The energetic cost of information

Maxwell's demon and Landauer's bound: from gedanken to real experiments

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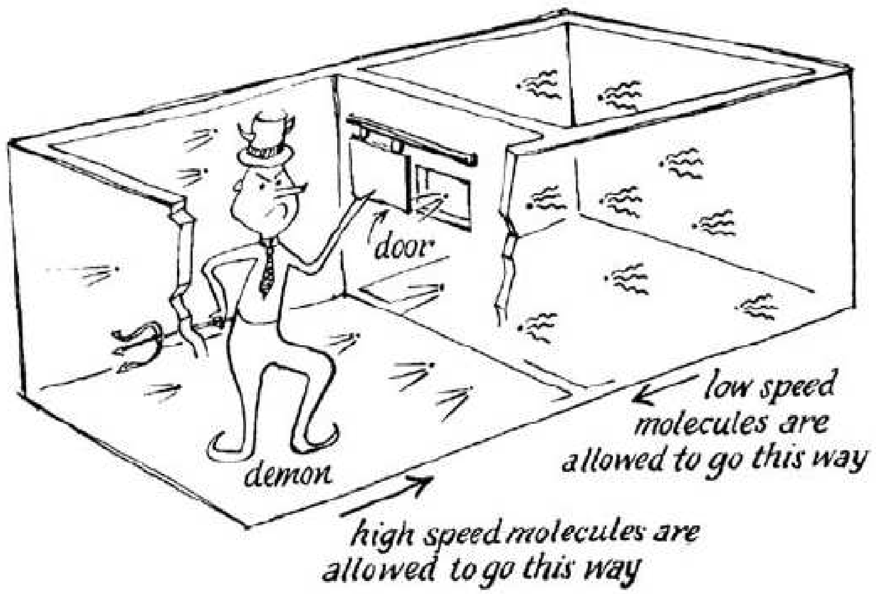
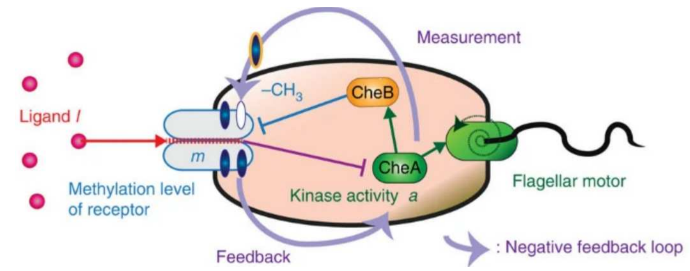
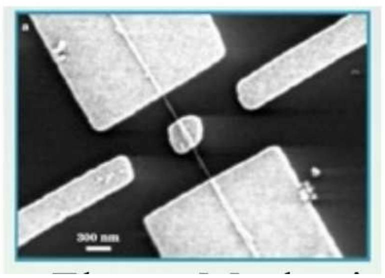




Biological systems

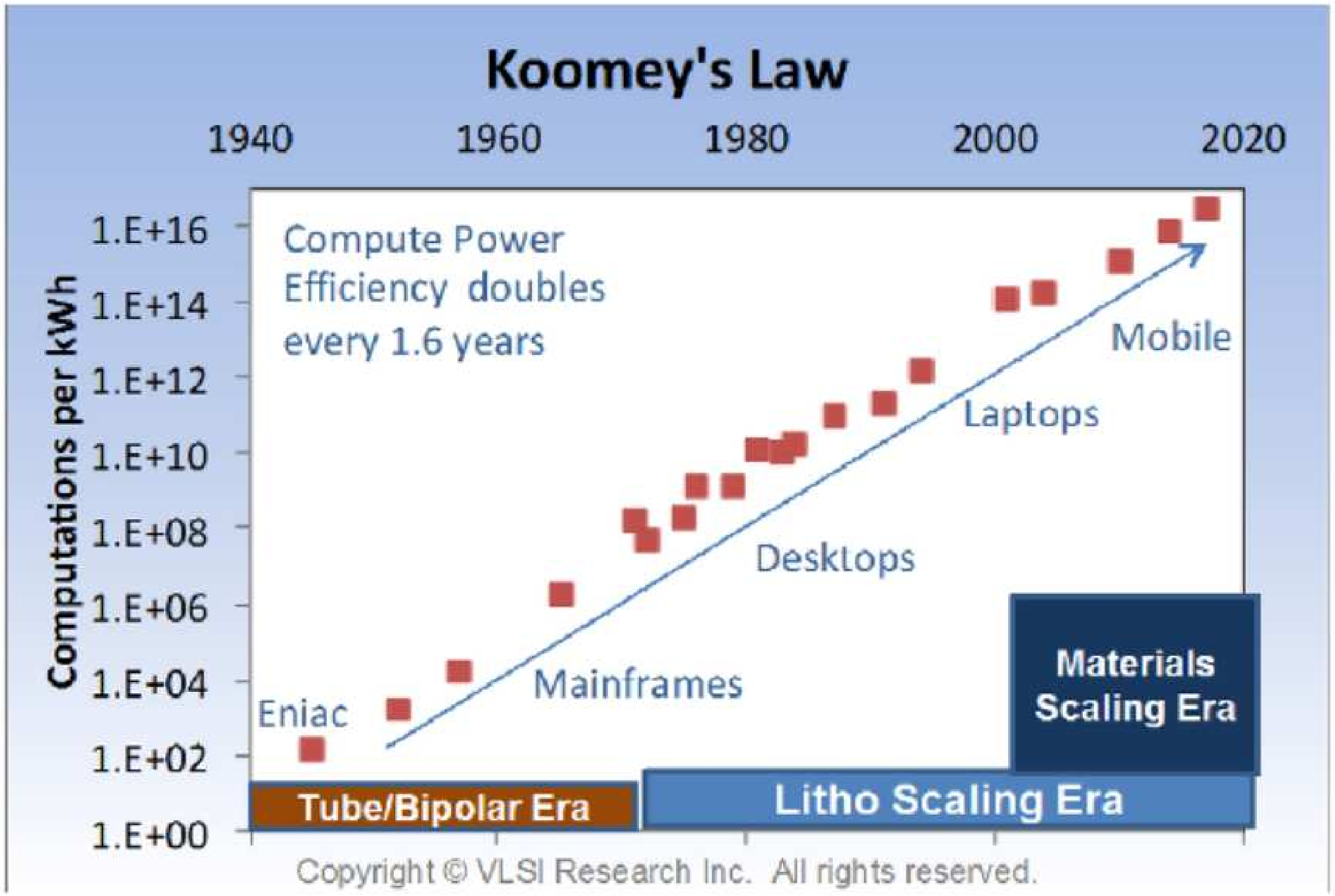


Microdevices
Electro-Mechanica

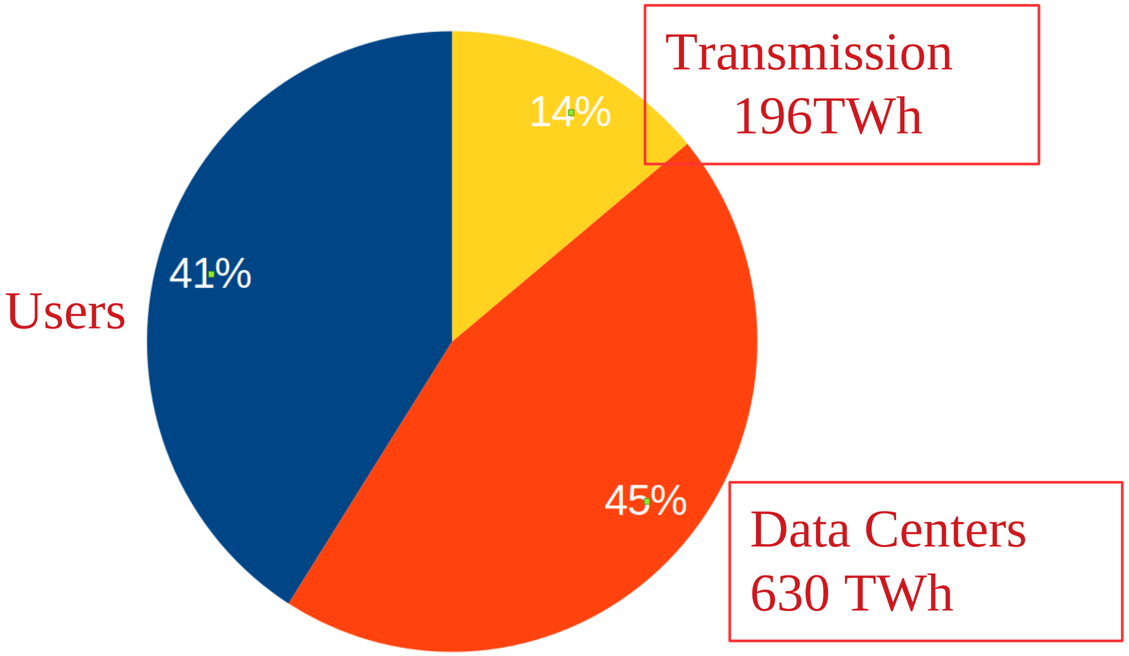
Google data center

The energetic cost of information

Computers costs are driven by Flops/W



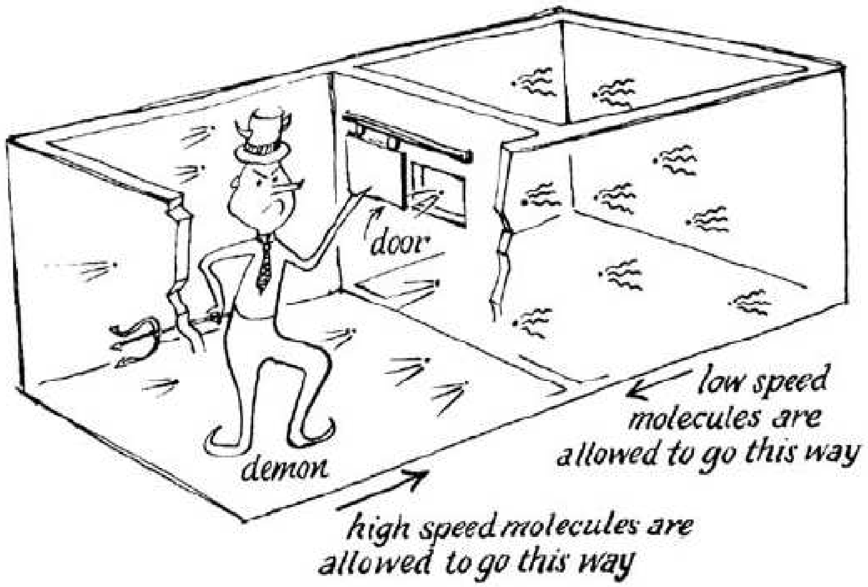
10% of total electric energy production is used by computers and internet



The energetic cost of information

Is there a fundamental law of physics (therdynamics) which connects Energy with Information ?

Can we perform logic operation with no energy cost ?



Several famous Physicists have been interested by these questions:

J.C.Maxwell(1870), L. Szillard(1930), L.Brillouin(1950), C.Shannon(1950), R.Landauer (1961).

1) Second principle of thermodynamics and Maxwell's demon

2) Maxwell's demon and Landauer's bound

3) Experiments and experimental results

4) The importance for applications

Main Laws of Thermodynamics I

Clausius

The **First Law of Thermodynamics** is a version of the **Law of Conservation of Energy**

Clausisus statement of **the First Law**

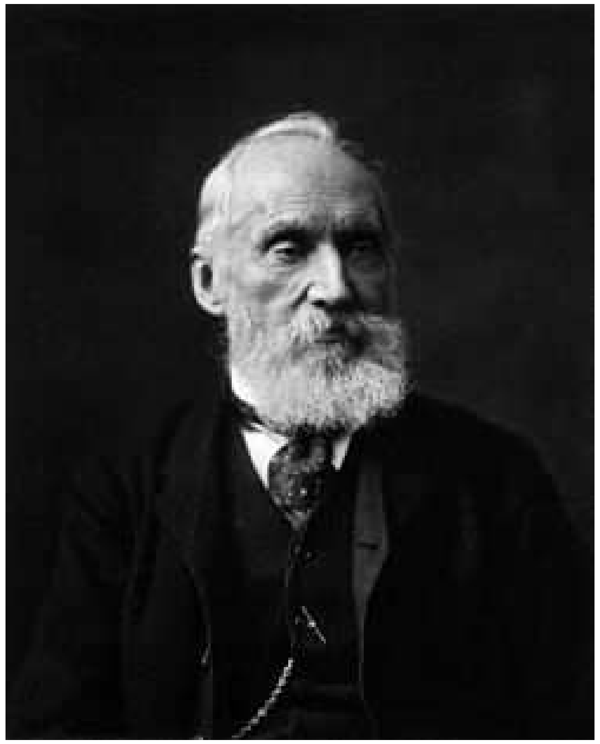
In a thermodynamic process, the increment in the internal energy of a system is equal to the difference between the heat exchanged by the system with the heat bath and the increment of work done on it.

$$\Delta U_{A,B} = W_{A,B} - Q$$

Main Laws of Thermodynamics II

The Second Law is a statement about irreversibility.
It is usually stated in physical terms of impossible processes.

Sadi Carnot was the first to give a formulation of this principle



Lord Kelvin

Lord Kelvin Statement of the Second Law

No process is possible in which the sole result is the absorption of heat from a reservoir and its complete conversion into work.

Main Laws of Thermodynamics III

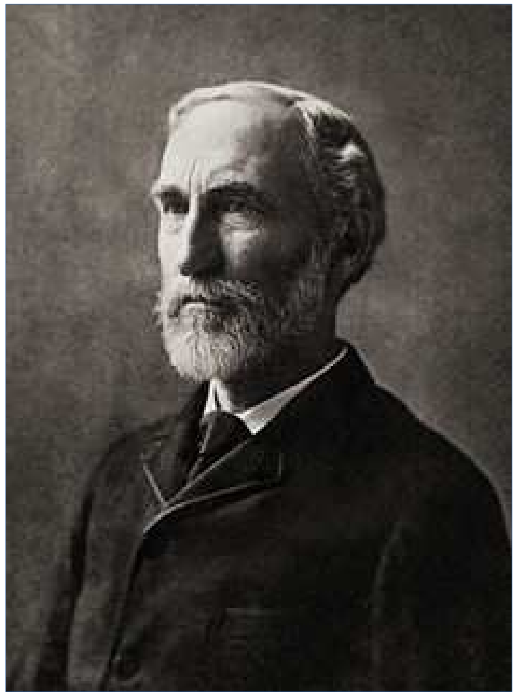
The **Second Law of Thermodynamics** is related to the concept of **Entropy**

$$\Delta S = \frac{Q}{T}$$

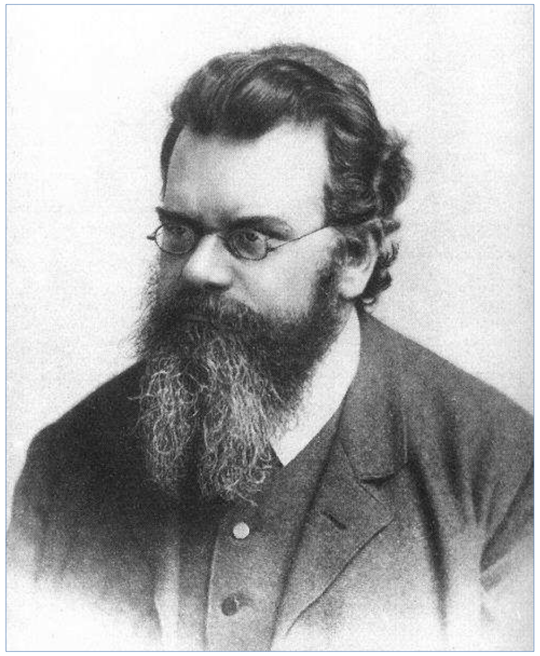
$$\Delta S_{tot} \geq 0$$

In **statistical mechanics**,**Entropy** is related to the probability of the microstates, correponding to a particular macrostate:

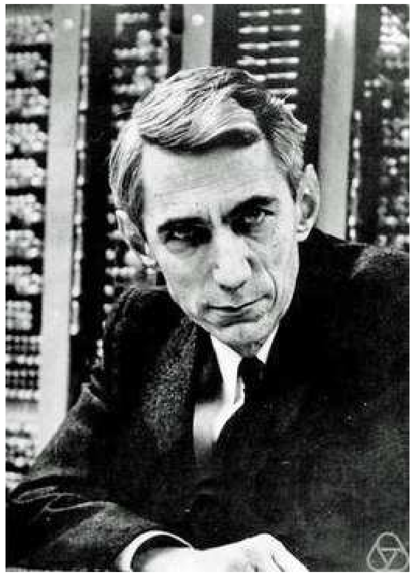
$$S = -k_B \Sigma_i p_i \log p_i$$



Gibbs



Boltzmann



C. Shannon

Shannon entropy

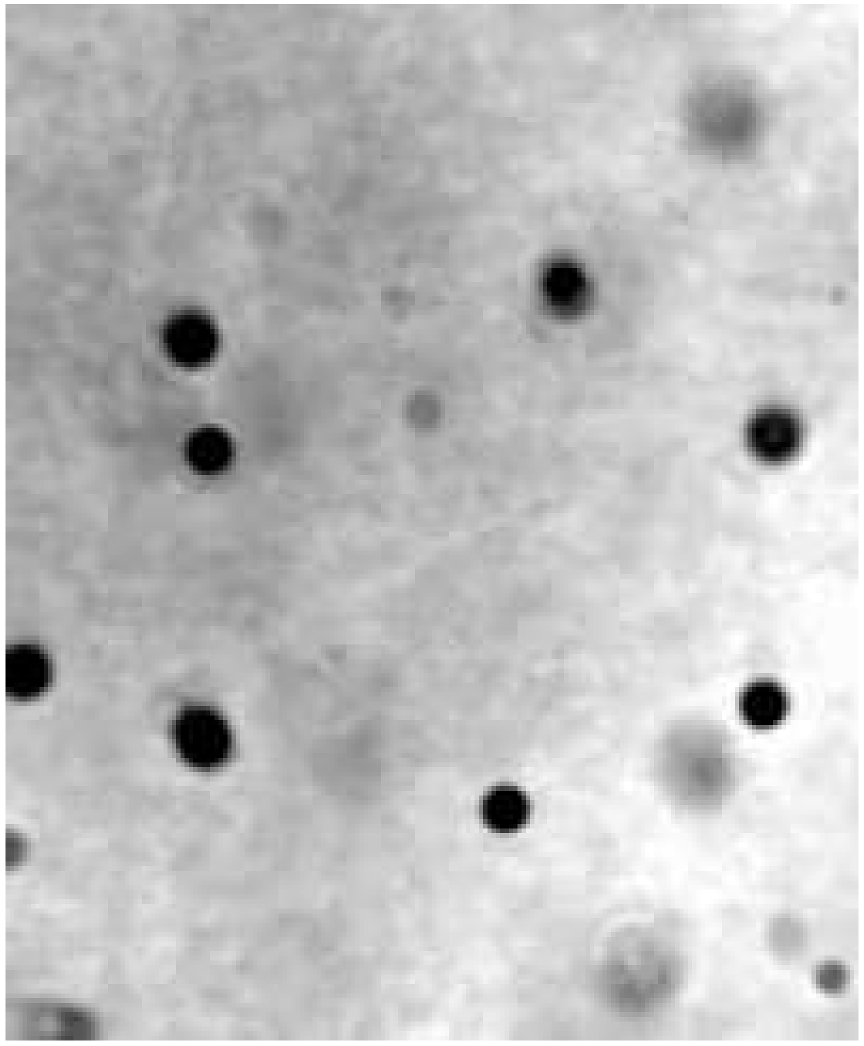
$$\mathbf{S}\text{=-}\sum_i p_i \log p_i$$

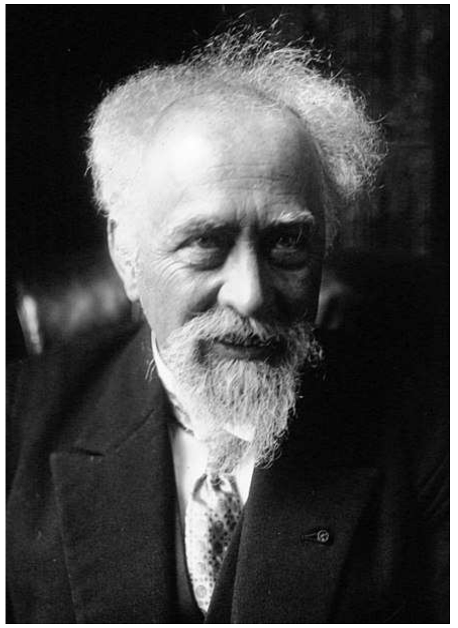
Measure the amount of information contained in a text, in a signal…..

