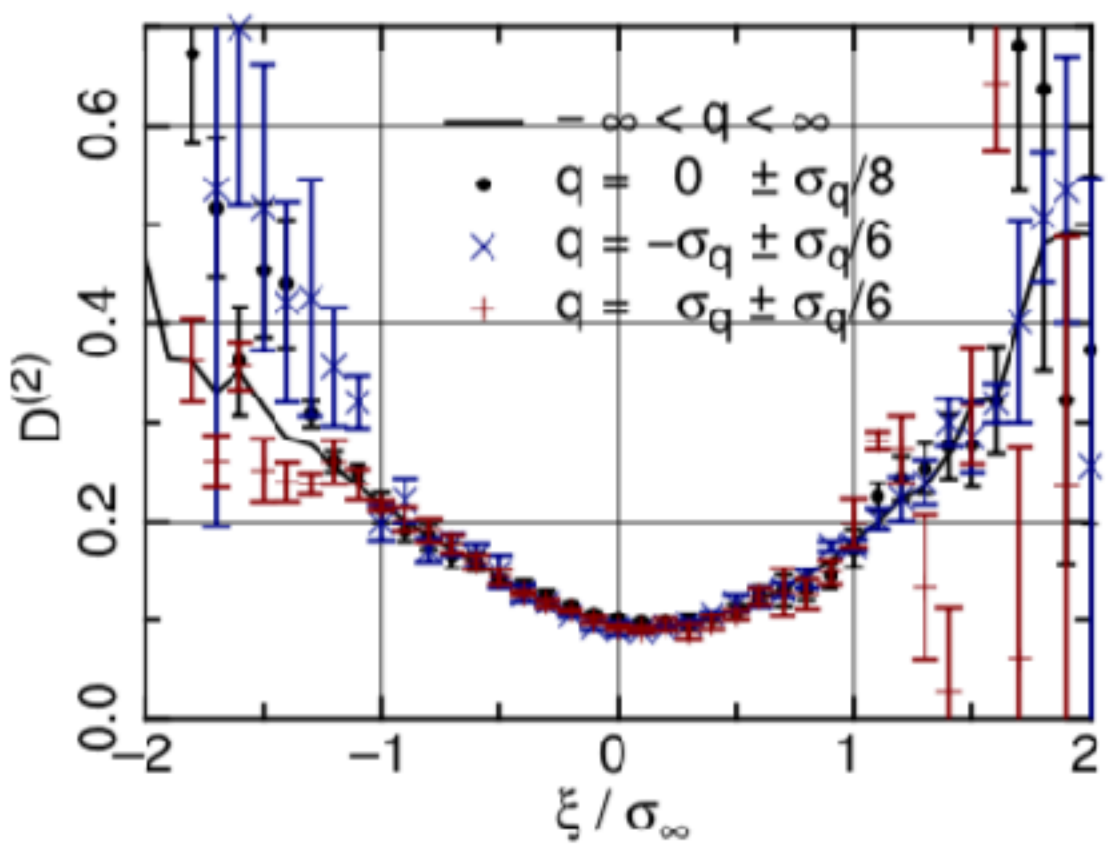
$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j}|u_{r_k},u(x_1)) = \{-\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j},r_j,u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j},r_j,u(x_1))\}\ p(u_{r_j}|u_{r_k},u(x_1))$$

Drift and Diffusion coefficient can be "measured"

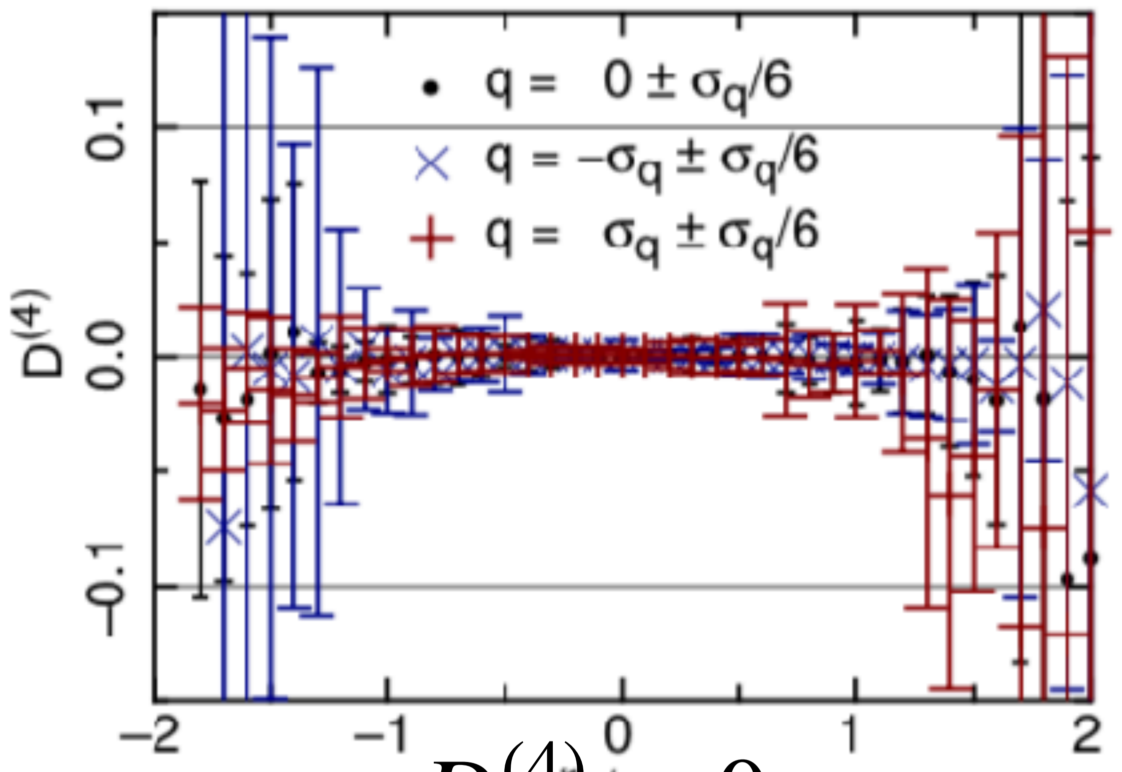
Drift - $D^{(1)}$



Diffusion - $D^{(2)}$



Higher order KM



$D^{(4)} = 0$

$$D^{(1)} \propto u_r$$

$$D^{(2)} \propto A + u_r^2$$

White noise - **Paula theorem**

=> Langevin equ

And Fokker-Planck equ.

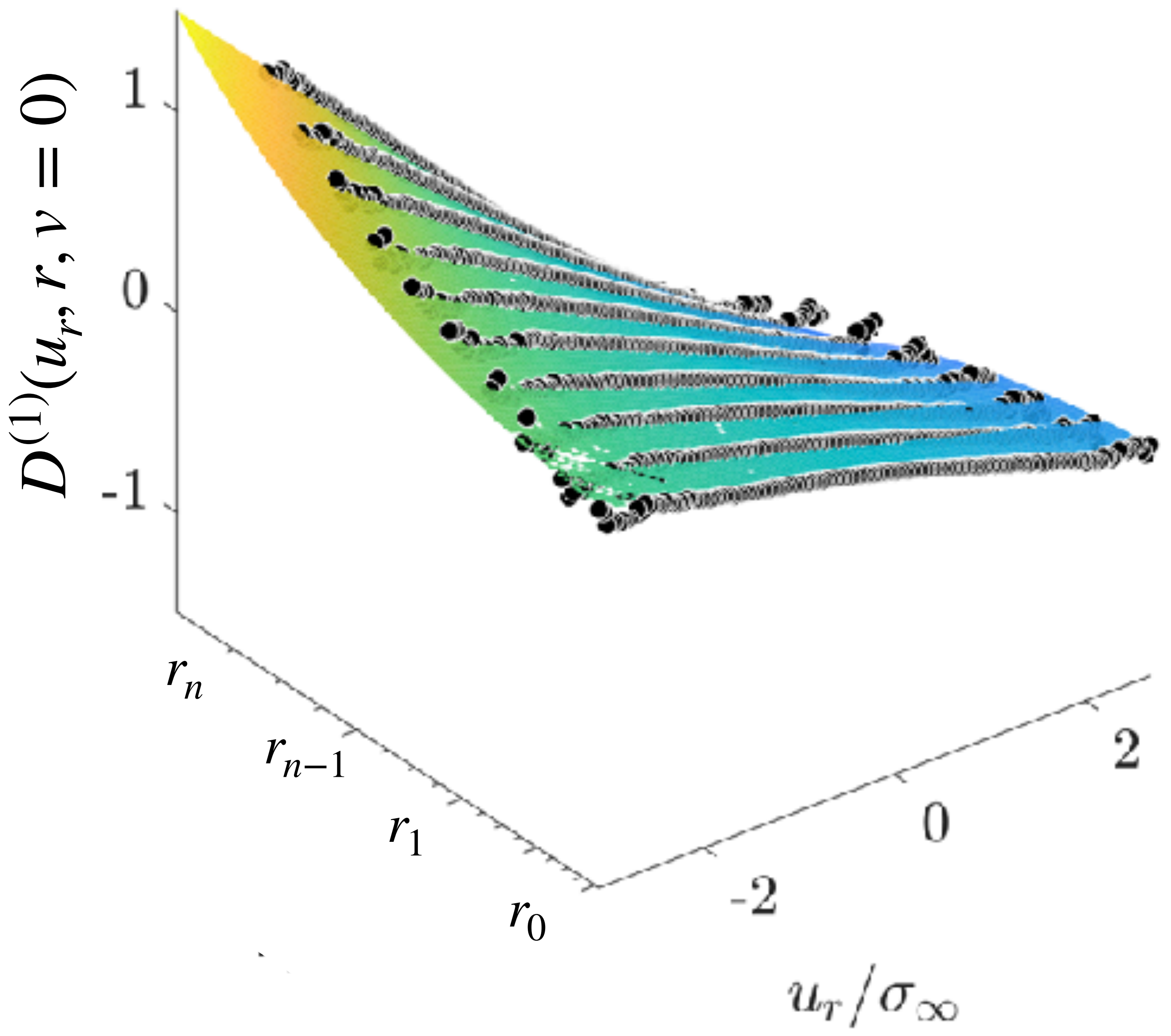
Stresing et.al. New Journal of Physics **12** (2010)

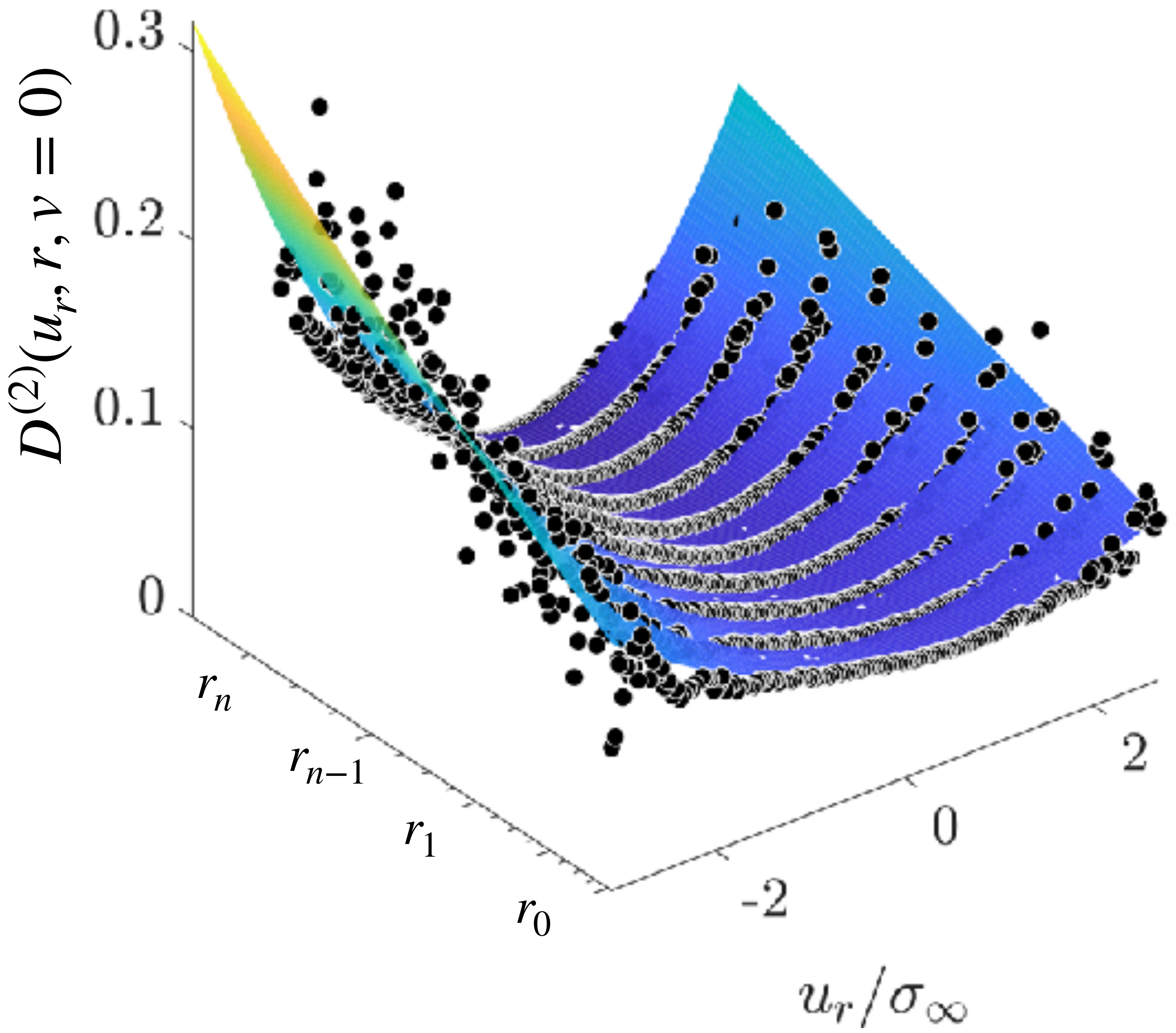
Renner et.al JFM (2001)

multi-point characterisation (FP for turbulence)

$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j}|u_{r_k}, u(x_1)) = \{-\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1))\}\, p(u_{r_j}|u_{r_k}, u(x_1))$$

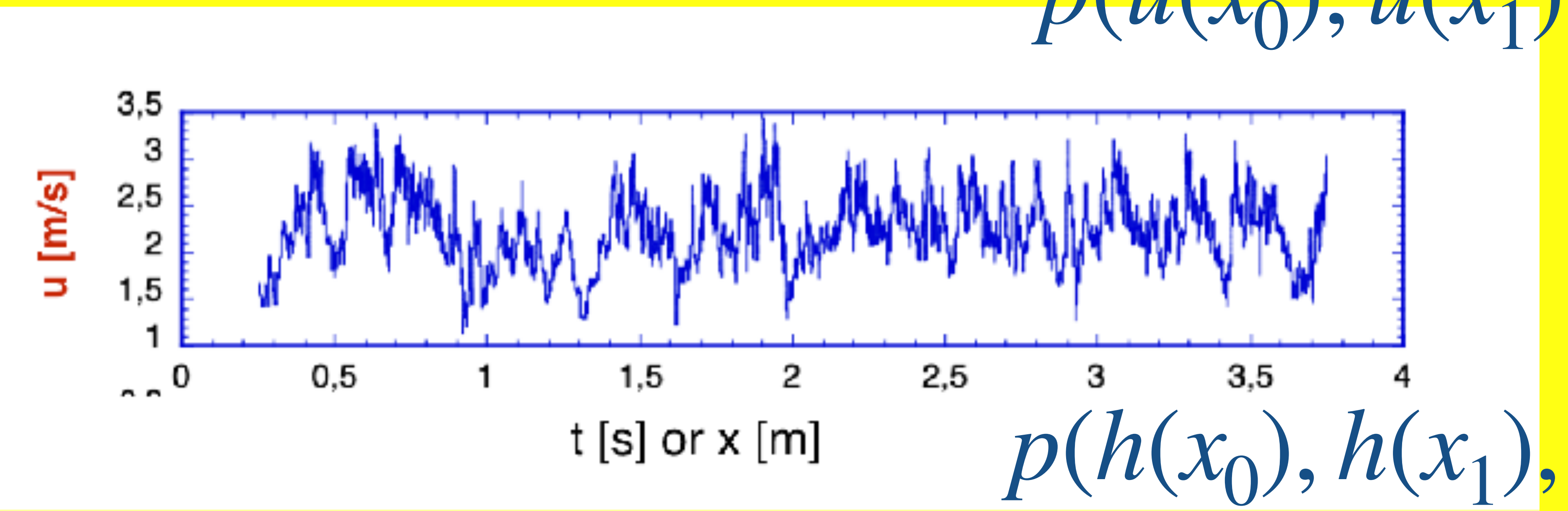
Drift and Diffusion coefficient as a function of r can be “measured”





Recipe for stochastic analysis

$p(u(x_0), u(x_1), \ldots, u(x_n))$



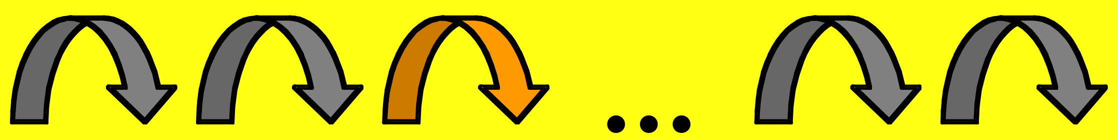
$p(h(x_0), h(x_1), \ldots, h(x_n))$

data

Joint - n-point statistics reduces to

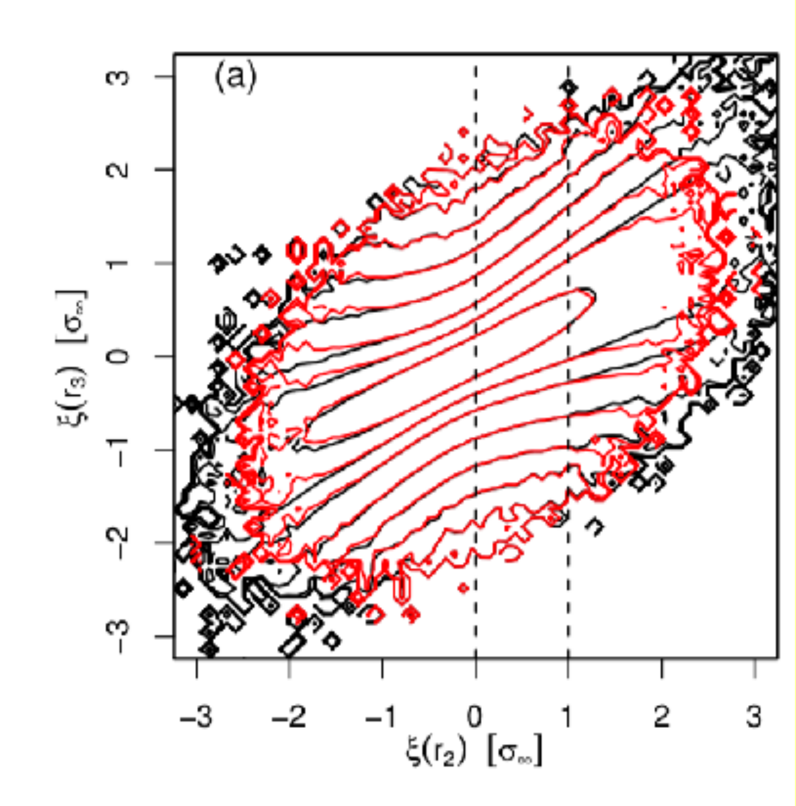
conditional pdfs

L ⟶ r ⟶ η



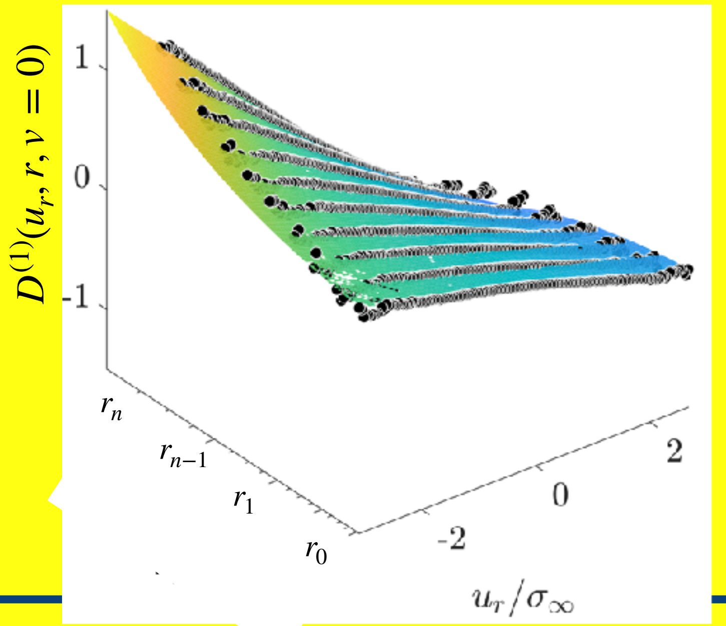
$p(u_{r_i} | u_{r_{i+1}}, u(x_n))$

$p(h_{r_i} | h_{r_{i+1}}, h(x_n))$

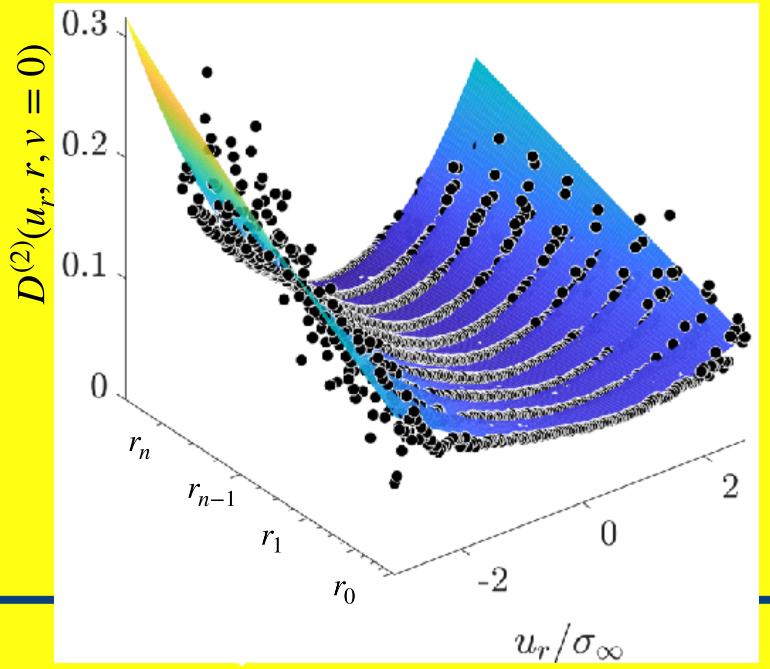


Fokker-Planck equation describes the cond- pdf

Drift - $D^{(1)}$



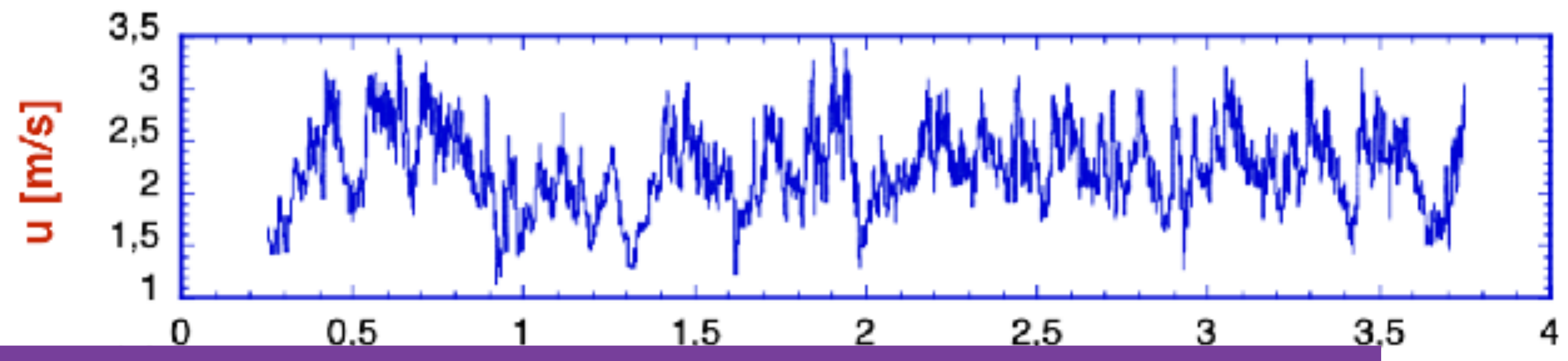
Diffusion - $D^{(2)}$



Recipe for stochastic analysis

$$p(u(x_0), u(x_1), \ldots, u(x_n))$$

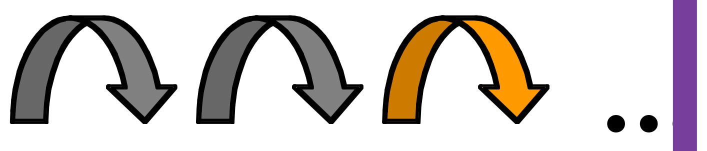
data

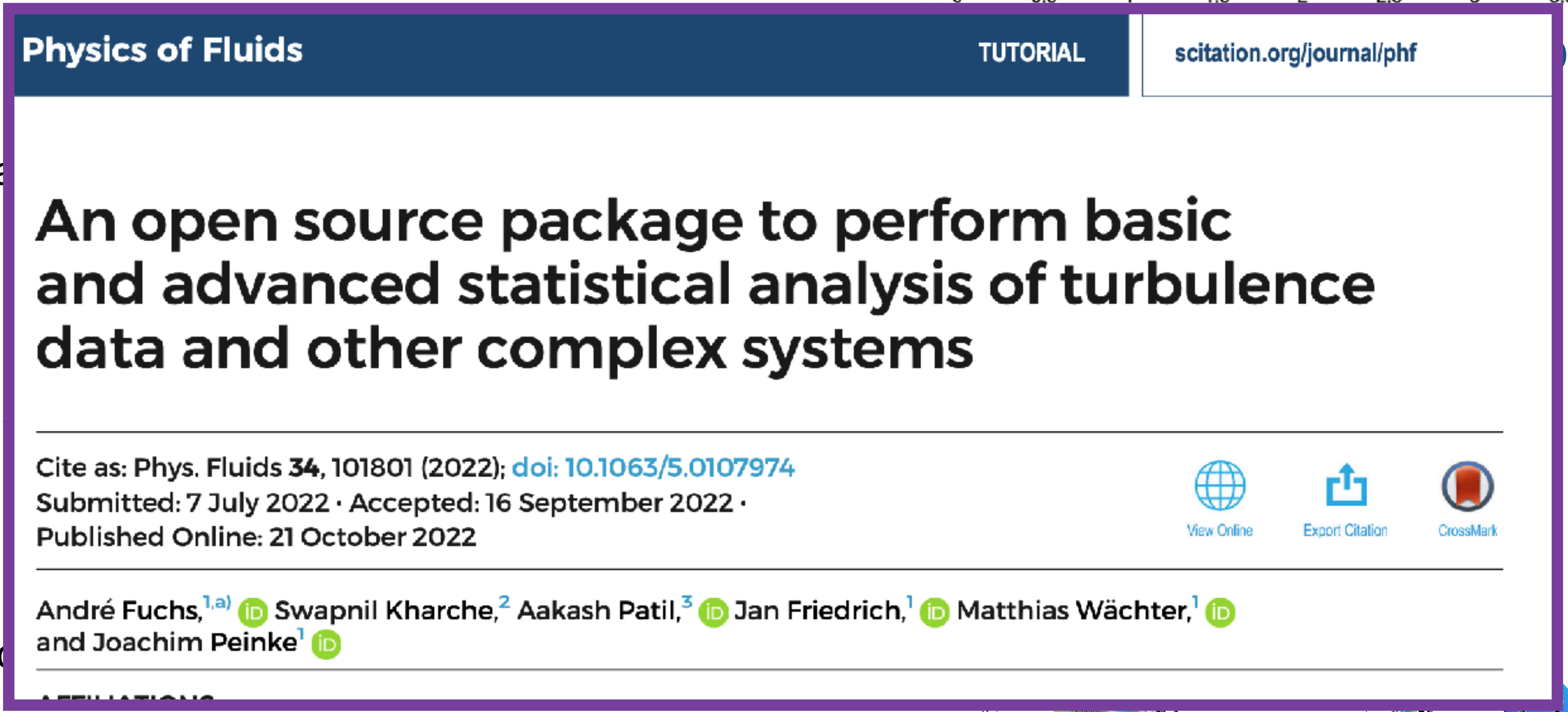


$$), h(x_1), \ldots, h(x_n))$$

Joint - n-point sta

L → r





Physics of Fluids TUTORIAL scitation.org/journal/phf

An open source package to perform basic and advanced statistical analysis of turbulence data and other complex systems

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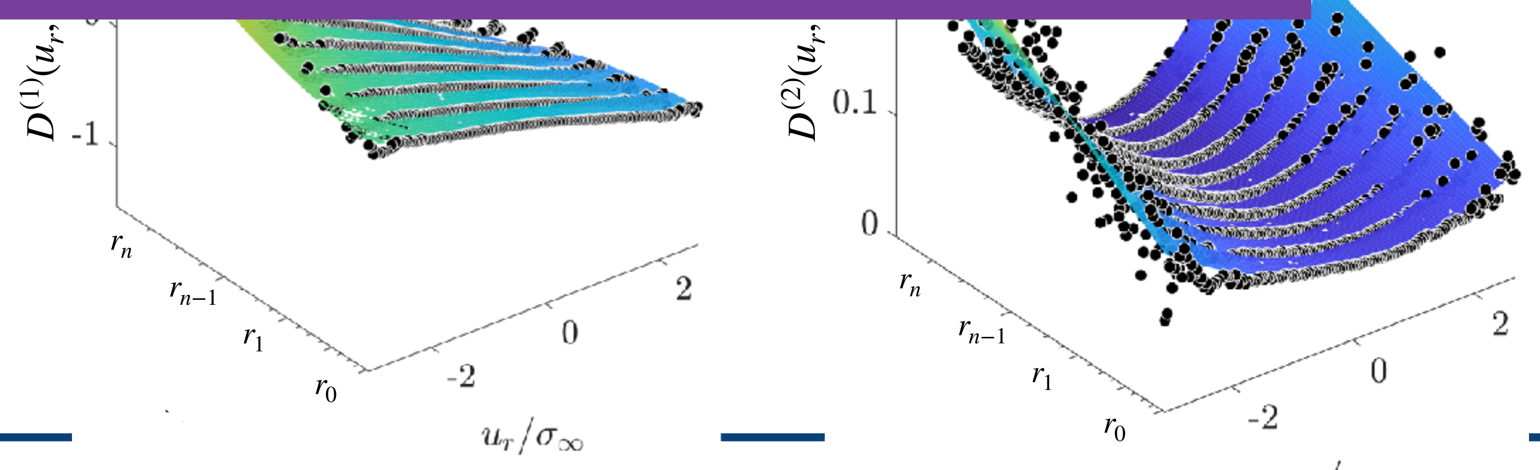
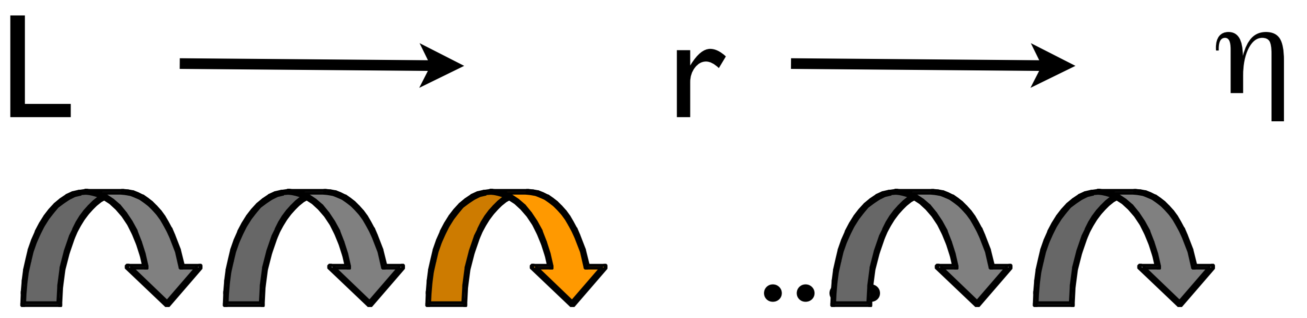
André Fuchs,[1,a)] Swapnil Kharche,[2] Aakash Patil,[3] Jan Friedrich,[1] Matthias Wächter,[1] and Joachim Peinke[1]

Fokker-Planck ec

$D^{(2)}$



stochastic cascade process: Fokker - Planck equation



$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j}|u_{r_k}, u(x_1)) = \{-\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1))\}\ p(u_{r_j}|u_{r_k}, u(x_1))$$

What can we do with this?

