Large Deviations in Active Matter

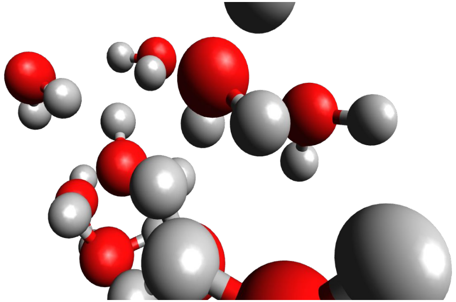
– Overview and Examples –

— Outline —

- Overview
  - What is «Active Matter»?
  - A variety of results
- Example 1: the Active Ornstein–Uhlenbeck Process (AOUP)
  - Steady-state and escape rate
  - Open questions
- Example 2: an active lattice gas model with non-Gaussian noise
  - Fluctuating Hydrodynamics
  - Dynamical Phase Transitions

– Overview – *What is «Active Matter»?*

What distinguishes the following systems?



molecules in solution

a sandstorm

a murmuration of starlings

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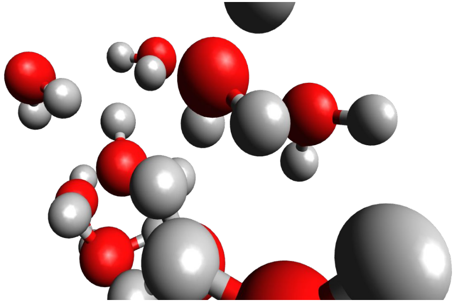

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Equilibrium

External drive

(External and) internal drive

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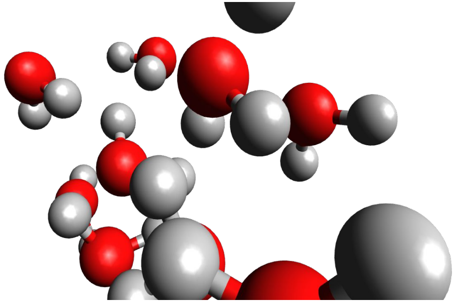

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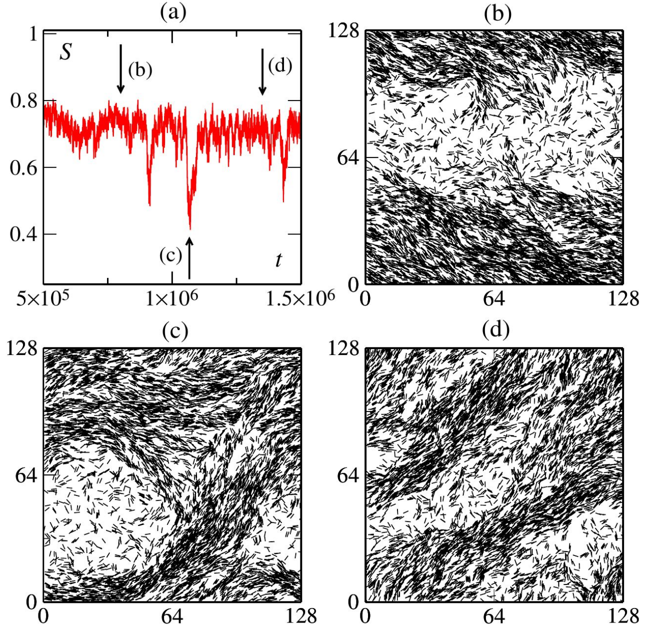
Recent review: J Tailleur, G Gompper, M C Marchetti, JM Yeomans & C Salomon, 2018 school "Active Matter and Nonequilibrium Statistical Physics", Lecture Notes of the Les Houches Summer School, OUP 2022

Examples and Motivations for studying Large Deviations

- *Giant number fluctuations in active fluids*

  Ramaswamy et al. [EPL **62** 196 (2003)], Chaté et al. [PRL **96** 180602 (2006)]