Question : is Thermodynamic entropy the same than information entropy ?

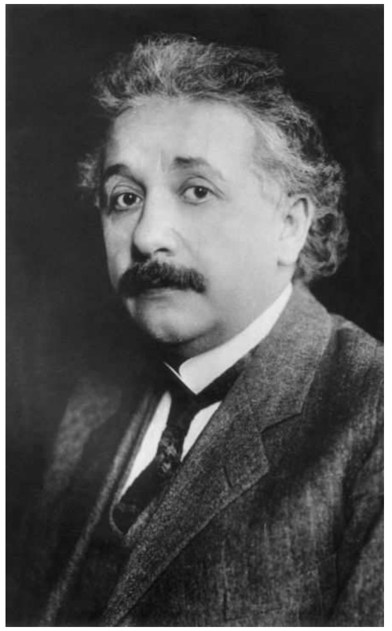
Brownian motion of micro particles in water

Observed by Mr. Brown in 1827





J. Perrin



A. Einstein

Characteristic energy of these thermal fluctuations is :
$k_B T$ ($4*10^{-21}$ Joule à 25°C)

This is the energy of molecules around us

It is 1000 times smaller than the energy used by a bacteria to swim into the water.

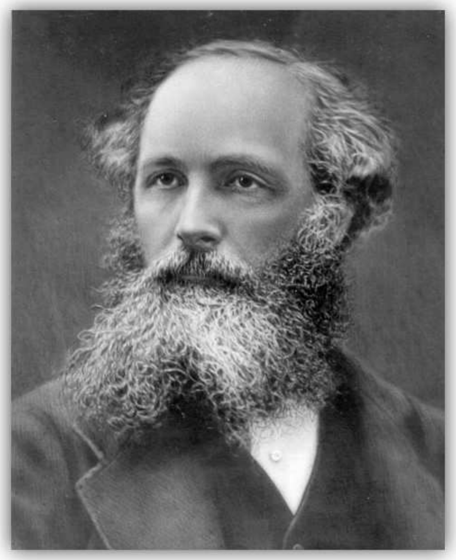
10^6 smaller than the energy used in computer to commute a bit.

(10 times faster)

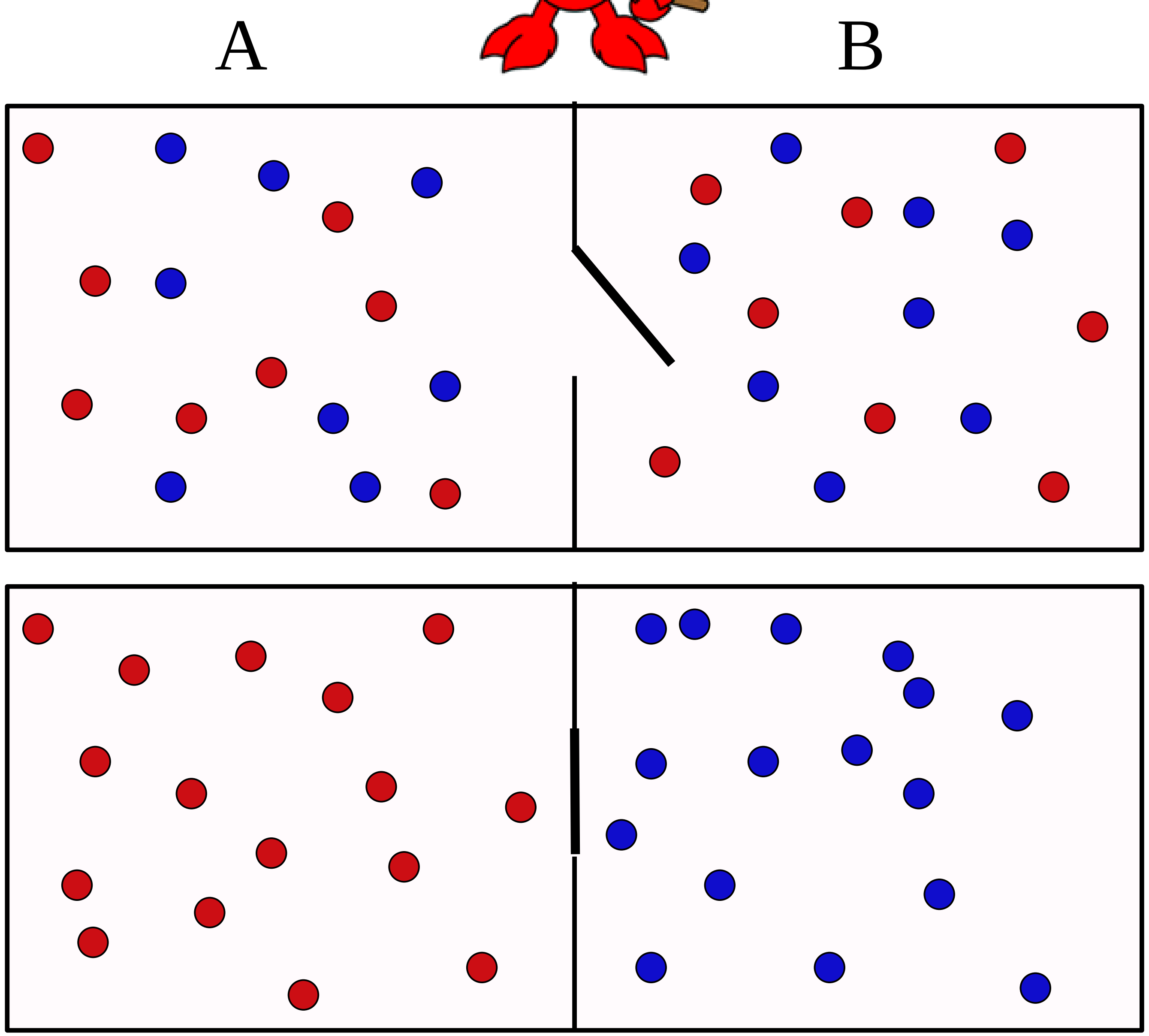
Particles size is
2 micron= 2 mm/1000

Maxwell's Demon

The first explanation came in 1929 by Leó Szilárd, and later by Léon Brillouin

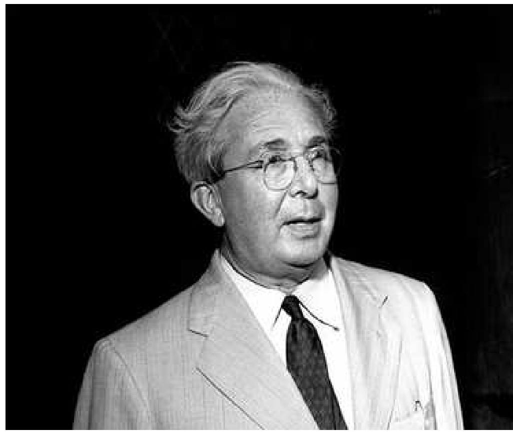


- slow molecules
- fast molecules



How to overcome the problem ?

Szillard's Engine and the connection between thermodynamics and information



Leó Szilárd

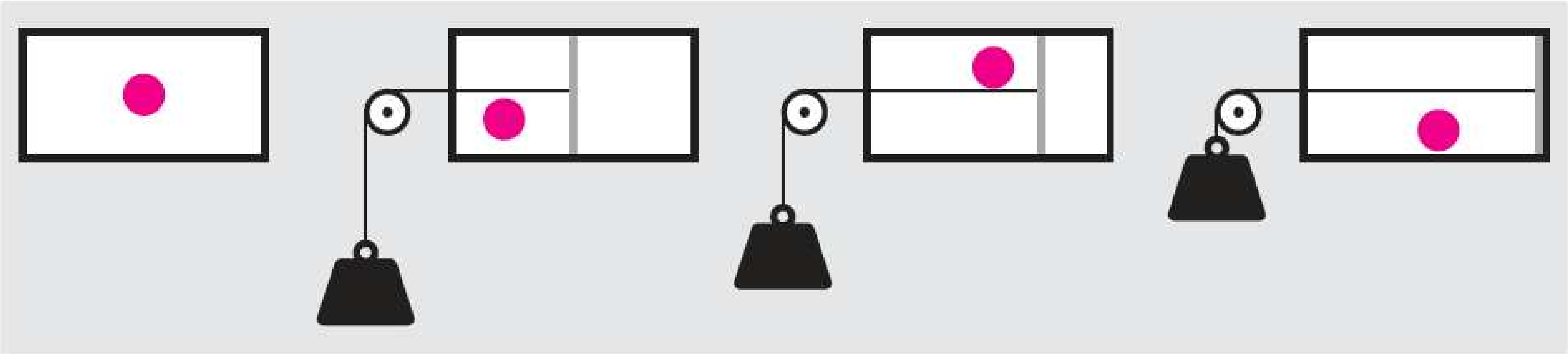


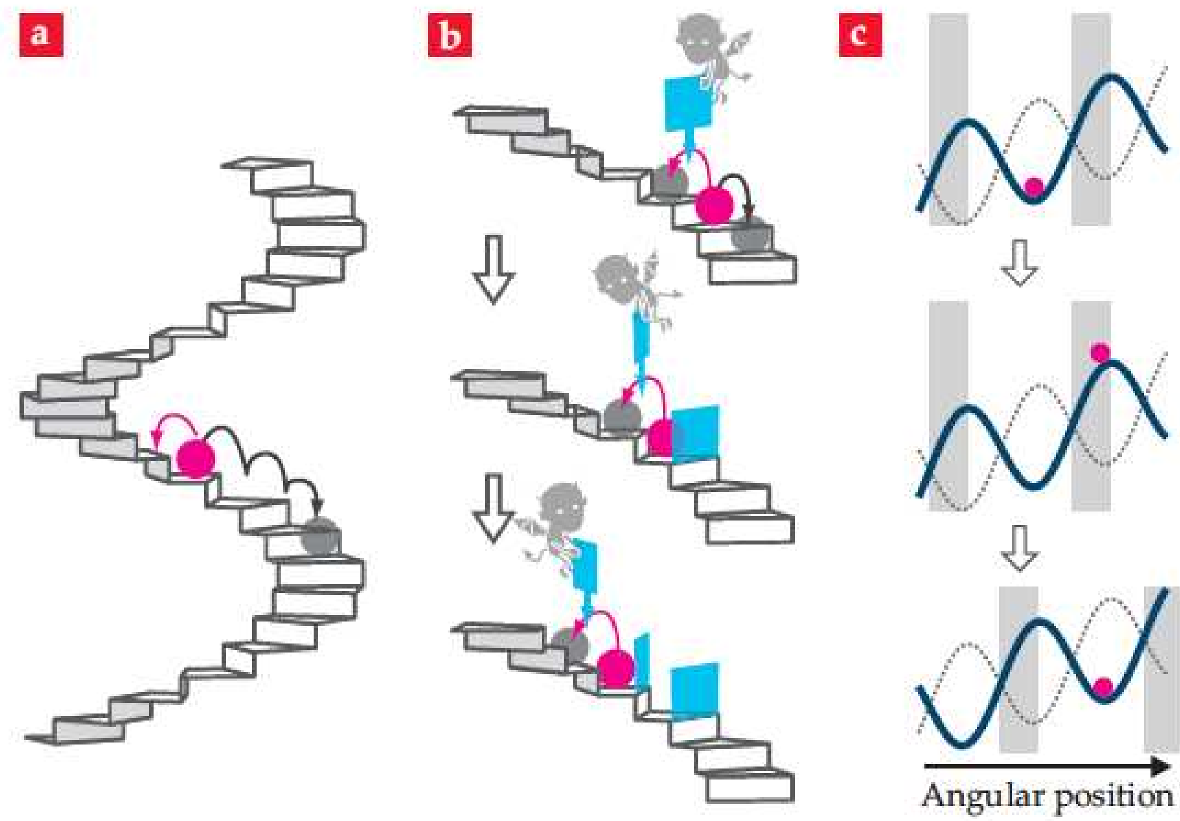
Figure 2. Szilard's engine. A crafty observer can turn a single particle in a box into an engine that converts information into mechanical work. If, say, the particle is found on the box's left-hand side, the observer inserts a movable wall and attaches a weight to its left side. The free expansion of the one-particle gas pushes the wall to the right, lifts the weight, and thereby performs work against gravity. (Adapted from ref. 12, J. V. Koski et al.)