Statistical non-equilibrium physics

IOP Publishing — Reports on Progress in Physics
Rep. Prog. Phys. **75** (2012) 126001 (58pp) doi:10.1088/0034-4885/75/12/126001

Stochastic thermodynamics, fluctuation theorems and molecular machines

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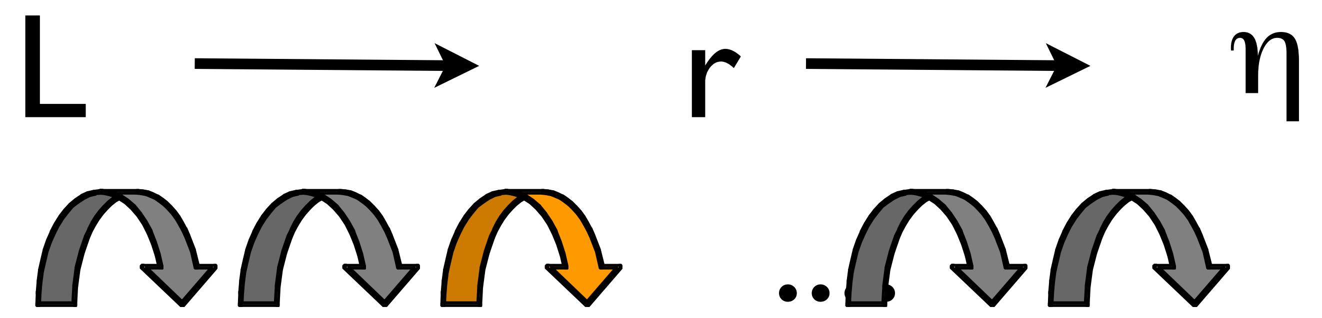
Published 20 November 2012
Online at stacks.iop.org/RoPP/75/126001

stochastic cascade process: Fokker - Planck equation



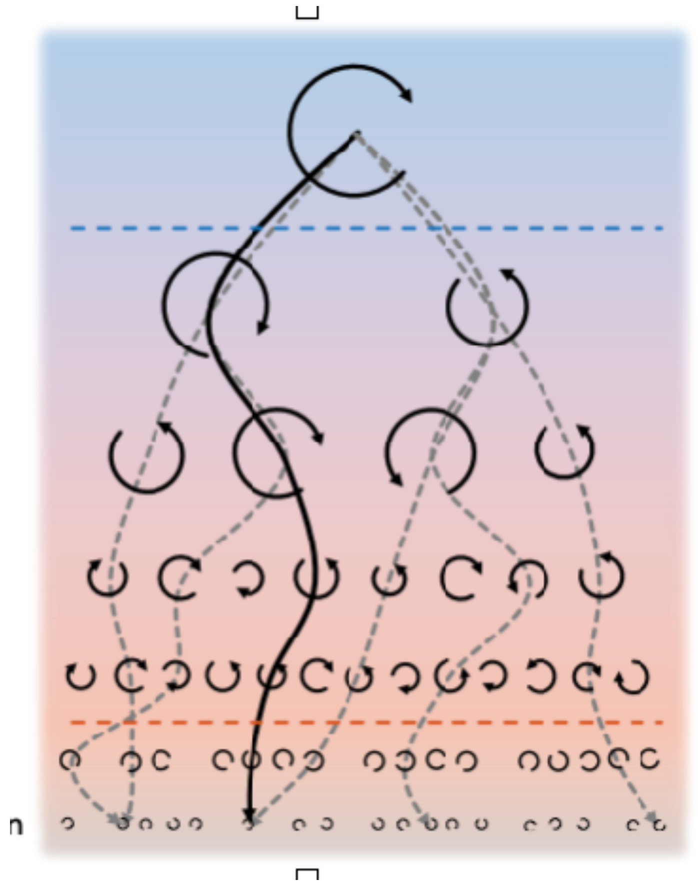
$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j} | u_{r_k}, u(x_1)) = \{ - \frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1)) \} \, p(u_{r_j} | u_{r_k}, u(x_1))$$

- **non equilibrium thermodynamics**
  - **Entropy** (Seifert 2005)
  - **Fluctuation Theorem**
  - Hamiltonian for cascade - instantones

entropy of cascade trajectories

2nd law : entropy balance

there are two contributions of the entropy (cf. Seifert 2005) to the subsystem



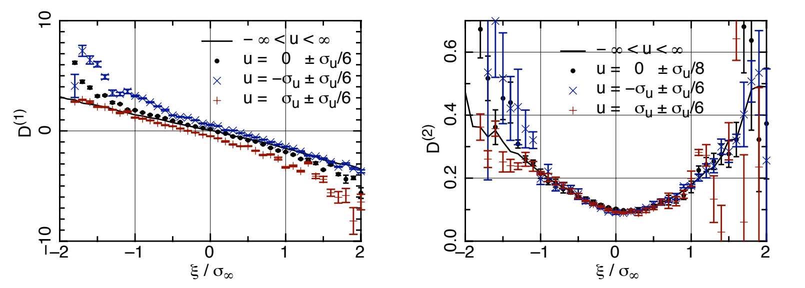
$$S_{tot}(u_r) = S_m(u_r) + \Delta S$$

(1) interaction with the medium results in **power done on the subsystem**

$$\dot{x}F = \dot{x}\partial_x \varphi$$

for the cascade path u_r

$$S_m(u_r) = -\int_L^\lambda \partial_r u_r \, \partial_u \varphi(u_r, r) dr$$



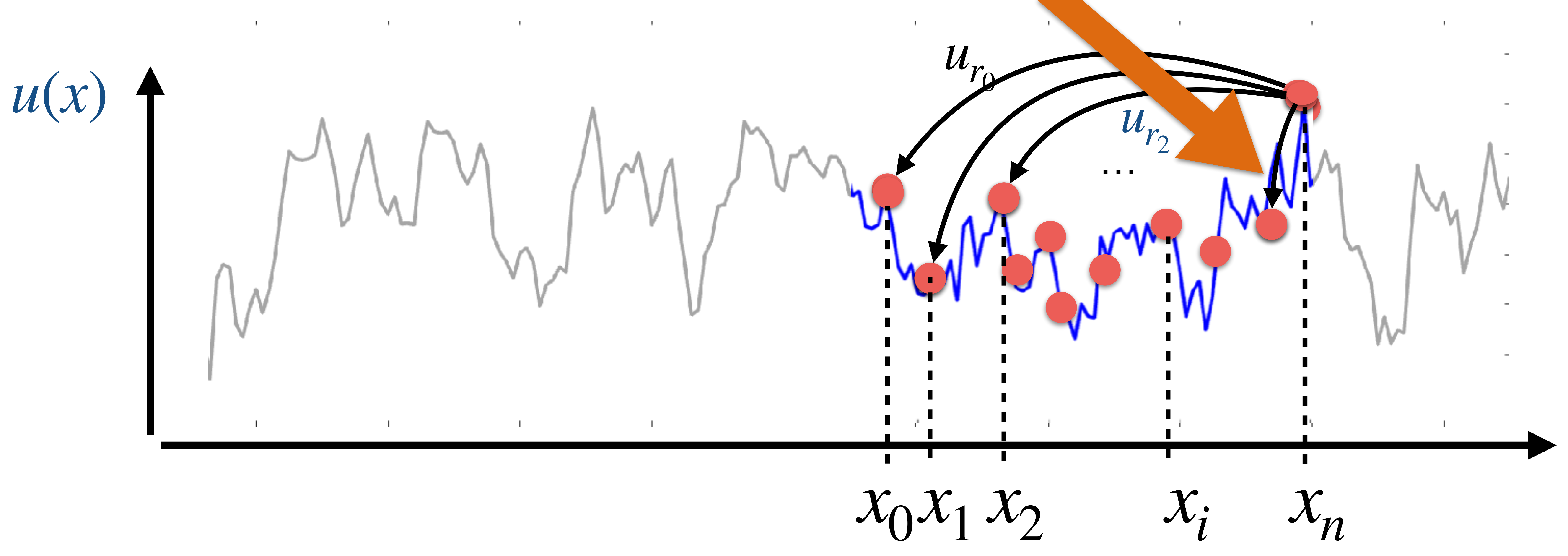
shift of drift function, no *u*-dependence of diffusion function

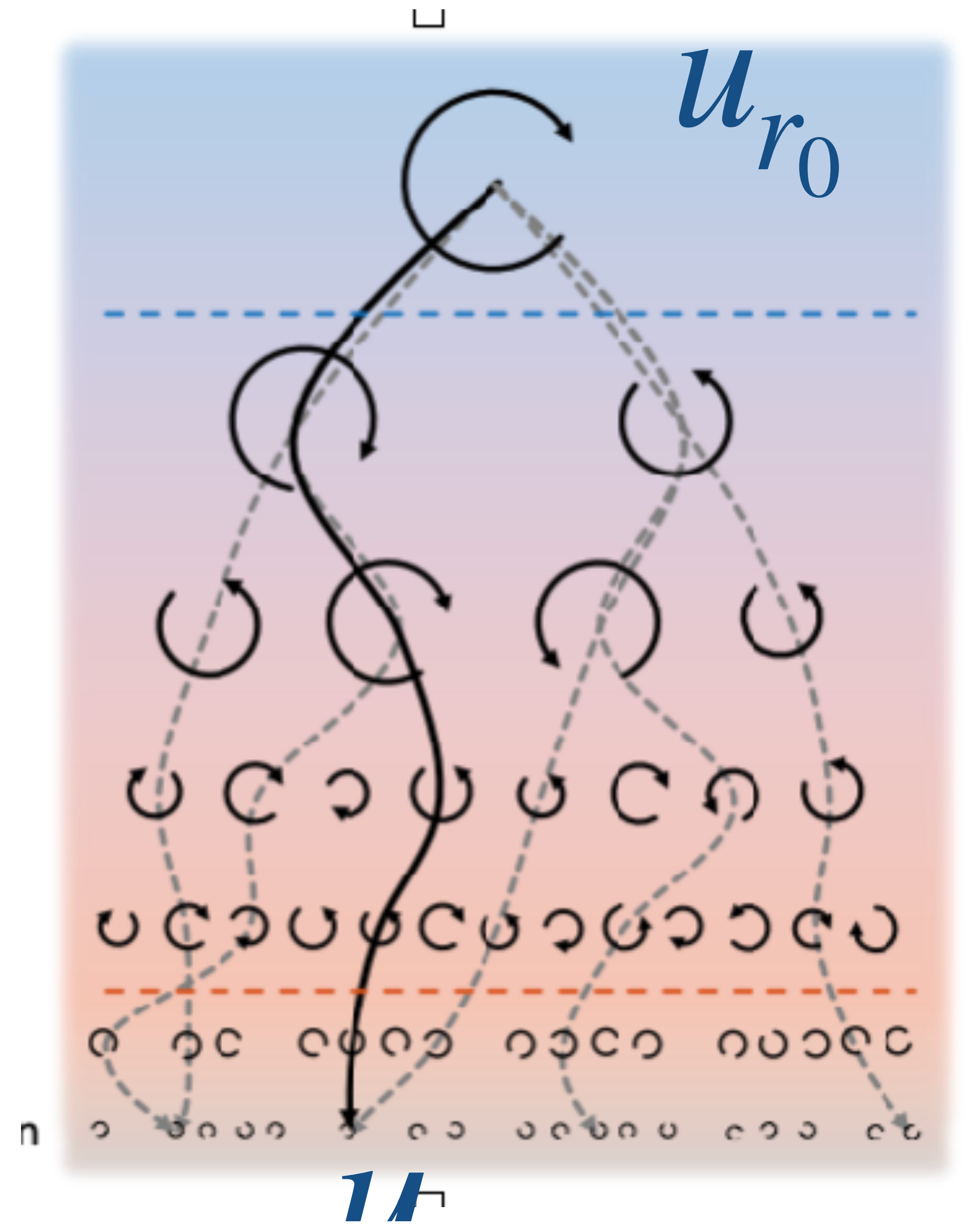
$$\varphi(u_r) = \ln D^{(2)}(u_r, r) - \int_{-\infty}^{u_r} \frac{D^{(1)}(u', r)}{D^{(2)}(u', r)} du'$$

$$\Delta S = -ln \frac{p_\lambda(u_\lambda)}{p_L(u_L)}$$

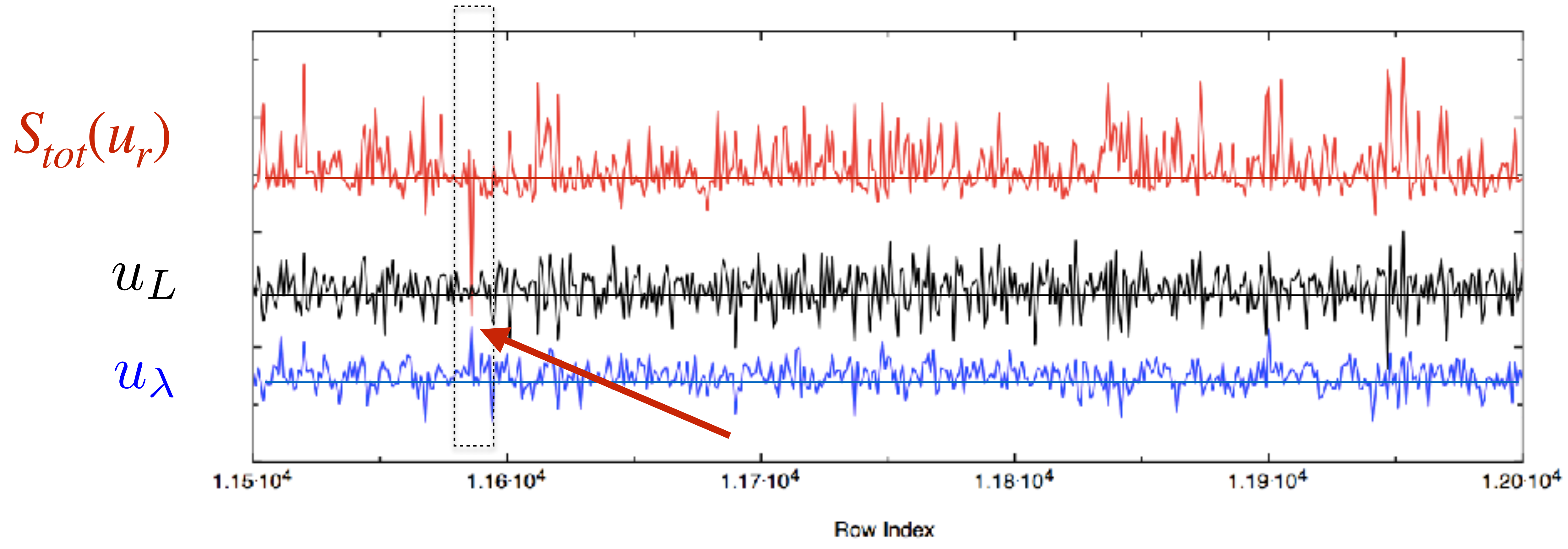
- Nickelsen Engel PRL 110 (2013)

For each trajectory we get an entropy production term





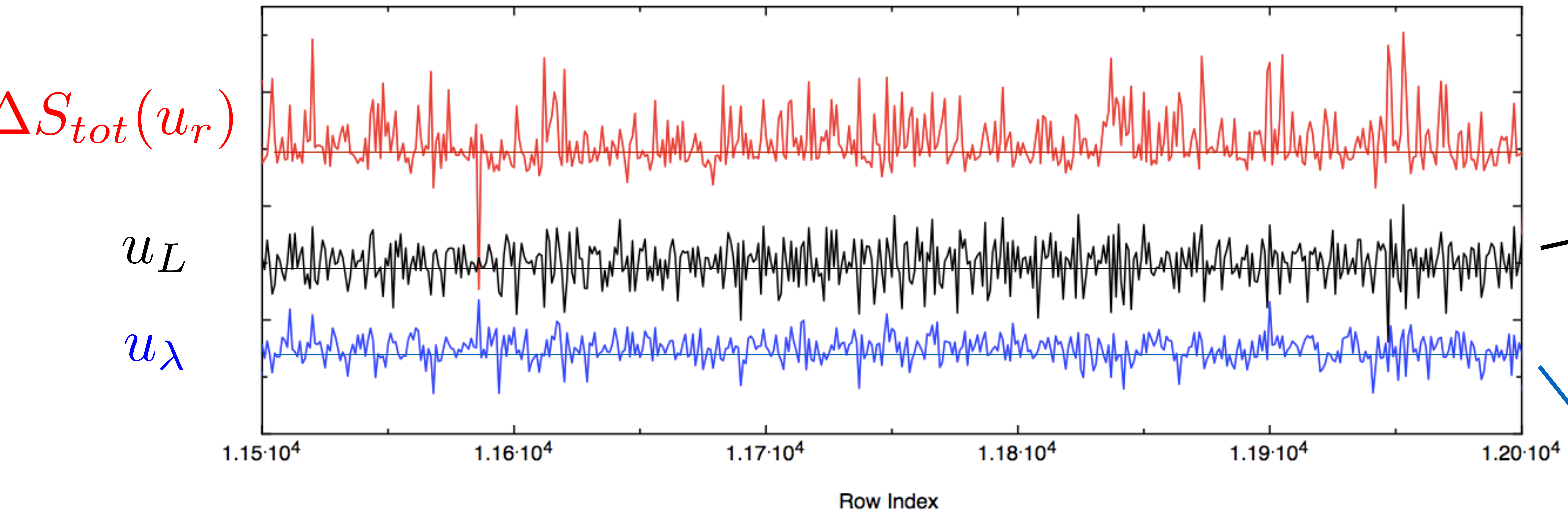
Cascade path $u(\cdot)$ $\quad S_{tot}(u_r) = S_m(u_r) + \Delta S$

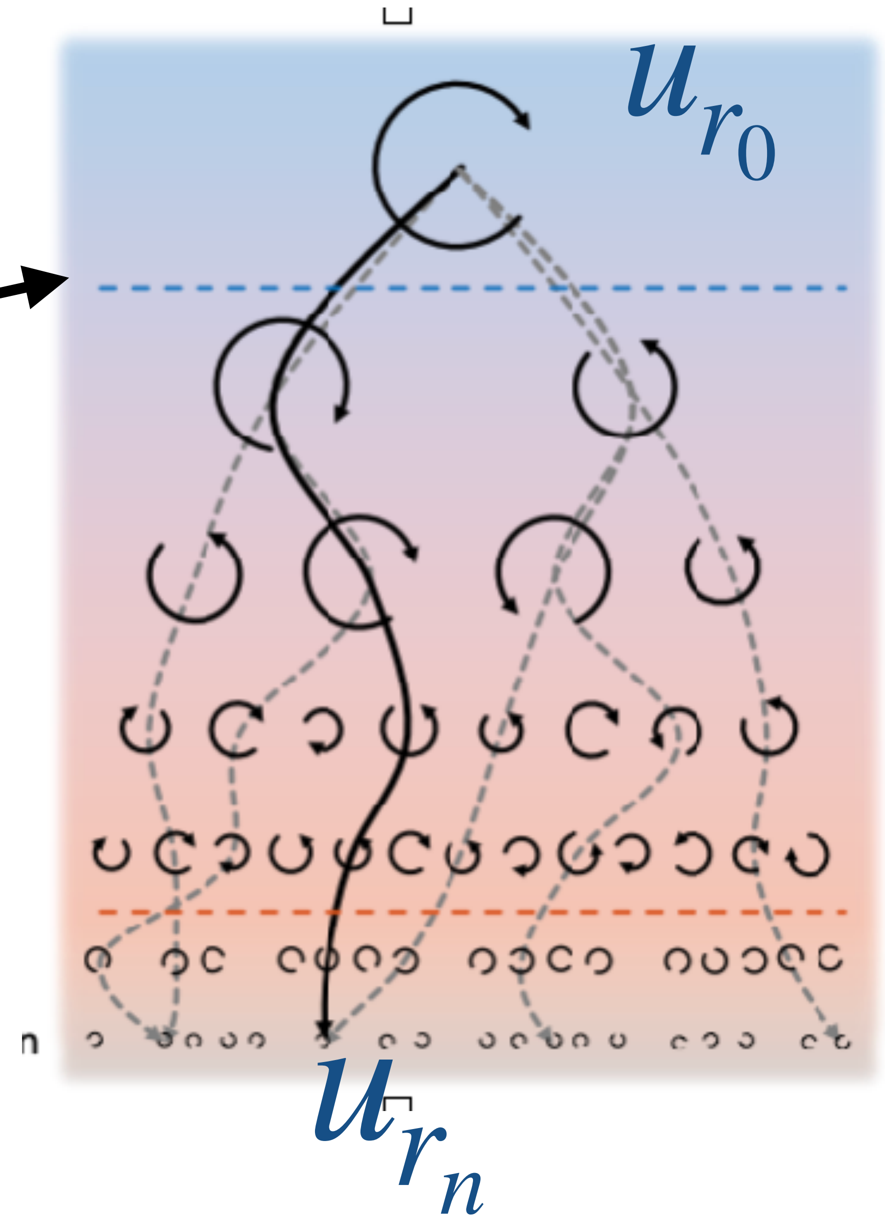


For each trajectory we get an entropy production term

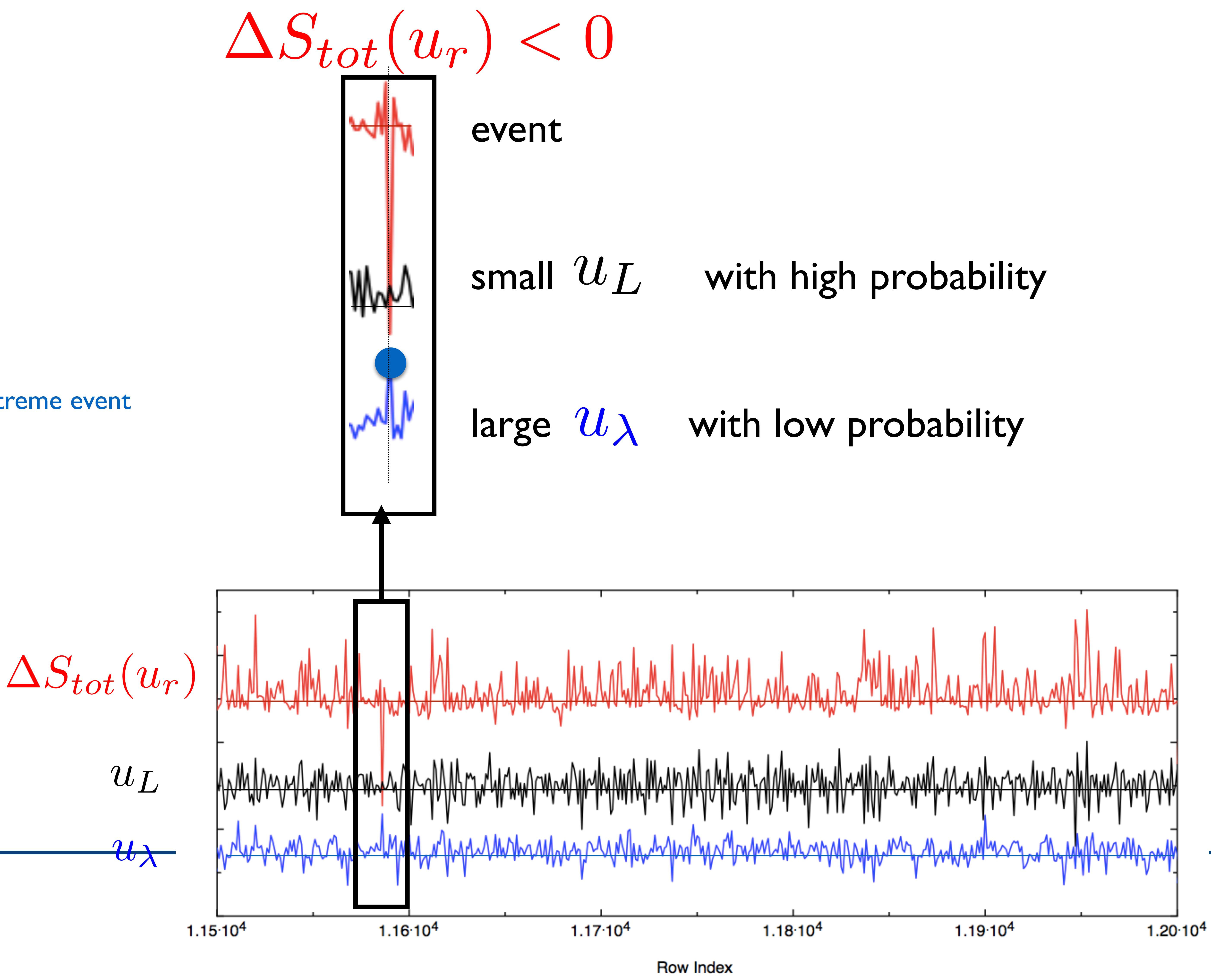
for each cascade path $u(\cdot)$: u_L -> u_r an entropy value is obtained

ΔS is a fluctuating quantity with positive and negative values

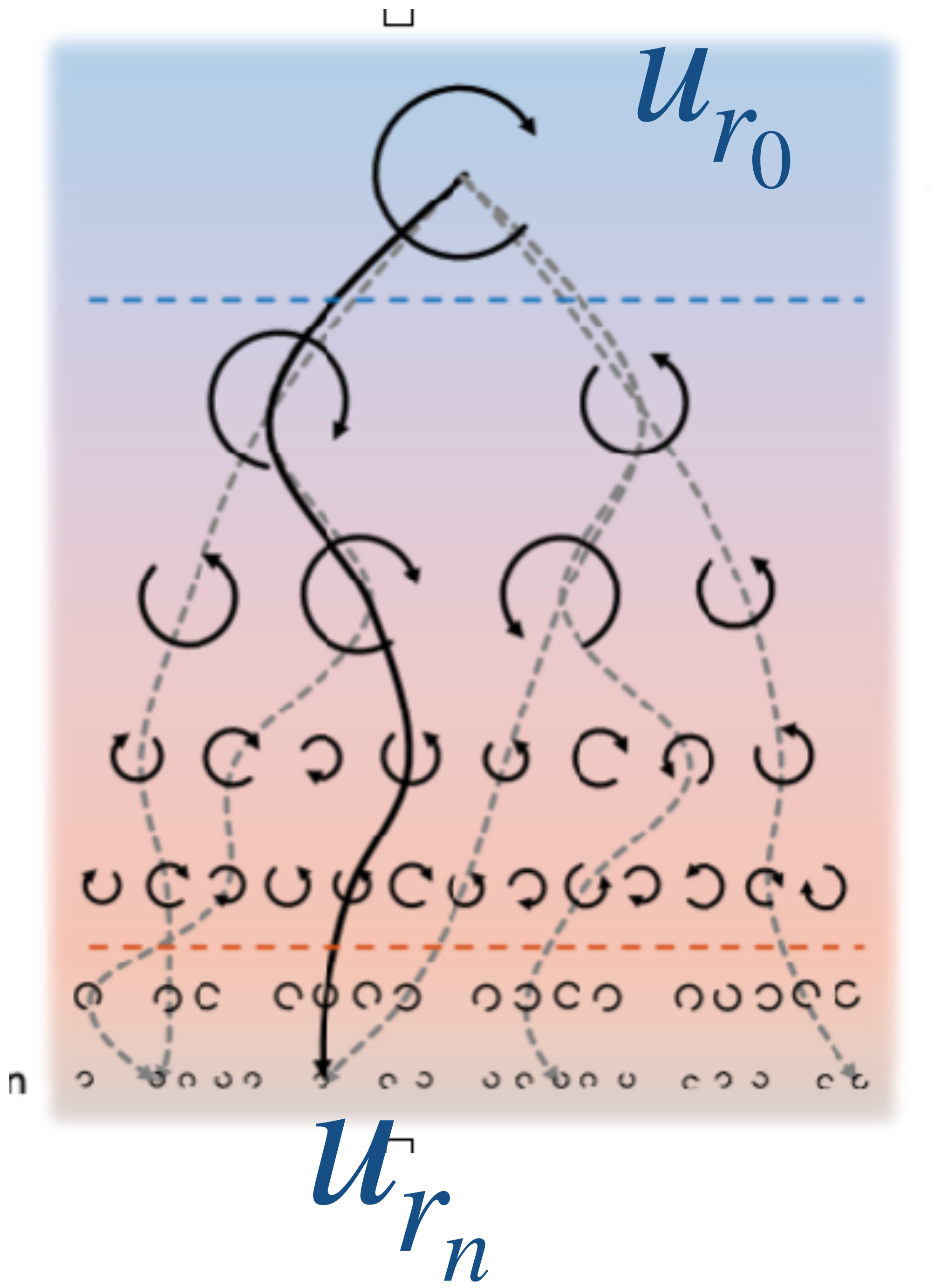




For each trajectory we get an entropy production term

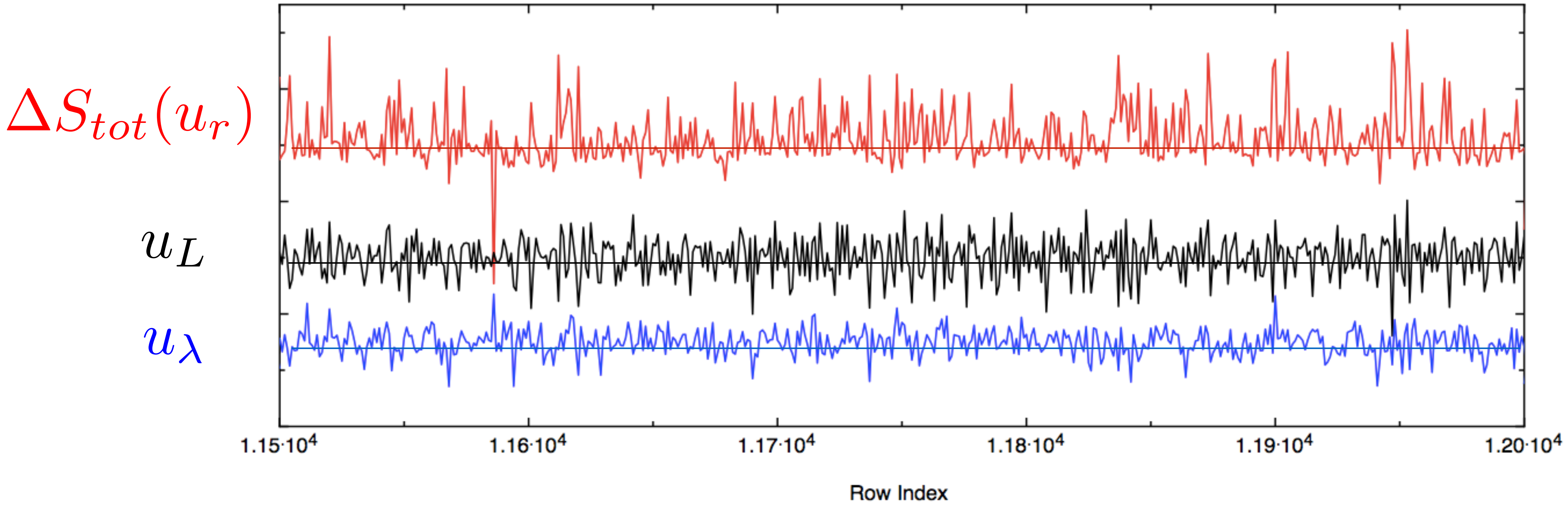


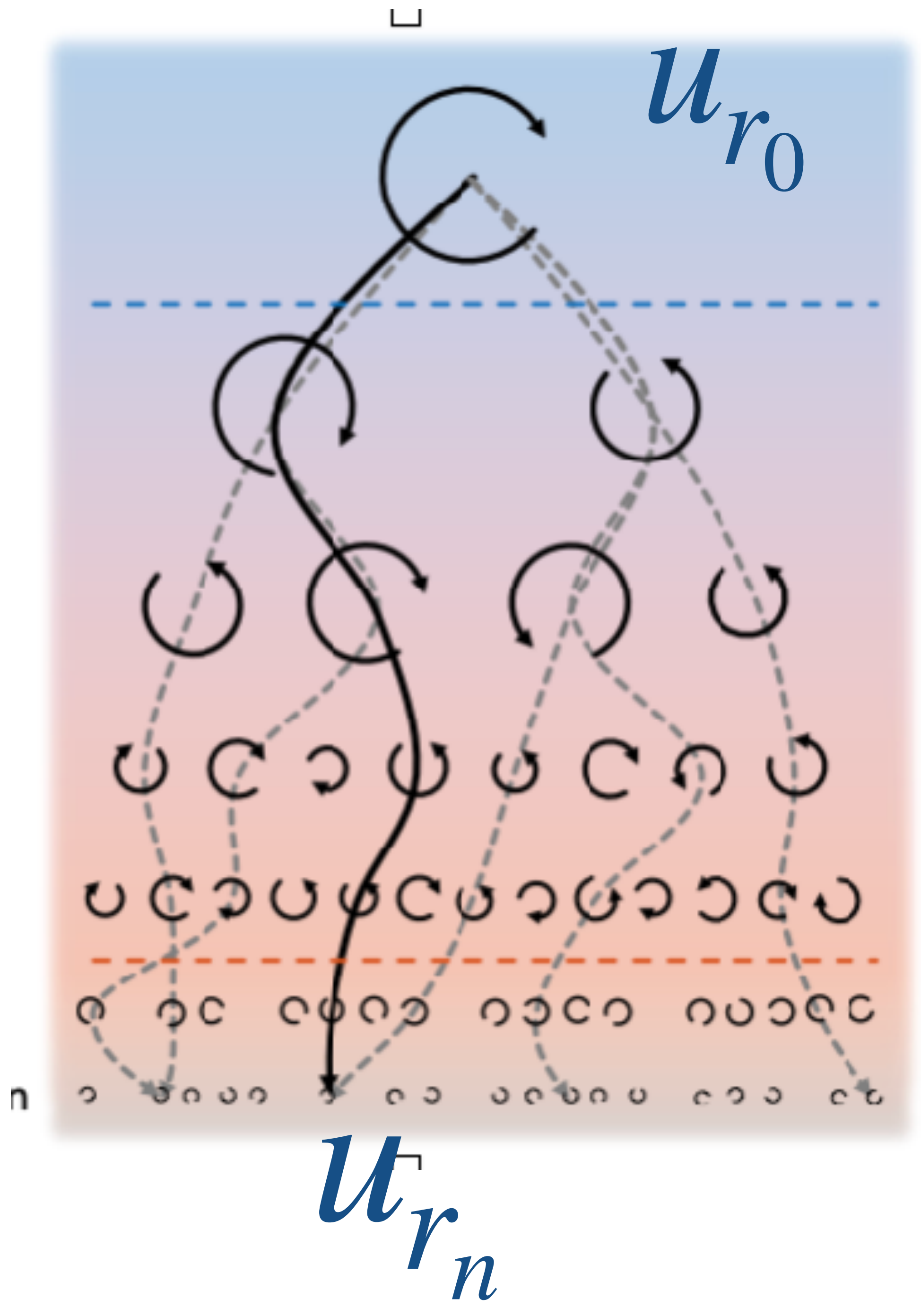
extreme event



For each trajectory we get an entropy production term

So what?