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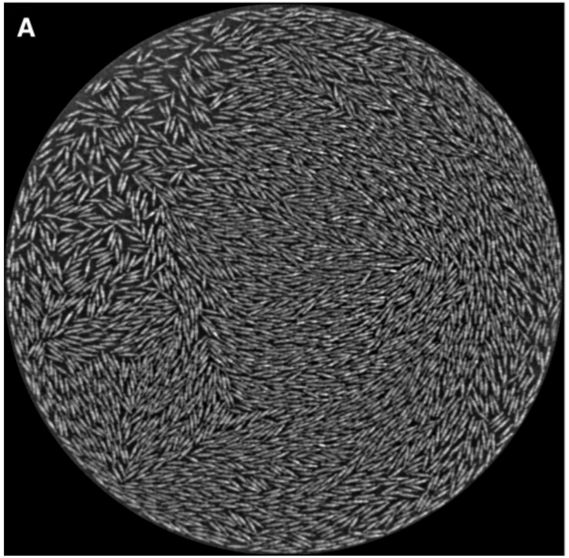
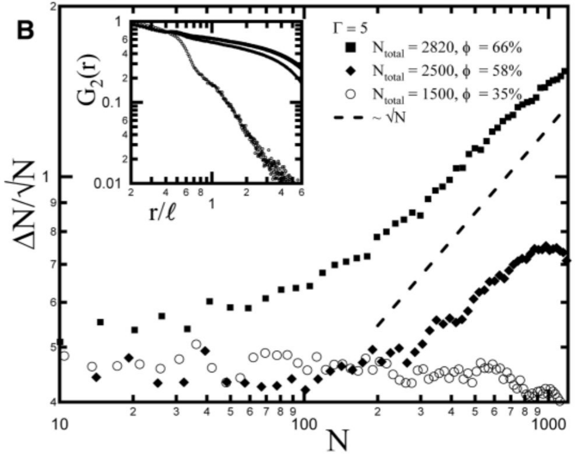
- Giant number fluctuations in active fluids

Narayan, Ramaswami & Menon [Science **317** 105 (2007)]





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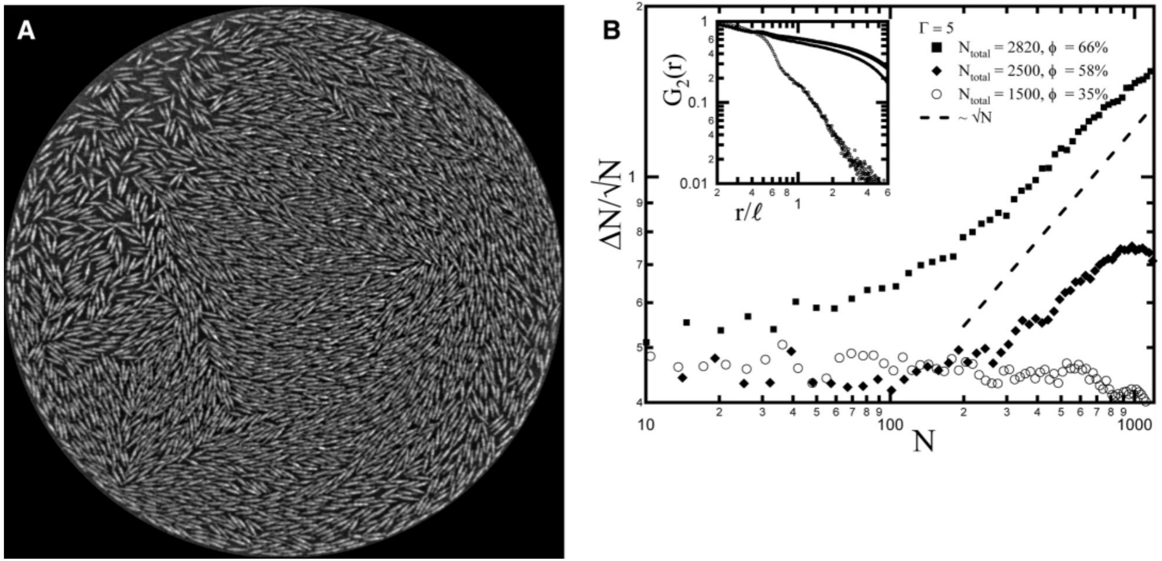