Brillouin, Landauer and Bennett

Modern technology and stochastic thermodynamics allow us to design real demons

The demon at work
From gedanken to real experiments

Experimental realisation of the Mawell’s demon



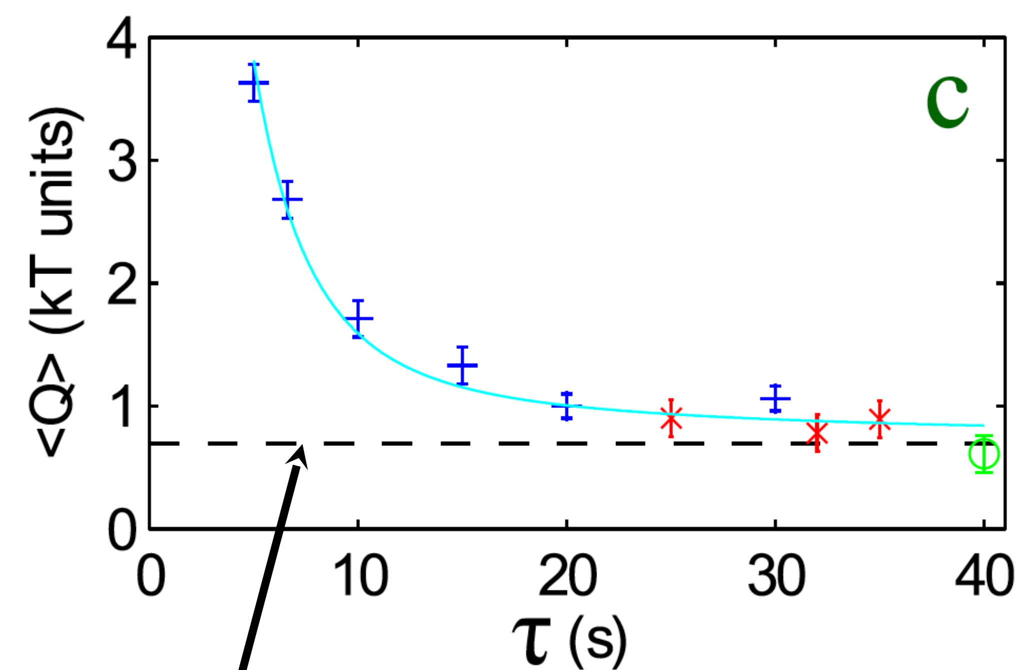
$$< e^{(-W+\Delta F-I)} >= 1$$

$$\langle W\rangle \geq \Delta F - k_B T\langle I\rangle.$$

$$I = -p\ln p - (1-p)\ln(1-p)$$

S. Toyabe et al., Nat. Phys. **6**, 988 (2010)

The measure of the Landauer’s bound



Landauer’s bound $k_B T \cdot \ln 2$

A. Bérut et al., Nature 483, 187 (2012).

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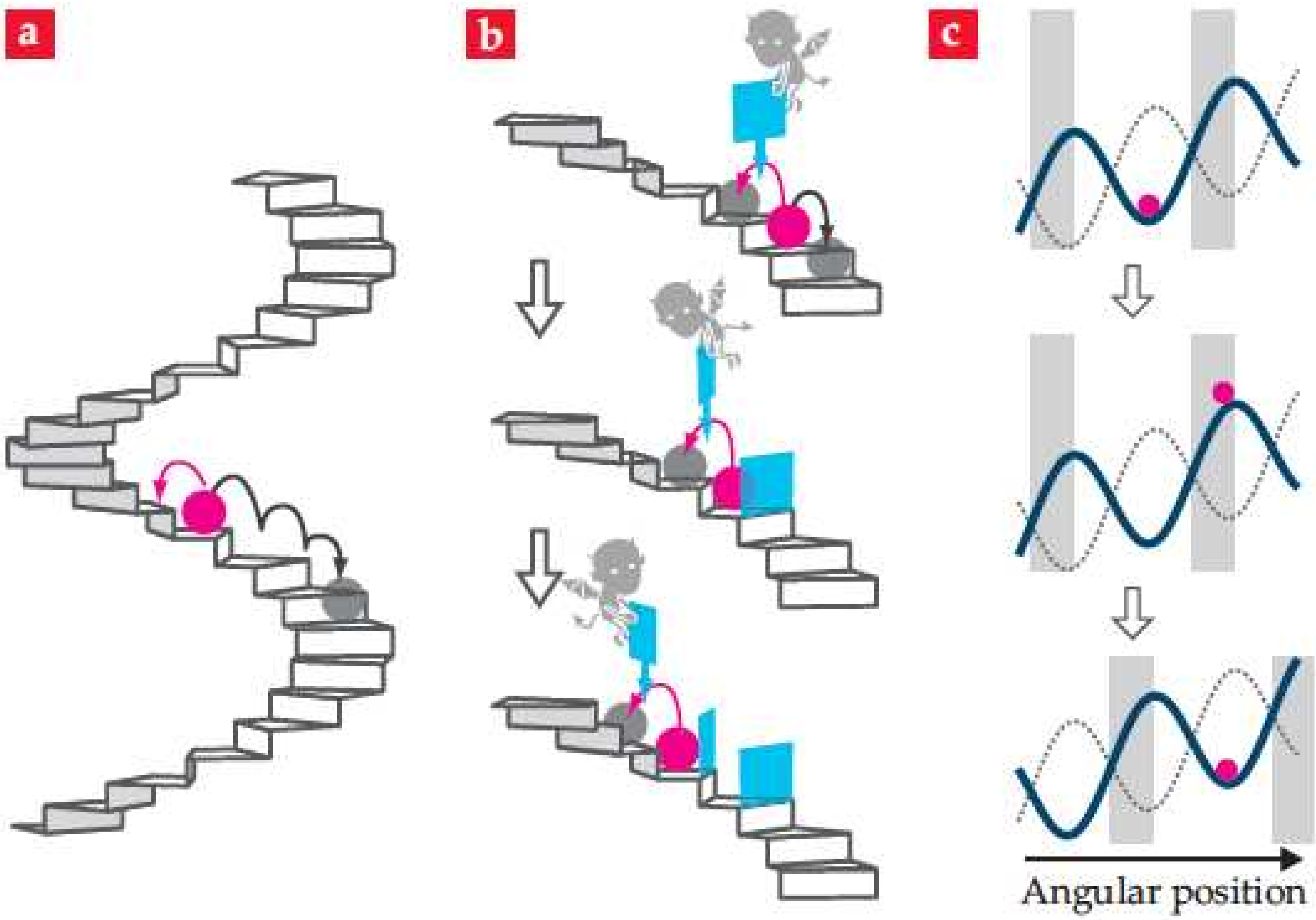

$$< e^{(-W+\Delta F-I)} >= 1$$

From Jensen inequality

$$\langle W \rangle \geq \Delta F - k_B T \langle I \rangle.$$

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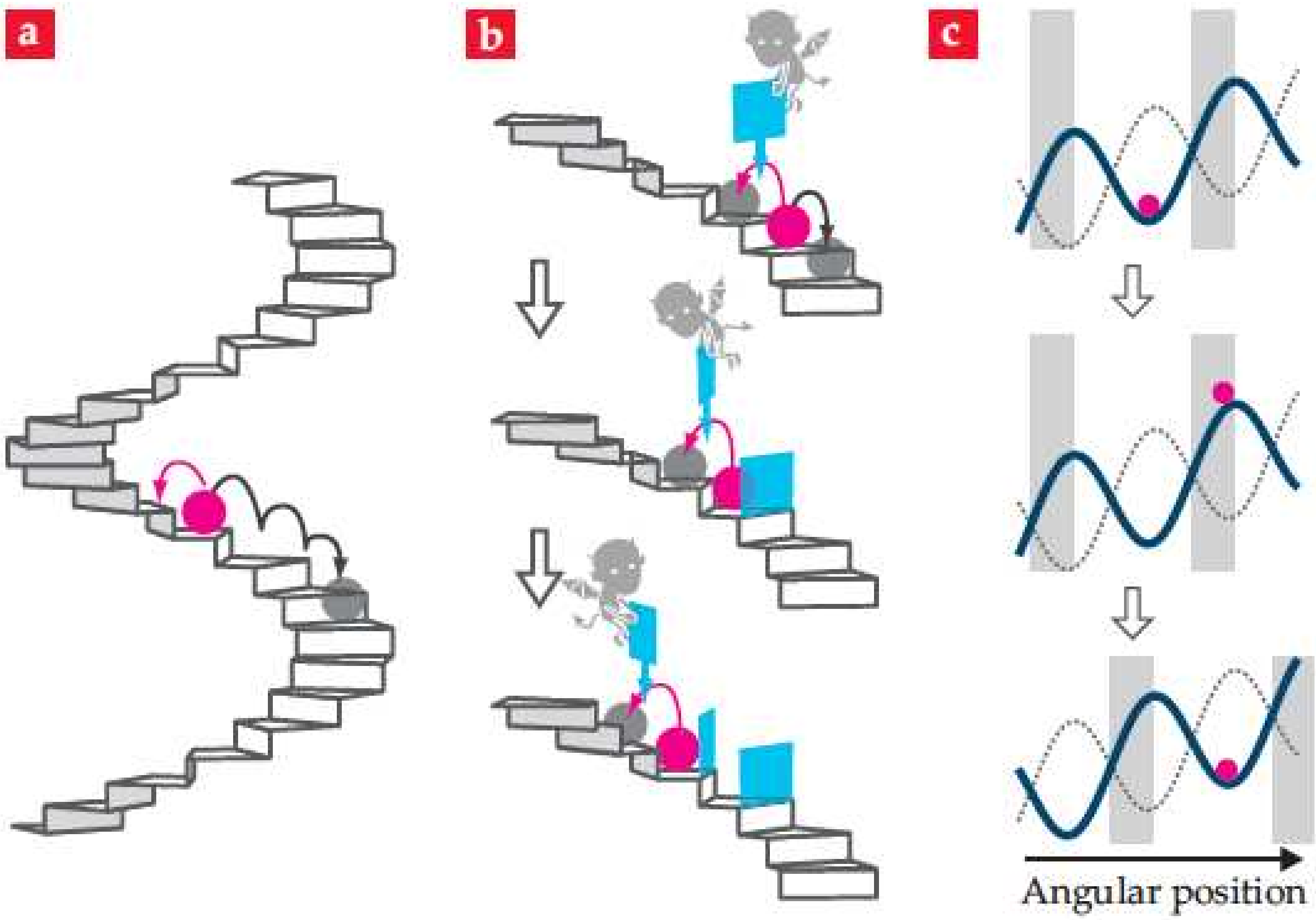

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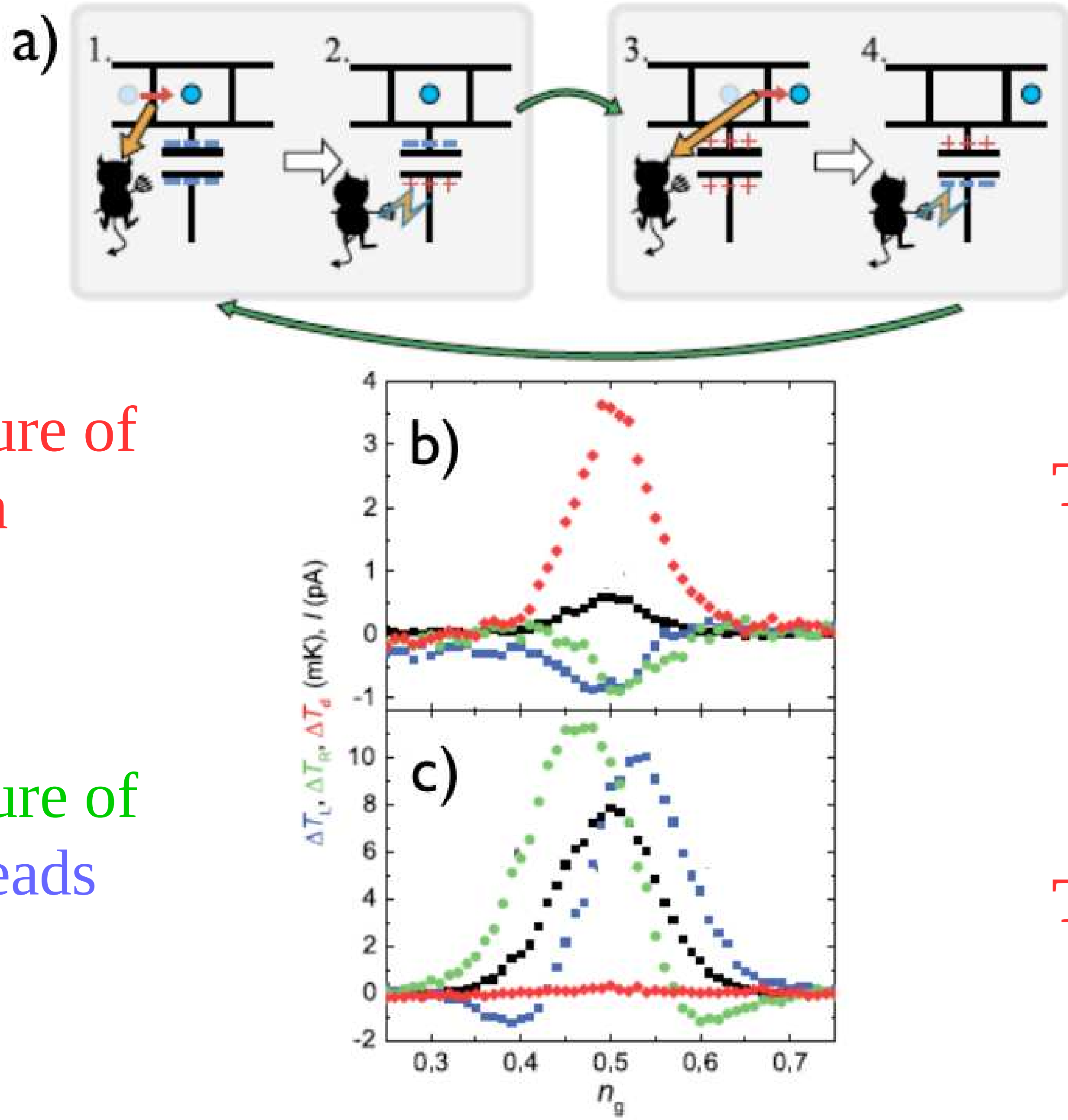
Cooling with a demon



Single electron transistor SET

Temperature of the demon

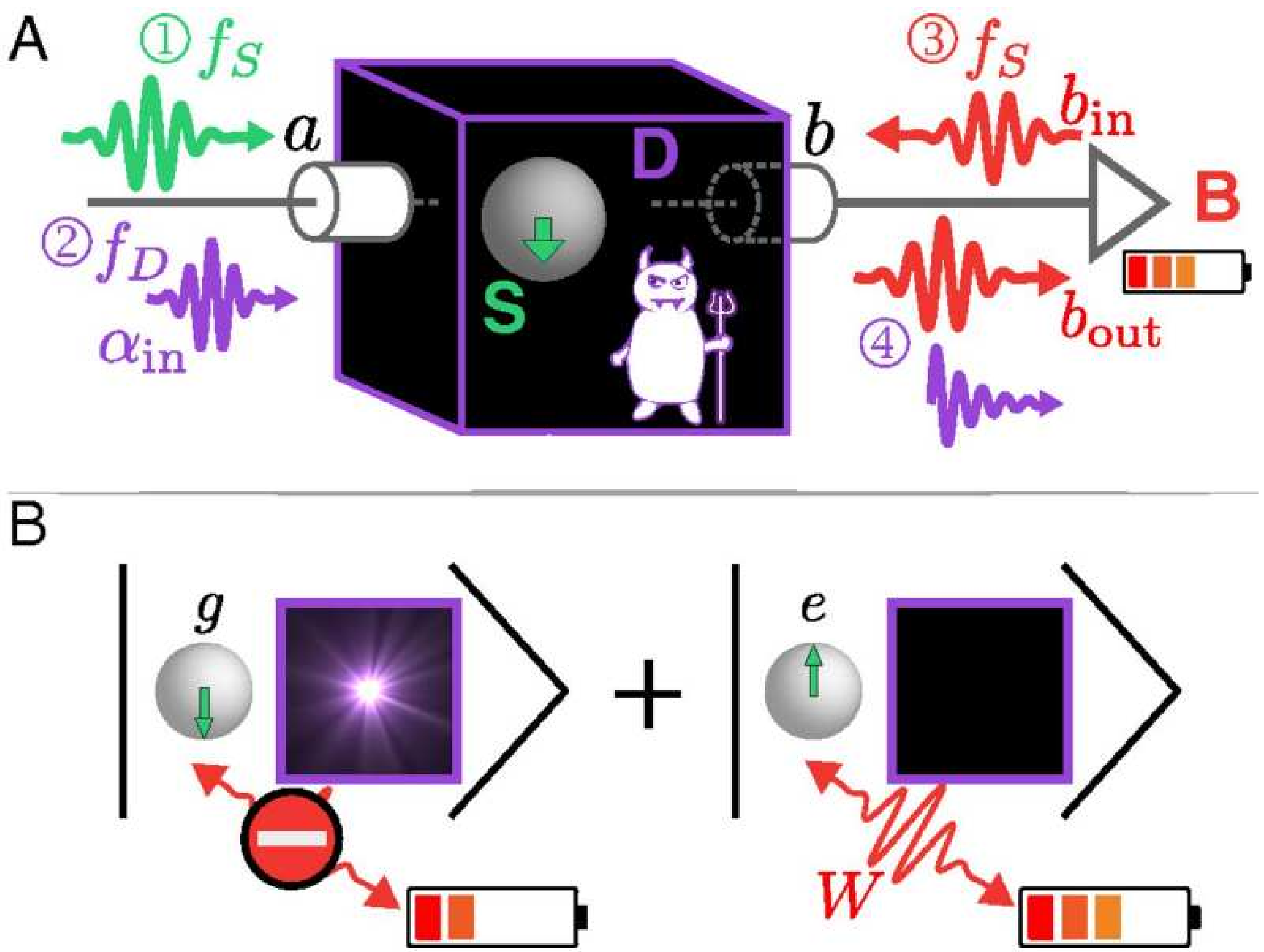
The demon is on

Temperature of the SET leads

The demon is off

J.V. Koski, A. Kutvonen, I. M. Khaymovich, T. Ala-Nissila, and J.P. Pekola. On-chip maxwell's demon as an information-powered refrigerator. Phys. Rev. Lett., 115:260602, 2015.

Observing a quantum Maxwell demon at work



The demon is a microwave cavity that encodes quantum information about a superconducting qubit and converts information to work
by powering a propagating microwave pulse
by stimulated emission

Nathanaël Cottet, Sébastien Jezouin, Landry Bretheau, Philippe Campagne-Ibarcq, Quentin Ficheux, Janet Anders, Alexia Auffèves, Rémi Azouit, Pierre Rouchon, and Benjamin Huard, PNAS, 114 (29) 7561-7564, (2017).