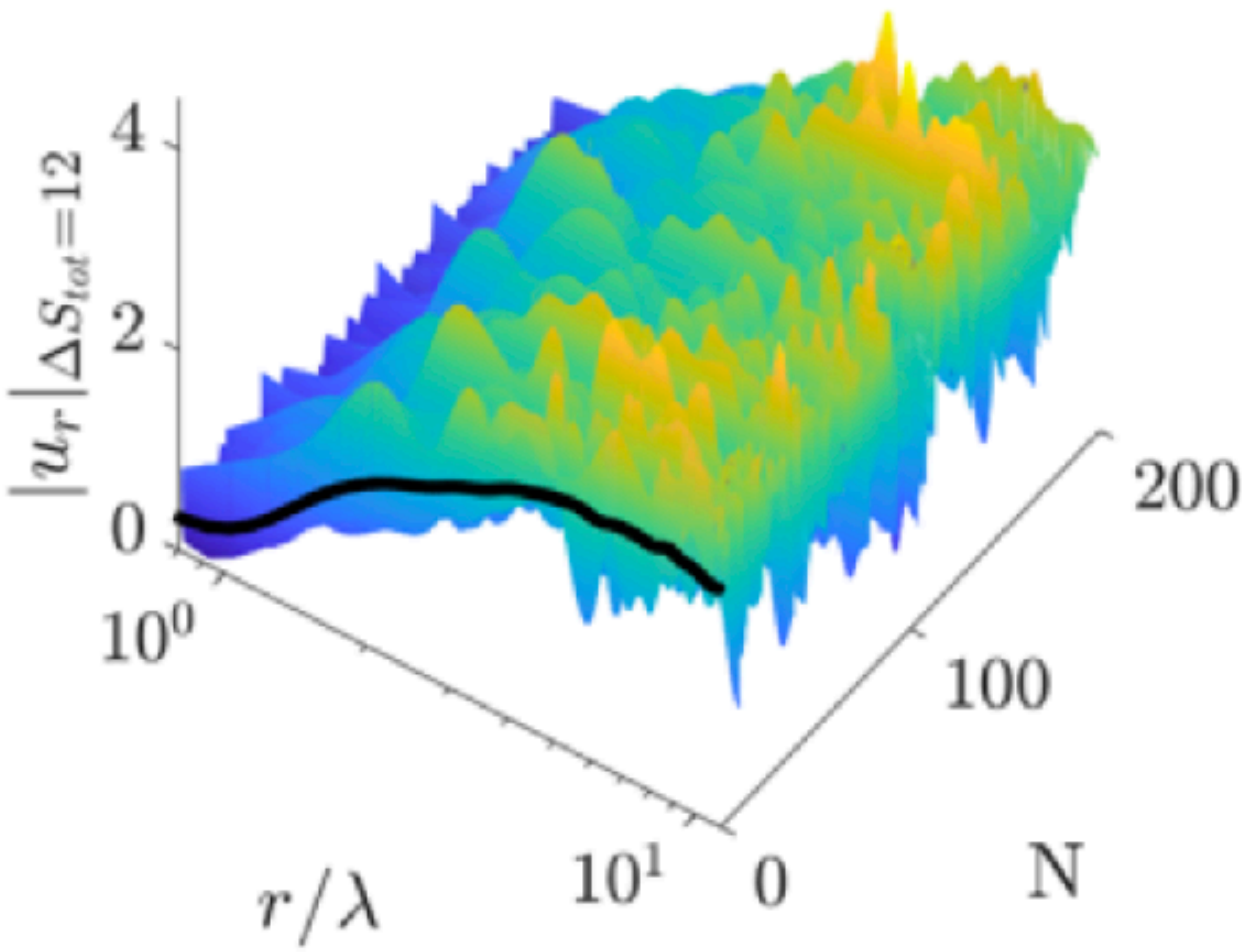




with positive entropy

$$u(\cdot)|_{S_{tot}>0}$$

(a)



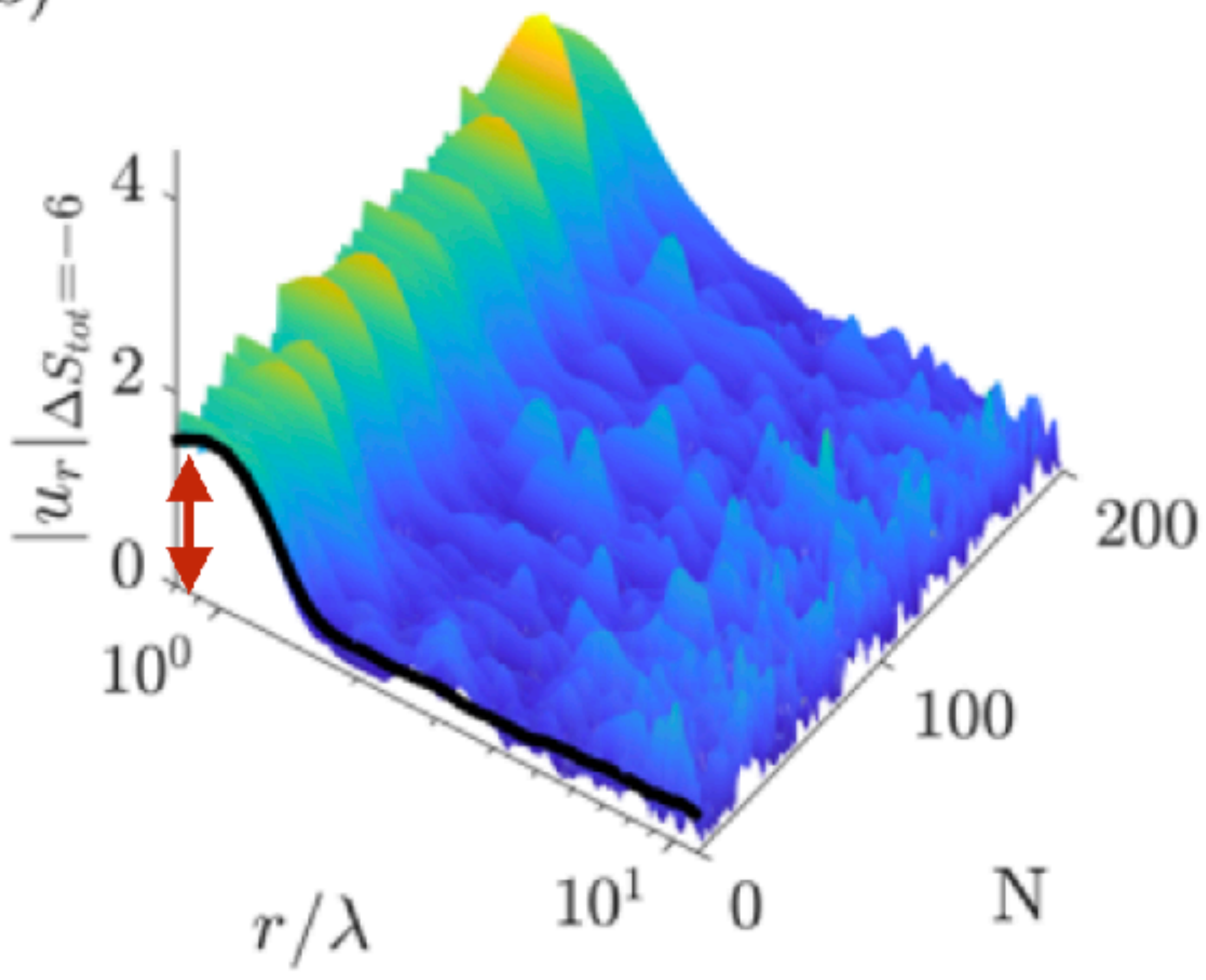
$$u_r \to 0 \text{ for } r \to 0$$

Kolmogorov 41 $\quad u_r \sim r^{1/3}$

with negative entropy

$$u(\cdot)|_{S_{tot}<0}$$

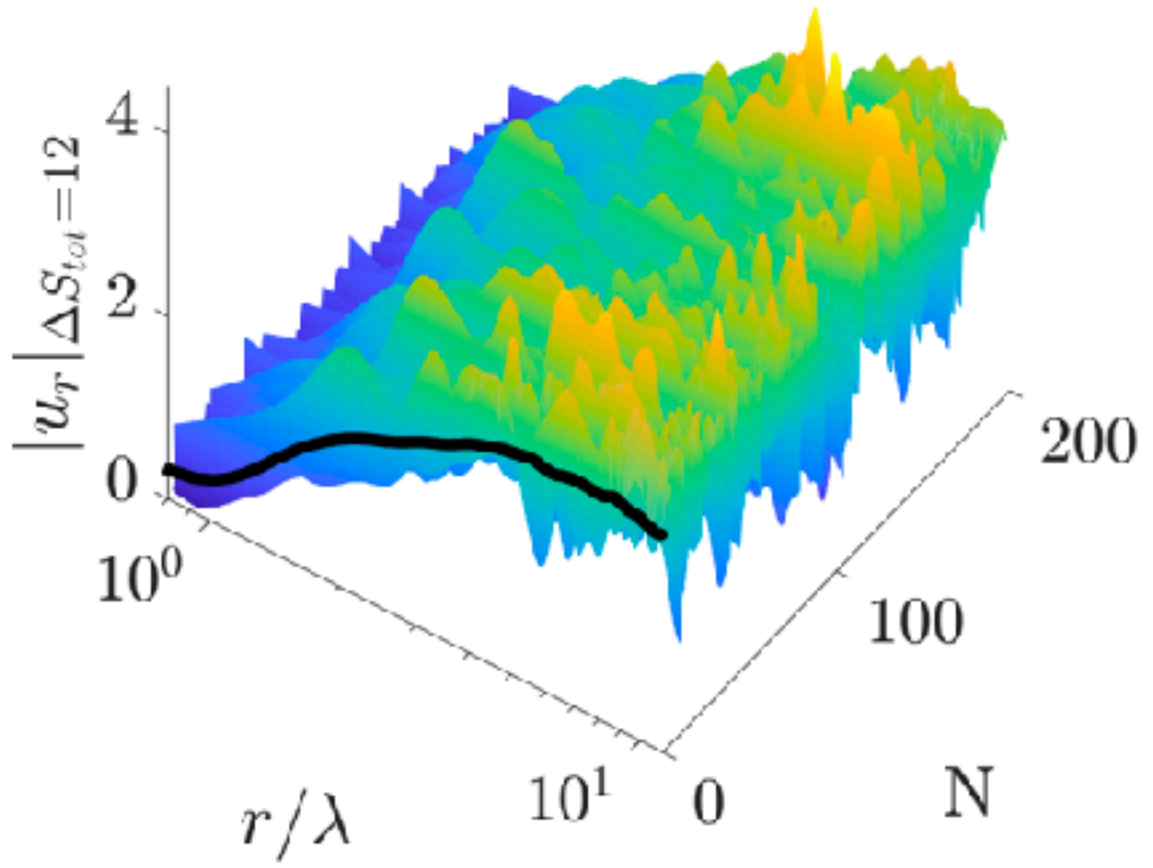
(b)

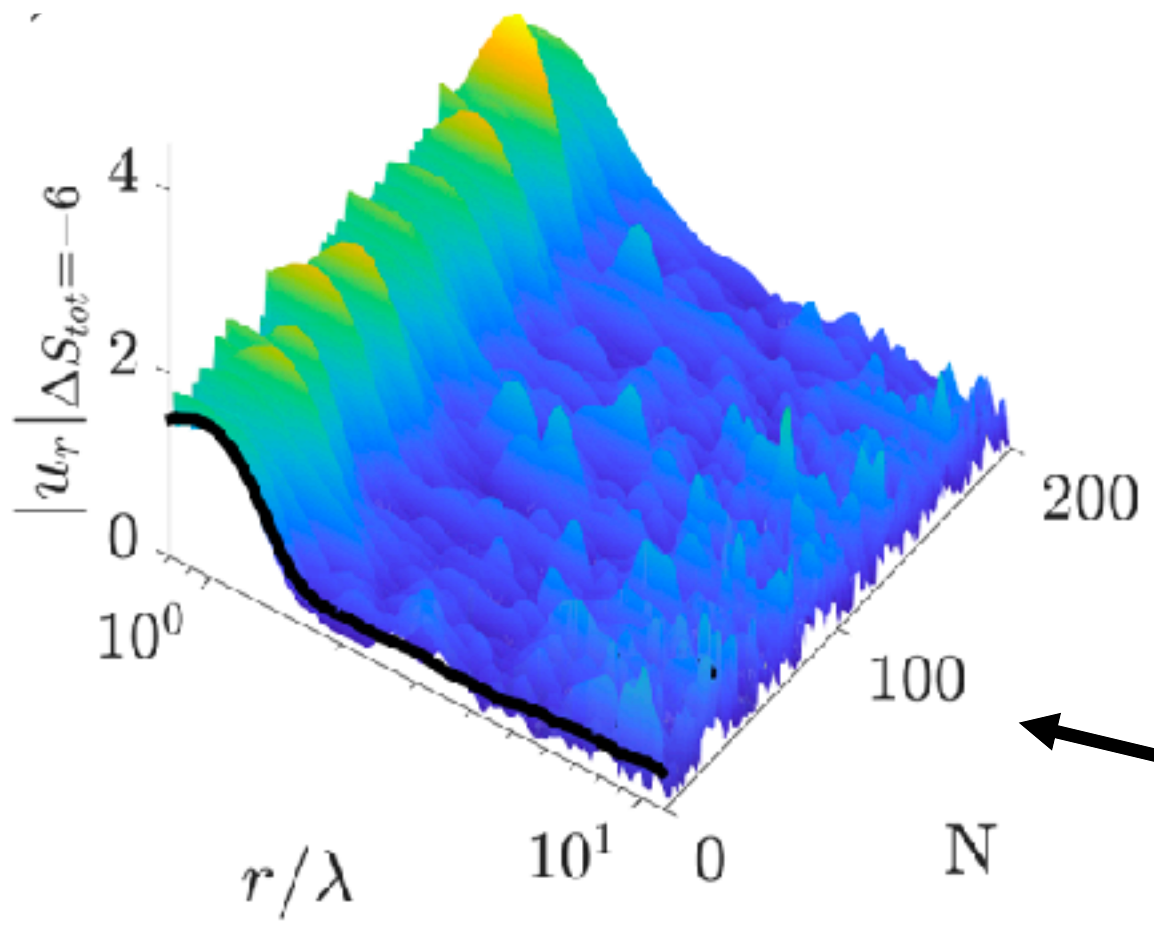


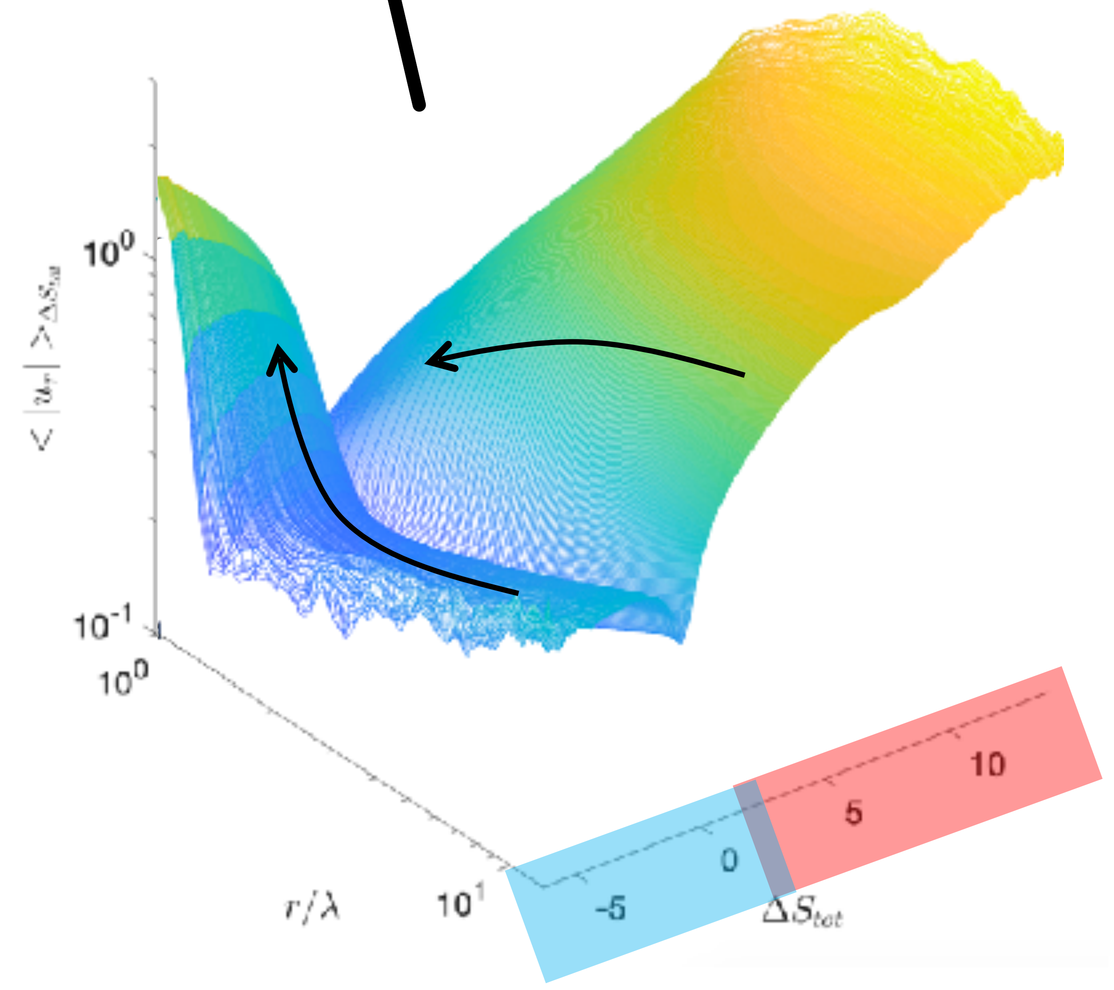
$$u_r > 0 \text{ for } r \to 0$$

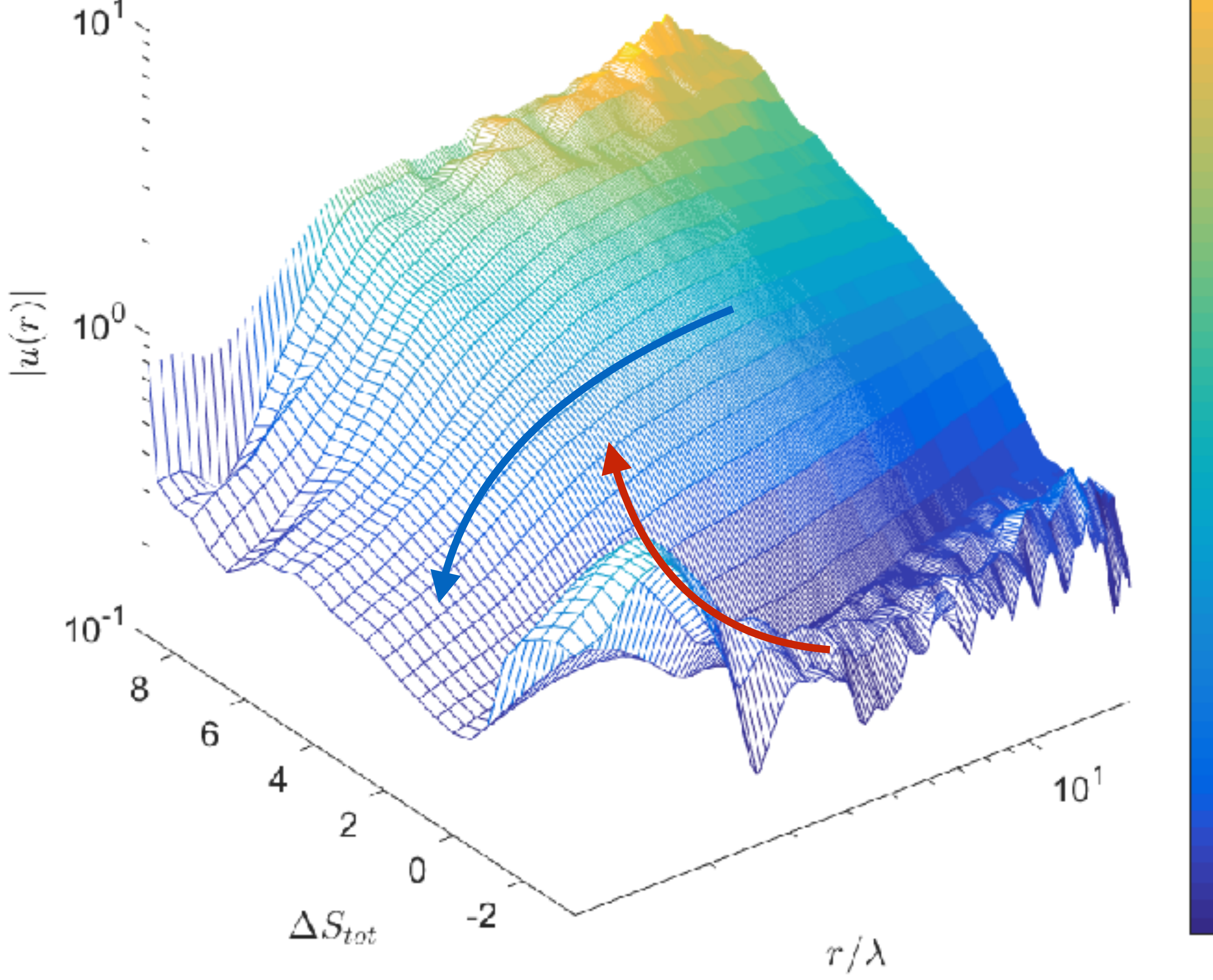
$\Delta S_{tot}\,[u(\cdot)]$ is linked to distinct local turbulent flow structures

K41: $u_r \propto r^{1/3}$

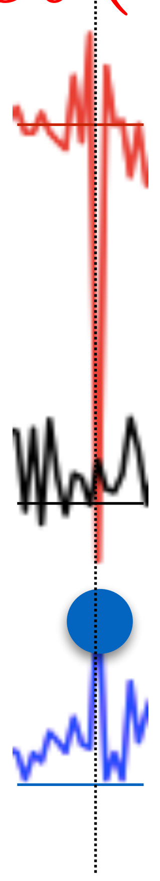








$$\Delta S_{tot}(u_r) < 0$$



event

small u_L with high probability

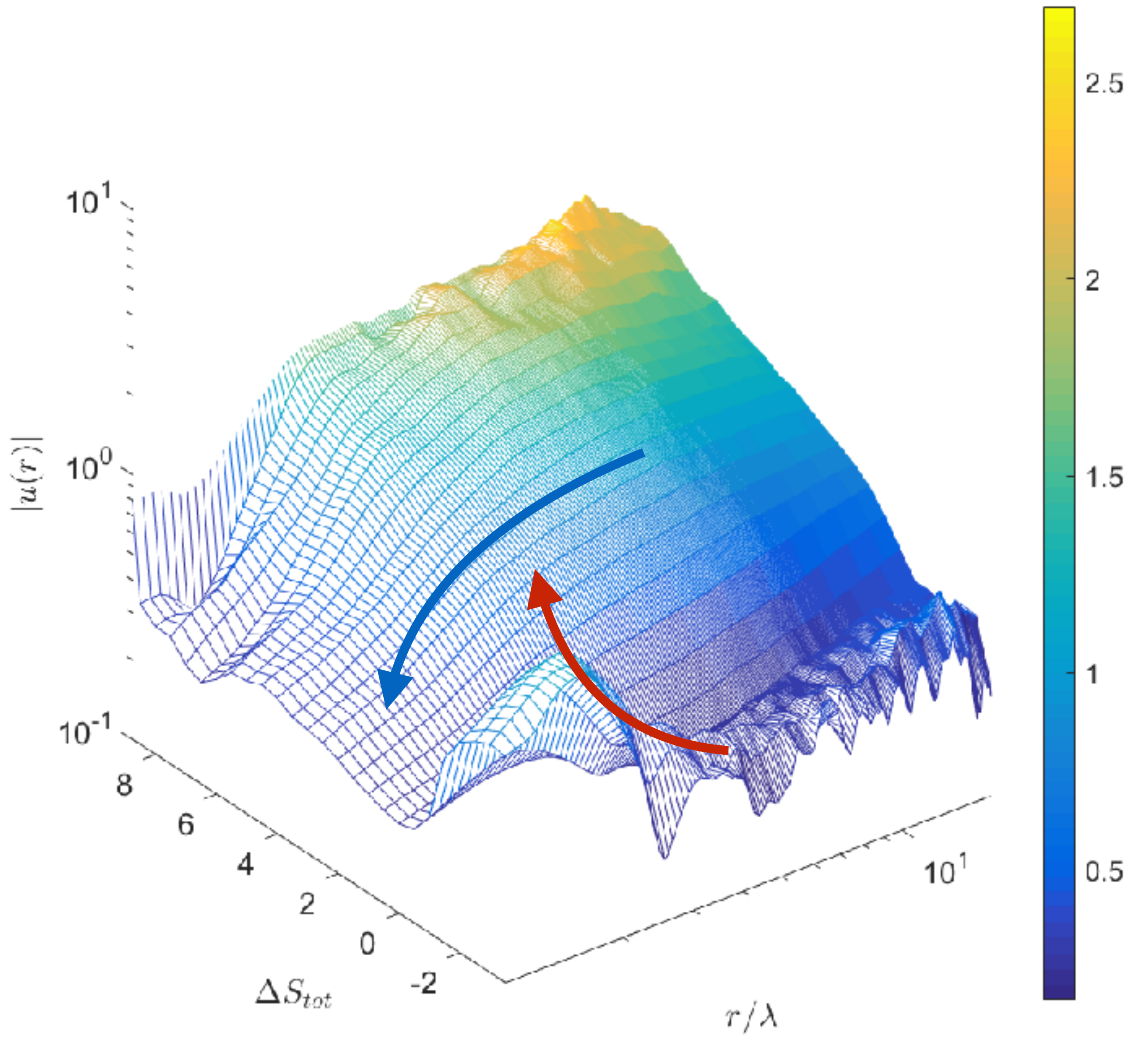
large u_λ with low probability

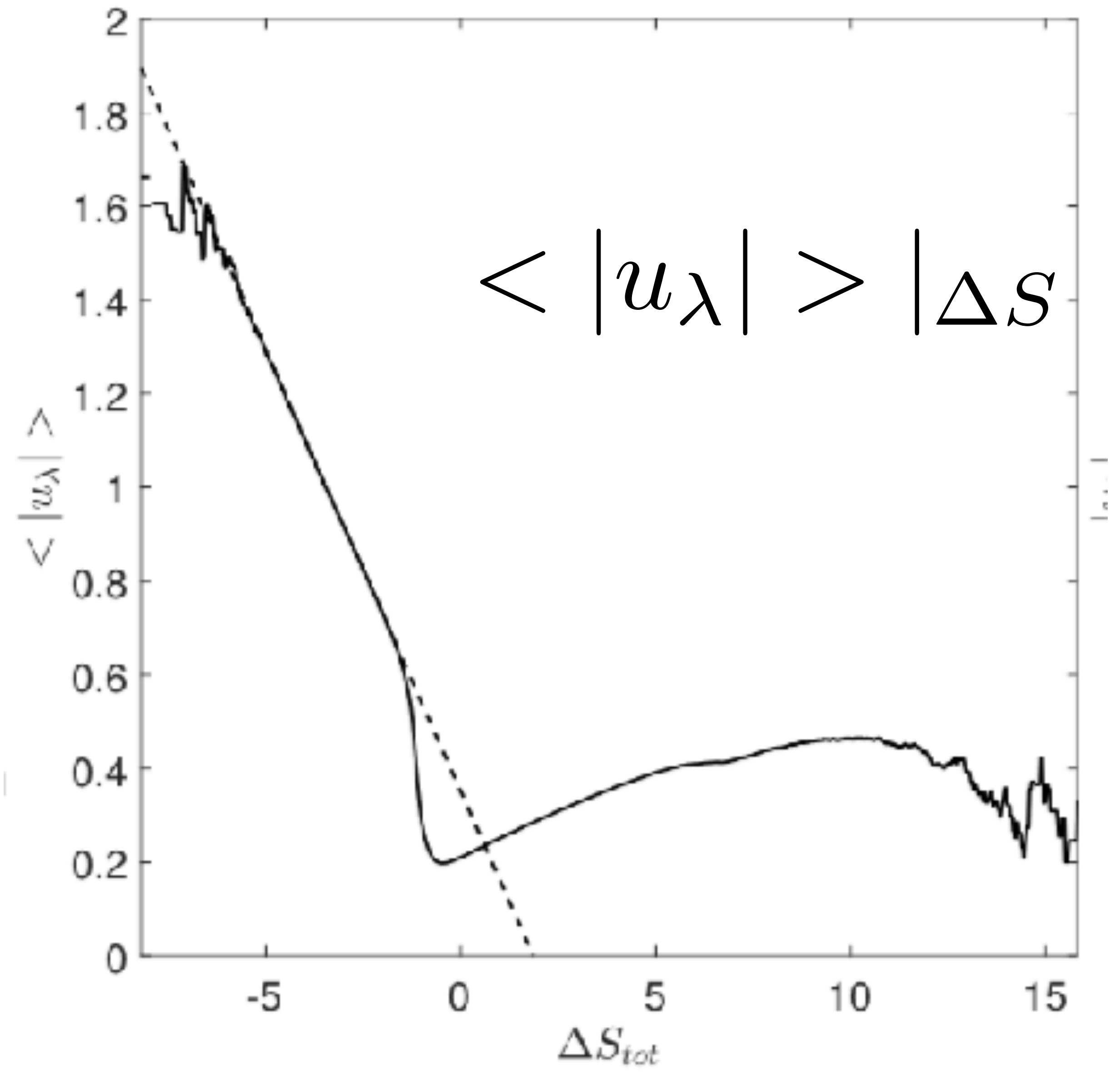
extreme event

Negative entropy events are small scale structures !!!!