$\Delta N \equiv [\langle N^2\rangle - \langle N\rangle^2]^{1/2}$

std dev of #particles

- $\Delta N \propto N^{1/2}$ «@ eq.»

- $\Delta N \propto N^{\alpha > 1/2}$ active

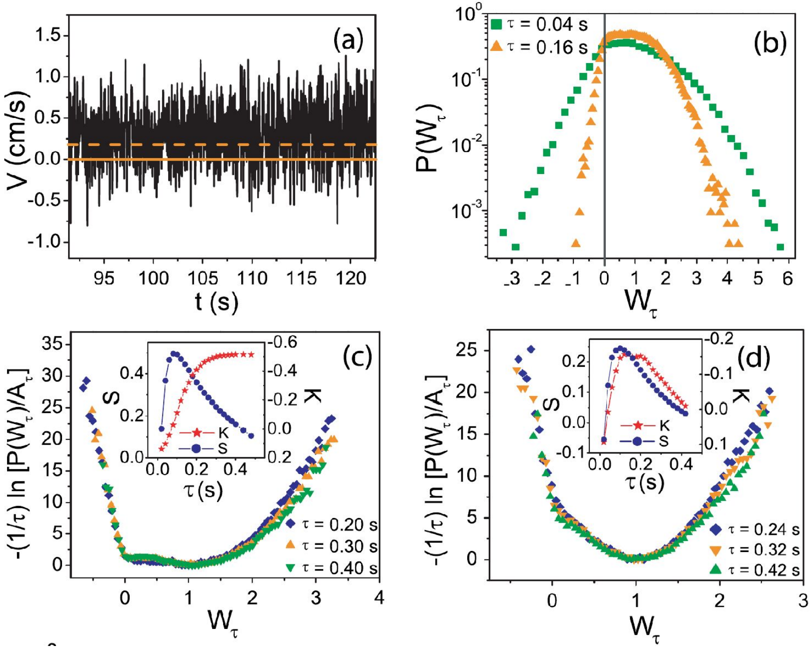
→ anomalous cumulant scaling

→ anomalous scaling for the cumulant generating function (CGF)?

Large Deviations in Active Matter – Example Results

- *An experiment: Distribution of time-averaged velocity for a gas polar granular rods confined in 2D*

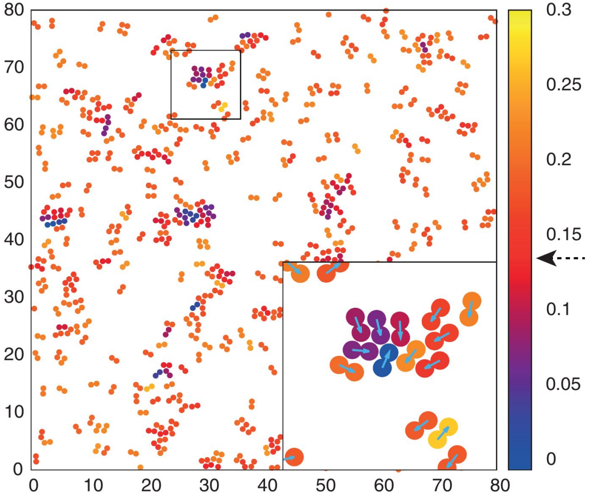
  *Kumar et al. [PRL **106** 118001 (2011)]*