The demon at work
From gedanken to real experiments

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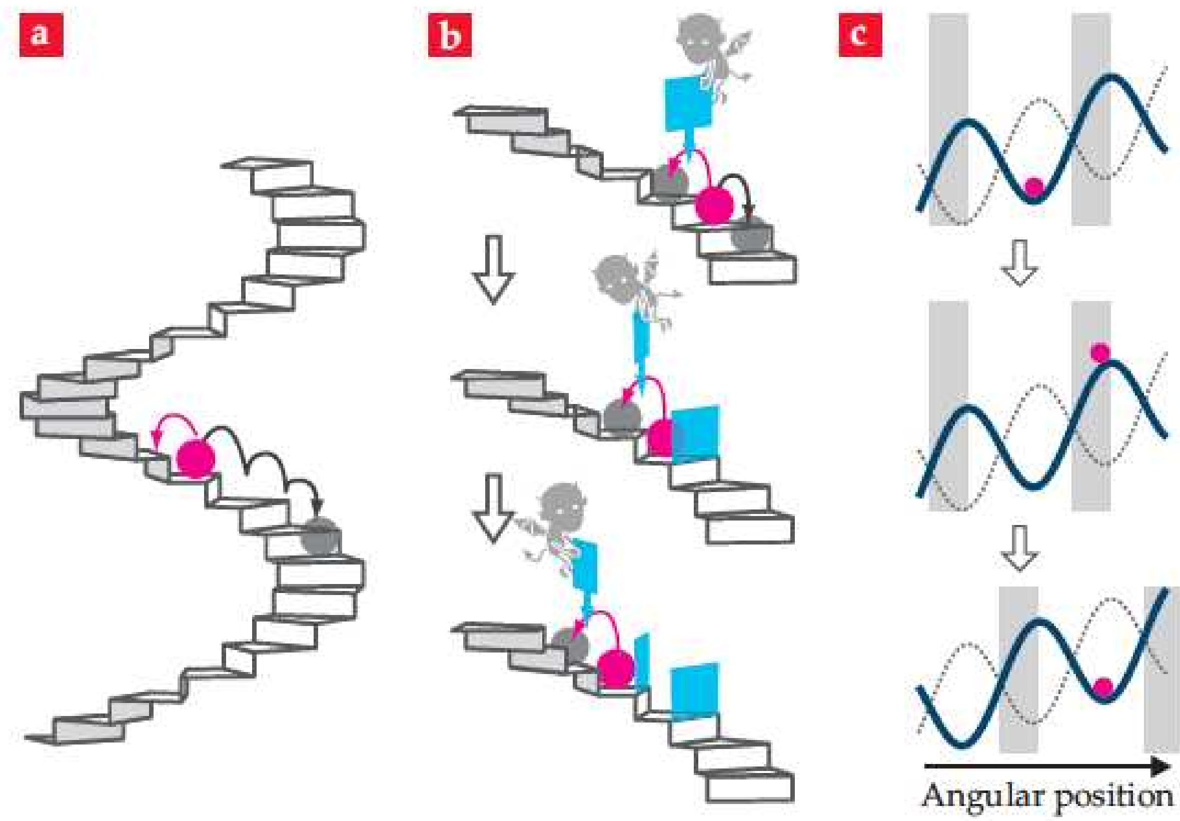

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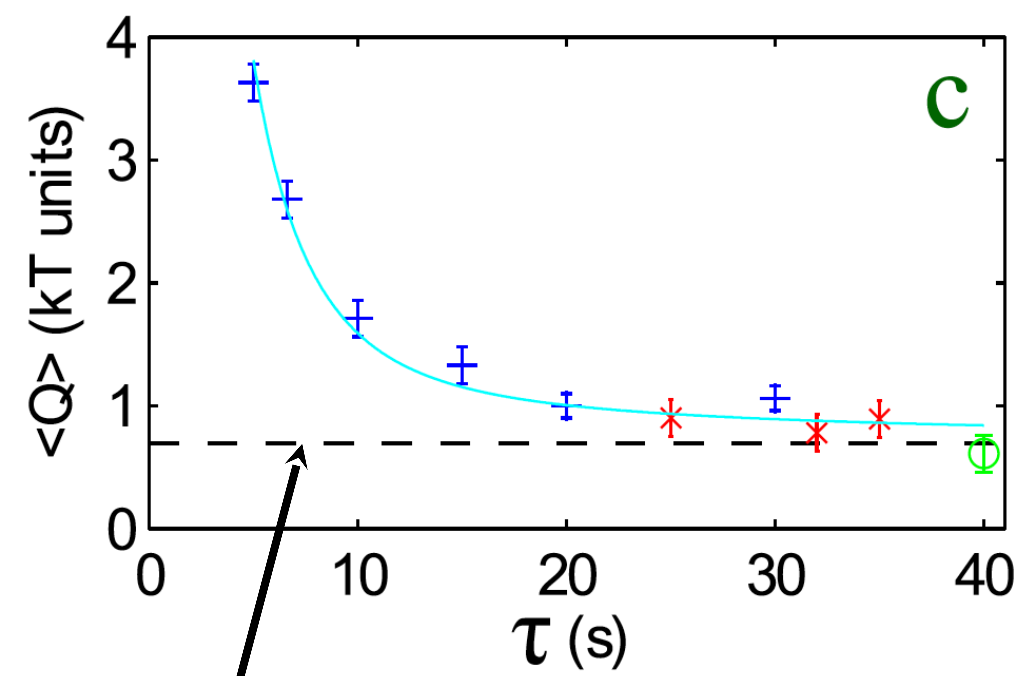


Landauer’s bound $k_B T \cdot \ln 2$

A. Bérut et al., Nature 483, 187 (2012).

ENS DE LYON

The Landauer's principle (I)

Any logically irreversible transformation of classical information is necessarily accompanied by the dissipation of at least $k_B T \cdot \ln 2$ of heat per lost bit (about $3 \cdot 10^{-21}$ Joules at room temperature)

Typical examples of logically irreversible transformations are Boolean functions such as AND, NAND, OR and NOR

They map several input states onto the same output state

The erasure of information, the RESET TO ONE operation, is logically irreversible and leads to an entropy production of $k_B \cdot \ln 2$ per erased bit

Landauer’s principle II

Landauer’s principle is a central result which exorcises the Maxwell’s demon

It has been criticized

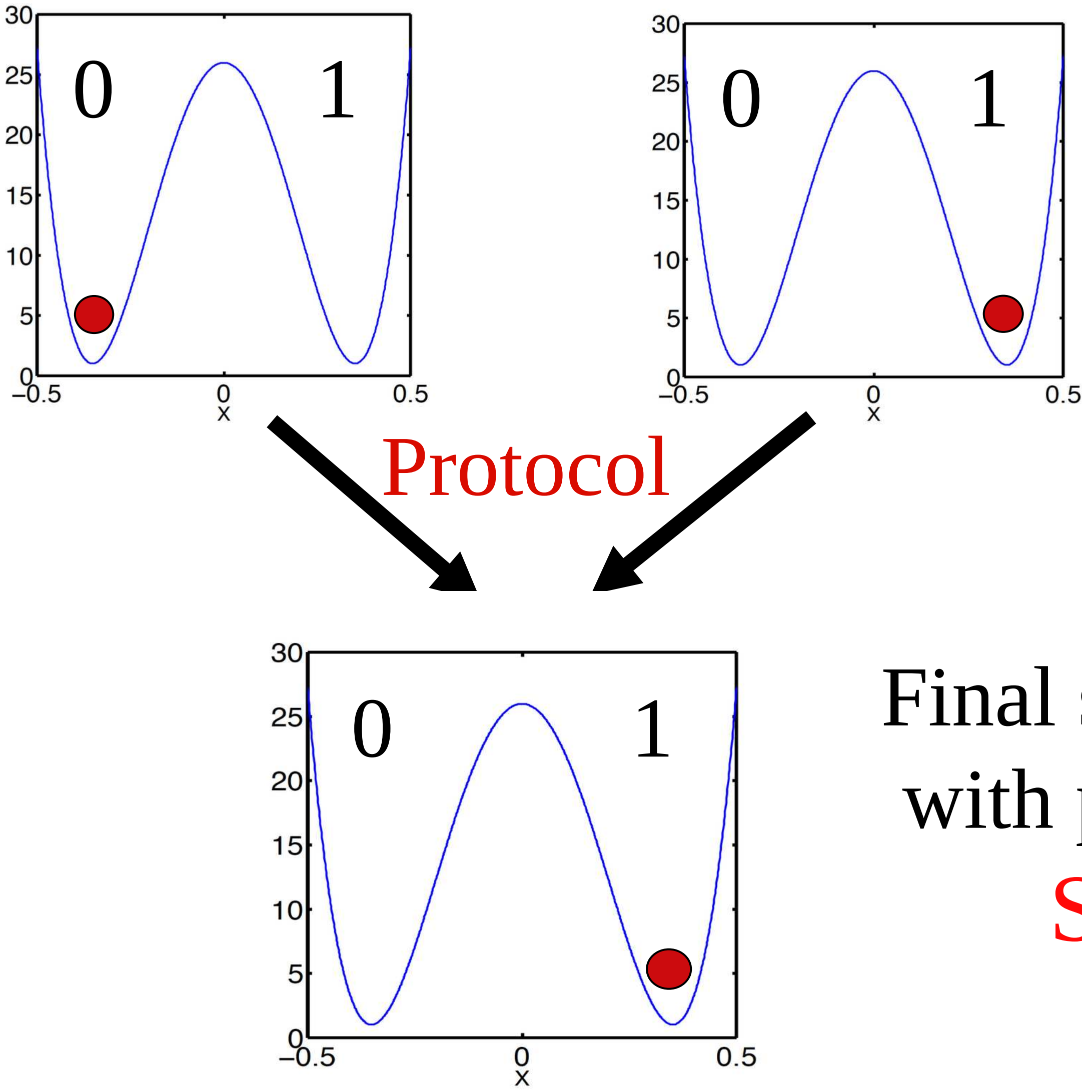
however it has been tested in several experiments

- How to reach Landauer’s bound in experiments ?
- Does any experimentally feasible protocol allow us to reach this bound ?
- Is there a strong connection with stochastic thermodynamics ?

Following Bennett the RESET to ONE operation is often used

Bennett, C. H. The thermodynamics of computation, a review. Int. J. Theor. Phys. 21, 905-940 (1982).

Bennett's erasure protocol



Initial state is 0 or 1 with
equal probability 1/2

$S_i = k_B \cdot \ln 2$

How can this protocol be experimentally realized ?

Final state is 1
with probability 1

$S_f = 0$

Thus $\Delta S_{min} = -k_B \cdot \ln 2$

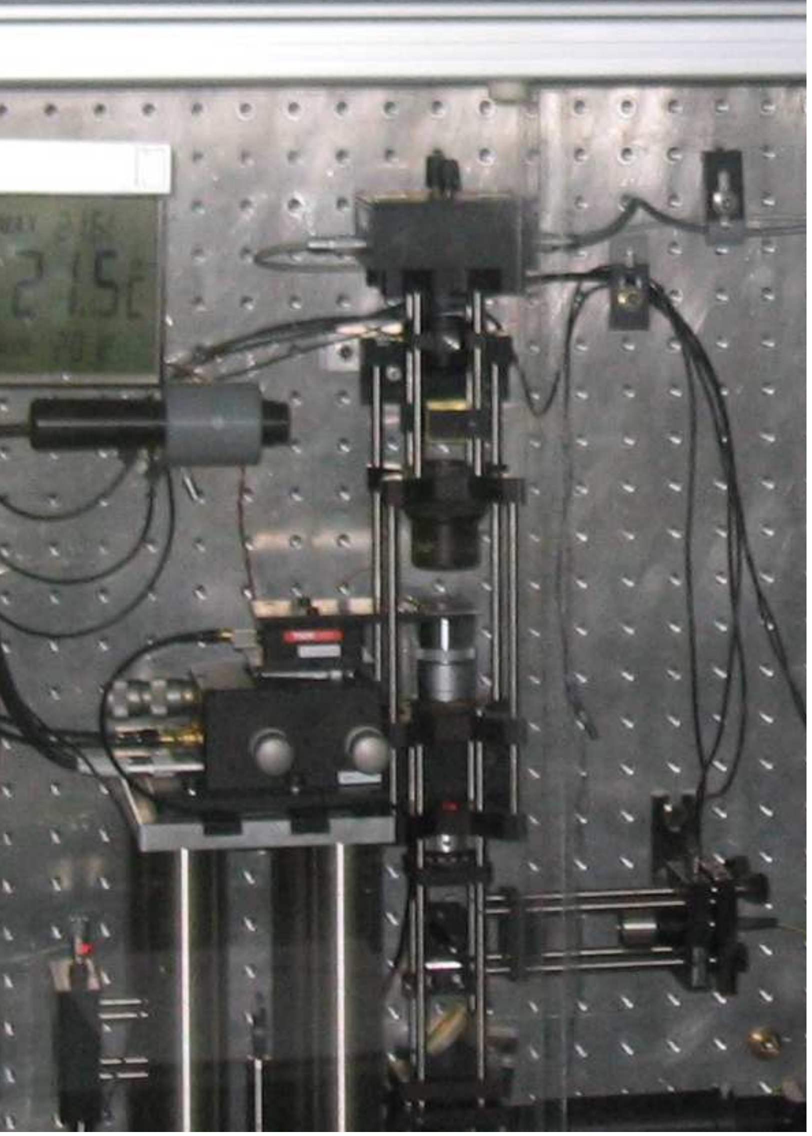
If the protocol is quasi static : $-T\Delta S=Q$

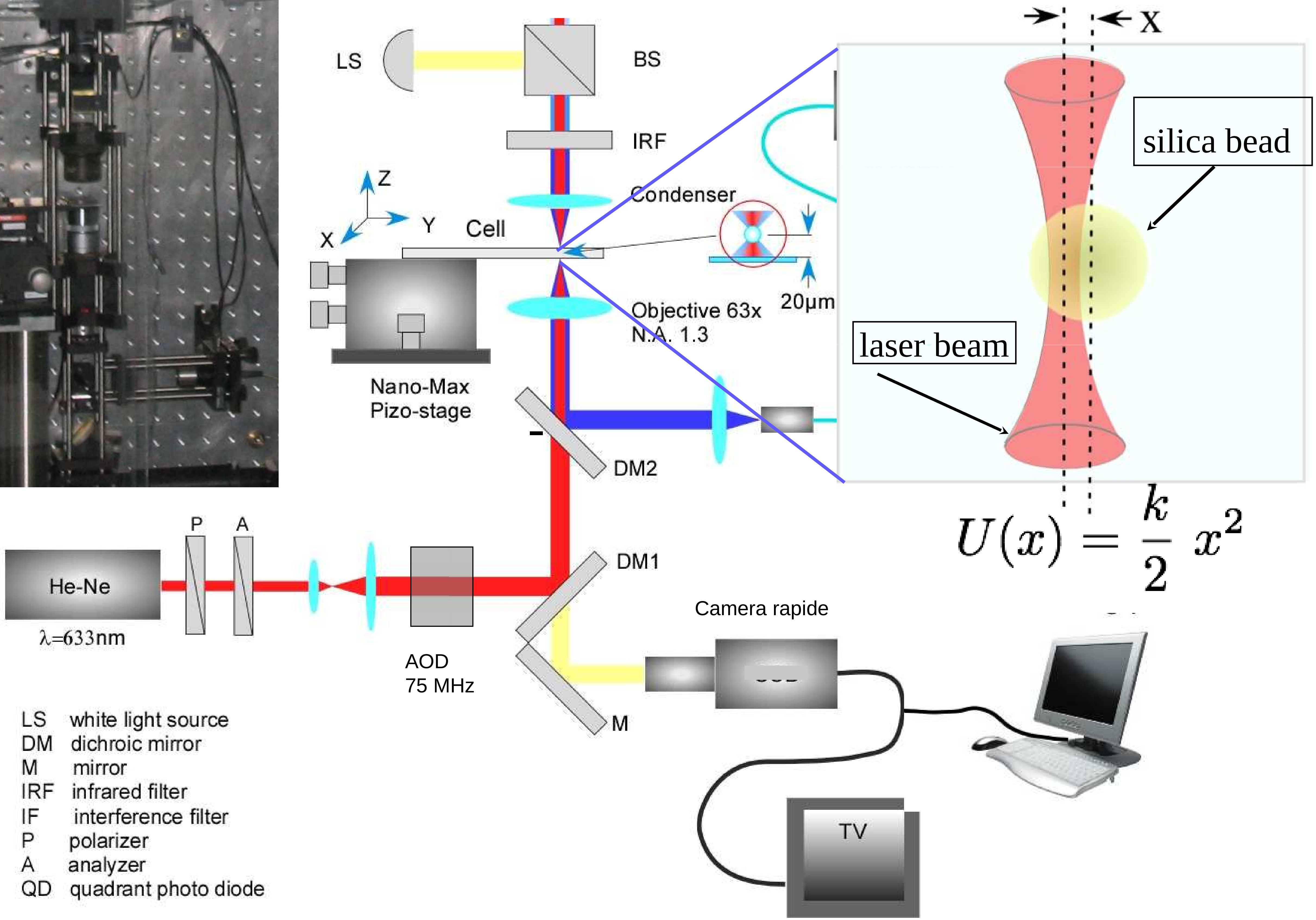
As the energy variation $\Delta U=0$ then from the first principle