Are these indication of shocks?

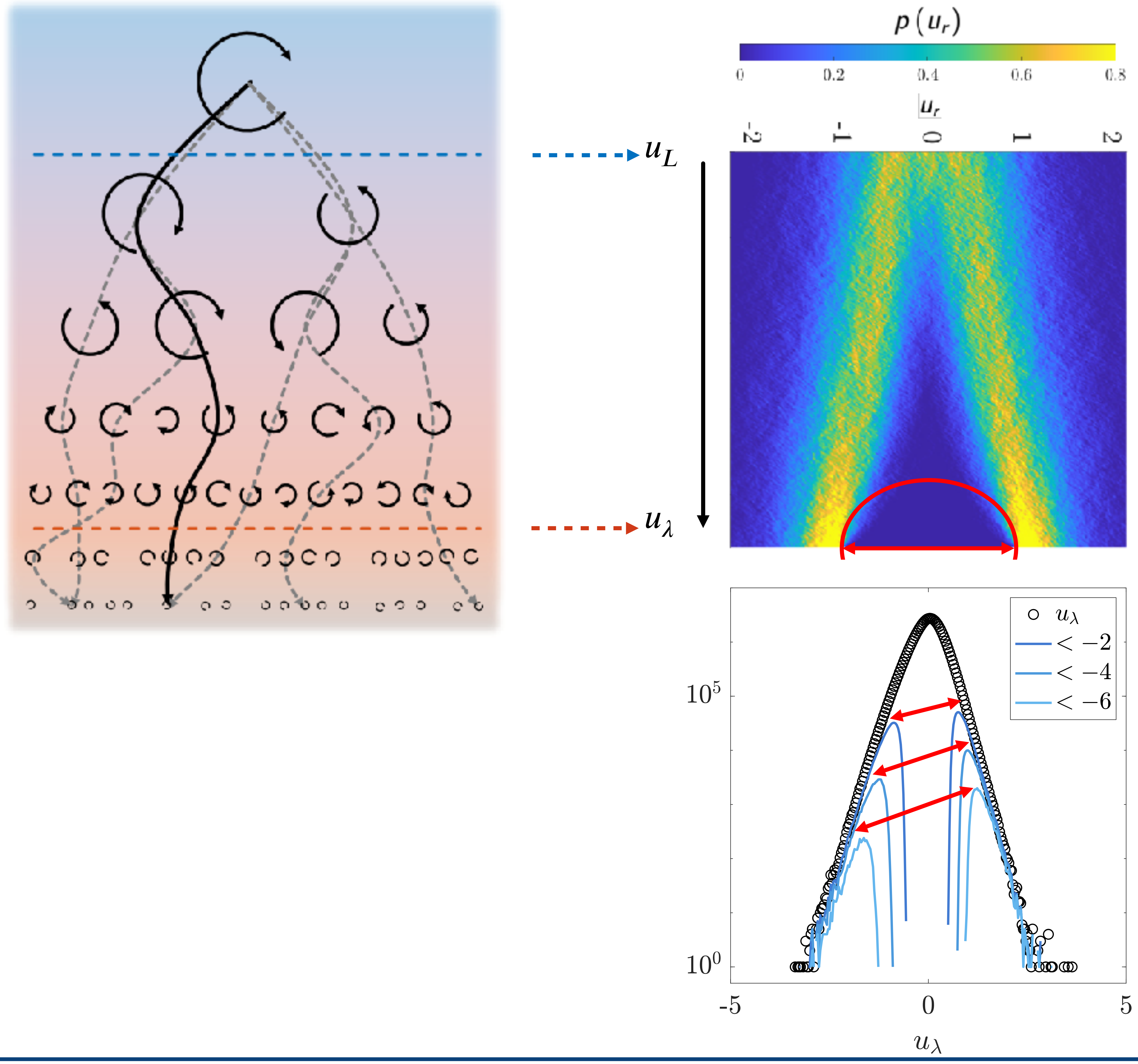
$$< |u_r| > |_{\Delta S}$$



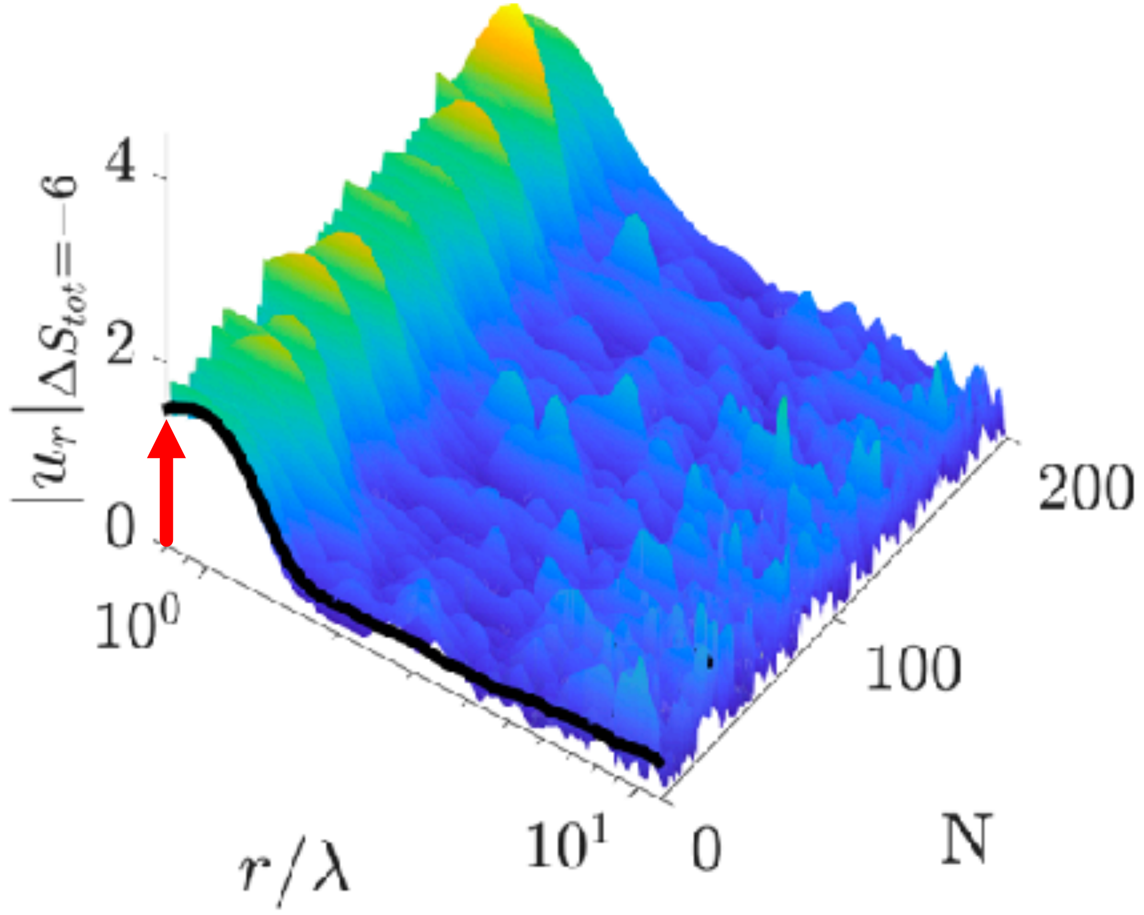


A. Fuchs et al Phys. Rev. F 5, 034602 (2020)

Negative entropy provides access to intermittency



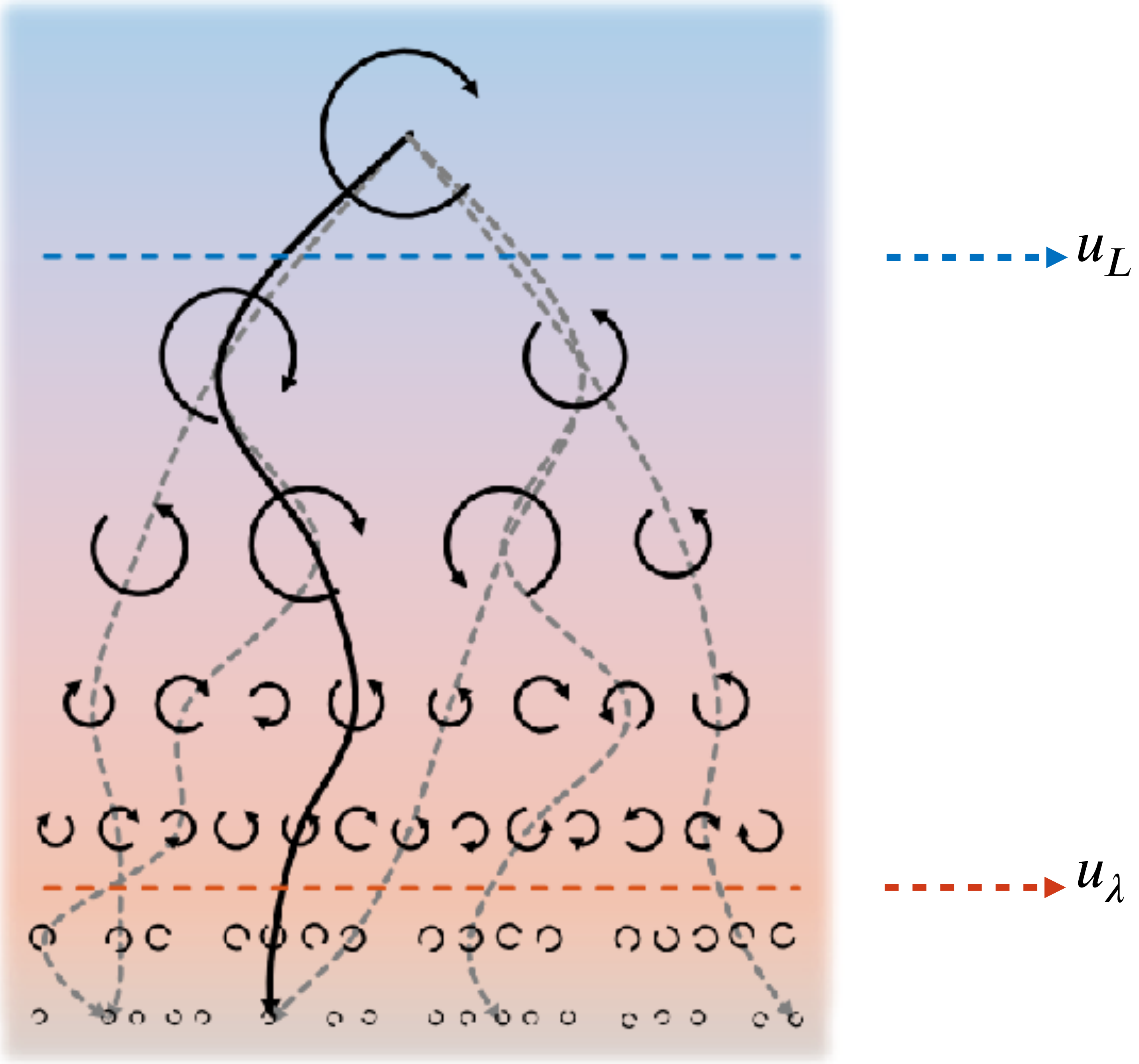
$\Delta S = -6$

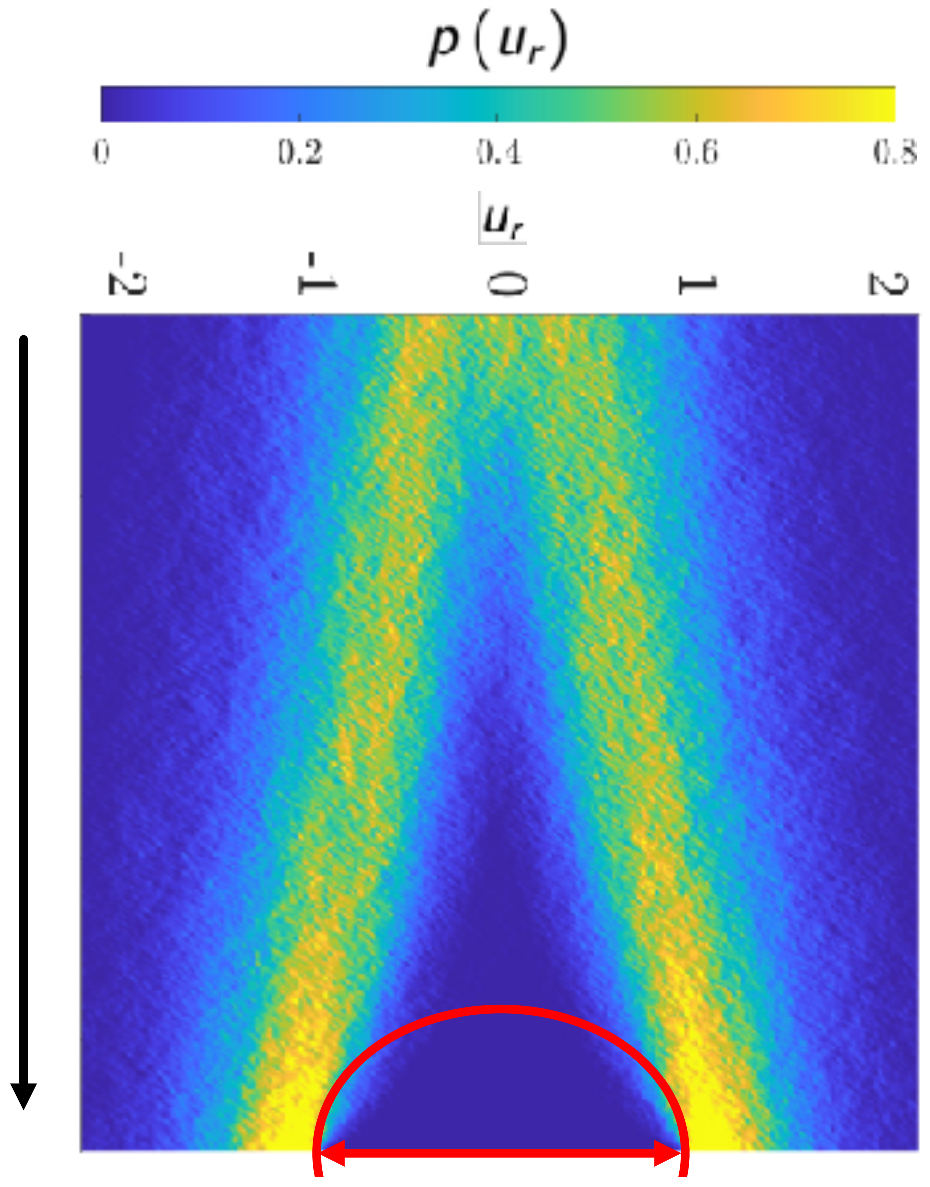


finite increment at small scales

$|u_r| > 0$ for $r \rightarrow \lambda$

Negative entropy provides access to intermittency





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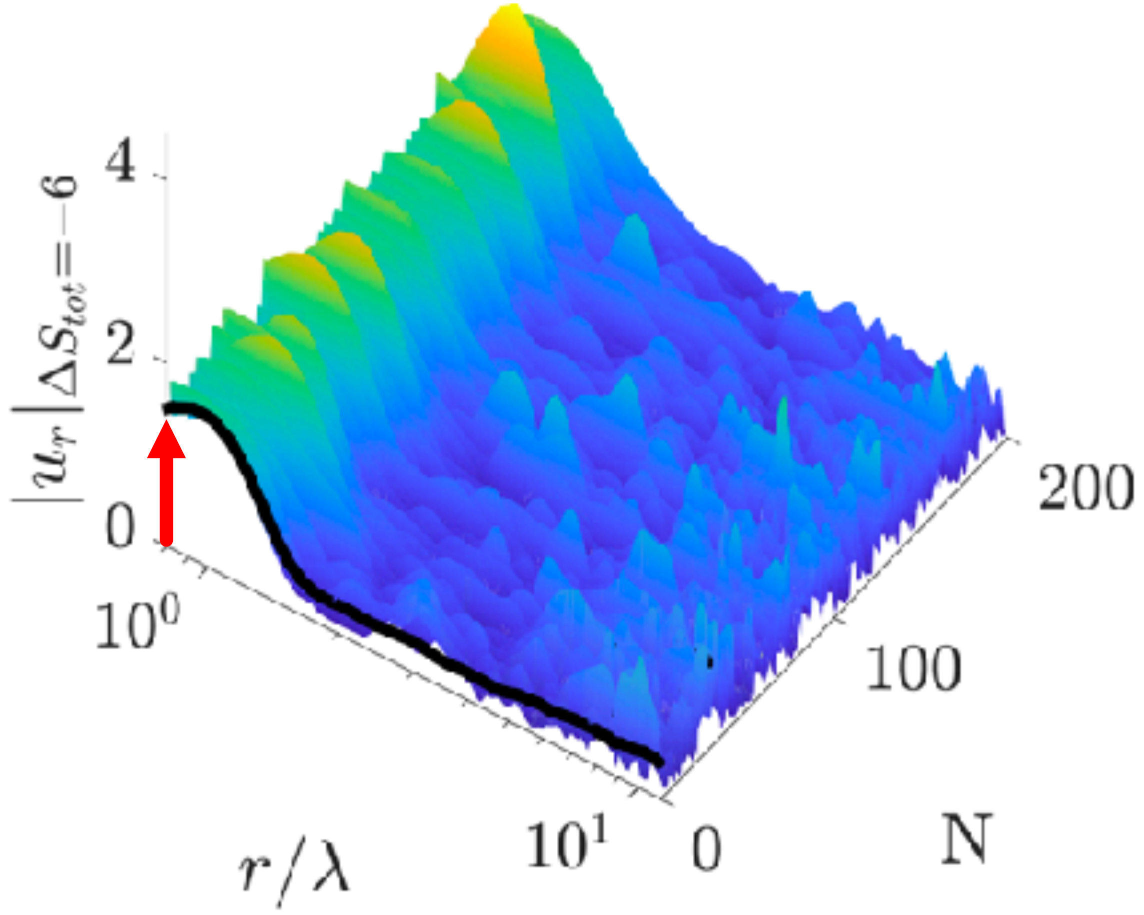

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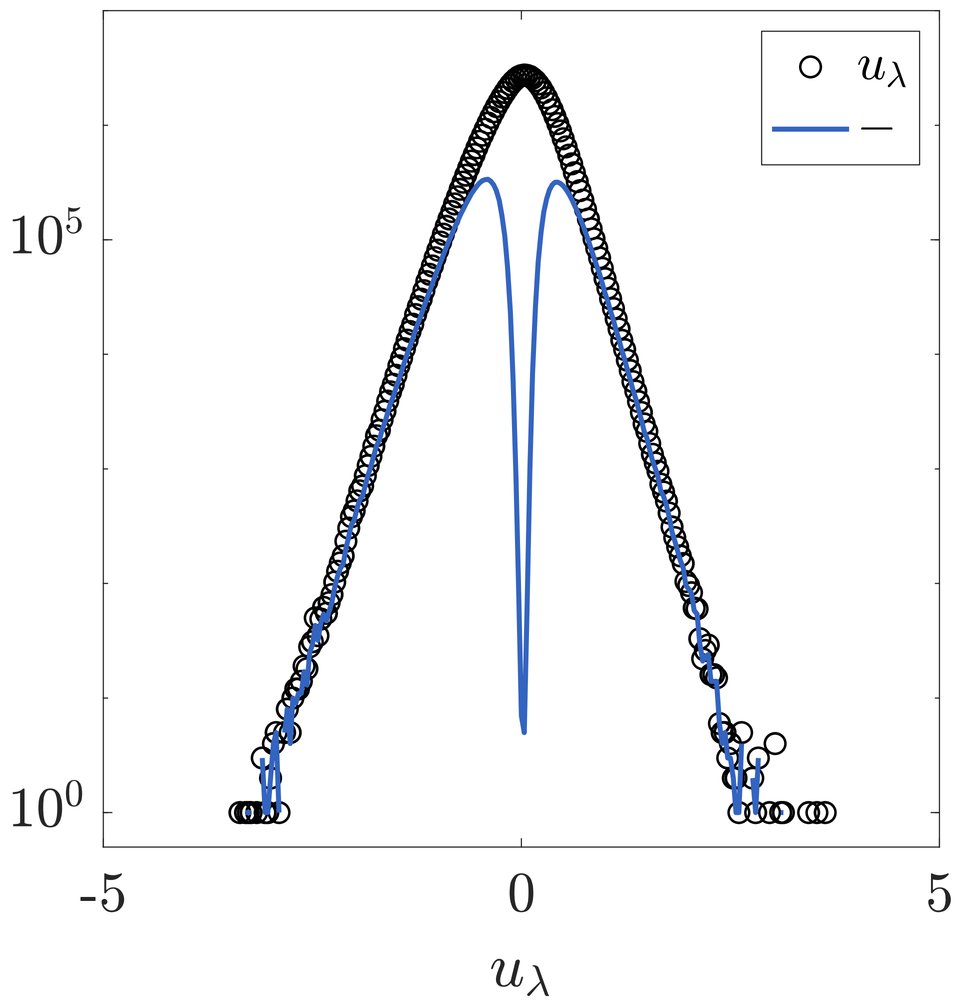
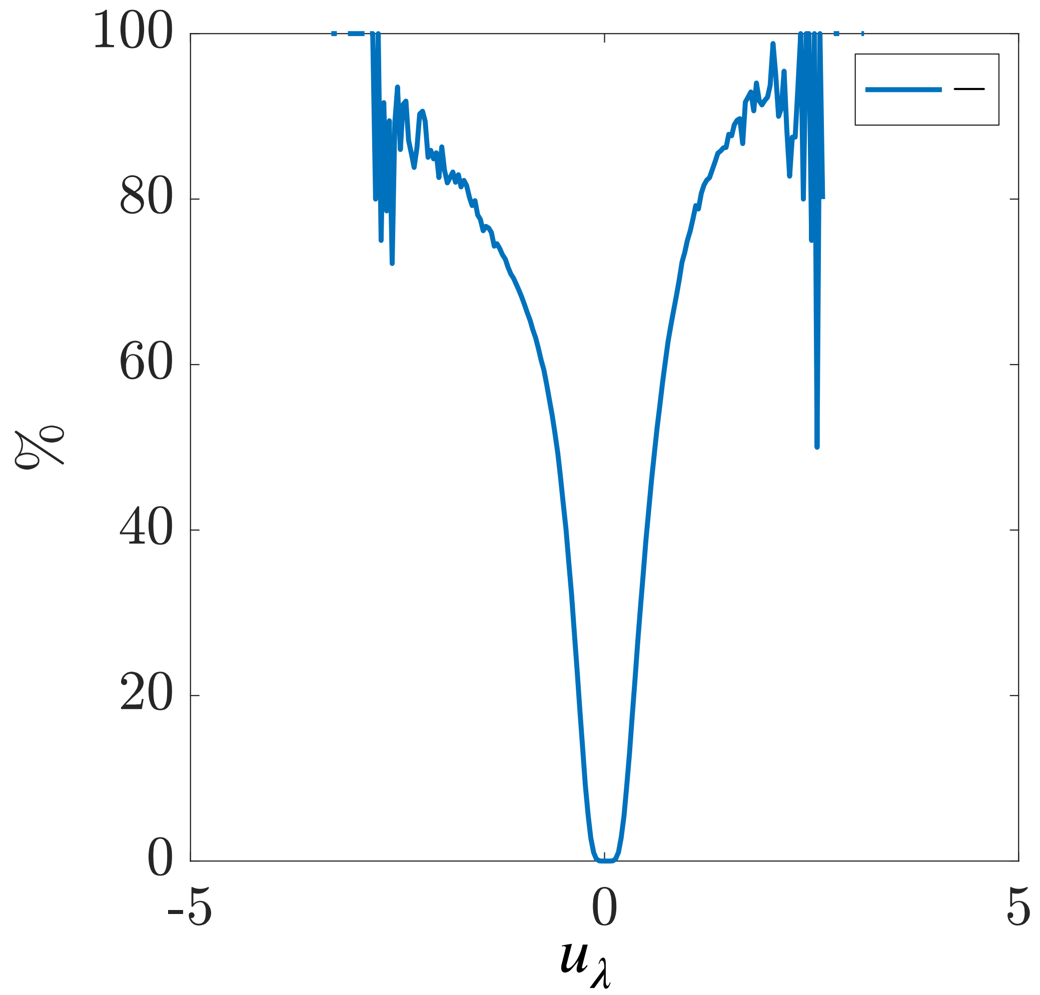
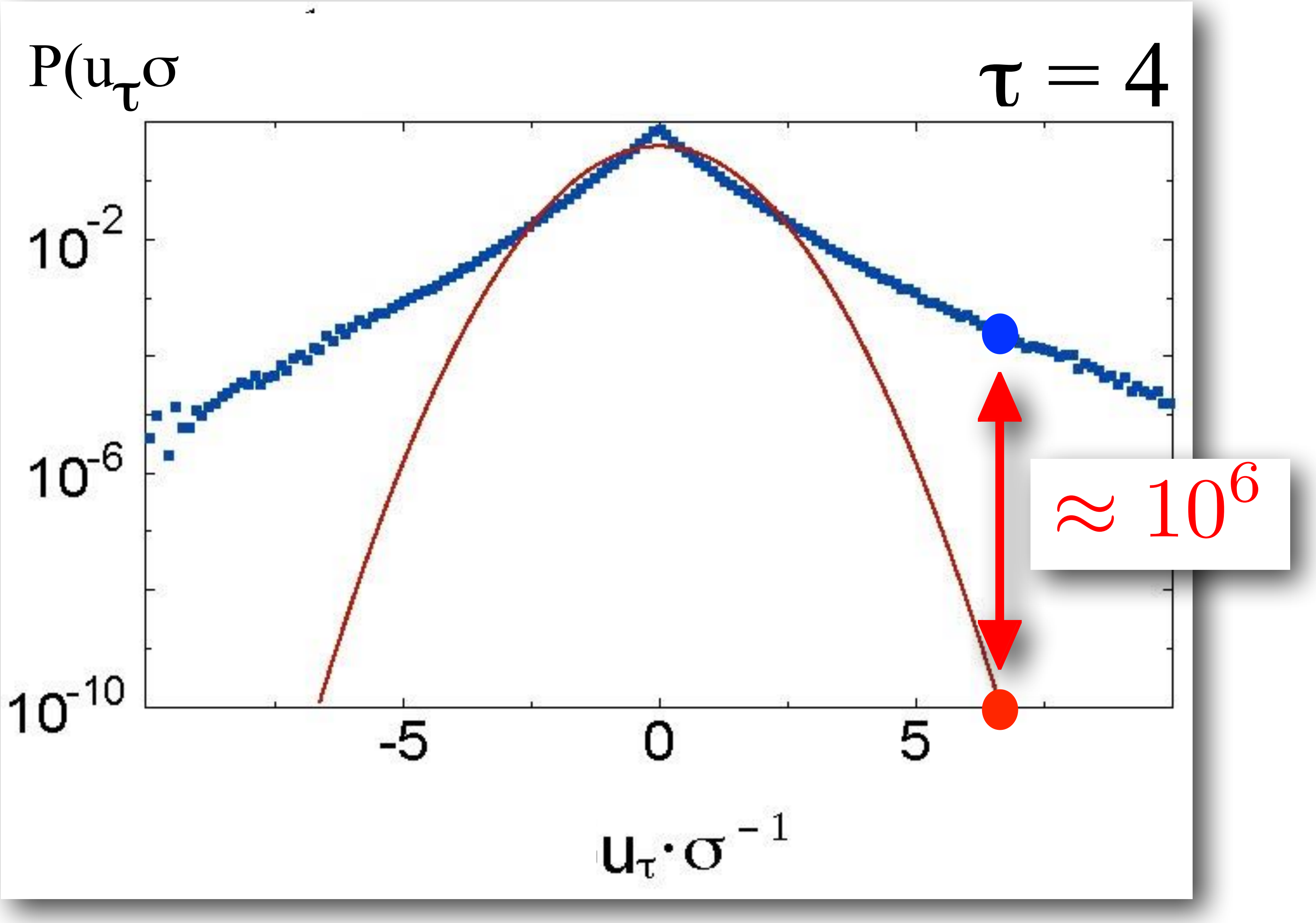




Negative entropy provides access to non-Gaussian statistics intermittency - small scale structures