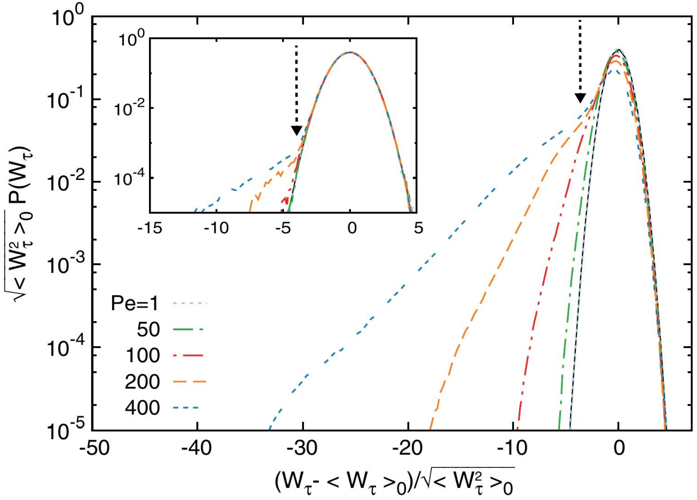


Large Deviations in Active Matter – Example Results

- *An example model: Distribution of "active work" for self-propelled dumbbell particles*

  Cagnetta et al. [PRL **119** 158002 (2017)]





Large Deviations in Active Matter – Example Results

- *An example model: Active Brownian Particles (APB)* $\dot{\mathbf{x}} = -\mu\nabla V + v\mathbf{u}(\theta) + \sqrt{2D}\boldsymbol{\xi}(t)$

Weak-noise description ($D \to 0$), for a single particle:

$$P(\mathbf{x}_2, t|\mathbf{x}_1, 0) = \int_{\mathbf{x}_1}^{\mathbf{x}_2} \mathcal{D}[\mathbf{x}(t), \theta(t)] e^{-(1/D)\mathcal{A}[\mathbf{x},\theta]} \mathcal{P}[\theta(t)]$$

$$\mathcal{A}[\mathbf{x}, \theta] = \frac{1}{4}\int_0^t \|\dot{\mathbf{x}} + \mu\nabla V(\mathbf{x}) - v\mathbf{u}(\theta)\|^2 dt'$$

$$\mathcal{P}[\theta(t)] \propto e^{-\int_0^t \dot{\theta}^2/(4\alpha)\mathrm{d}t'}$$

[Woillez et al. PRL **122** 258001 (2019)]

Large Deviations in Active Matter – Example Results

- *An example model: Active Brownian Particles (APB)* $\dot{\mathbf{x}} = -\mu\nabla V + v\mathbf{u}(\theta) + \sqrt{2D}\boldsymbol{\xi}(t)$

Weak-noise description @ $D \to 0$

$$P(\mathbf{x}_2, t|\mathbf{x}_1, 0) = \int_{\mathbf{x}_1}^{\mathbf{x}_2} \mathcal{D}[\mathbf{x}(t), \theta(t)] e^{-(1/D)\mathcal{A}[\mathbf{x},\theta]} \mathcal{P}[\theta(t)]$$

Minimizing w.r.t. $\theta(t)$ [contraction principle]:

$$\int \mathcal{D}[\theta(t)] e^{-(1/D)\mathcal{A}[\mathbf{x},\theta]} \mathcal{P}[\theta(t)] \underset{D\to 0}{\asymp} e^{-(1/D)\mathcal{A}[\mathbf{x},\tilde{\theta}]}$$

$$\mathcal{A}[\mathbf{x}, \tilde{\theta}] = \inf_{\theta}\left(\frac{1}{4}\int_0^t \|\dot{\mathbf{x}} + \mu\nabla V(\mathbf{x}) - v\mathbf{u}(\theta)\|^2 dt'\right)$$

Optimal angle trajectory: $\mathbf{u}(\tilde{\theta}) = \dfrac{\dot{\mathbf{x}} + \mu\nabla V(\mathbf{x})}{\|\dot{\mathbf{x}} + \mu\nabla V(\mathbf{x})\|}$