$$\langle W \rangle = \langle Q \rangle = -T\, \Delta S \geq k_B T \ln(2)$$

A minimum work is needed to erase a bit

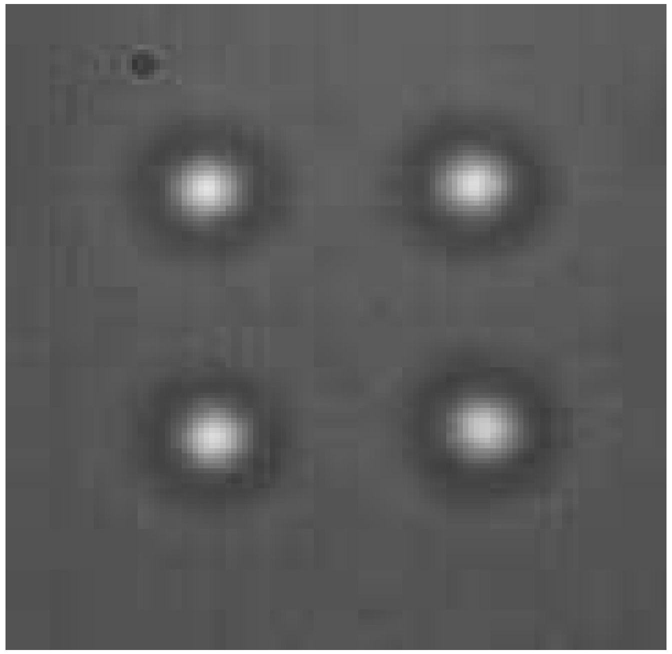
Experimental set-up Optical trap

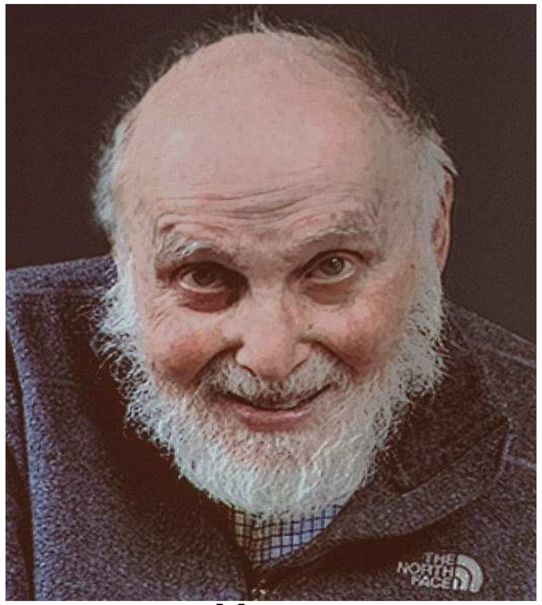




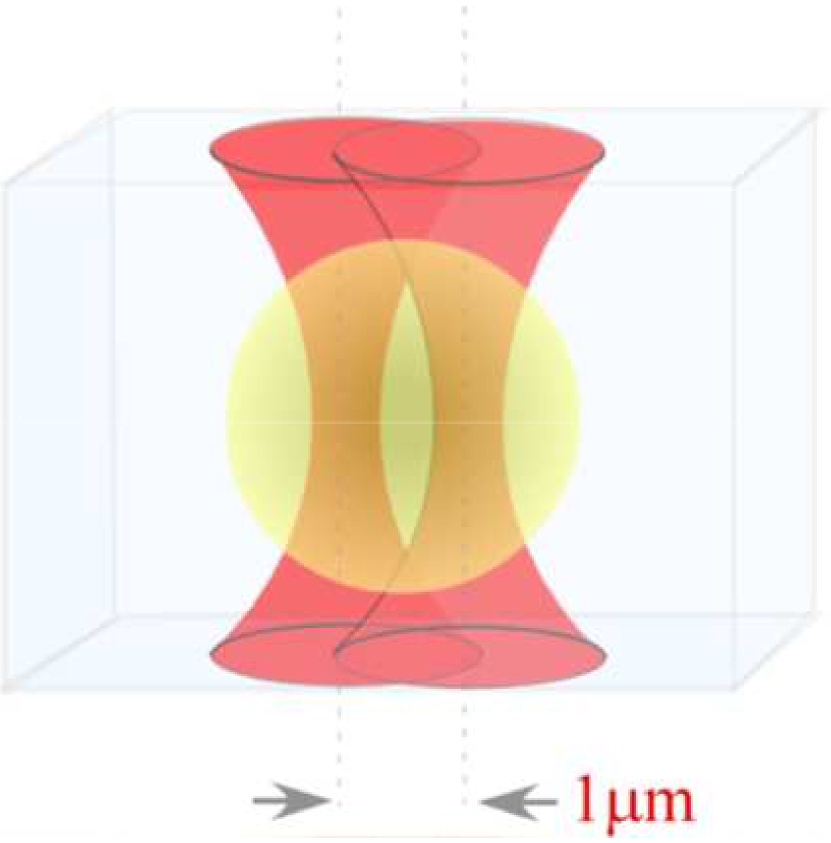
$$U(x) = \frac{k}{2} x^2$$

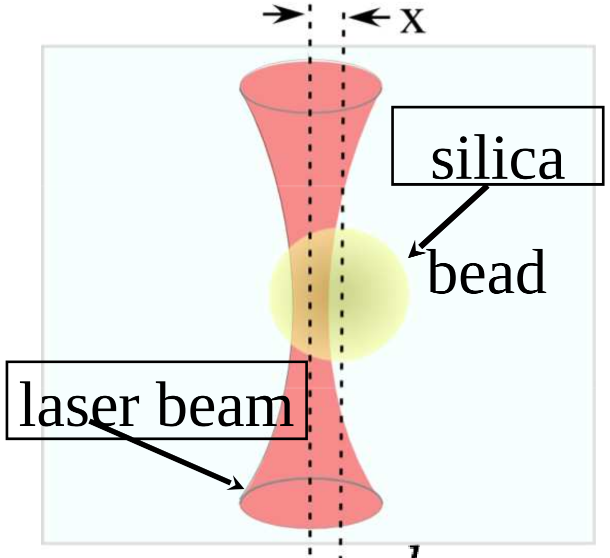
Examples of traps



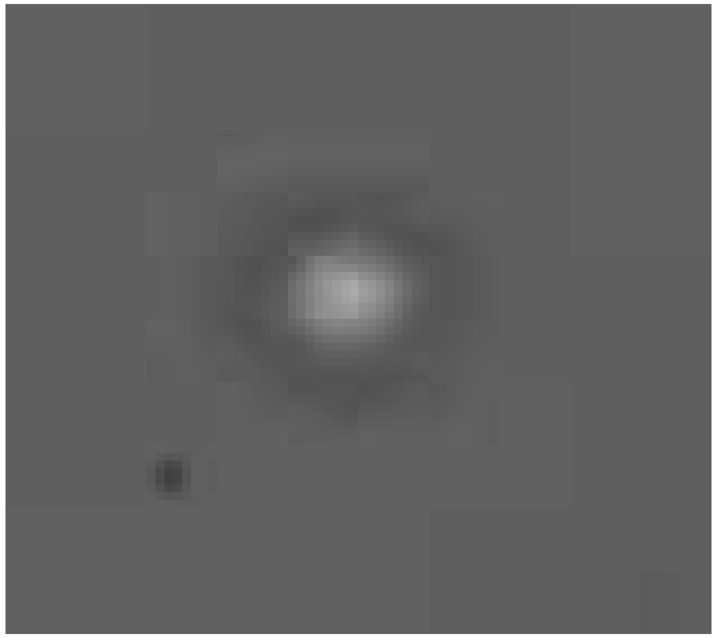


A Ashkin
Physics Nobel Price in 2018 for the invention of optical tweezers

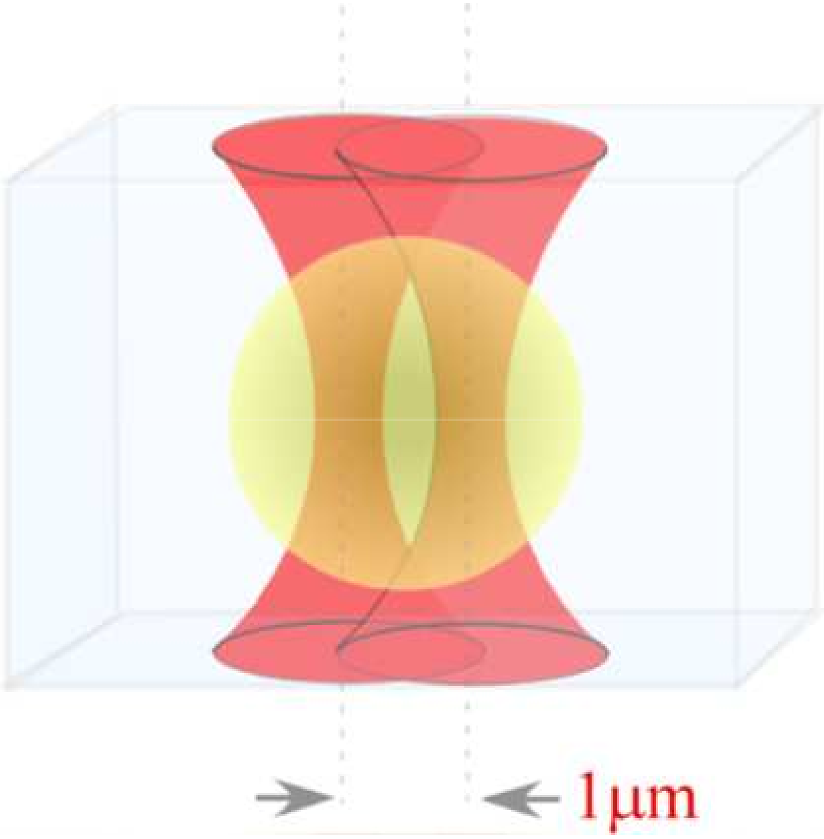


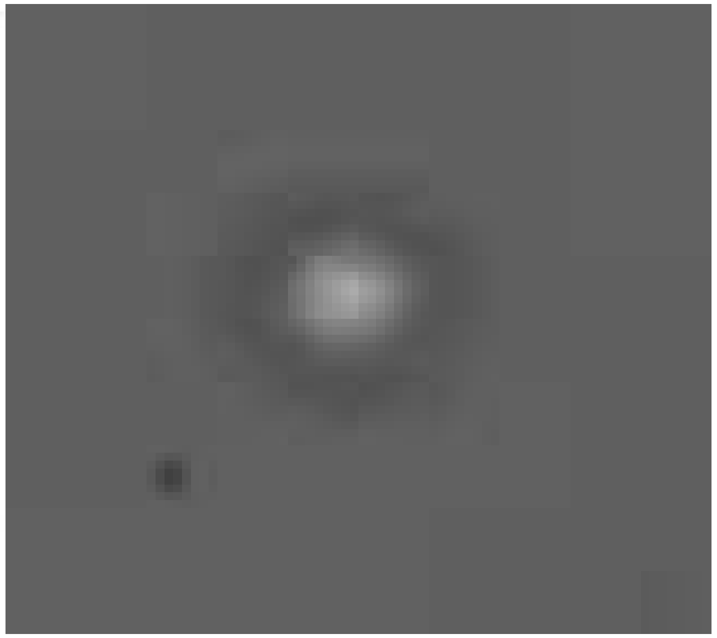


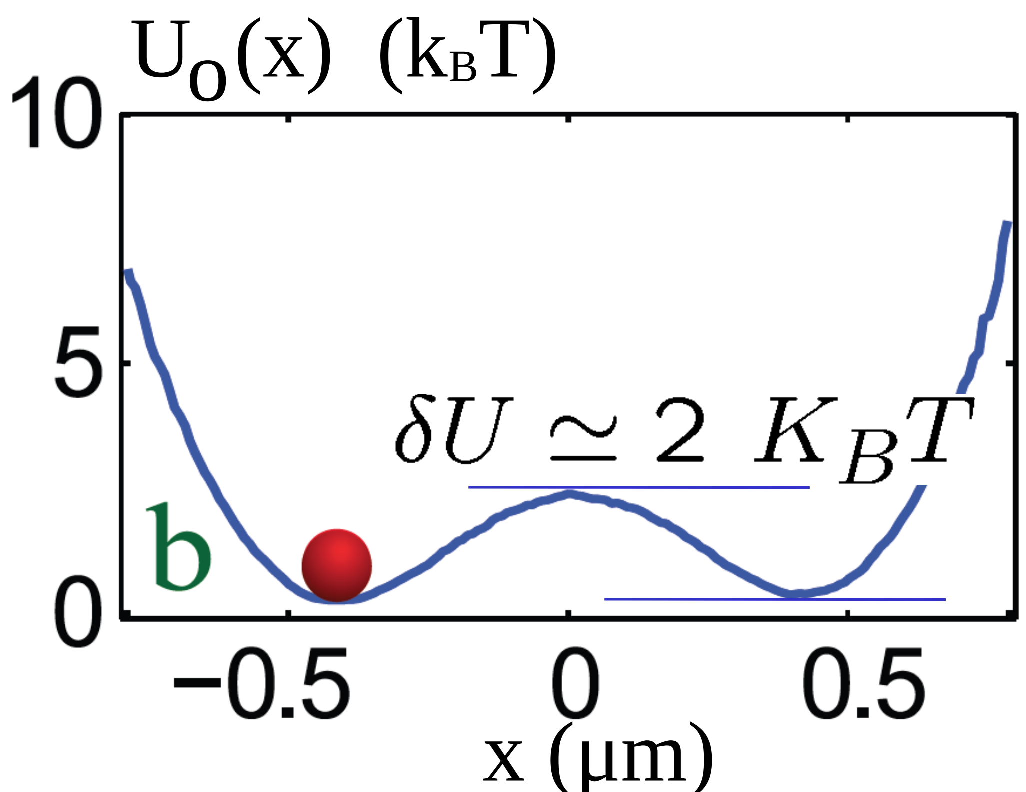
$$U(x) = \frac{k}{2}\, x^2$$



Brownian particle trapped by two laser beams







$$U_o(x) = a\ x^4 - b\ x^2\ +\ d\ x$$

The Kramers time

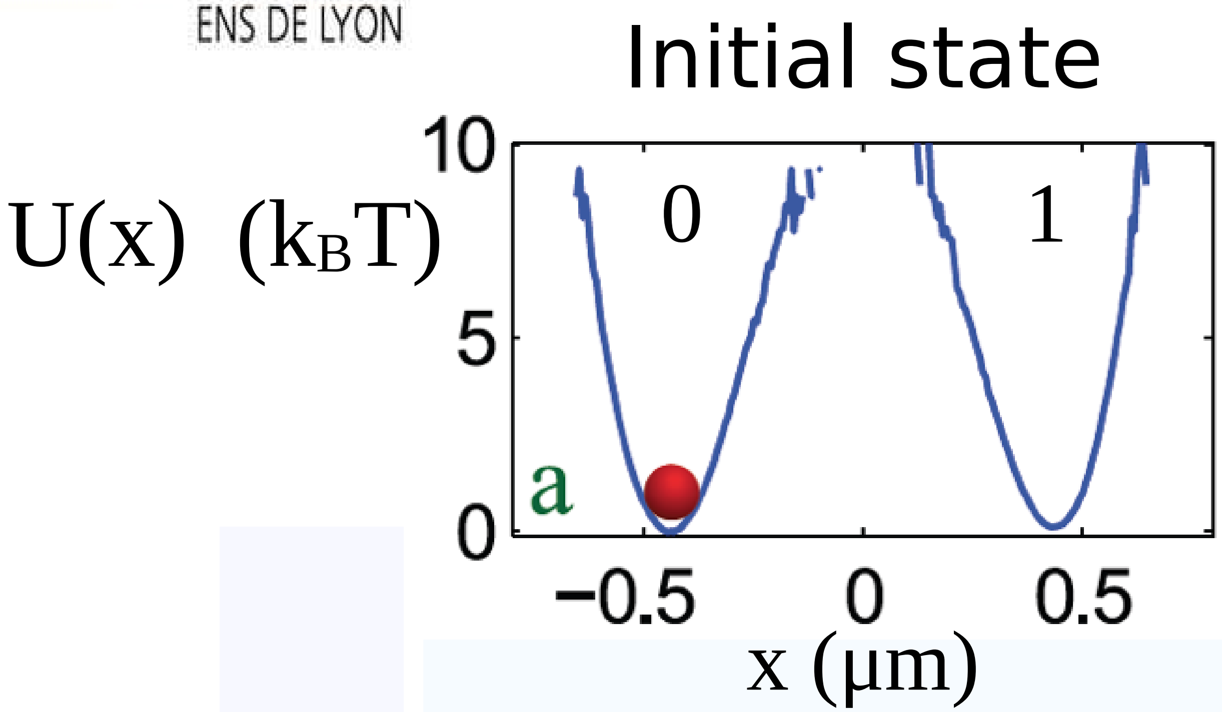
$$\tau_K = \tau_o\ \exp[\frac{\delta U}{k_B T}]$$

with $\tau_o = 1\ s$

Potential measured using the probability density function of $x(t)$

$$P(x) \propto \exp\left(\frac{-U(x)}{k_B T}\right)$$

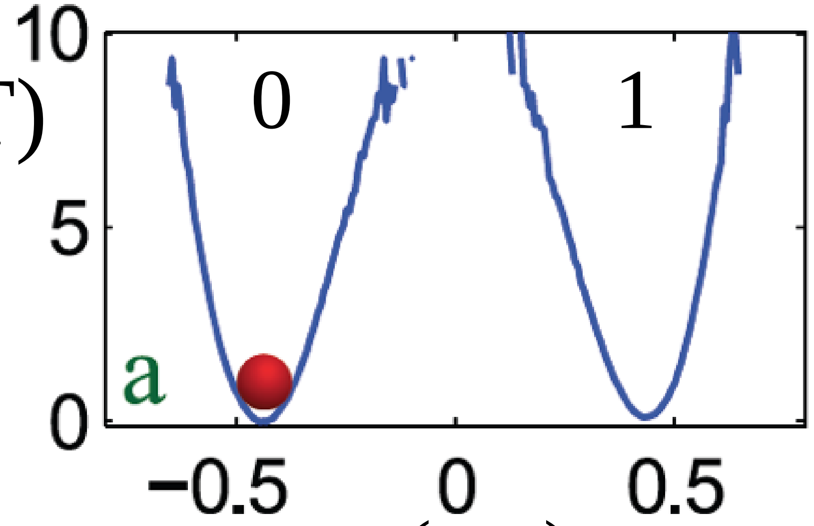
The Erasure Protocol



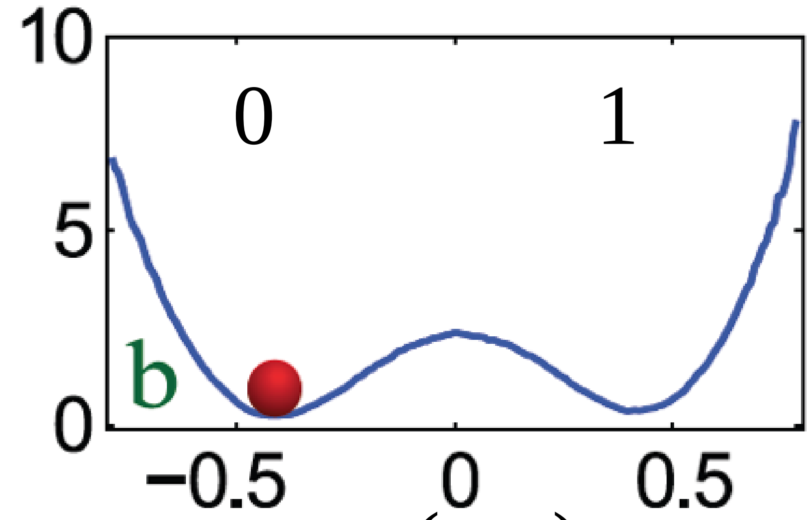
The Erasure Protocol

Initial state

U(x) (k$_B$T)