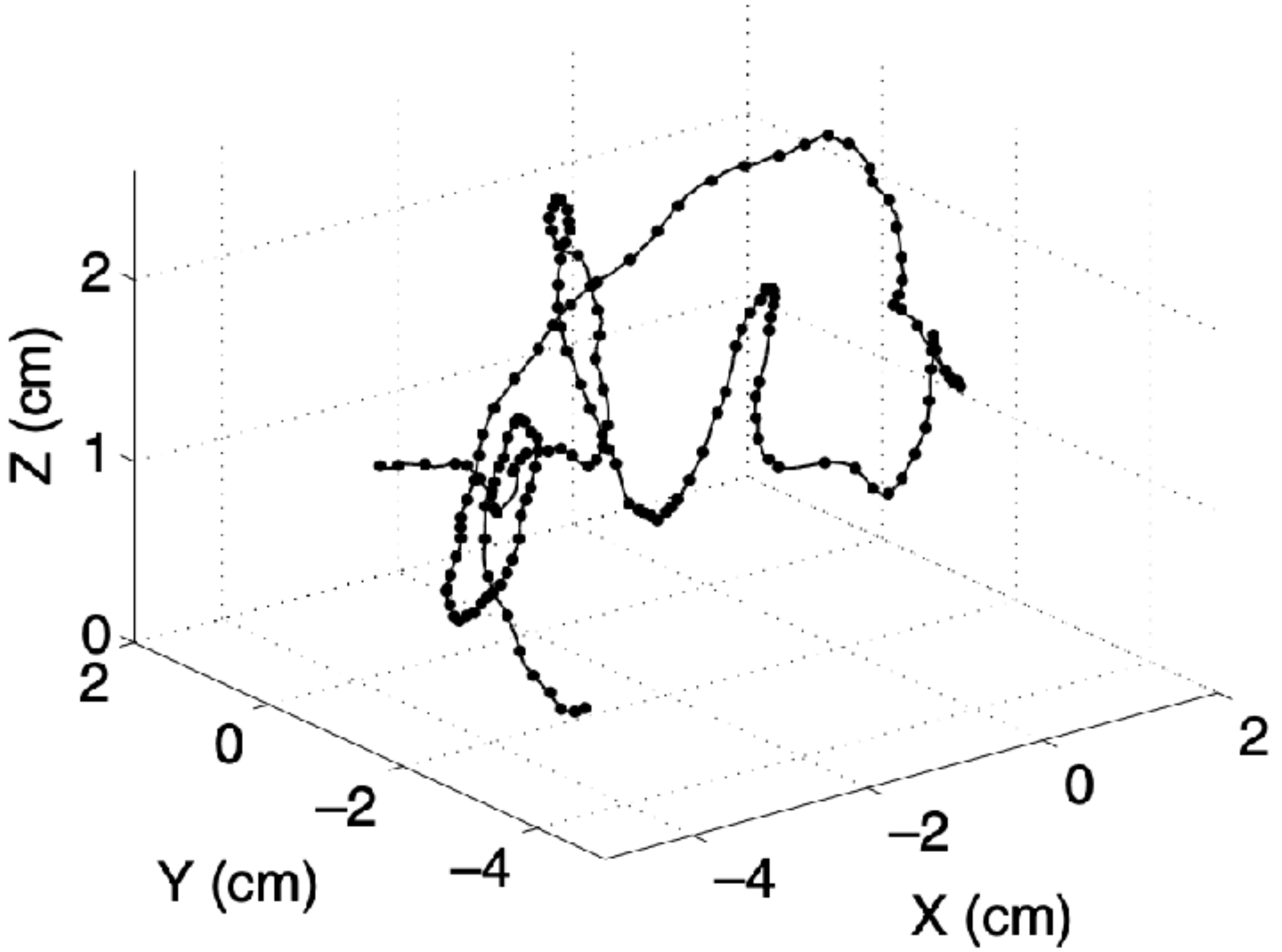
Further examples -

Waves and Lagrange turbulence

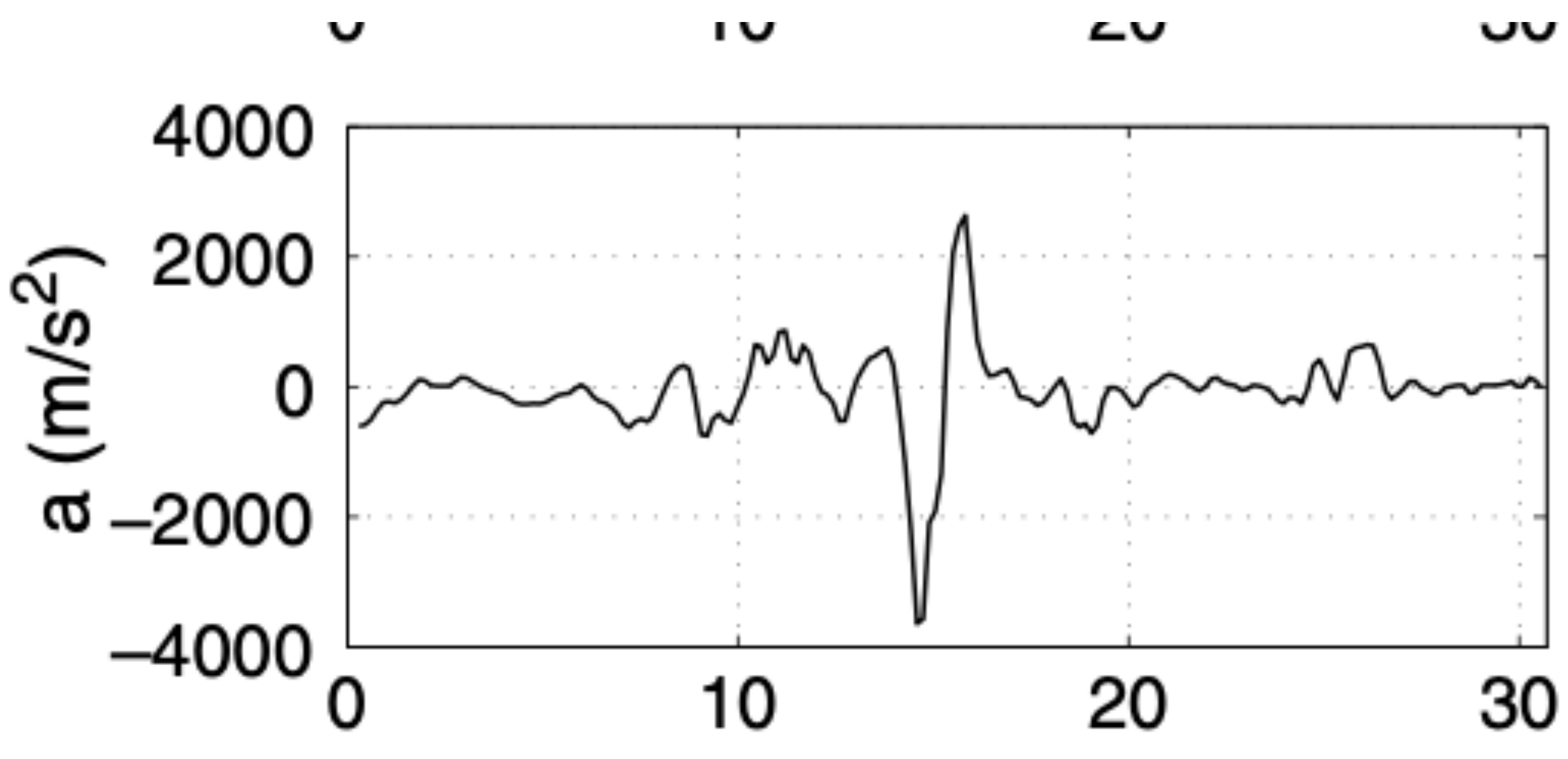


u_λ

Lagrangian particles in turbulent flow - old challenge



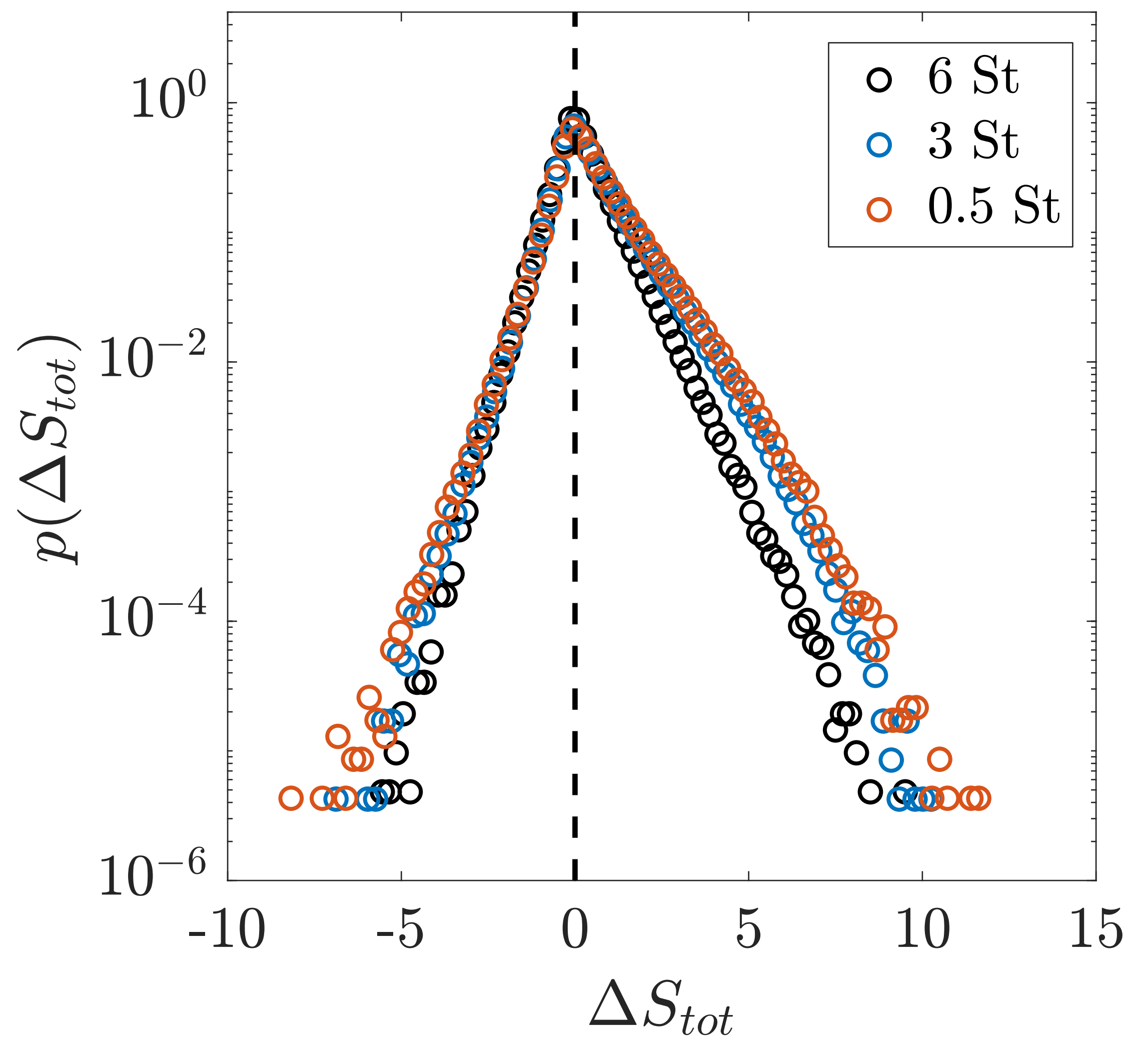
Typical trajectories



Events of high particle accelerations

Nicolas Mordant et al. New Journal of Physics 6 (2004) 000

Consequences of stochastic process of u_τ

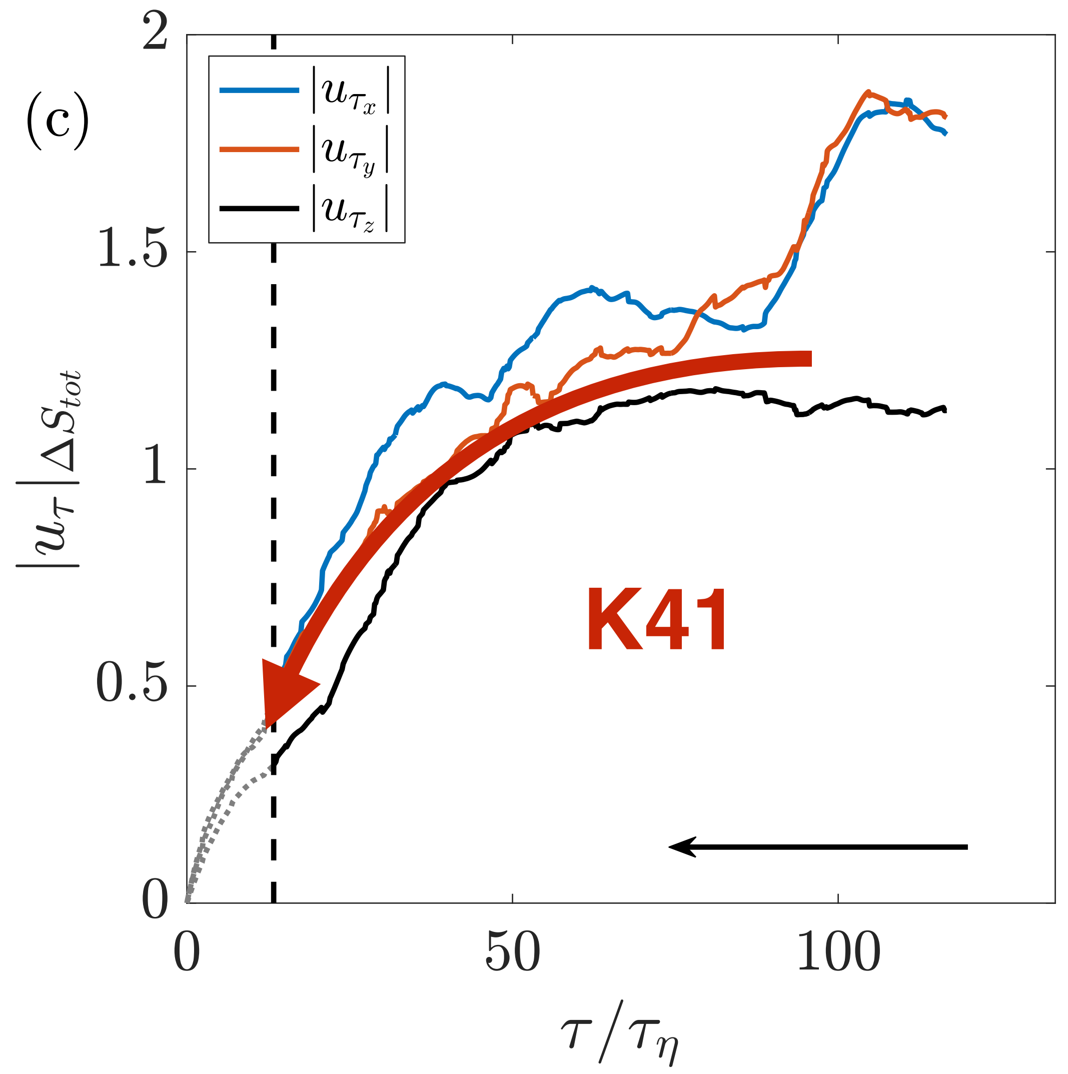


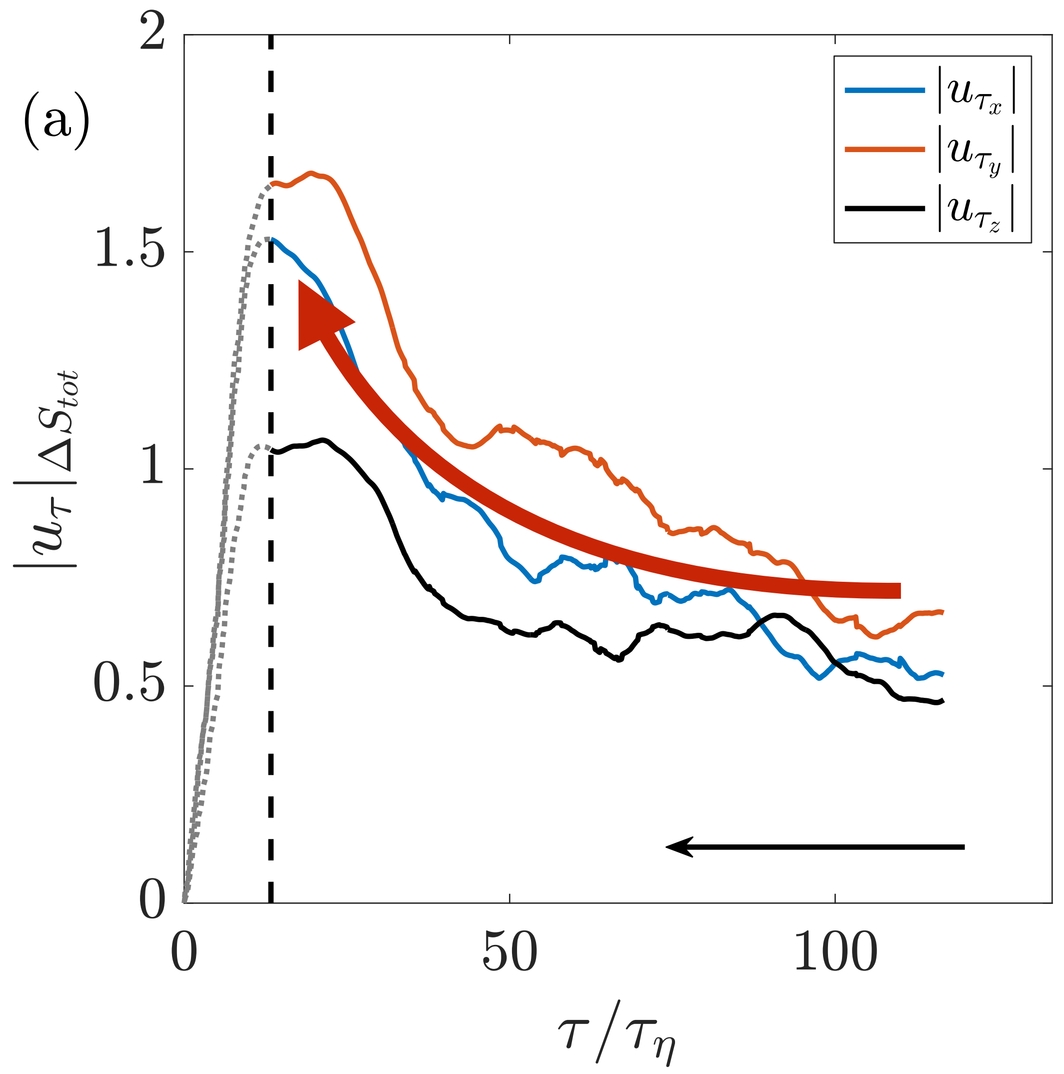
- For path $u(\cdot)$ an **entropy** can be defines, fluctuating quantity

$$\Delta S_{tot}\left[u(\cdot)\right] = \Delta S_{sys}\left[u(\cdot)\right] + \Delta S_{med}\left[u(\cdot)\right]$$

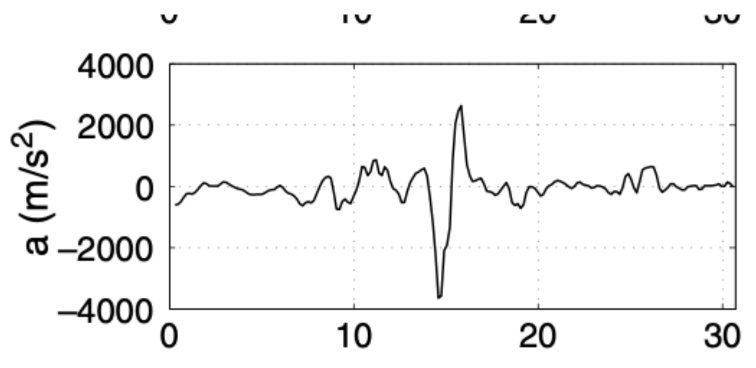
  - Negative entropy events -> biggest $u_{\Delta_{EM}}$ (or acceleration: $a = u_{\Delta_{EM}}/\Delta_{EM}$ events)
  - Positive entropy events -> small $u_{\Delta_{EM}}$

➡ **Entropy are connected with structures of Lagrangian turbulence**





Anormal - intermittent



Negative entropy provides access to intermittency - small scale structures

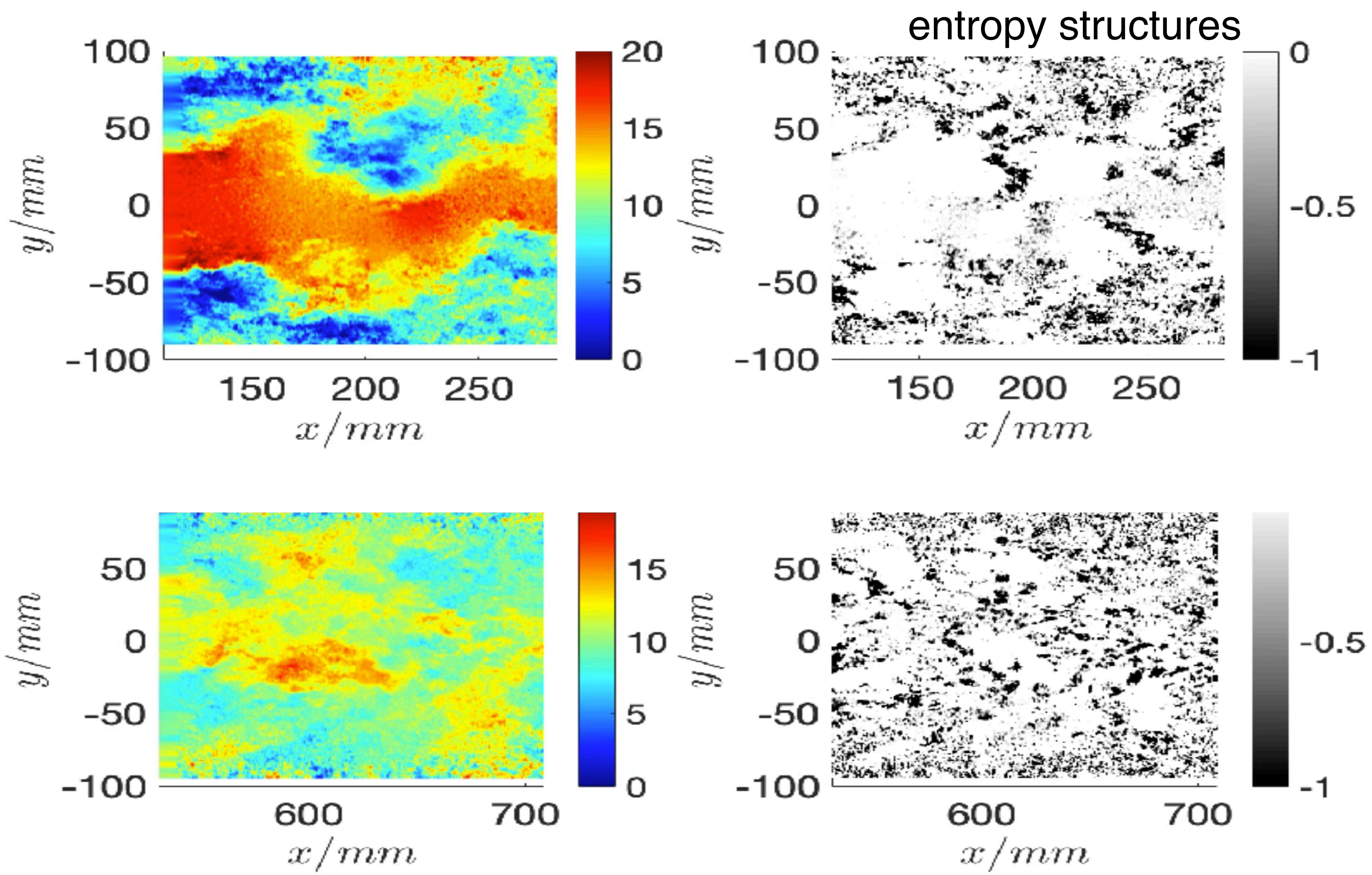
Further examples -

Waves and Lagrange turbulence

2-dim flow structures

u_λ

entropy and structures
application to high speed PIV measurements:



Negative entropy provides access to intermittency - small scale structures

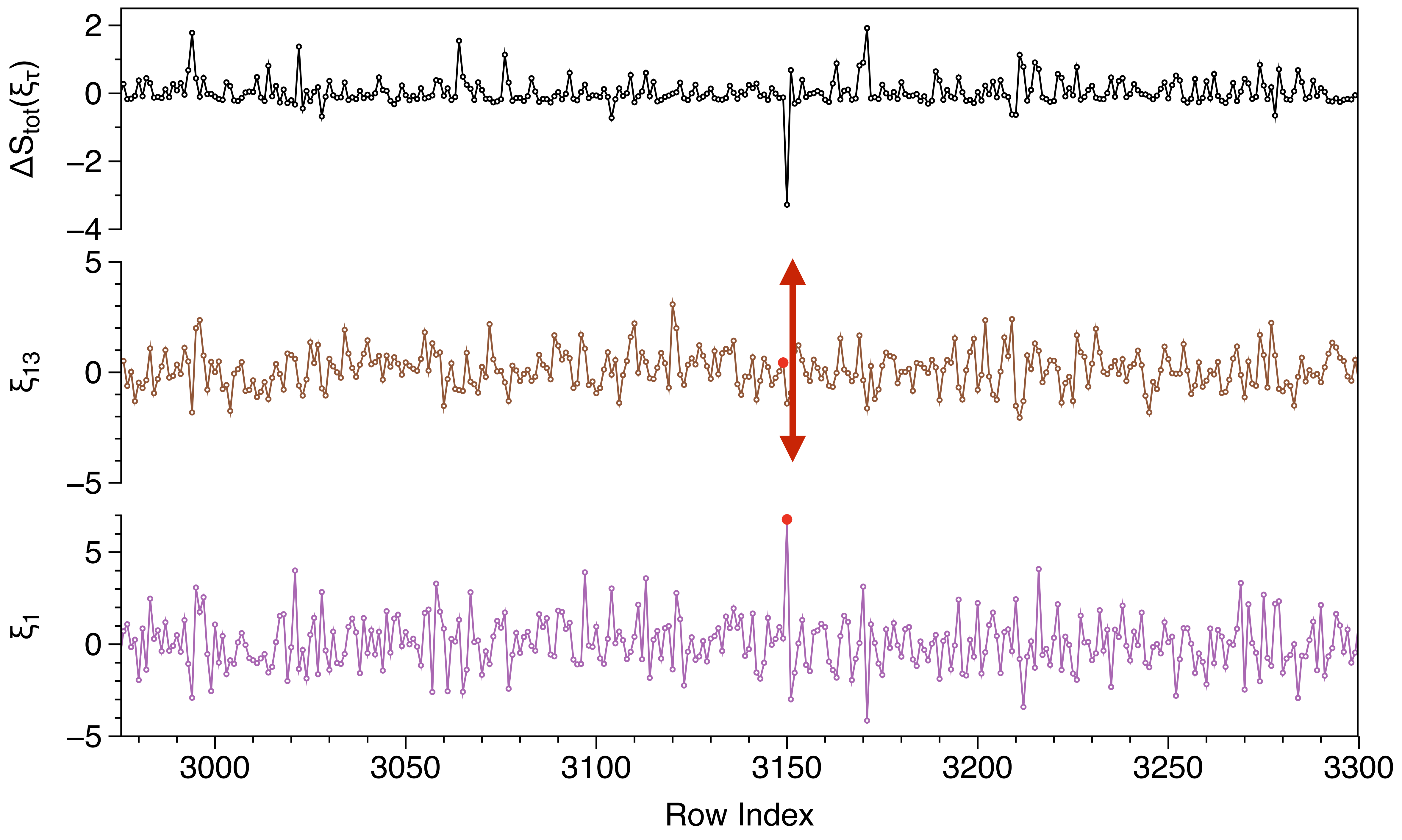
Further examples -

Waves and Lagrange turbulence

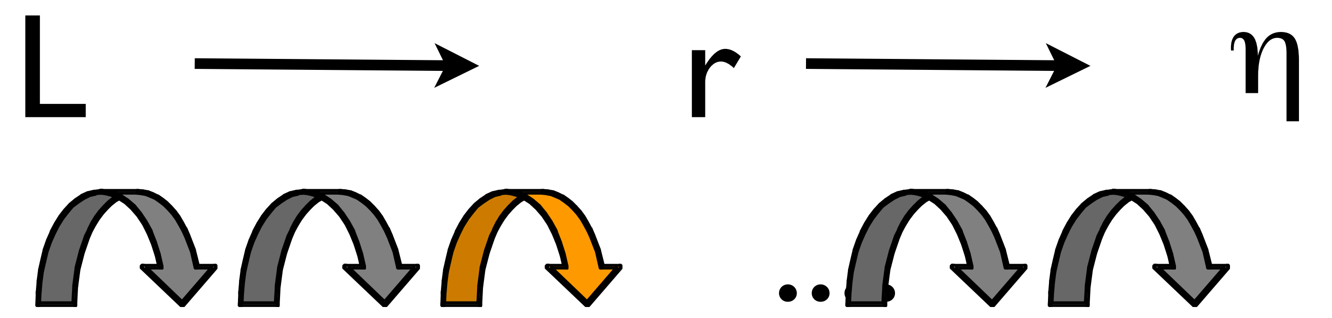
2-dim flow structures

u_λ

rogue wave: event of negative entropy production



stochastic cascade process: Fokker - Planck equation



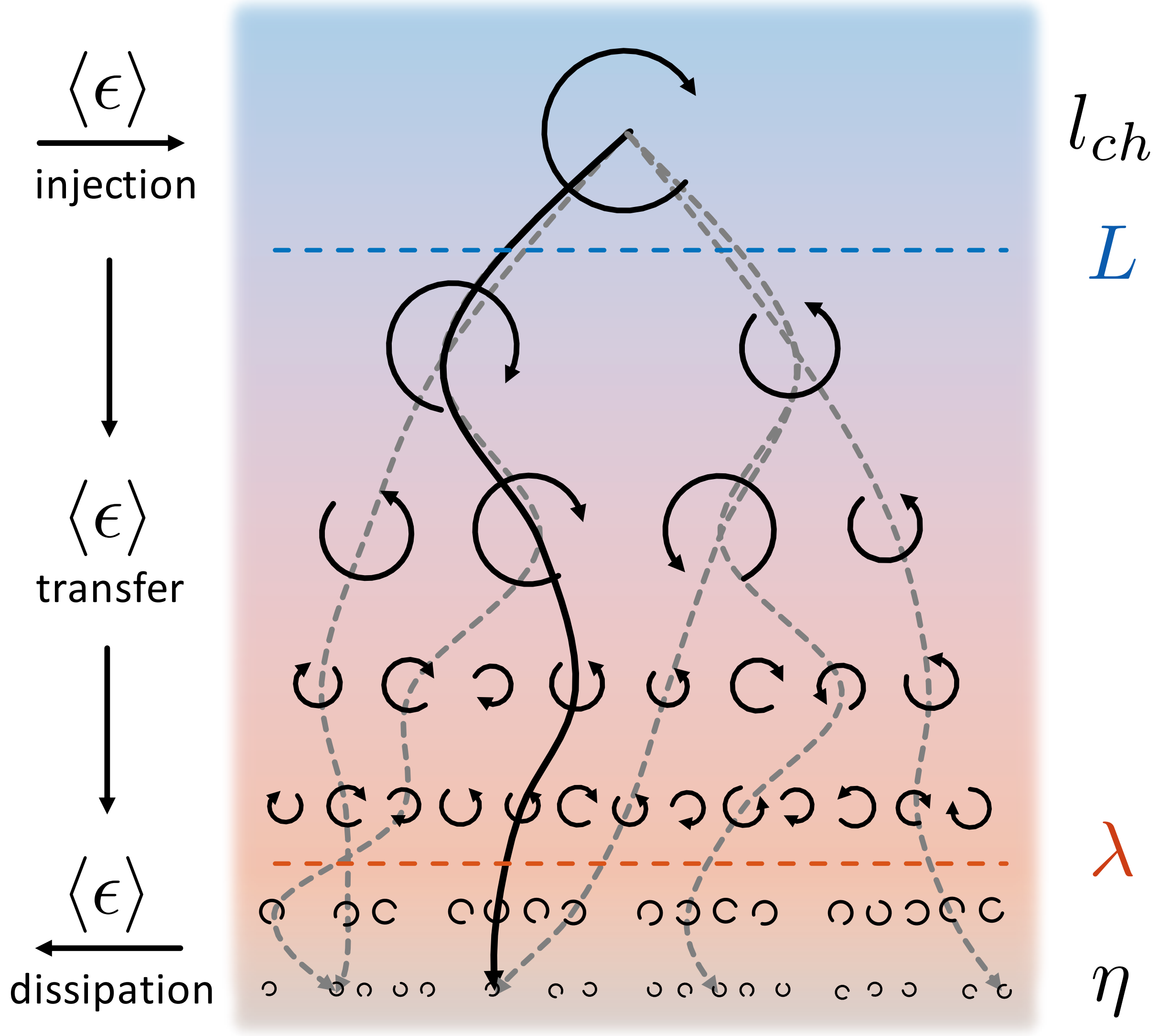
$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j}|u_{r_k}, u(x_1)) = \{-\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1))\} \, p(u_{r_j}|u_{r_k}, u(x_1))$$

Statistical approach: multipoint statistics, **new data sets**

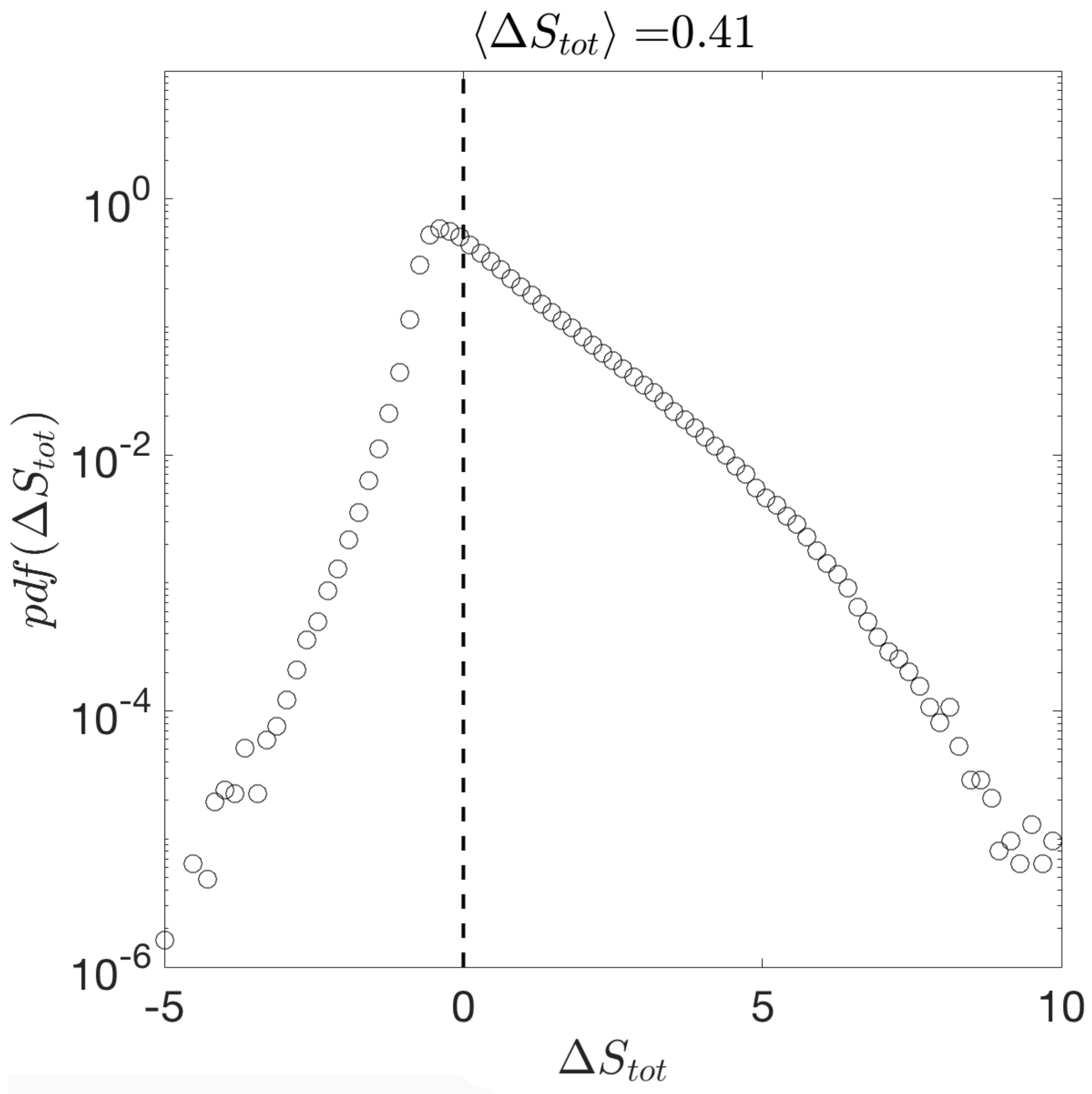
- **non equilibrium thermodynamics**
  - **Entropy** (Seifert 2005)
  - **Fluctuation Theorem**
  - Hamiltonian for cascade - instantones
  -