[Woillez et al. PRL **122** 258001 (2019)]

Large Deviations in Active Matter – Example Results

- *An example model: Active Brownian Particles (APB)* $\dot{\mathbf{x}} = -\mu \nabla V + v\mathbf{u}(\theta) + \sqrt{2D}\boldsymbol{\xi}(t)$

Substituting, we arrive at

$$\mathcal{A}[\mathbf{x}] = \frac{1}{4}\int (\|\dot{\mathbf{x}} + \mu\nabla V(\mathbf{x})\| - v)^2 dt'$$

Escape problem: mean escape time $\langle \tau \rangle$ *between from domain* C_1 *to domain* C_2 *given by*

$$\langle \tau \rangle \underset{D\to 0}{\asymp} e^{\frac{\phi}{D}},$$

$$\phi = \inf_{\{\mathbf{x}_1 \in \mathbf{C}_1, \mathbf{x}_2 \in \mathbf{C}_2\}} \inf_{\mathbf{x}(t)} \mathcal{A}[\mathbf{x}(t)]$$

[Woillez et al. PRL **122** 258001 (2019)]

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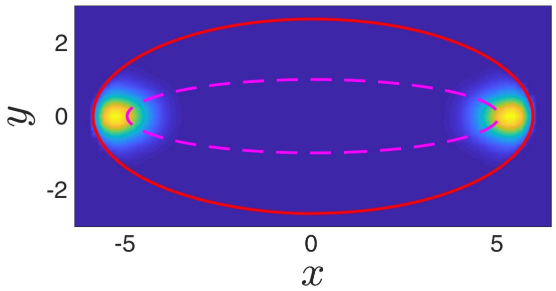
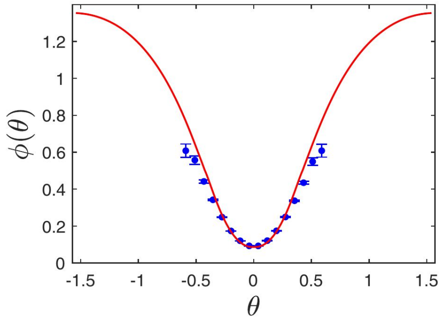
Escape from an elliptic trap in 2D



rate of escape from an elliptic trap V = ½x·Ax (pink to purple)

the escape time would be uniform for a passive particle (@D⟶0)



comparison between analytical prediction and numerics

[Woillez et al. PRL **122** 258001 (2019)]

Large Deviations in Active Matter – Example Results

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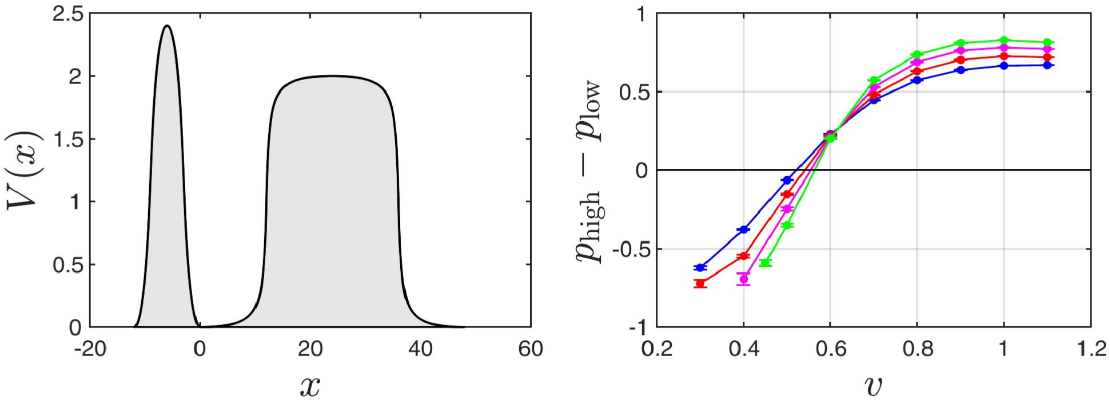
From barrier of potential to barrier of force