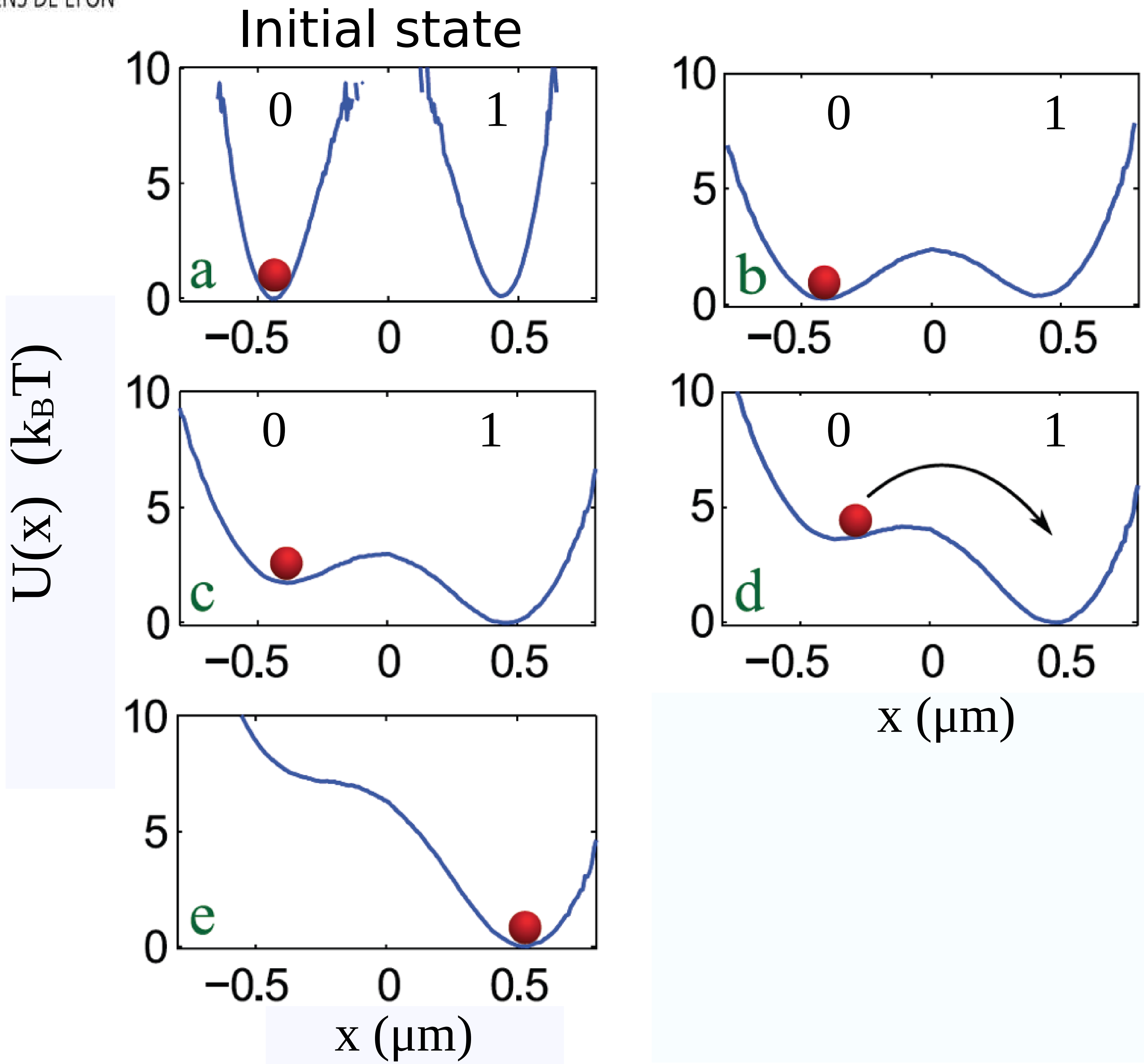
x (µm)



x (µm)