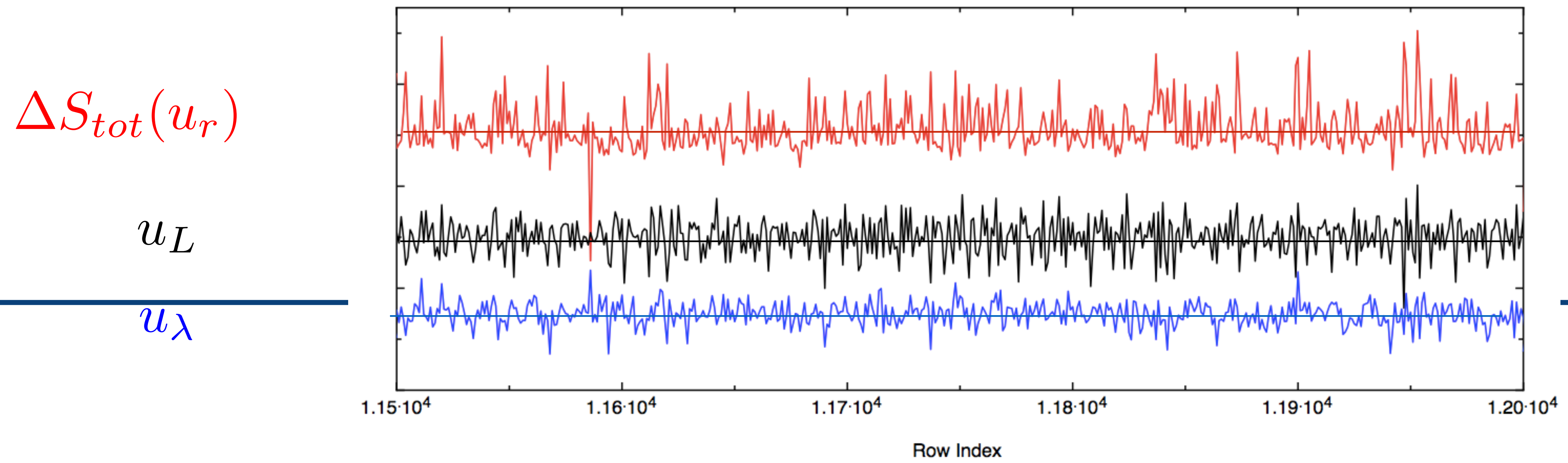
entropy of cascade trajectories - non-equilibrium thermodynamics

Cascade trajectories **u(·)** set of increments for fixed x and changing r, $L > r > \eta$



- **entropy of cascade trajectories**
- **Statistics of entropy**
- **Fluctuations**





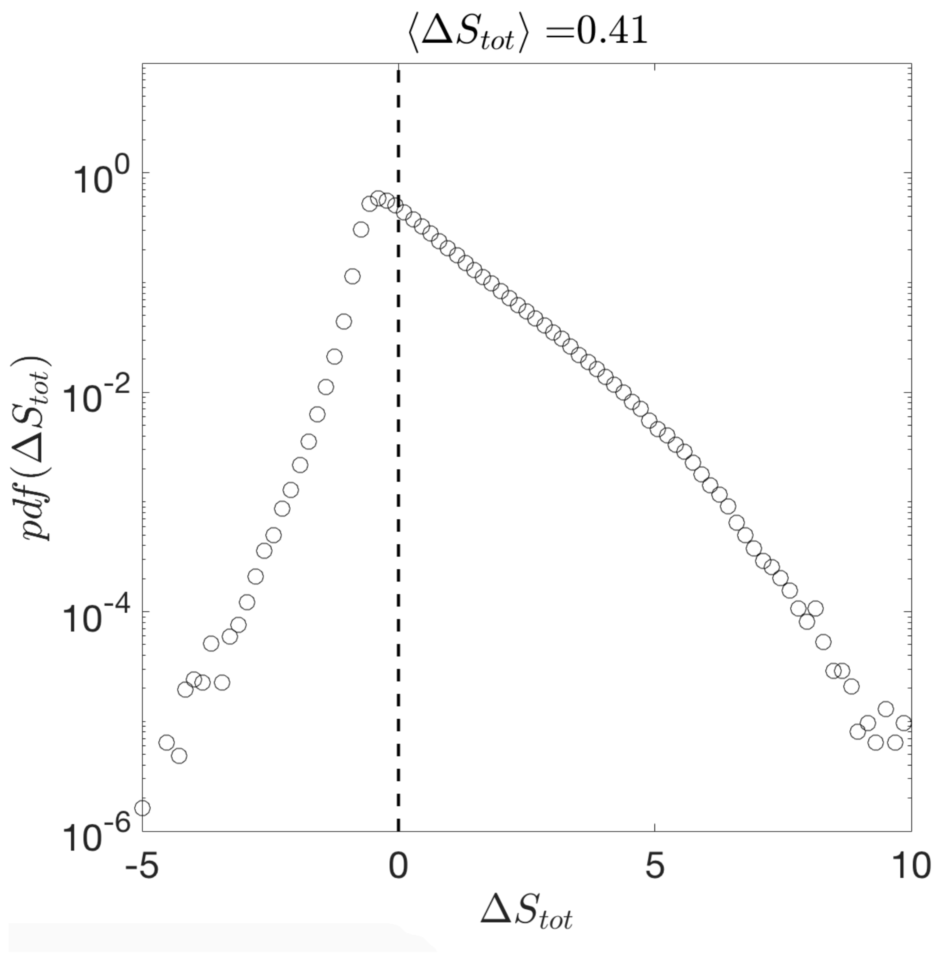
integral fluctuations theorem a new precision law

nonequilibrium thermodynamics

- a new rigorous entropy law for turbulence

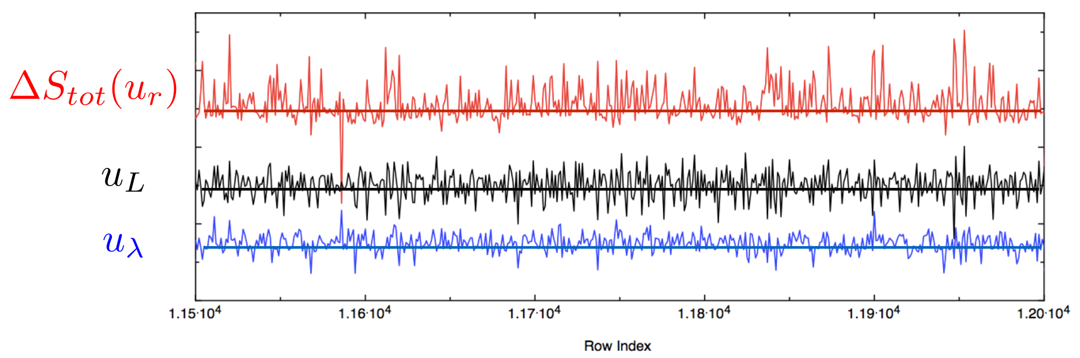
$$\langle e^{-\Delta S_{tot}} \rangle = 1$$

integral fluctuation theorem
Seifert (2005) (and Jarzyski)

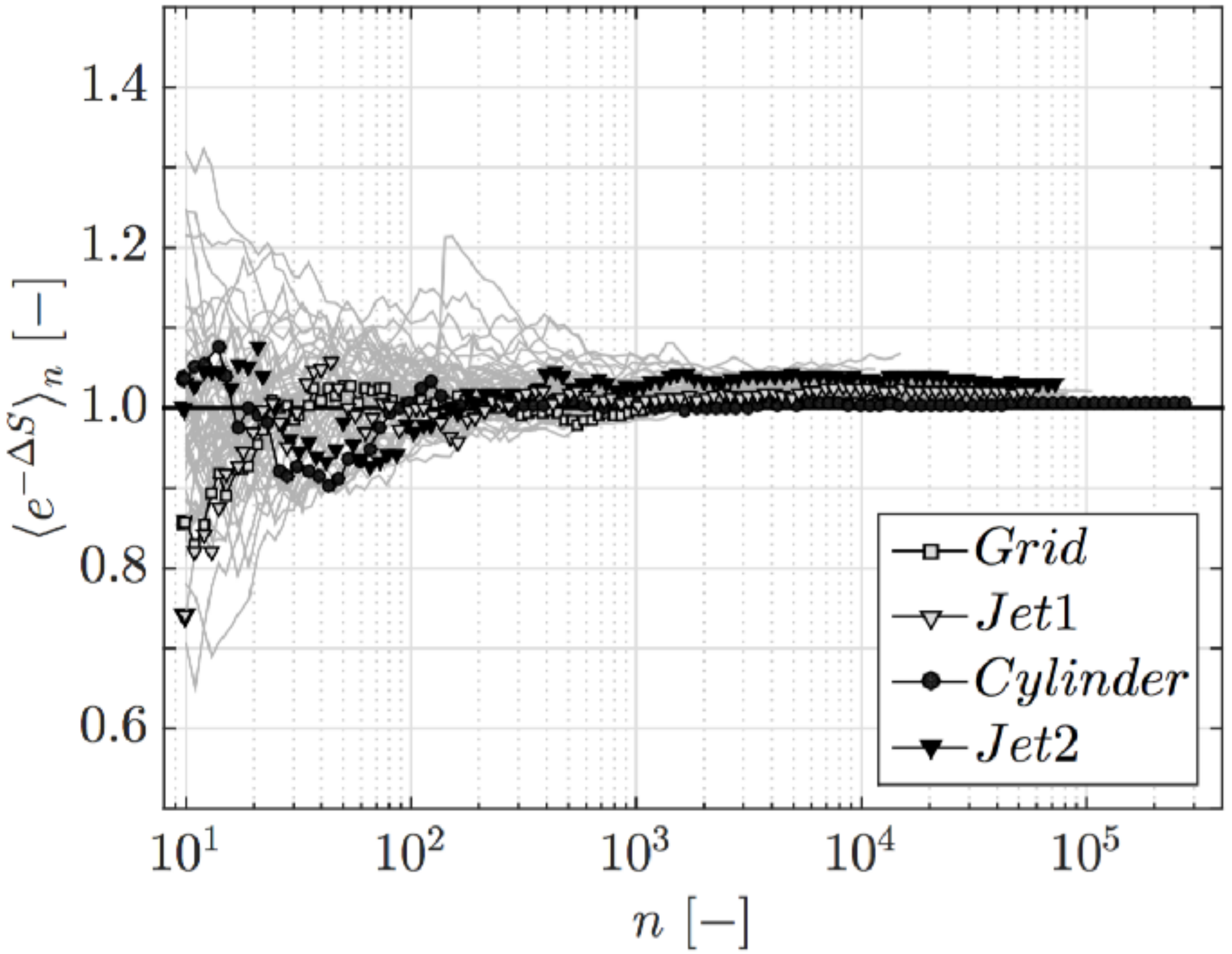


- should hold, if the non-equilibrium thermodynamics is given by a Fokker_Planck equation

integral fluctuations theorem a new precision law
IFT for 60 different data sets



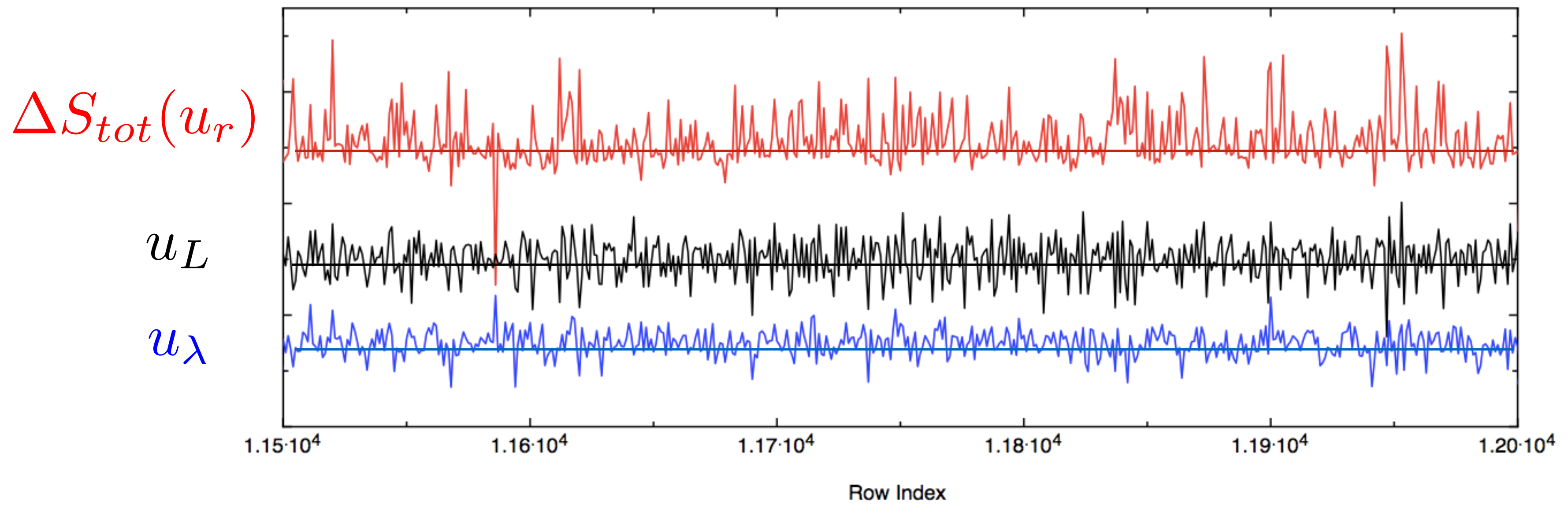
$$< e^{-S_{tot}(u_r)} >= 1.01 \pm 0.01$$



Dataset	Re_λ [-]	$L[mm]$	λ [mm]
i) grid	153	25.0	2.61
iii) cylinder	894	25.2	3.35
iv) jet (2)	996	1.95	0.186
vii) jet (1)	166	62.5	5.84

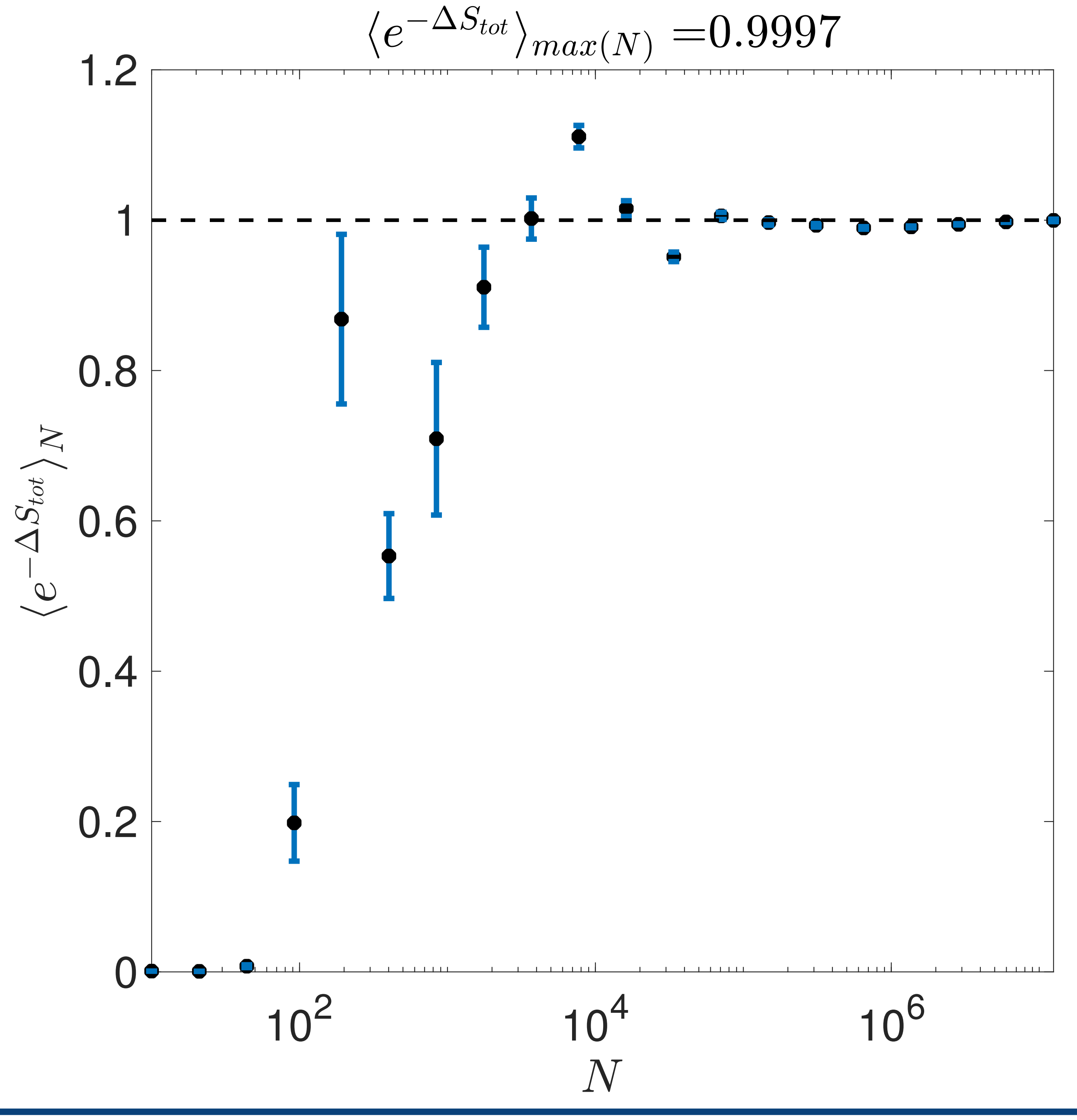
N. Reinke et al. JFM 2018

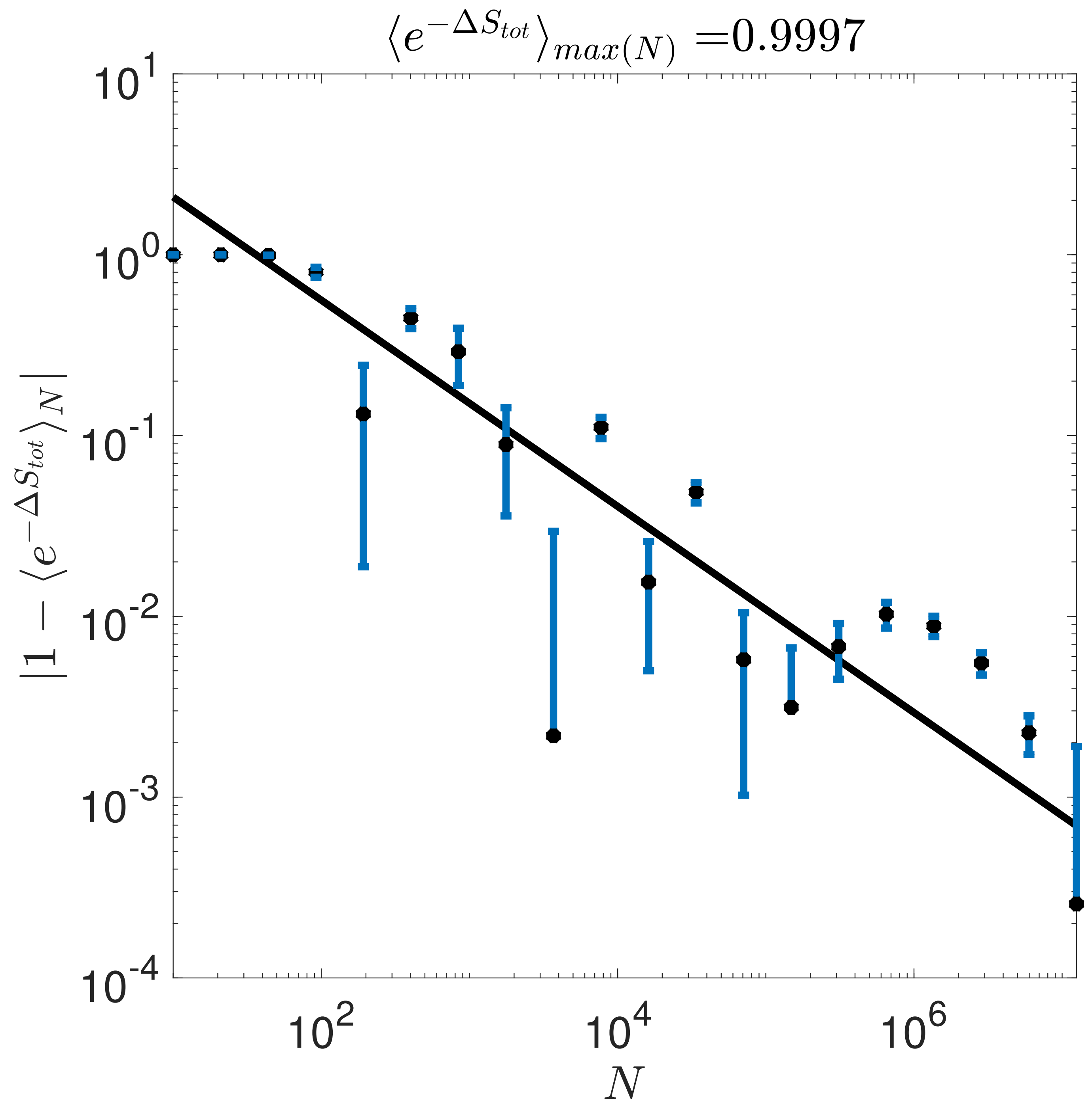
integral fluctuations theorem a new precision law



$$\langle e^{-S_{tot}(u_r)} \rangle = 1$$

fulfilled to 10^{-3}





Data Renner et.al J. Fluid Mech. 433, pp. 383–409 (2001)

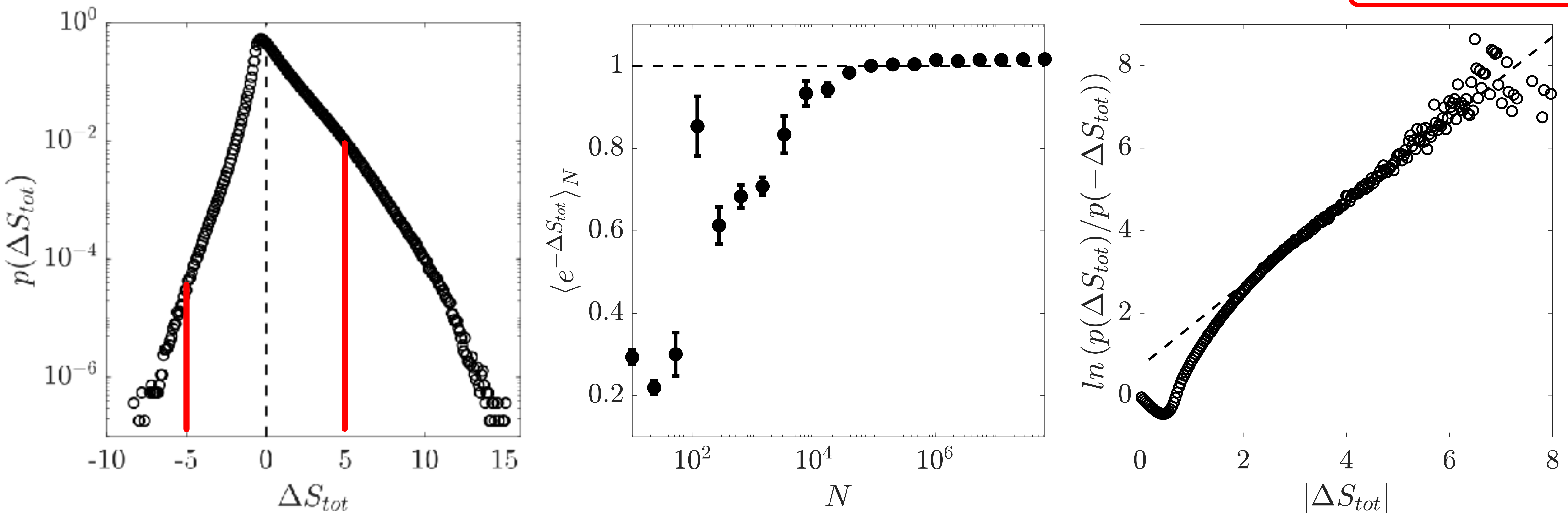
A. Fuchs et al, A Rigorous Entropy Law for the Turbulent Cascade Turbulent Cascades II. Springer, Cham, 2019. 17-25.

Entropy balance for Eulerian/Lagrangian turbulence

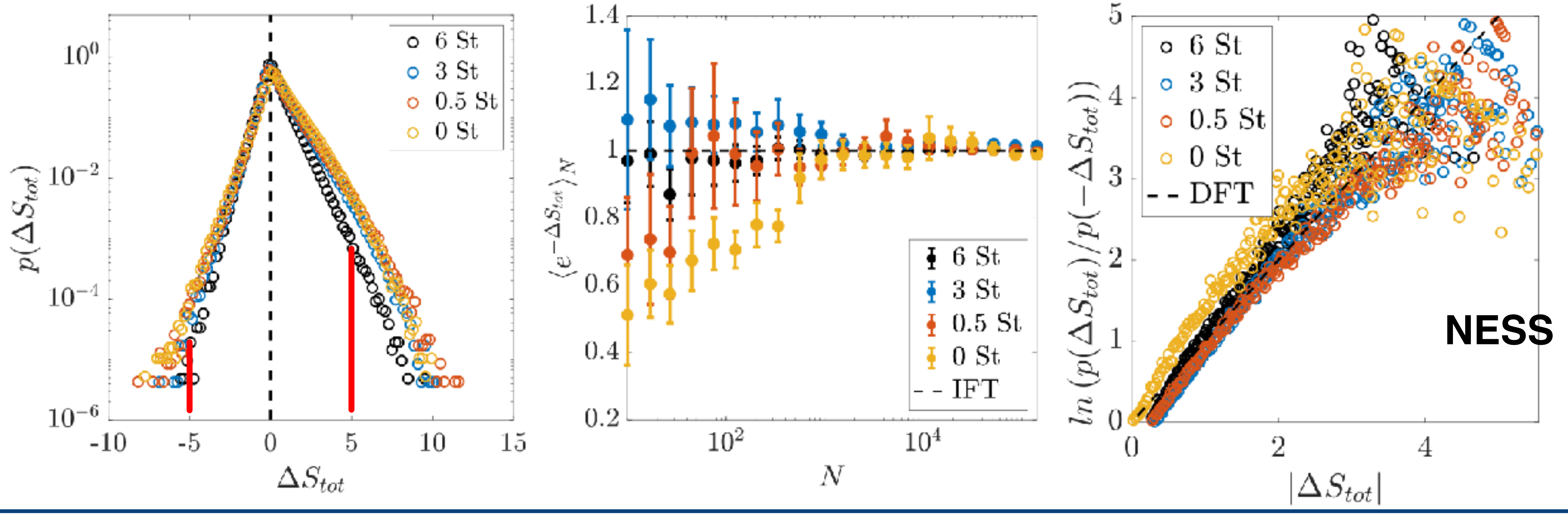
$$\ln\left(\frac{p(\Delta S_{tot})}{p(-\Delta S_{tot})}\right) \propto \Delta S_{tot}$$

$$p(\Delta S_{tot}) \propto p(-\Delta S_{tot})\, e^{\Delta S_{tot}}$$

Eulerian



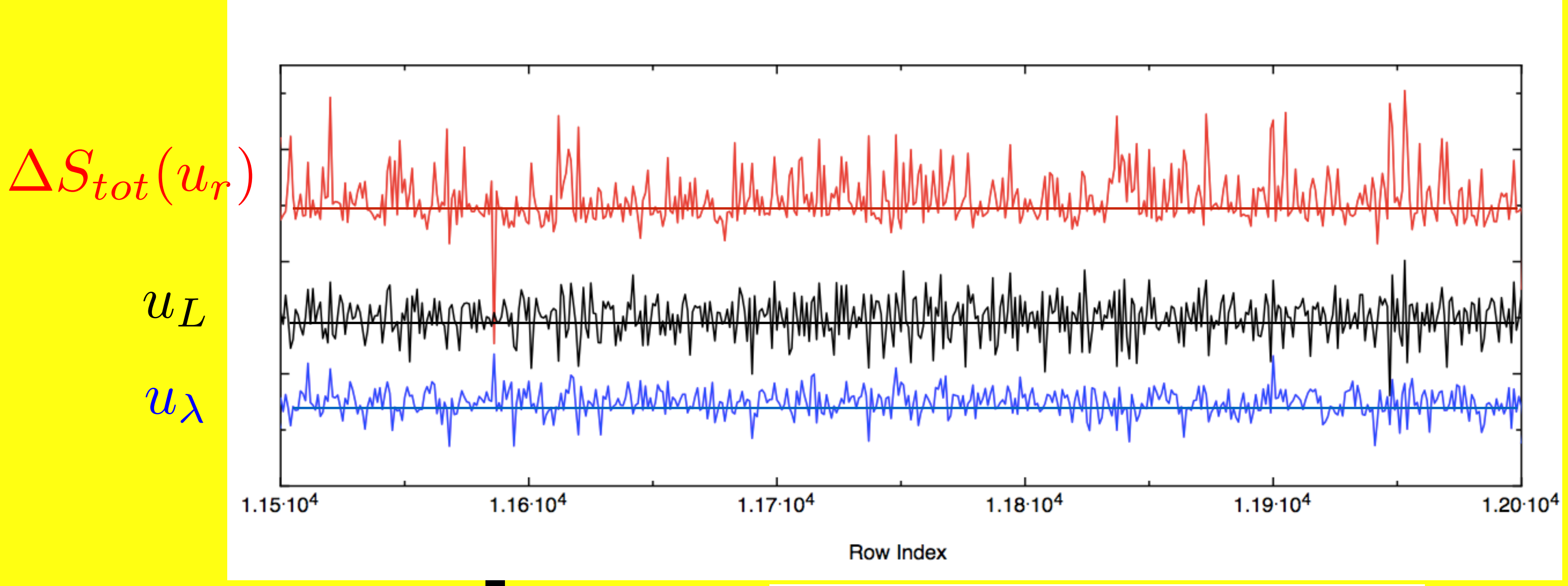
Lagrangian



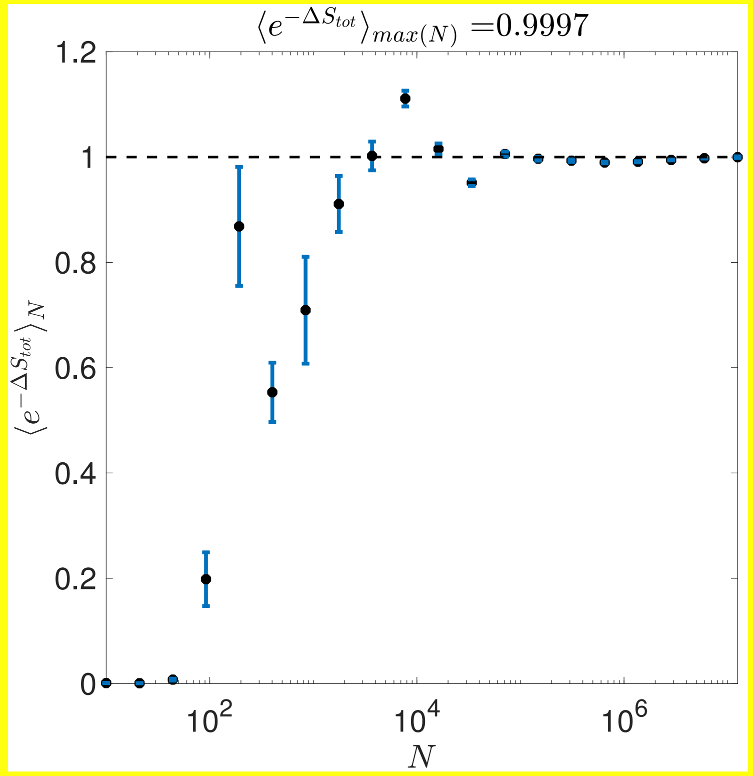
A. Fuchs, M. Obligado, M. Bourgoin, M. Gibert, P.D. Mininni, and J. Peinke :

EPL 137, 53001 (2022)

Integral and detailed fluctuations theorem



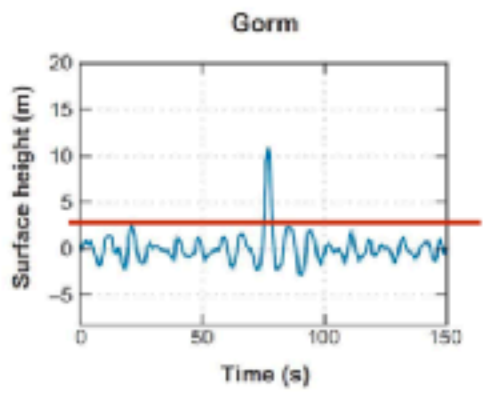
Give balance of negative & positive entropy events
And „explain“ non-Gaussian statistics



Introduction

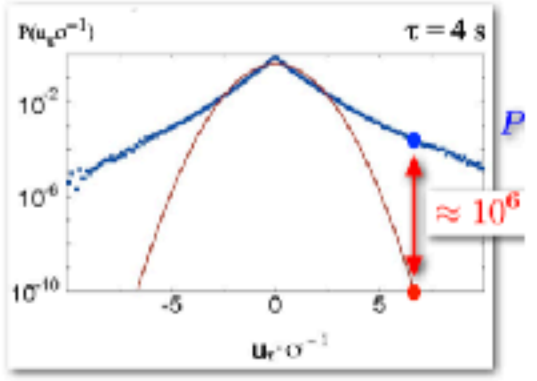
- monster wave

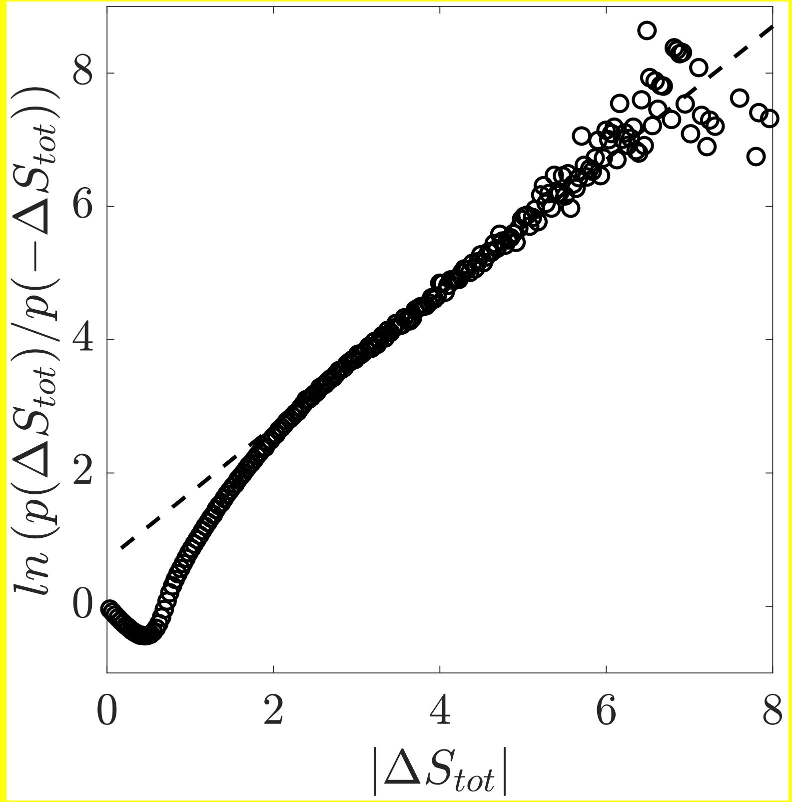
Are extreme wave (extreme structures) part of the system or additional structures?