particles



$p_{high,low}$ = fraction of

crossing the high,low

barrier

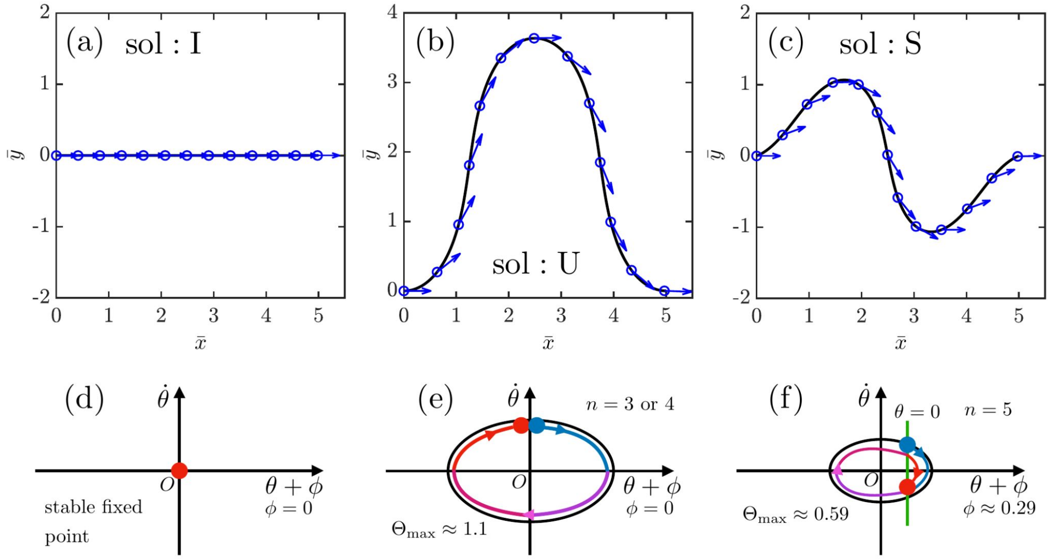
*At large activity v, the particle prefers to cross the barrier with the smaller maximal **force**.*

[Woillez et al. PRL **122** 258001 (2019)]

Large Deviations in Active Matter – Example Results

- *An example model: Active Brownian Particles (APB)* $\dot{\mathbf{x}} = -\mu\nabla V + v\mathbf{u}(\theta) + \sqrt{2D}\boldsymbol{\xi}(t)$

More generic optimal paths in 2D: depending on boundary conditions



[Yasuda & Ishimoto, PRE **106** 064120 (2022)]

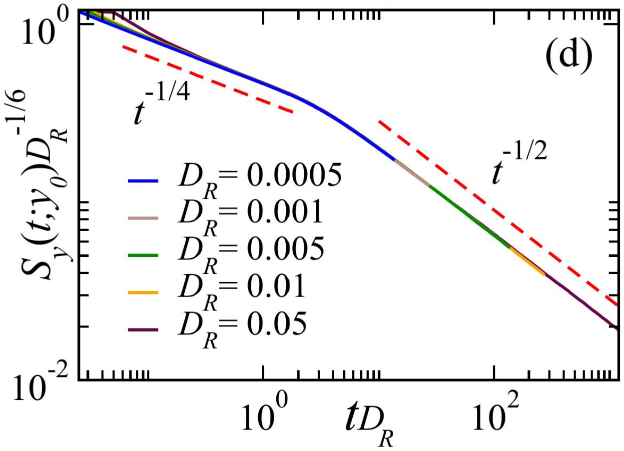
Large Deviations in Active Matter – Example Results

- *An example model: Active Brownian Particles (APB)* $\dot{\mathbf{x}} = -\mu\nabla V + v\mathbf{u}(\theta) + \sqrt{2D}\boldsymbol{\xi}(t)$