reduction
of the barrier

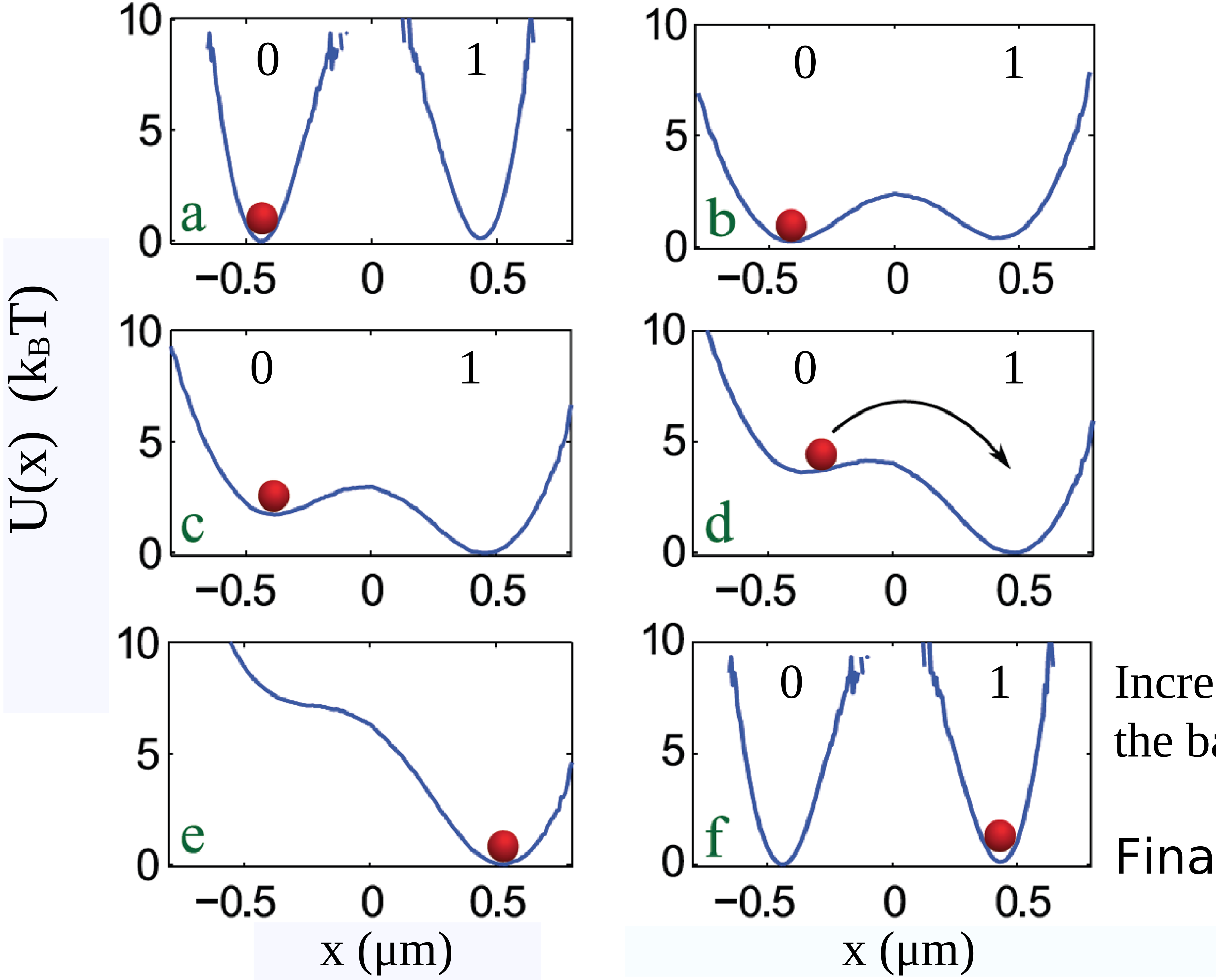
The Erasure Protocol



Progressive tilt of the potential

The Erasure Procedure

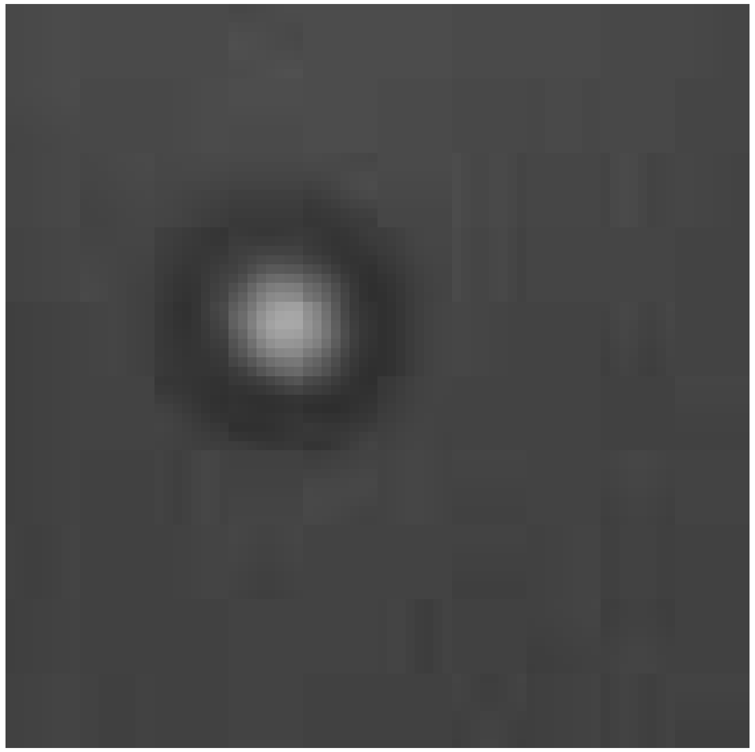
Initial state

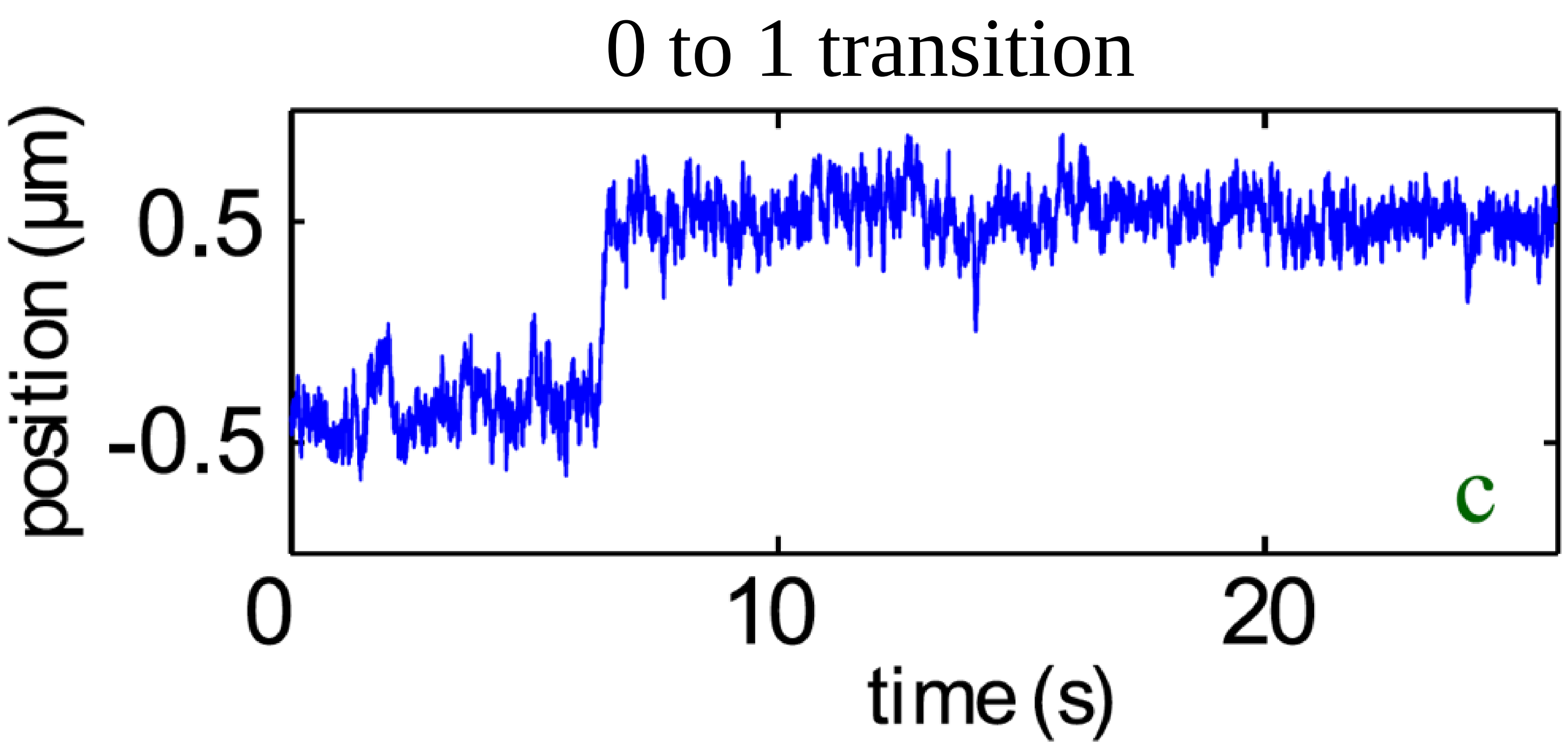


Increasing of the barrier

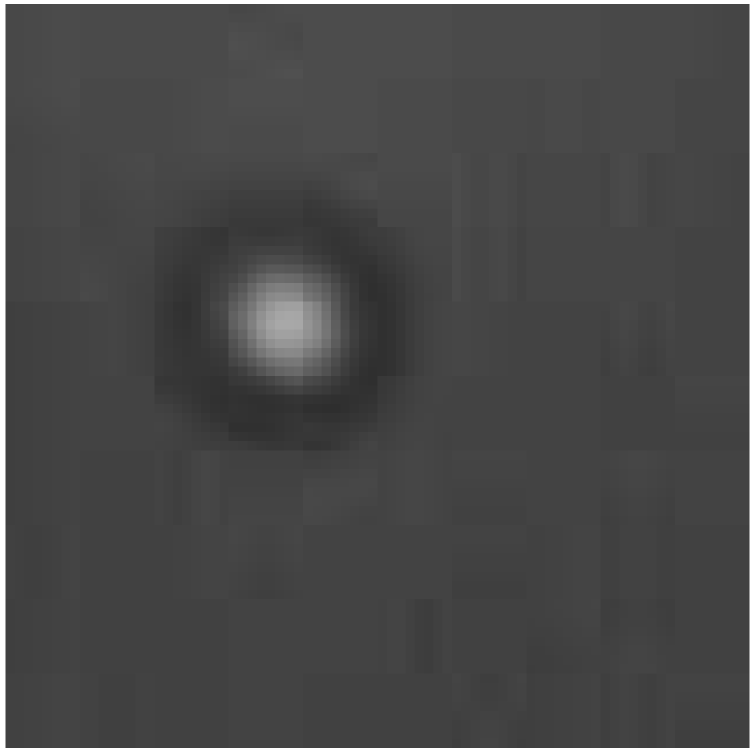
Final state

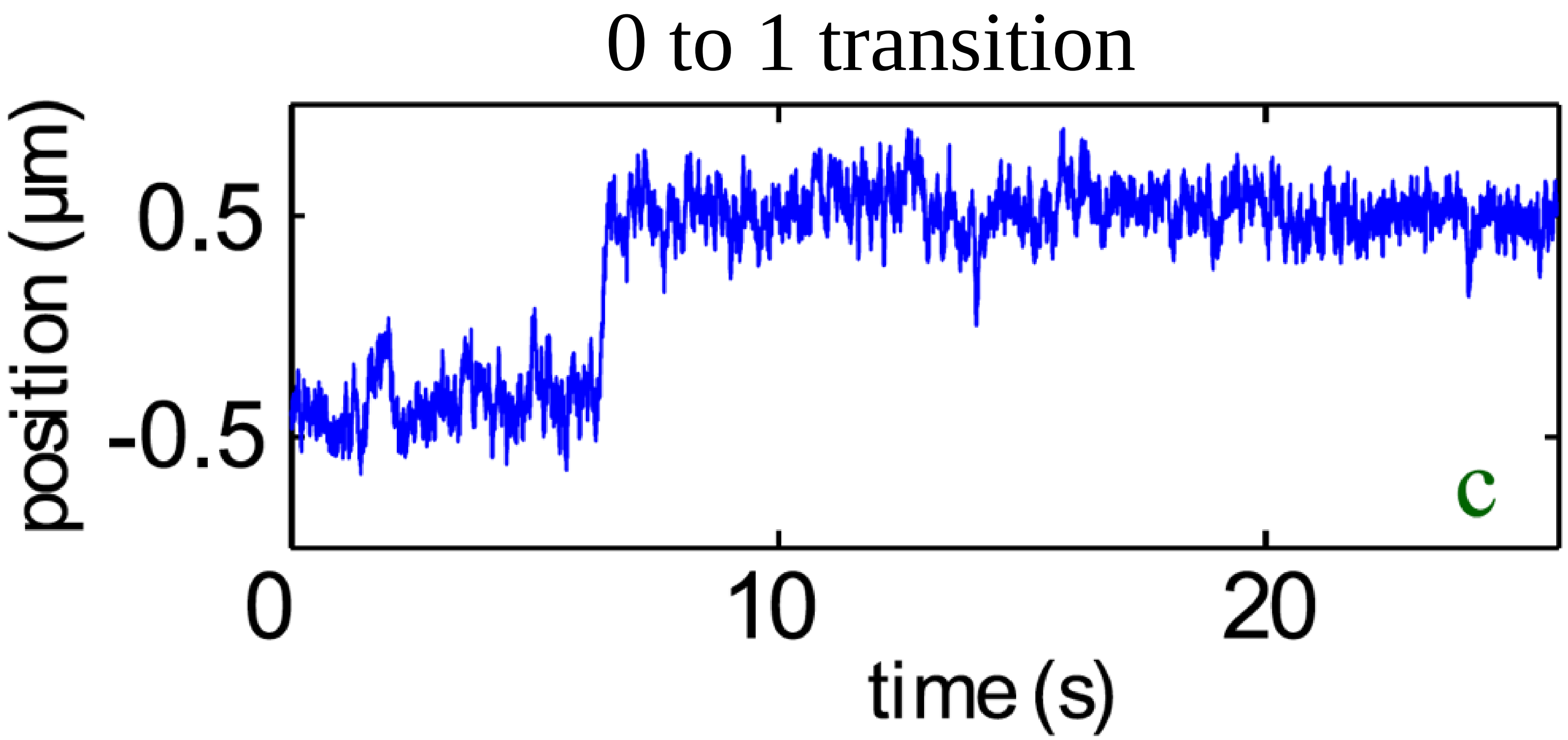
Bead trajectories



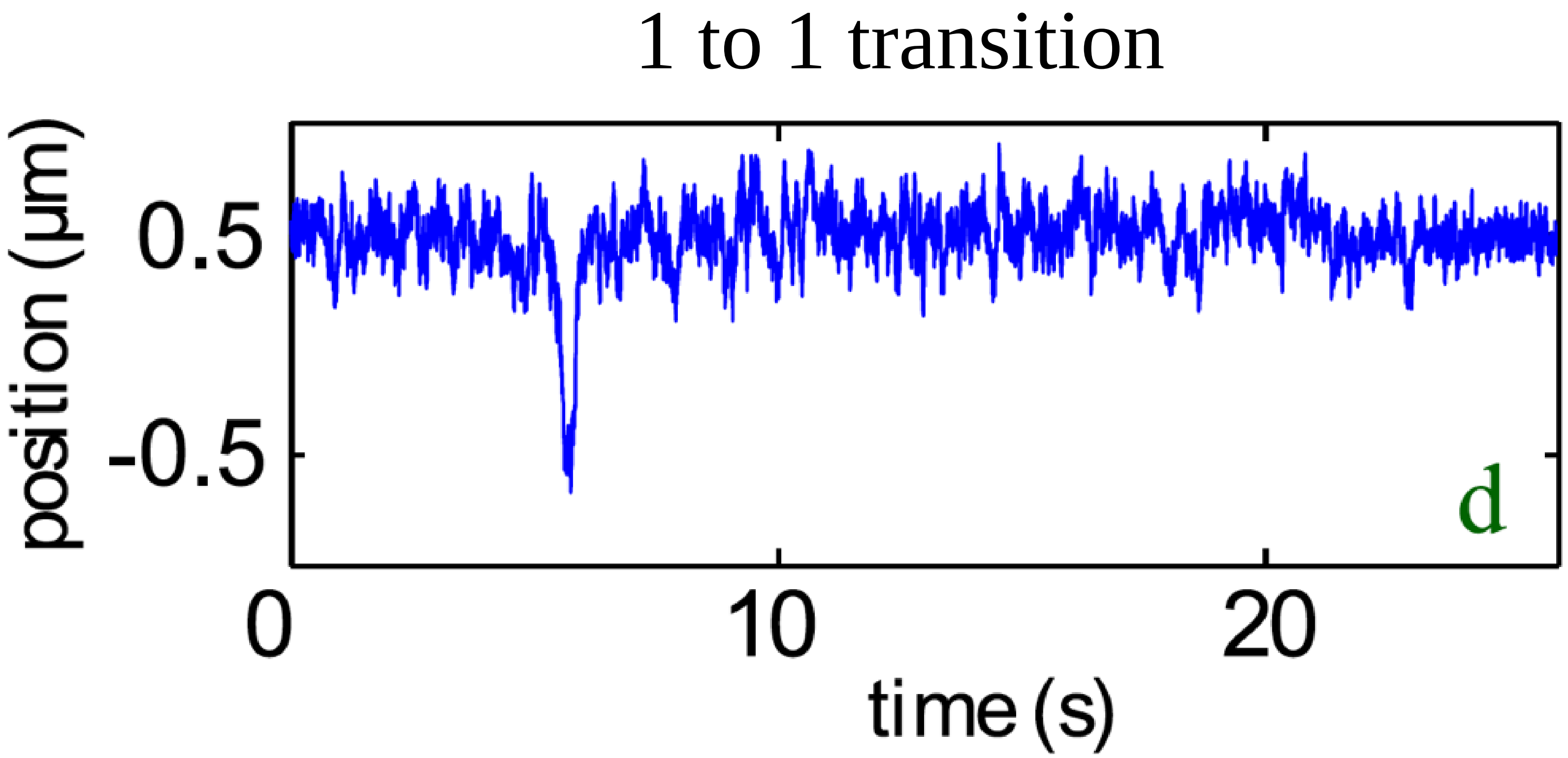


Bead trajectories





From the measure of the applied forces and of the particles trajectories we can estimate the work needed to erase the memory



The work on the erasure cycle

$$\nu \dot{x} = -\frac{\partial U_o(x,t)}{\partial x} + f(t) + \eta$$

multiplying by ẋ and integrating for a time τ we get :

$$\Delta U_\tau = W_\tau - Q_\tau$$

Stochastic thermodynamics

$$\Delta U_\tau = -\int_0^\tau \frac{\partial U_o}{\partial x}\, \dot{x}\; dt \qquad W_\tau = \int_0^\tau f\; \dot{x}\; dt$$

$$Q_\tau = \int_0^\tau \nu \dot{x}^2\; dt - \int_0^\tau \eta \dot{x}\; dt$$

Sekimoto K, Progress of Theoretical Phys. supplement (130), 17 (1998).

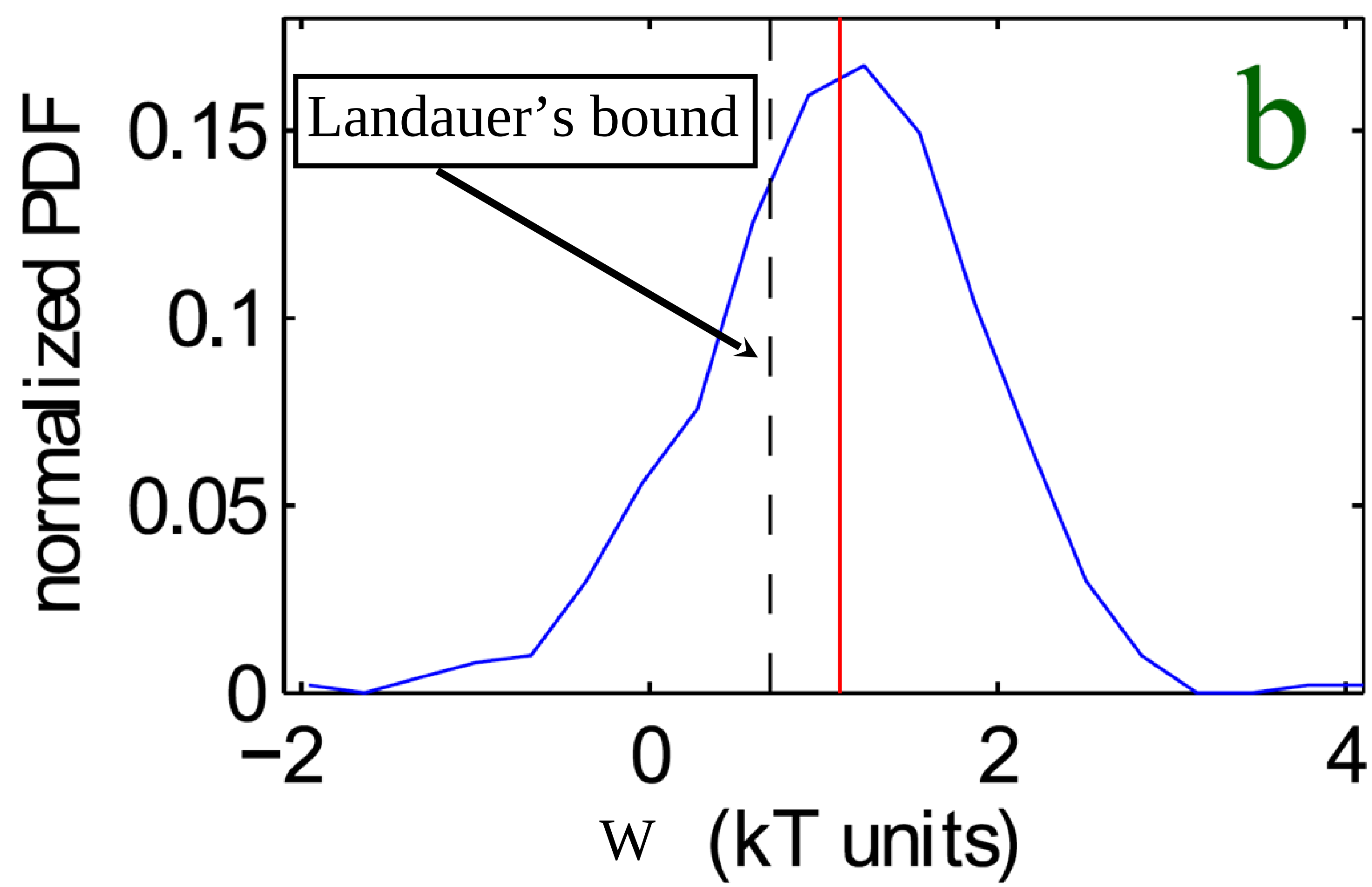
The work on the erasure cycle

The two erasure cycles have been considered

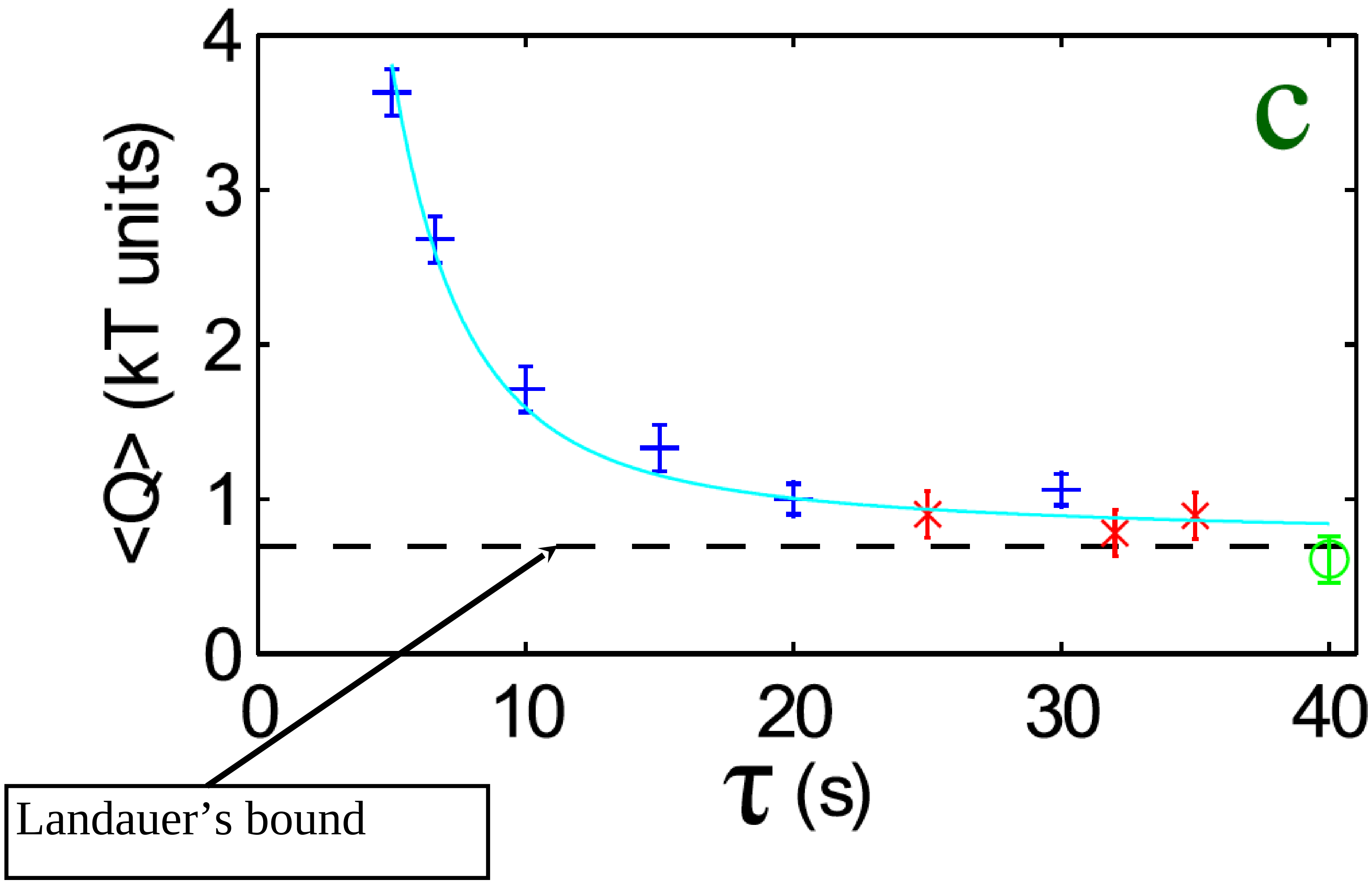
$$\begin{matrix} 0 \searrow & \\ & 1 \\ 1 \nearrow & \end{matrix} \qquad \text{and} \qquad \begin{matrix} 0 \searrow & \\ & 0 \\ 1 \nearrow & \end{matrix}$$

$$W_F = -\int_0^{\tau_{cycle}} \nu \; v(t)\dot{x} \; dt = \int_0^{\tau_{cycle}} F_{max} \frac{t}{\tau} \dot{x} \; dt$$

$$\xcancel{\Delta U_\tau = -\int_0^{\tau_{cycle}} \frac{\partial U_o}{\partial x} \; \dot{x} \; dt}$$



Landauer’s bound



The work approaches the bound but its values are larger than

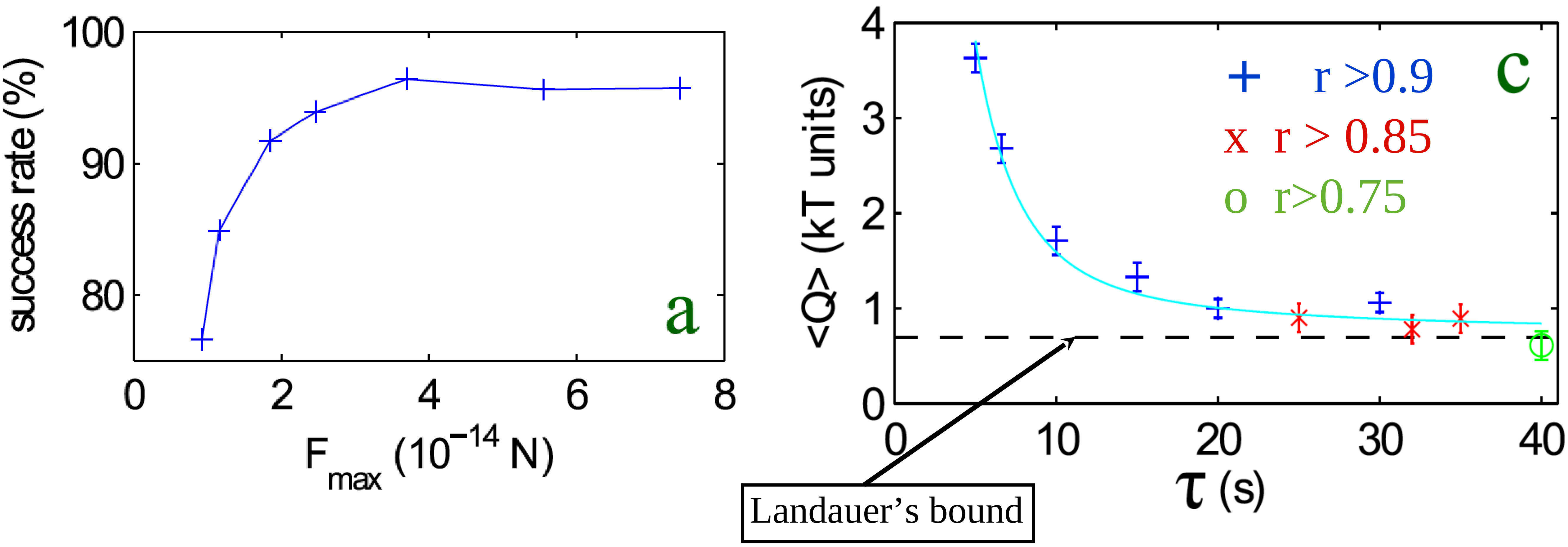
$k_B T \ln 2$

Results of the erasure procedure

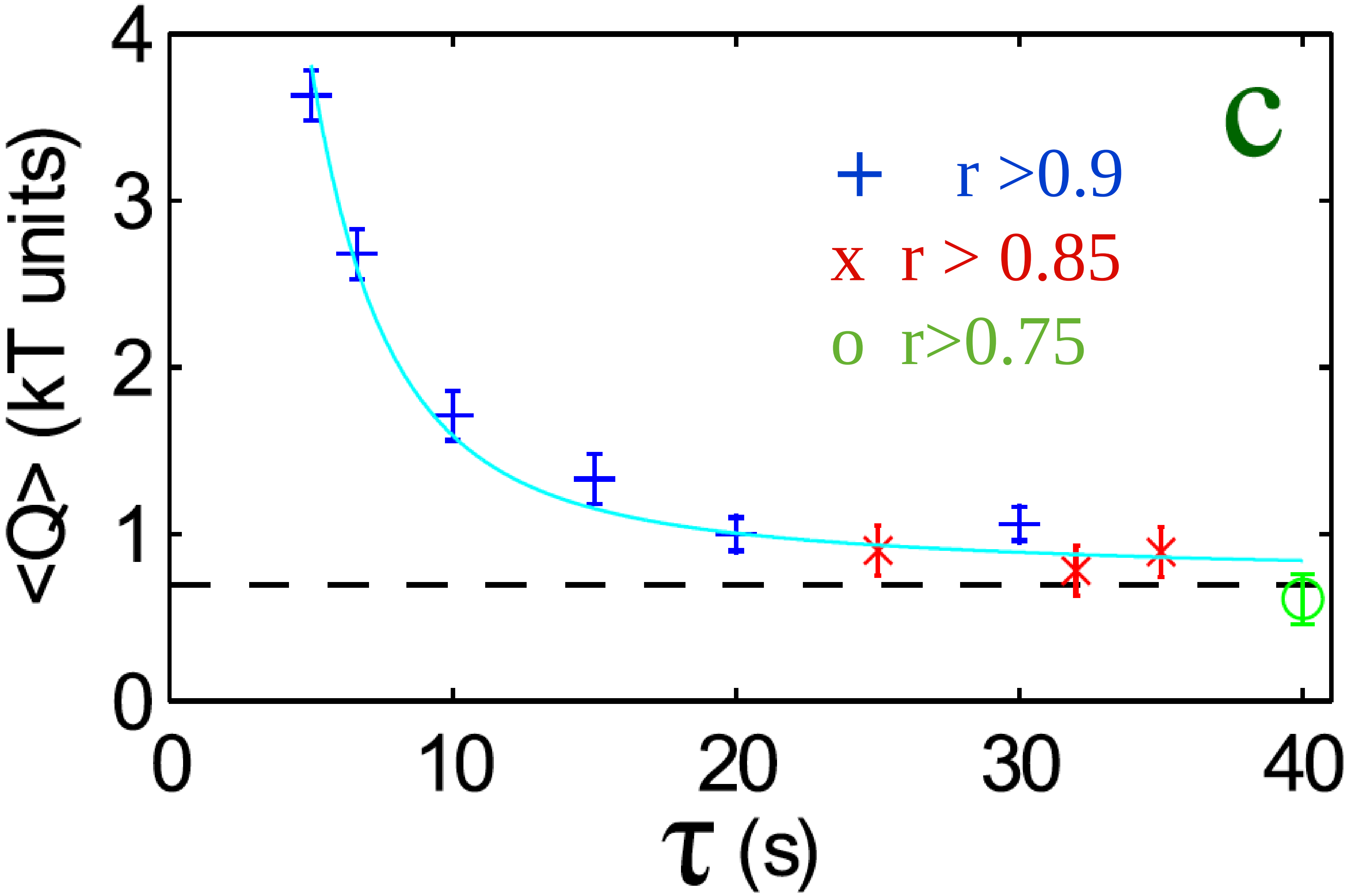
$$\text{Success rate } r = \frac{\text{number of successful cycles}}{\text{Total number of cycle}}$$