- wind gusts

Non- Gaussian statistics of wind fluctuations





stochastic cascade process: Fokker - Planck equation

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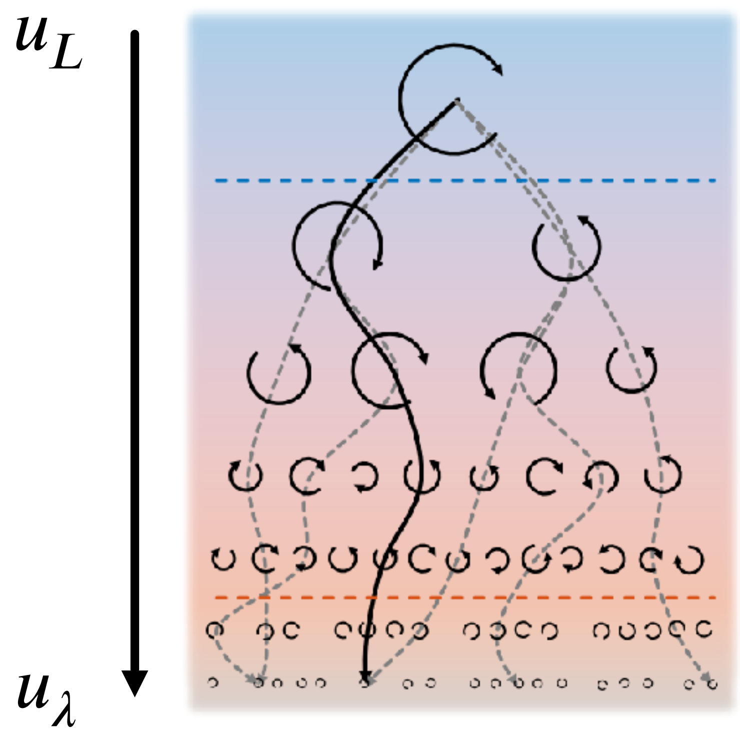
Statistical approach: multipoint statistics, **new data sets**

- **non equilibrium thermodynamics**
  - **Entropy** (Seifert 2005)
  - **Fluctuation Theorem**
  - **Hamiltonian for cascade - instantones & entropons**
  -

A.Fuchs , C. Herbert , J. Rolland , F. Bouchet , PRL 129 (2022)

Entropons: Instantons conditioned on entropy



Langevin equation for trajectories (stochastic process for cascade) $-\partial_r u_r = D^{(1)}(u_r, r) + \sqrt{D^{(2)}(u_r, r)}\Gamma(r)$

- Path integral formalism: probability of a trajectory is proportional to $e^{-\mathcal{A}[u(\cdot)]}$
- $\mathcal{A}$ is the Onsager-Machlup action given by

$$\mathcal{A}[u(\cdot)] = \int_L^\lambda \left[\frac{\left(\dot{u}_r - D^{(1)} + D'^{(2)}/2\right)^2}{4D^{(2)}} + \frac{D'^{(1)}}{2} \right] dr$$

Instantons are paths linking u_L and u_λ, given by the variational problem

$$A(u_L, u_\lambda) = \inf_u \{\mathcal{A}[u(\cdot)] \mid u(L) = u_L, u(\lambda) = u_\lambda\}$$

Entropons: instanton for fixed entropy -> constrained variational problem

$$A(S) = \inf_{u_L} \{A(u_L, \pm\tilde{u}_\lambda(S, u_L)) + f(u_L)\}$$

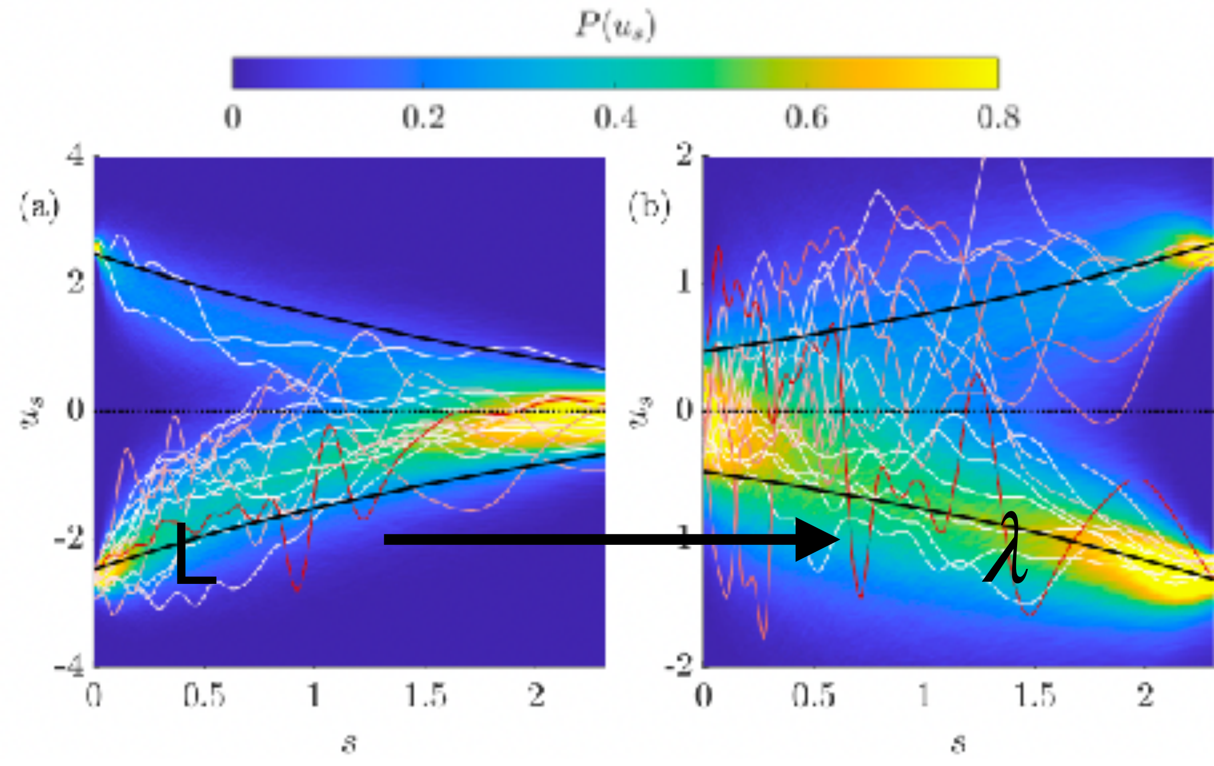
Entropons: Instantons for given entropy

path integral formalism probability of a trajectory is proportional to $exp\left[-\mathscr{A}\left[u(\cdot)\right]\right]$, where $\mathscr{A}$ in an **action**

and given by

$$\mathscr{A}\left[u(\cdot)\right] = \int_{s_i}^{s_f}\left[\frac{\left(\dot{u}_s - D^{(1)} + D'^{(2)}/2\right)^2}{4D^{(2)}} + \frac{D'^{(1)}}{2}\right] ds$$

L ⟶ λ



For path u_0 to u_f instantons are given by the variational problem

$$A(u_0, u_f) = \inf_u \left\{\mathscr{A}\left[u(\cdot)\right] \mid u(s_i) = u_0, u(s_f) = u_f\right\}$$

Which leads to the **Hamiltonian**

$$H = D^{(2)}p^2 + \left(D^{(1)} - \frac{D'^{(2)}}{2}\right)p - \frac{D'^{(1)}}{2},$$

$$= (\beta + \gamma u^2)p^2 - (\alpha + 2\gamma)up + \frac{\alpha + \gamma}{2}$$

J. Rolland

C. Herbert

F. Bouchet

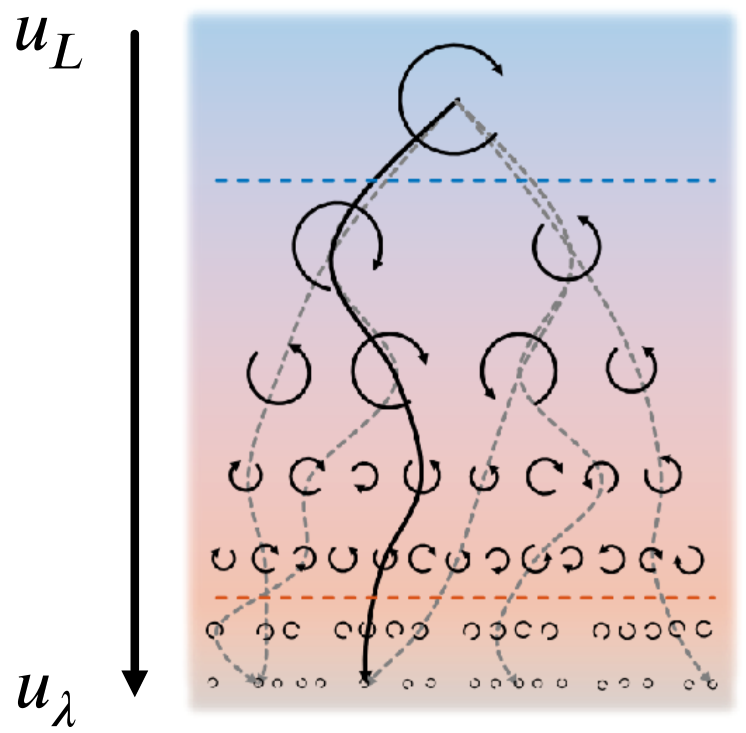
For given entropy of the trajectory we get

$$A(S) = \inf_u \left\{\mathscr{A}\left[u(\cdot)\right] + f(u_0) \mid \Delta S_{med}\left[u(\cdot)\right] = S\right\}$$

From what is follows -

$$\tilde{u}_f(S, u_0) = \sqrt{\frac{\beta}{\gamma}\left(e^{-\frac{2\gamma S}{\alpha + 2\gamma}} - 1\right) + u_0^2 e^{-\frac{2\gamma S}{\alpha + 2\gamma}}}$$

Entropons: Instantons conditioned on entropy



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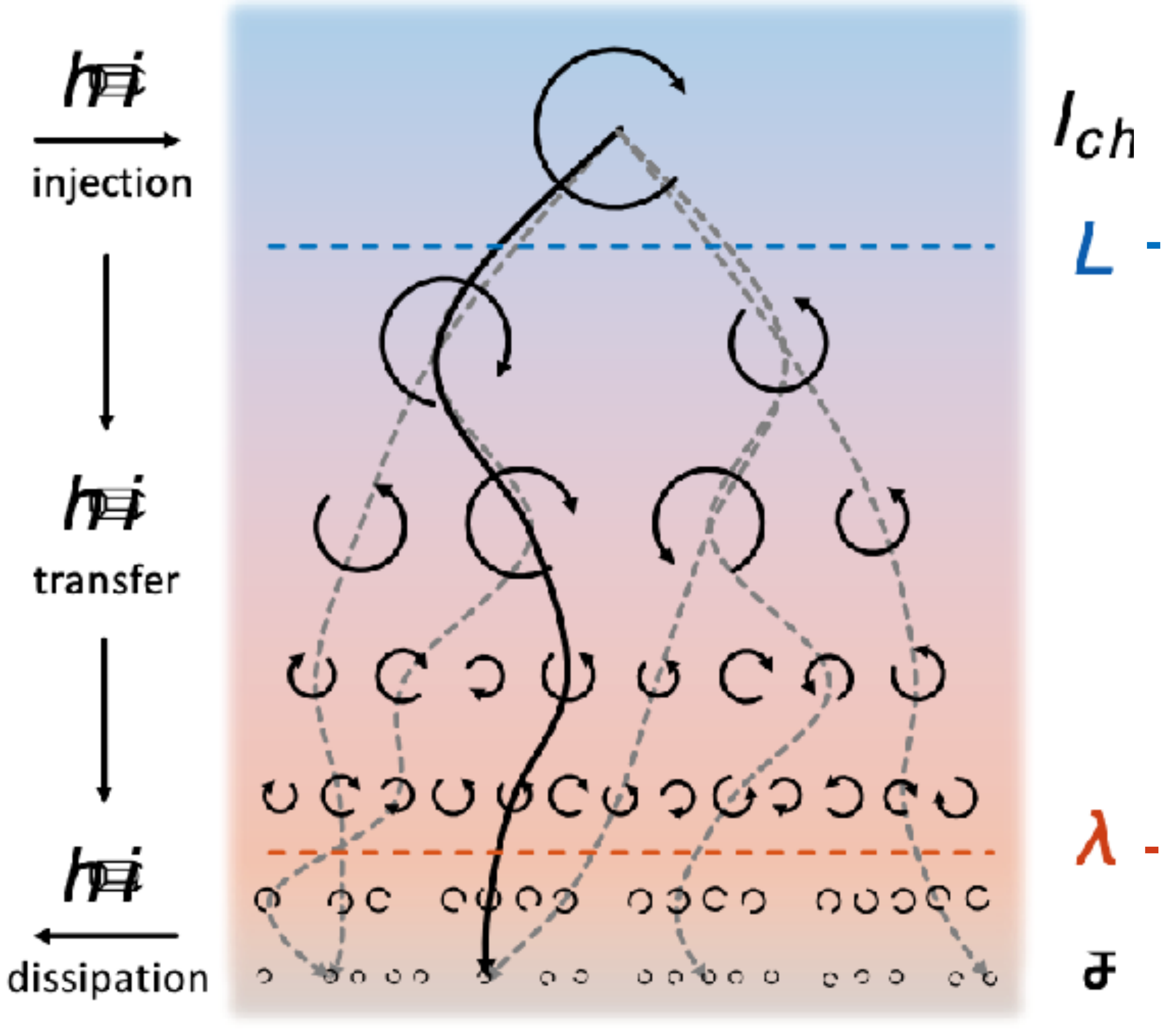
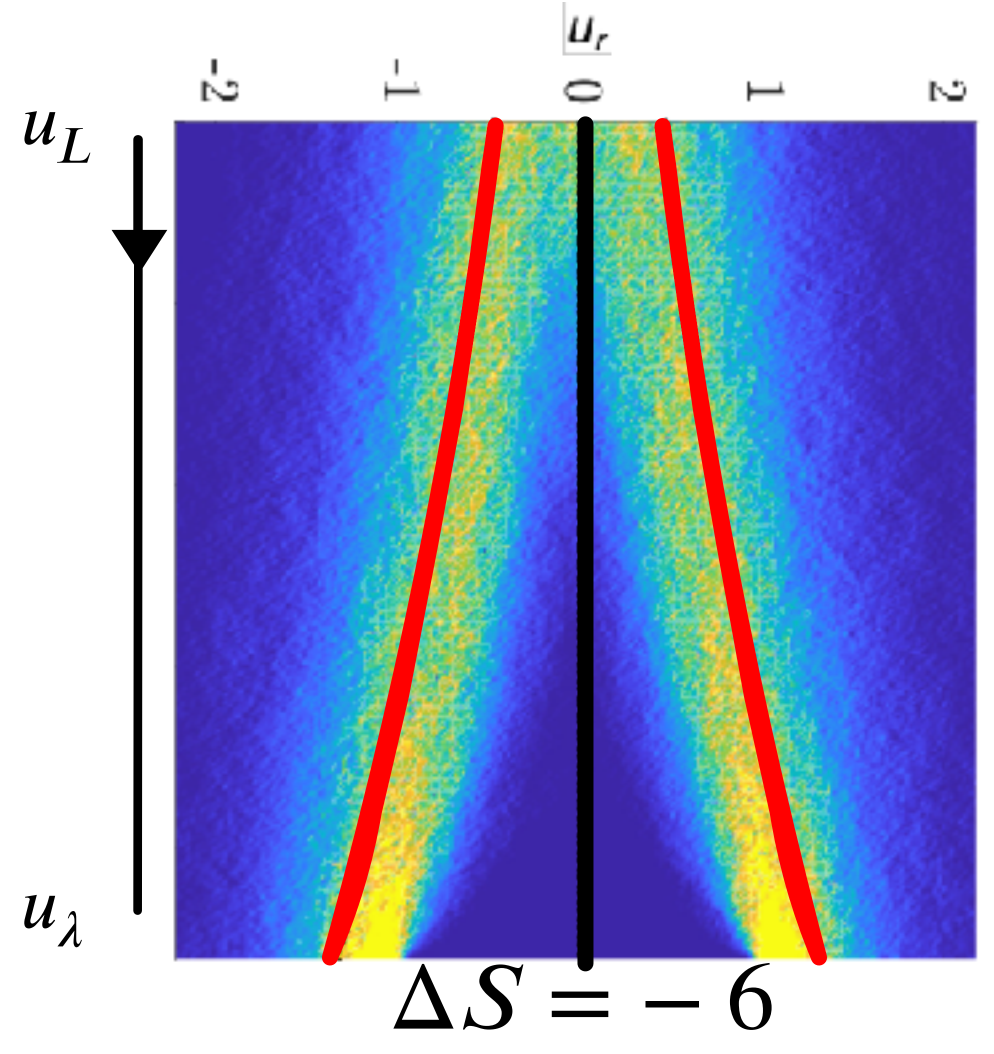
Simplified process

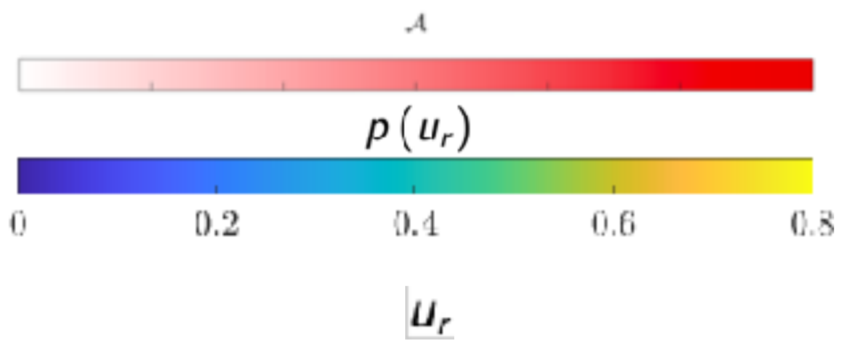
$$-\partial_r u_r = D^{(1)}(u_r) + \sqrt{D^{(2)}(u_r)}\Gamma(r)$$

$$D^{(1)}(u_r) = -(\alpha+\gamma)\, u_r$$
$$D^{(2)}(u_r) = \beta + \gamma u_r^2$$

Entropons are an underlying order within turbulence







Simplified process

$$D^{(1)}(u_r) = -(\alpha+\gamma)\,u_r$$
$$D^{(2)}(u_r) = \beta + \gamma u_r^2$$

$(\beta = 0)$ -> Kolmogorov 62 scaling

$(\beta \neq 0)$ -> Intermittency correction

instantons with $\beta \neq 0$ and $\Delta S < 0$ condition contribute to the non-Gaussian statistics at small scales

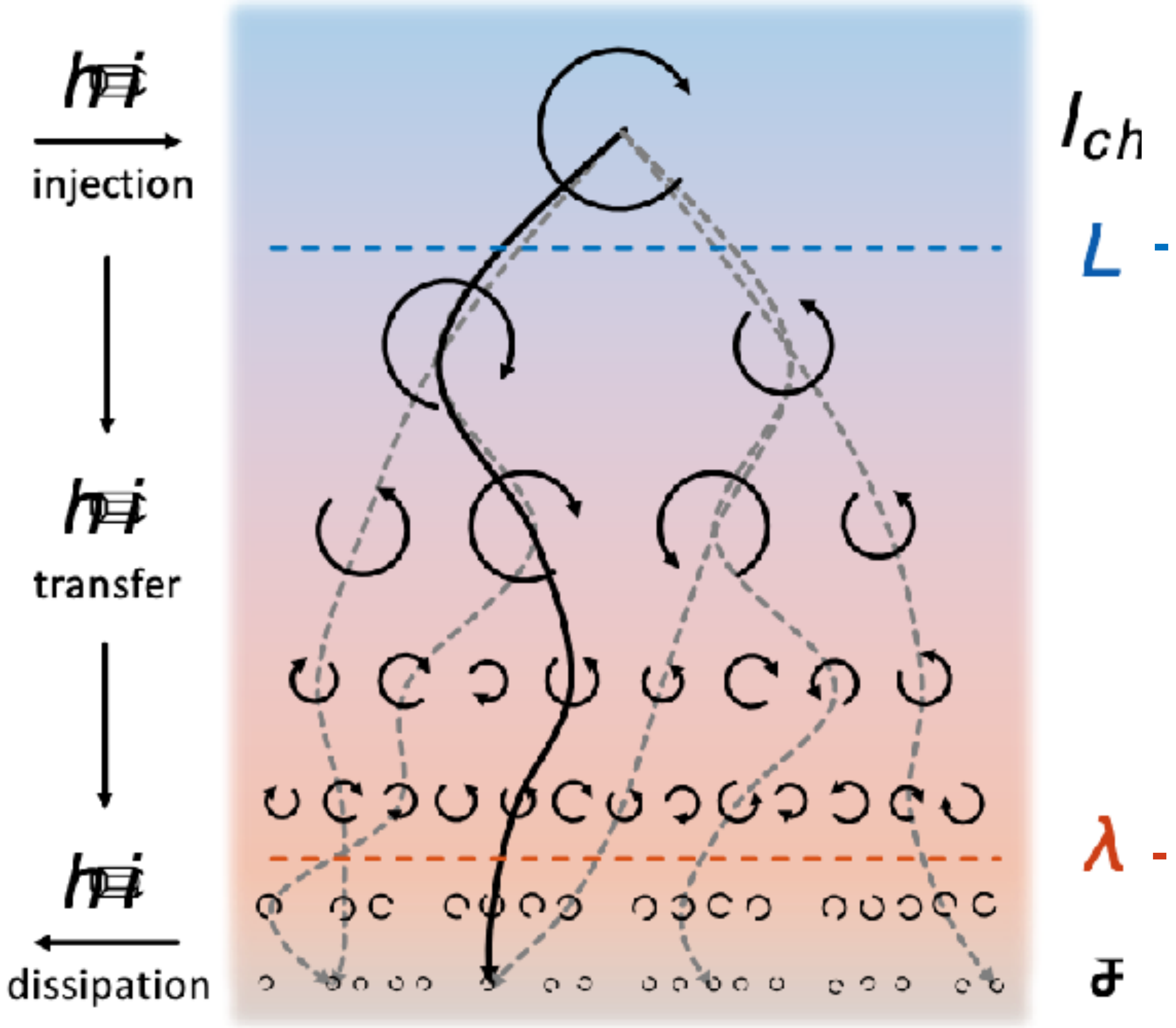
For given entropy of the trajectory we get

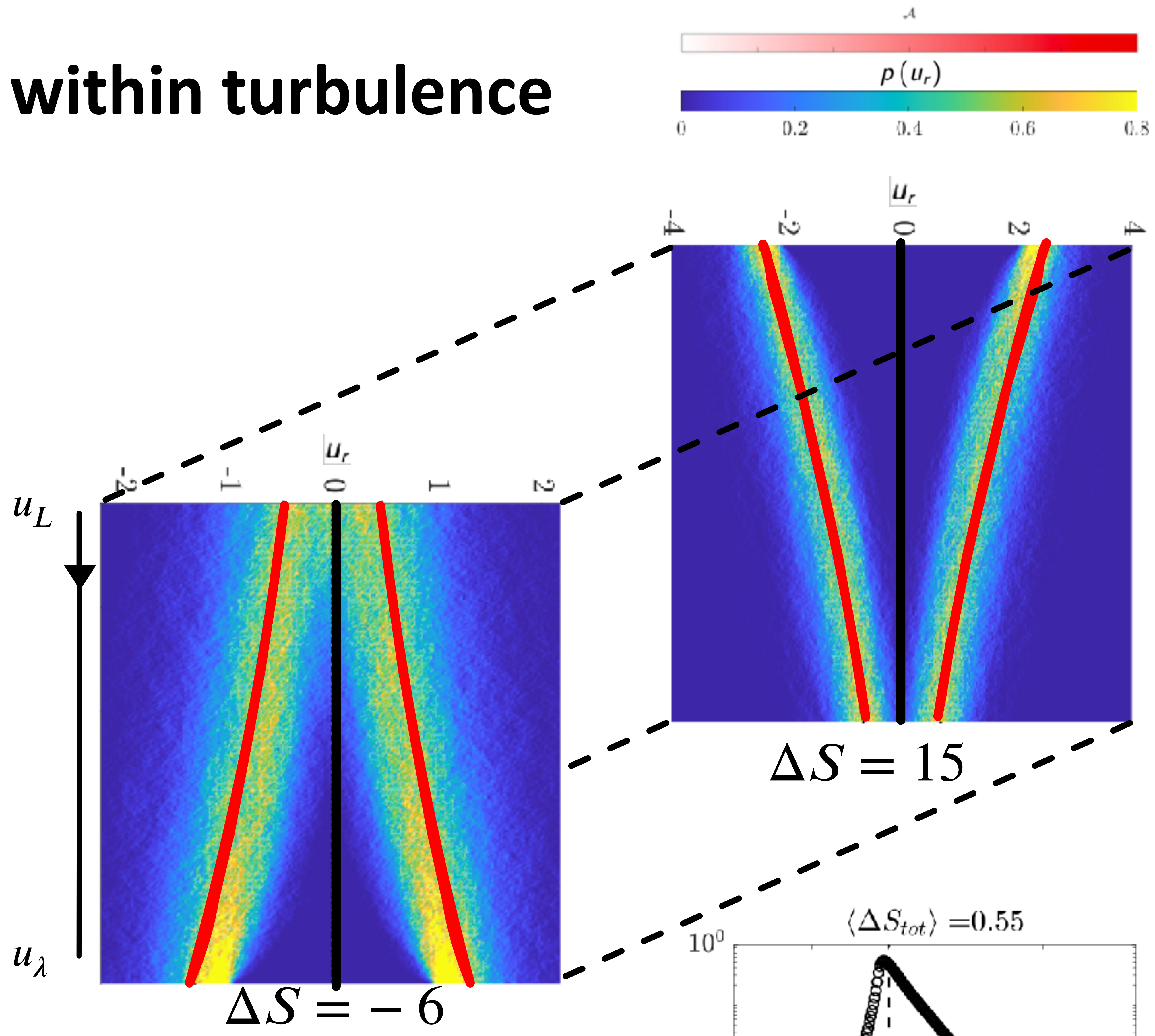
$$A(S) = \inf_u \left\{ \mathscr{A}\left[u(\cdot)\right] + f(u_0) \mid \Delta S_{med}\left[u(\cdot)\right] \right\}$$

From what is follows -

$$\tilde{u}_f(S, u_0) = \sqrt{\frac{\beta}{\gamma}\left(e^{-\frac{2\gamma S}{\alpha+2\gamma}} - 1\right) + u_0^2 e^{-\frac{2\gamma S}{\alpha+2\gamma}}}$$

Entropons are an underlying order within turbulence





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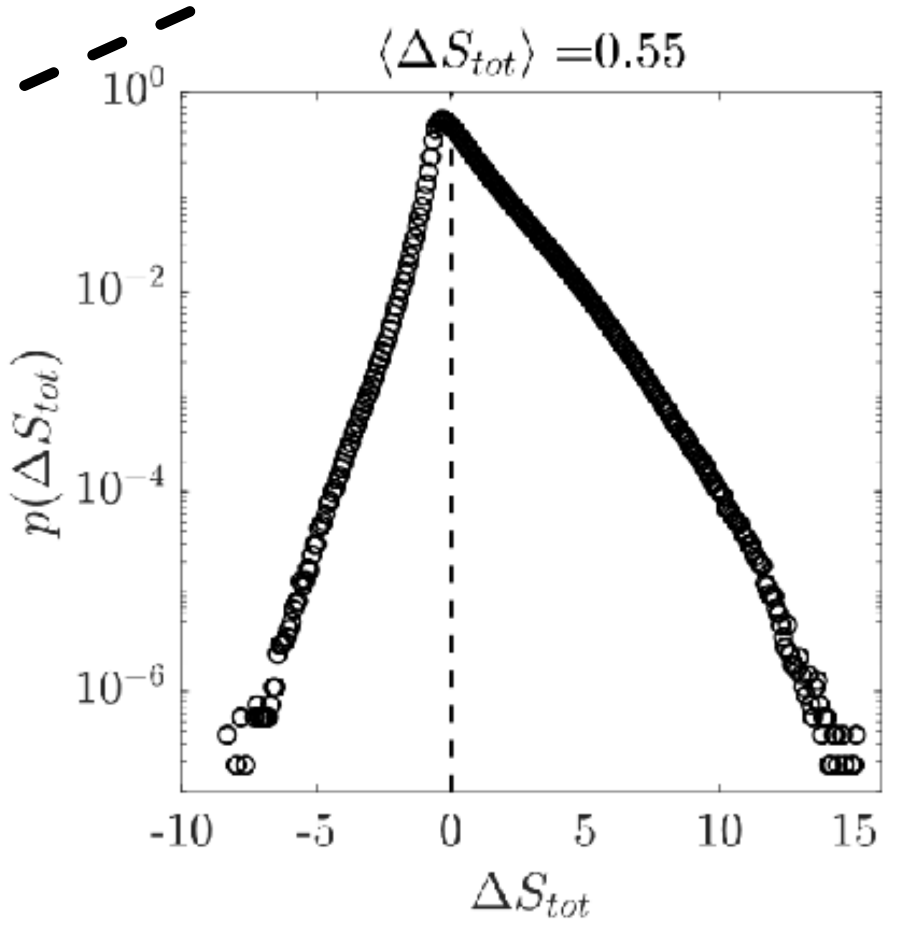