2D case without potential:

- Anomalous scaling of the marginal distributions at short times
- Anomalous scaling of 1^{st}-passage time at short times
- Fingerprint of the short-time dynamics at large times

Survival probability:

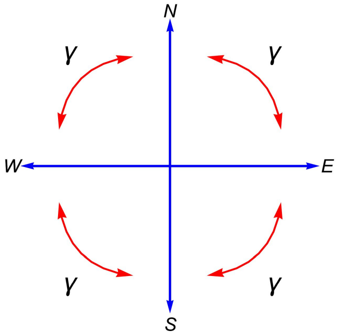


[Majumdar & Meerson, PRE **102** 022113 (2020)] [Basu, Majumdar, Rosso & Schehr, PRE **98** 062121 (2018)]

Large Deviations in Active Matter – Example Results

- *An example model*: Active Brownian Particles (APB) $\dot{\mathbf{x}} = -\mu\nabla V + v\mathbf{u}(\theta) + \sqrt{2D}\boldsymbol{\xi}(t)$
  2D case with a harmonic potential:
  Exact solution at all times [Caraglio & Franosch, PRL **129** 158001 (2022)]

- *Another important model*: Run-and-Tumble (RT)
  2D case with a harmonic potential:
  Exact solution at all times [Smith, Le Doussal, Majumdar & Schehr PRE **106** 054133 (2022)]



- How to extract the large-time and/or small-noise asymptotics?

  Do we learn generic features of active particles from this?

Large Deviations in Active Matter – Example Results

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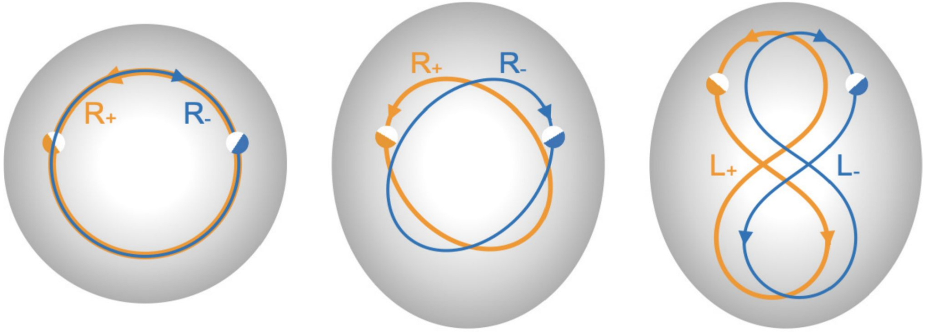
2D case with a torque:

- *radially symmetric or elliptic confinement*
- *different types of optimal trajectories*



[Damascena & de Souza Silva, PRE **108** 044605 (2023)]

Large Deviations in Active Matter – Example Results

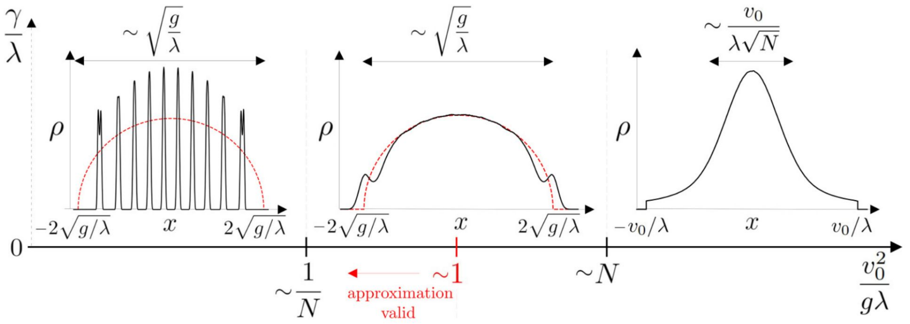
- *<u>An example model</u>: Active Brownian Dyson model*

N particles in 1D, non-intersecting, with a logarithmic repulsion

$$\dot{x}_i(t) = -\lambda x_i(t) + \frac{2}{N}\sum_{j\neq i}\frac{g_{\sigma_i(t),\sigma_j(t)}}{x_i(t)-x_j(t)} + v_0\sigma_i(t) + \sqrt{\frac{2T}{N}}\xi_i(t).$$

$\sigma_i(t) = \pm 1$ *with flips at rate* γ

Large-deviation scalings for $N \gg 1$



[Touzo, Le Doussal & Schehr, EPL **142** 61004(2023); PRE **109** 014136 (2024)]

Large Deviations in Active Matter – Example Results

- A variety of large-deviation scalings: a run-and-tumble model in a parabolic potential

$$m\ddot{x}(t) + \Gamma\dot{x}(t) + kx(t) = \Sigma(t)$$

  - $\Sigma(t)$ is a telegraphic noise of typical correlation time $\mathcal{T}_c$
  - Large-deviation scaling for the steady-state as $\mathcal{T}_c \to 0$

$$P_{\mathrm{st}}(X) \sim e^{-s(X)/\tau_c} \qquad \tau_c = \sqrt{k/m}\,\mathcal{T}_c$$

[Smith & Farago, PRE **106** 054118 (2022)]

Large Deviations in Active Matter – Example Methodology

- "Effective" / "auxiliary" forces as a tool

From Hugo Touchette's lecture: if A[trajectory] is extensive in the duration t of the trajectory

$$P[A/t = a] \asymp \exp[\, t\, I(a)\,] \quad \Leftrightarrow \quad \langle e^{sA} \rangle \asymp \exp[t\, \psi(s)] \qquad \text{"biased" ensemble} \quad (\psi(s) = \text{scaled CGF})$$

$\Leftrightarrow$ modified dynamics with s-dependent "effective forces"

Methodology: if the biased ensemble presents interesting features (clustering, oscillations, etc.), use the effective forces to design protocols to control the system

[Keta et al., PRE **103** 022603 (2021)]
[Fodor et al., Ann Rev CondMat Phys **13** 215 (2022)]
[Lamtyugina et al., PRL **129** 128002 (2022)]
[Piñeros & Fodor, arXiv:2403.16961 (2024)]

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  - A variety of results
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The AOUP – References

A. J. Bray and A. J. McKane, *Instanton Calculation of the Escape Rate for Activation over a Potential Barrier Driven by Colored Noise*, Phys. Rev. Lett. **62**(5), 493 (1989), doi:10.1103/PhysRevLett.62.493.

A. J. McKane, *Noise-induced escape rate over a potential barrier: Results for general noise*, Phys. Rev. A **40**(7), 4050 (1989), doi:10.1103/PhysRevA.40.4050.

H. C. Luckock and A. J. McKane, *Path integrals and non-Markov processes. III. Calculation of the escape-rate prefactor in the weak-noise limit*, Phys. Rev. A **42**(4), 1982 (1990), doi:10.1103/PhysRevA.42.1982.

M. M. Kłosek-Dygas, B. J. Matkowsky and Z. Schuss, *Colored Noise in Dynamical Systems*, SIAM J. Appl. Math. **48**(2), 425 (1988), doi:10.1137/0148023.

J. F. Luciani and A. D. Verga, *Functional Integral Approach to Bistability in the Presence of Correlated Noise*, EPL **4**(3), 255 (1987), doi:10.1209/0295-5075/4/3/001.

H. S. Wio, P. Colet, M. San Miguel, L. Pesquera and M. A. Rodríguez, *Path-integral formulation for stochastic processes driven by colored noise*, Phys. Rev. A **40**(12), 7312 (1989), doi:10.1103/PhysRevA.40.7312.

Blackboard lecture based on [E Woillez, Y Kafri & VL, J Stat Mech 063204 (2020)]