Simplified process

$$D^{(1)}(u_r) = -(\alpha + \gamma)\, u_r$$
$$D^{(2)}(u_r) = \beta + \gamma u_r^2$$

$(\beta = 0)$ -> Kolmogorov 62 scaling

$(\beta \neq 0)$ -> Intermittency correction

IFT + DFT:
Statistical balance of entropy events

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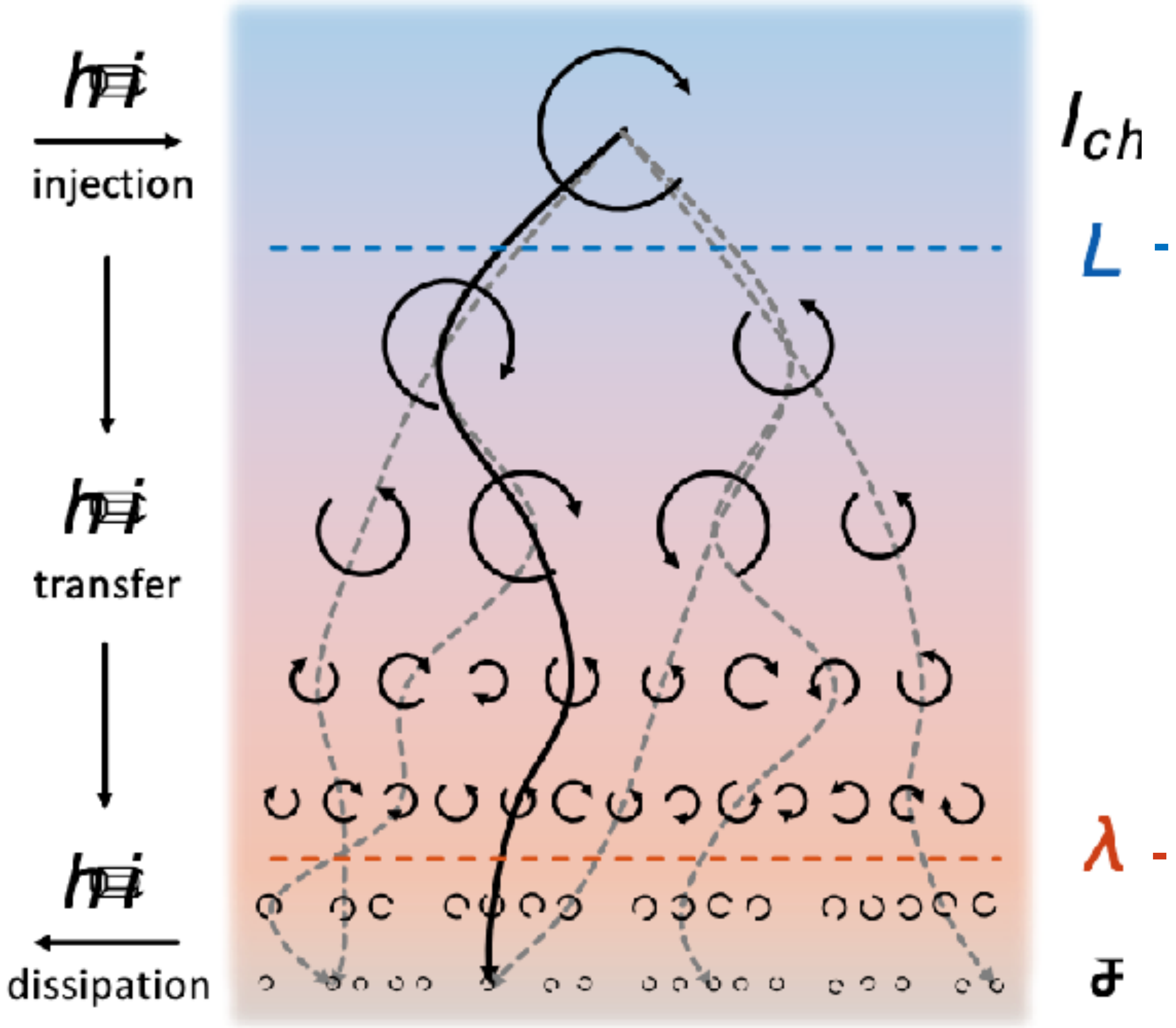
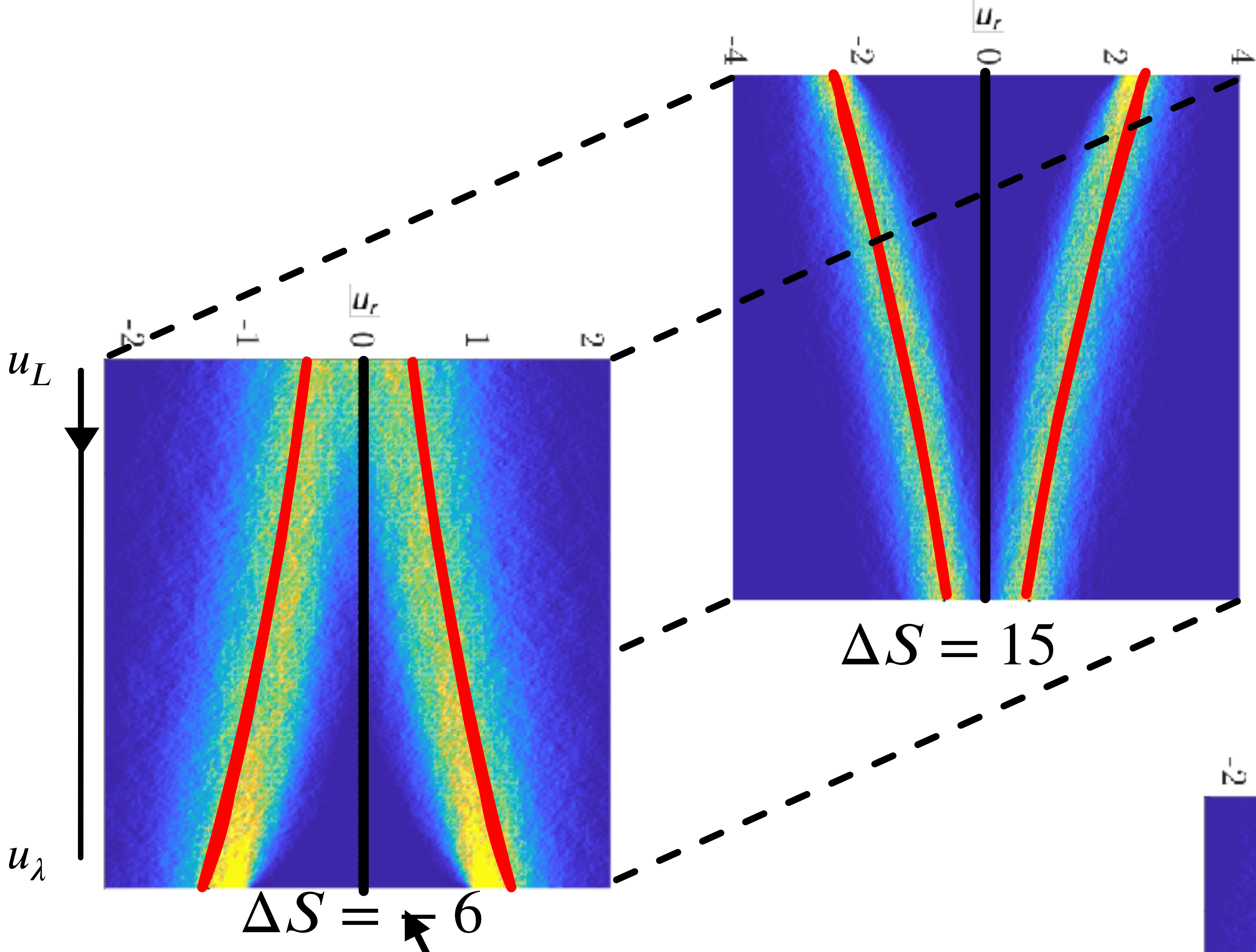
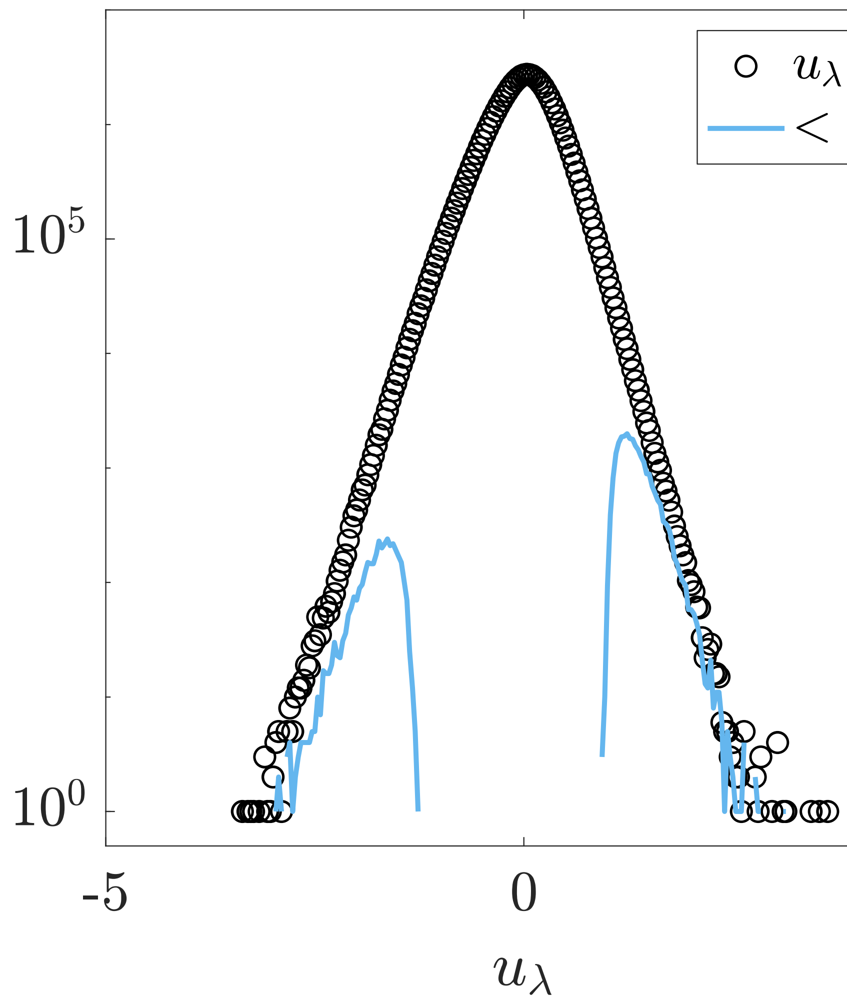
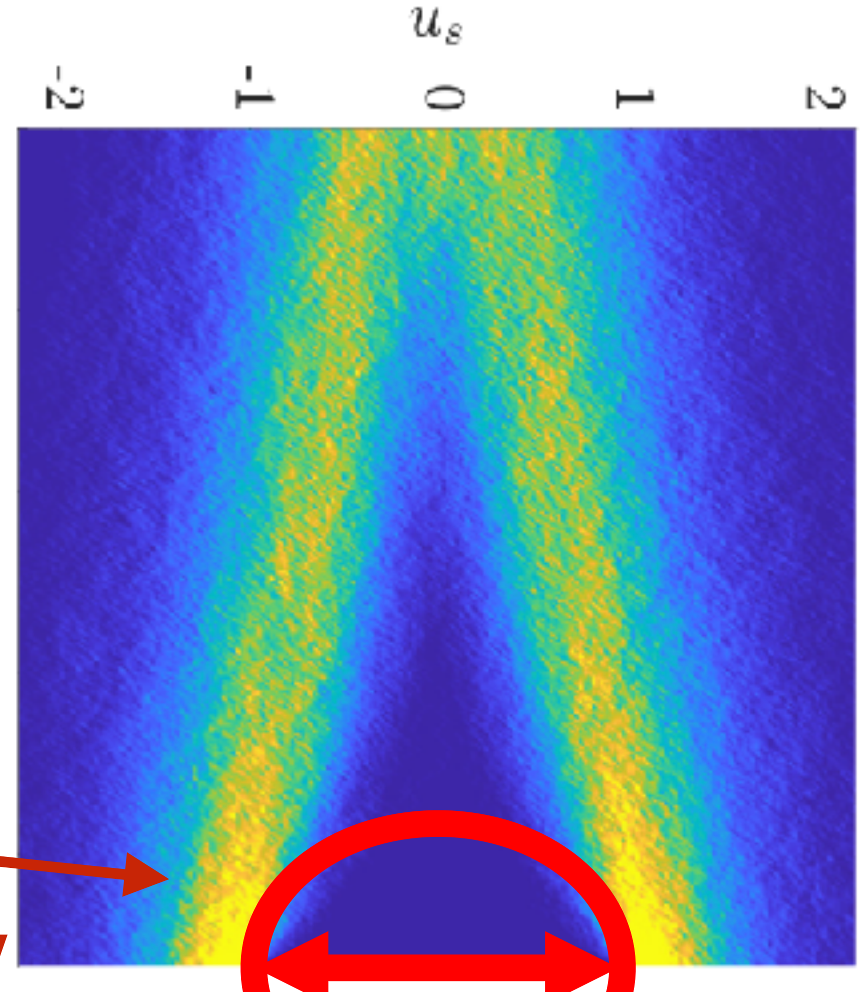
Entropons are an underlying order within turbulence







Exp data



$$
\begin{aligned}
D^{(1)}(u_r) &= -(\alpha+\gamma)\, u_r \\
D^{(2)}(u_r) &= \beta + \gamma u_r^2
\end{aligned}
$$

Forbidden region

Due to $\beta \neq 0$

=> Not K62 - breaking scaling symmetry

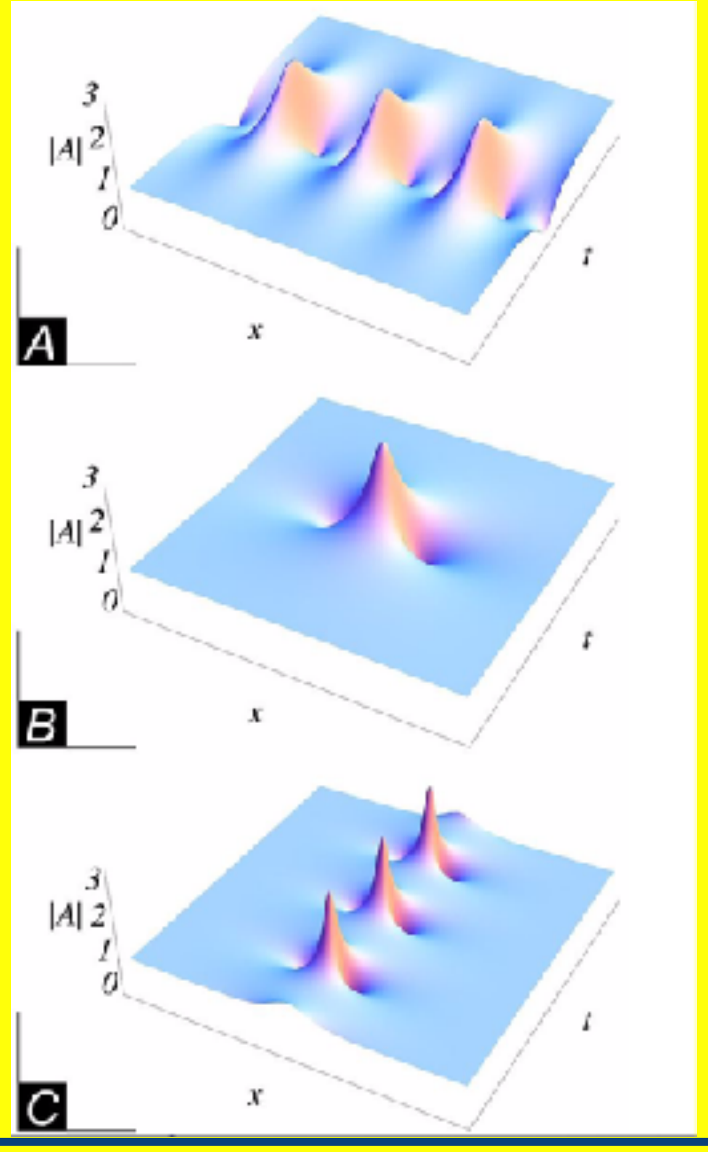
Turbulence and rogue waves

Nonlinear Schrödinger equation (NLSE)

$$i\frac{\partial\psi}{\partial x}+\frac{1}{2}\frac{\partial^2\psi}{\partial t^2}+|\psi|^2\psi=0$$

Navier Stokes equation (NSE)

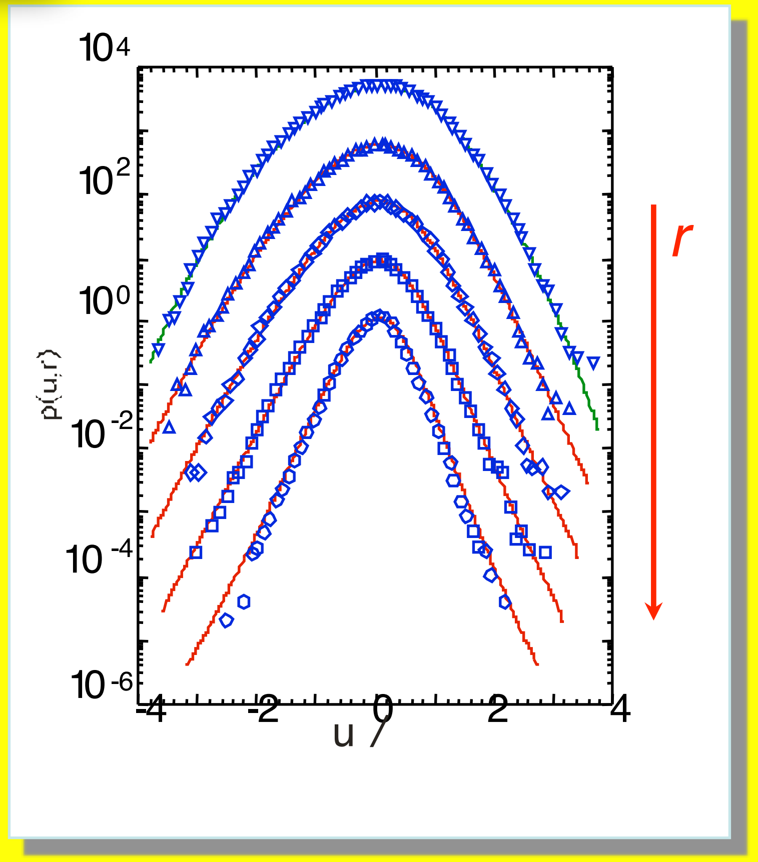
$$\partial_t\vec{u}+\left(\vec{u}\,\nabla\right)\vec{u}=-\frac{1}{\varrho}\nabla p+\nu\Delta\vec{u}\,,\qquad \nabla\vec{u}=0$$



How to approach?

coherent structures (Instabilities)

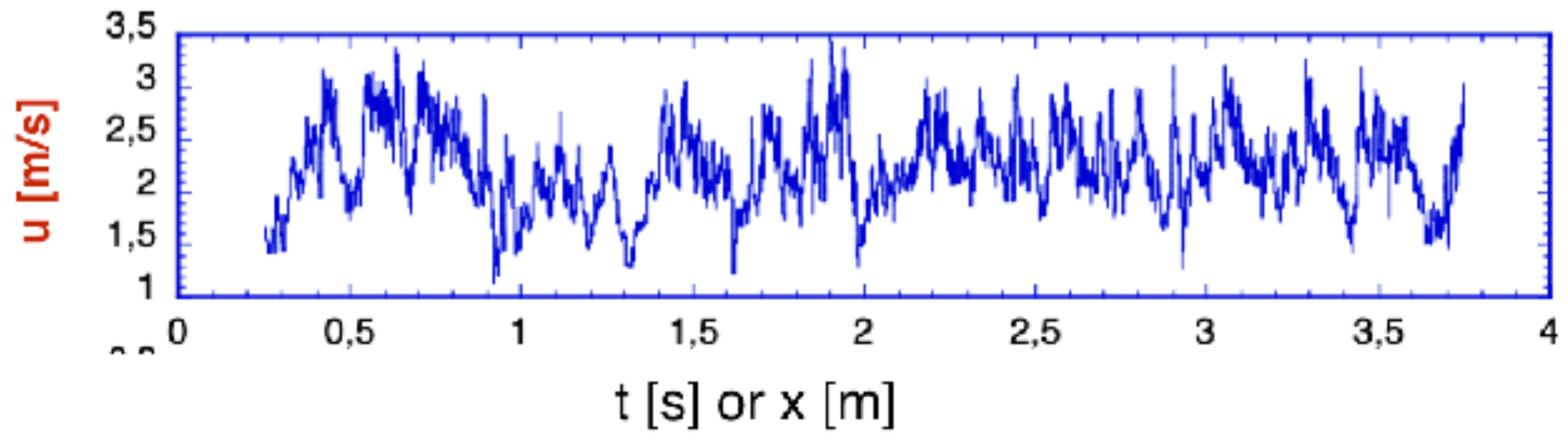
statistical approach- extreme events



Aim - general joint multipoint statistics combines both sides

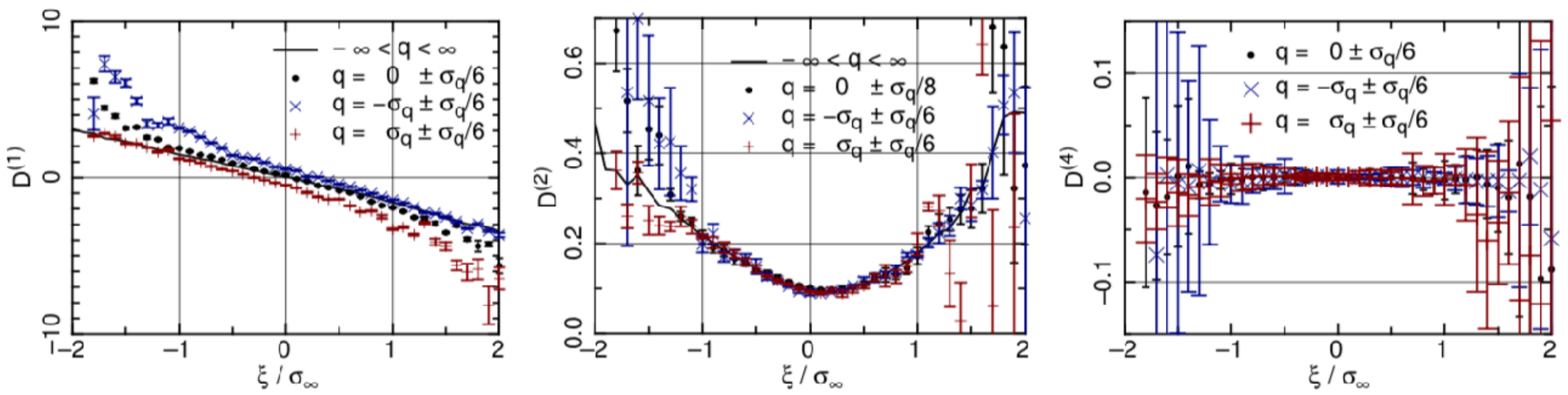
$$-r_j\frac{\partial}{\partial r_j}p(u_{r_j}|u_{r_k},u(x_1))=\{-\frac{\partial}{\partial u_{r_j}}D^{(1)}(u_{r_j},r_j,u(x_1))+\frac{\partial^2}{\partial u_{r_j}^2}D^{(2)}(u_{r_j},r_j,u(x_1))\}\,p(u_{r_j}|u_{r_k},u(x_1))$$

data



$$p(u(x_0), u(x_1), \ldots, u(x_n))$$

Data analysis - ML (Machine learning) of stochastic equations



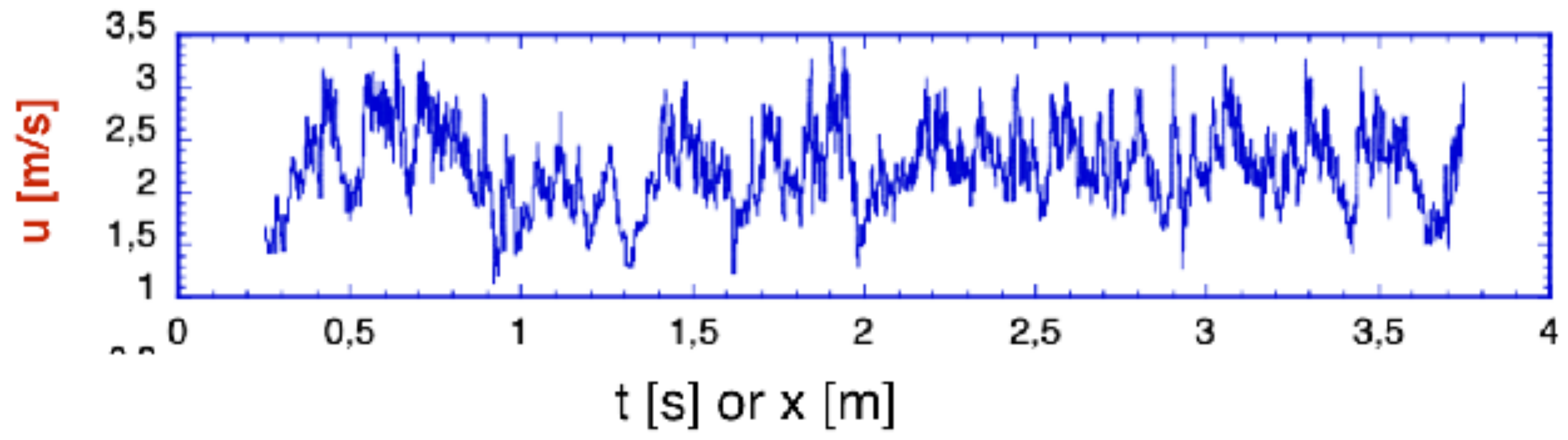
Fokker-Planck equation

$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j}|u_{r_k}, u(x_1)) = \{-\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1))\}\, p(u_{r_j}|u_{r_k}, u(x_1))$$

Langevin equation

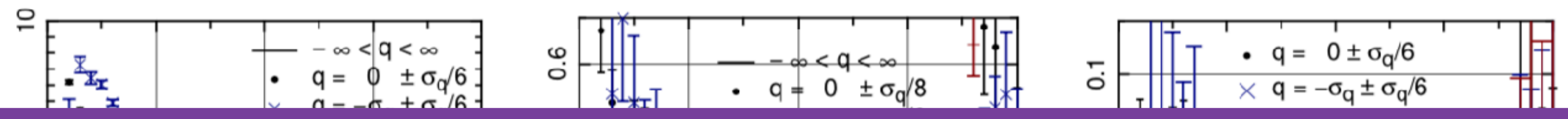
$$-r \frac{\partial}{\partial r} u_r = D^{(1)}(u_r, r) + \sqrt{D^{(2)}(u_r, r)}\,\eta(r)$$

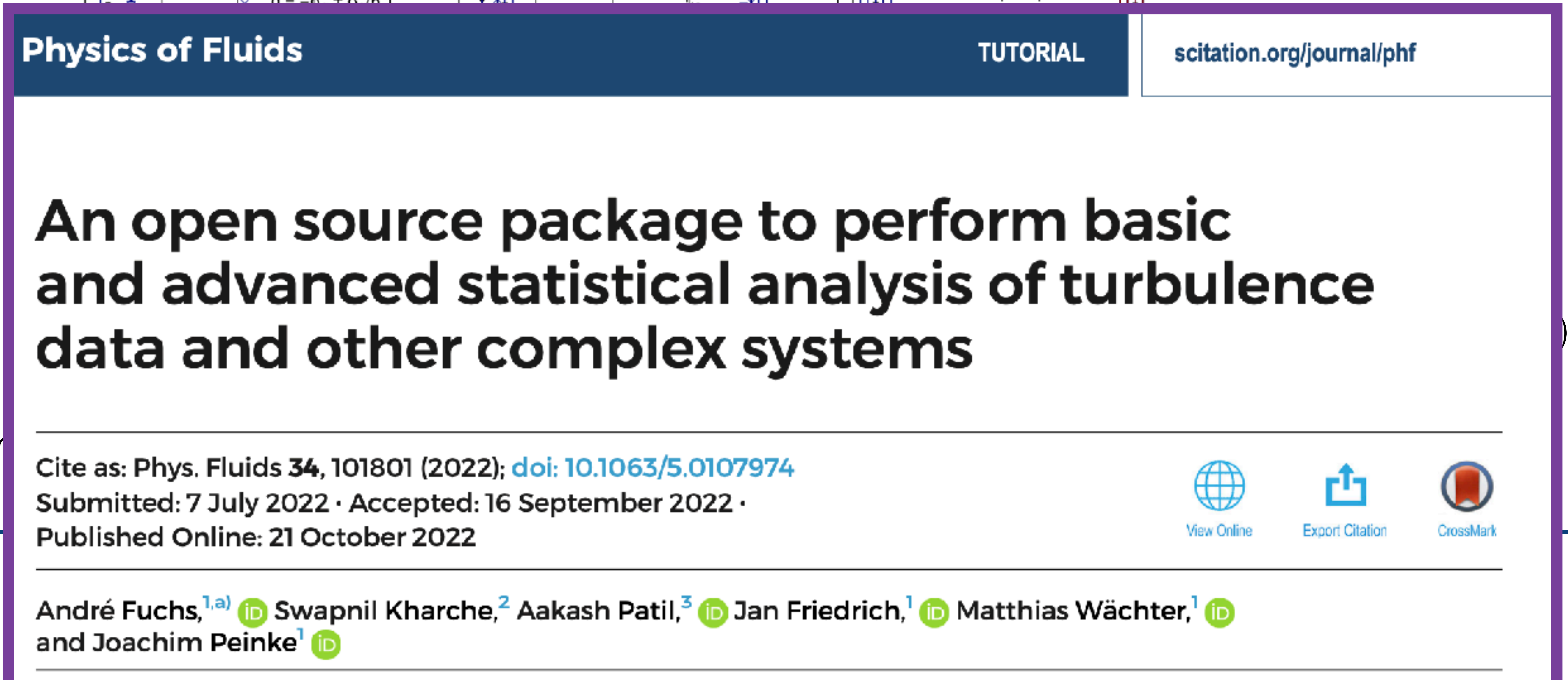
data



$$p(u(x_0), u(x_1), \ldots, u(x_n))$$

Data analysis - ML (Machine learning) of stochastic equations

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An open source package to perform basic and advanced statistical analysis of turbulence data and other complex systems

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Fokker-Planck eq

Langevin equatio

$$))\} \; p(u_{r_j} | u_{r_k}, u(x_1))$$

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Open questions

- Are nonlinear partial diff. equation equivalent to cascade Fokker-Planck equations?

$$\partial_t \vec{u} + (\vec{u}\,\nabla)\,\vec{u} = -\frac{1}{\rho}\nabla p + \nu \Delta \vec{u}\,, \qquad \nabla \vec{u} = 0$$

$$i\frac{\partial \psi}{\partial x} + \frac{1}{2}\frac{\partial^2 \psi}{\partial t^2} + |\psi|^2 \psi = 0$$

$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j}|u_{r_k}, u(x_1)) = \{-\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1))\}\, p(u_{r_j}|u_{r_k}, u(x_1))$$

- Is the fluctuation theorem of entropy a new constraint of extreme events (instabilities/structures)?
  - Entropons show the entropy is linked to structures - big increments

$$\langle e^{-\Delta S_{tot}} \rangle = 1$$

- what is the role of noise? For turbulence it is an intrinsic part of the system - for solitons?

Diffusion - $D^{(2)} \propto u_r^2$

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DLR Deutsches Zentrum für Luft- und Raumfahrt

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Forschungsbau Oldenburg

References

[1] J. Peinke, M. R. R. Tabar, and M. Wächter, Annual Review of Condensed Matter Physics 10, 107-132 (2019)

[2] A. Fuchs, C. Herbert, J. Rolland, M. Wächter, F. Bouchet, and J. Peinke, Phys. Rev. Lett. 129, 034502 (2022)

[3] A. Fuchs, S. Kharche, M Wächter, and Joachim Peinke, Physics of Fluids 34 (10), 101801 (2022)