Qualitative observations :

- At constant τ : W and r increase with F_{max}
- At constant F_{max} : W decreases and r increases for increasing τ



Landauer's bound



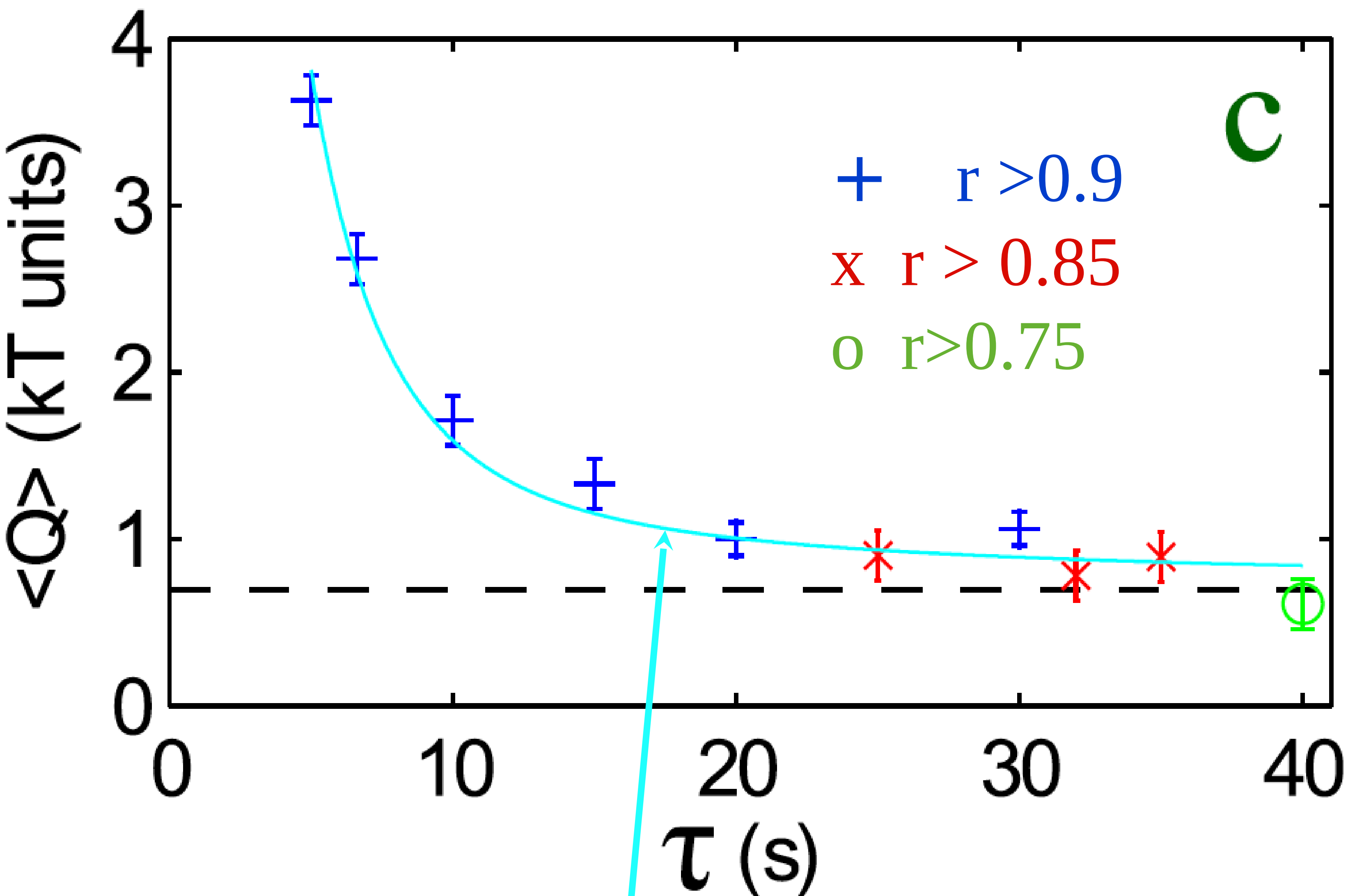
Landauer's bound as a function of r

$$< Q >^{r}_{\mathrm{Landauer}} = kT[\ln 2 + r\ln r + (1-r)\ln(1-r)]$$

At r=0.5 $< Q >^{r}_{\mathrm{Landauer}} = 0$

Indeed the Erasure Procedure left the initial state unchanged

Landauer's limit



Asymptotic behaviour

$$\tau \to \infty$$

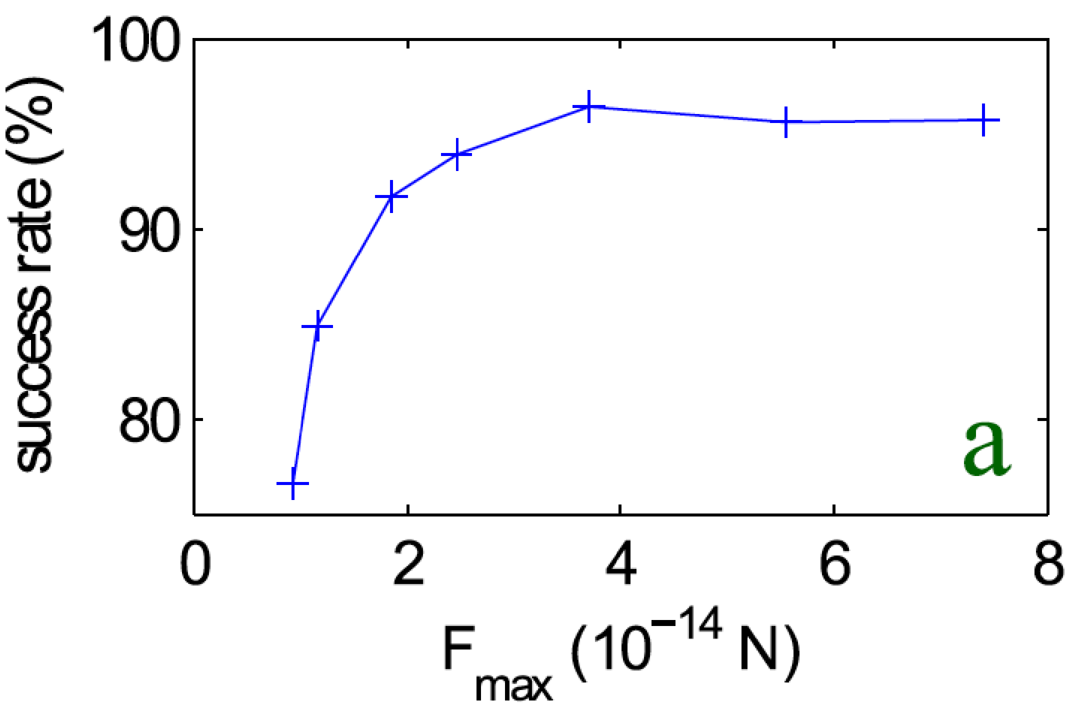
Sekimoto -Sasa J. Phys. Soc. Jpn. 66, 3326 (1997).

$$< W > \simeq \Delta F + B/\tau$$

As $< \Delta U >= 0$ then $\Delta F = -T\Delta S$ and $< Q >=< W > \simeq kT \ln 2 + B/\tau$

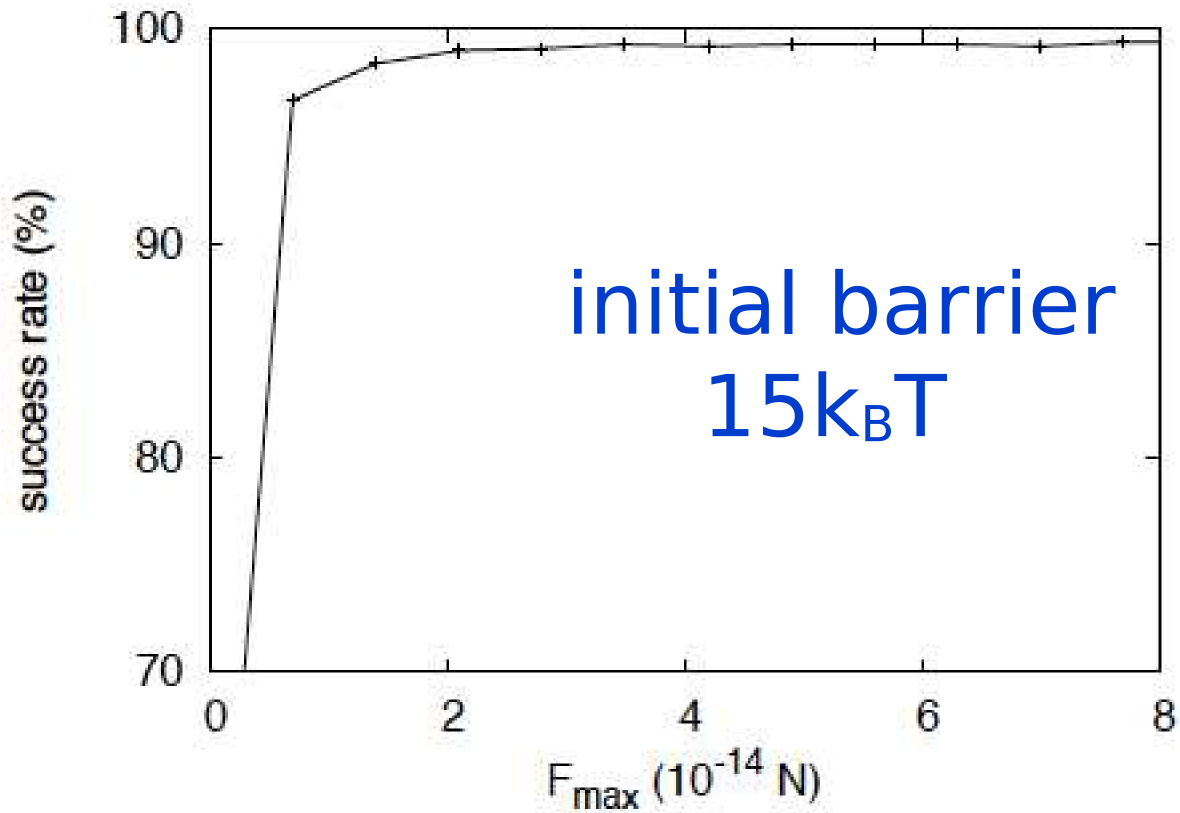
$$< Q >=< Q >_{\mathrm{Landauer}} + B/\tau$$

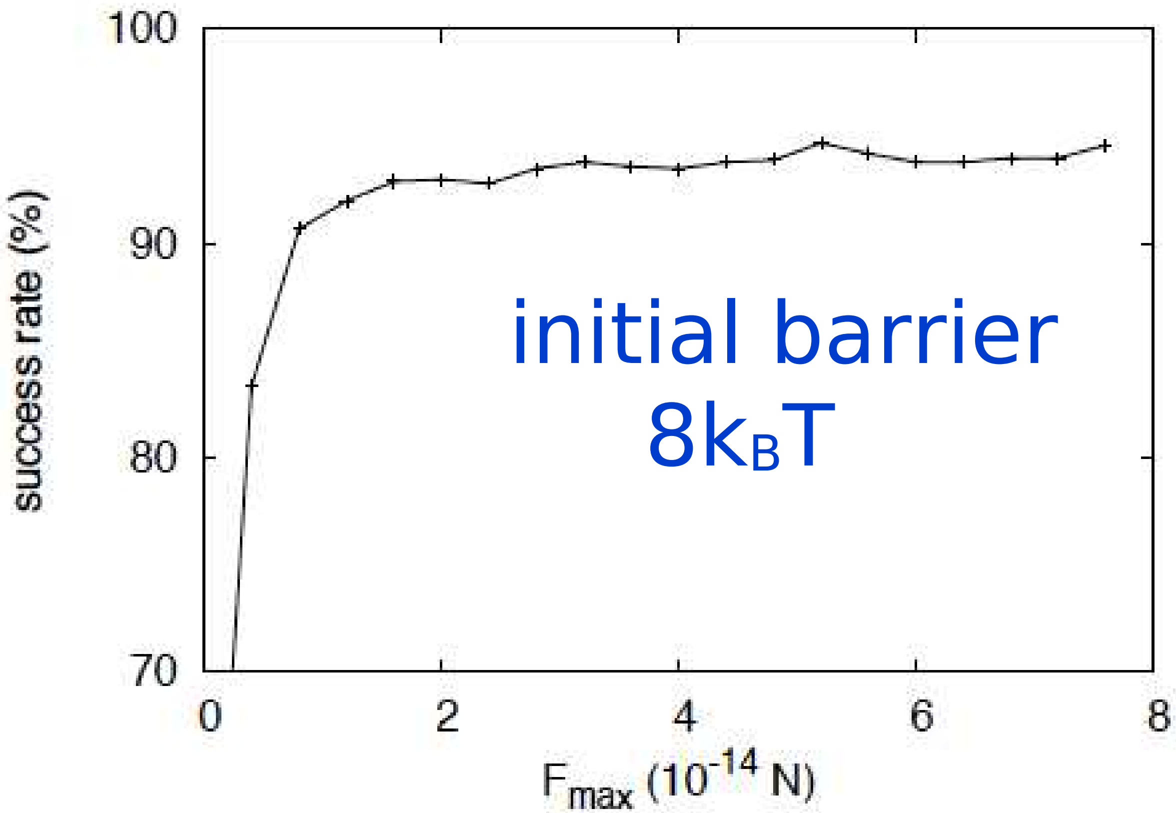
The success rate r



Why in the experiment r< 1 ?

The finite height of the initial and final barrier is responsible of r<1





Summary of the results

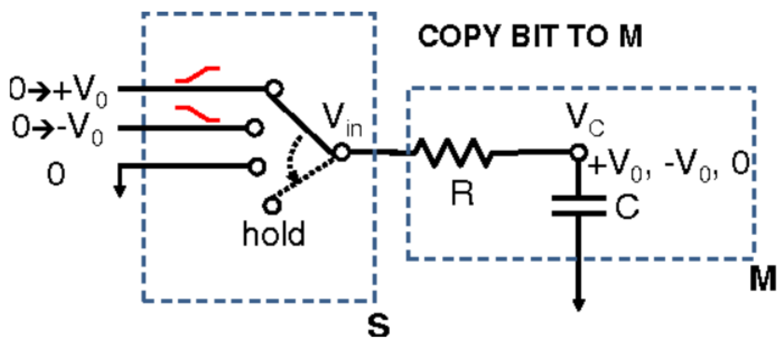
- Our experimental results indicate that the thermodynamic limit to information erasure, the Landauer bound, can be approached in the quasistatic regime, but not exceeded.

- The asymptotic limit is reached in $1/\tau$

- The fact that $r<1$ is due to the finite height of the initial barrier

- Thermal fluctuations play an important role to reach the limit

Other experiments on Landauer's bound in classical systems



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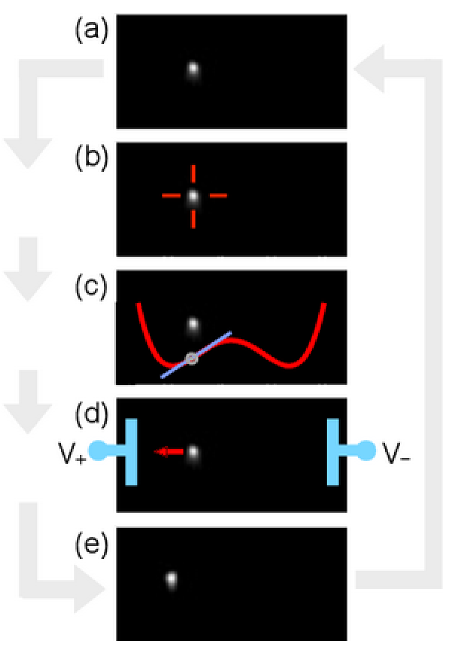

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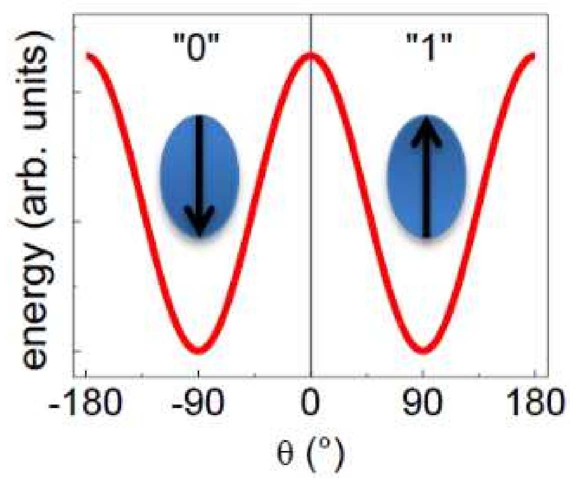

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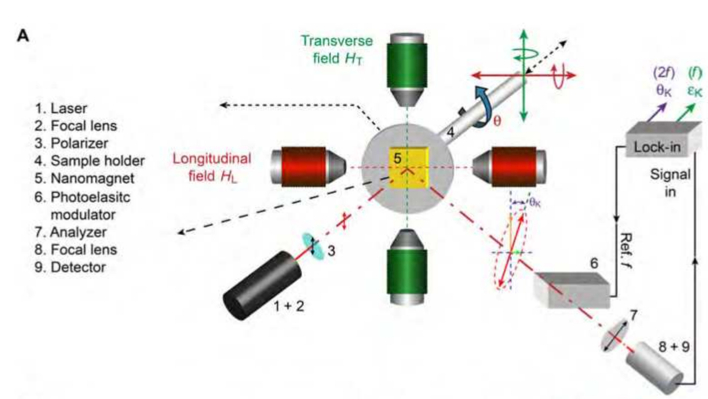

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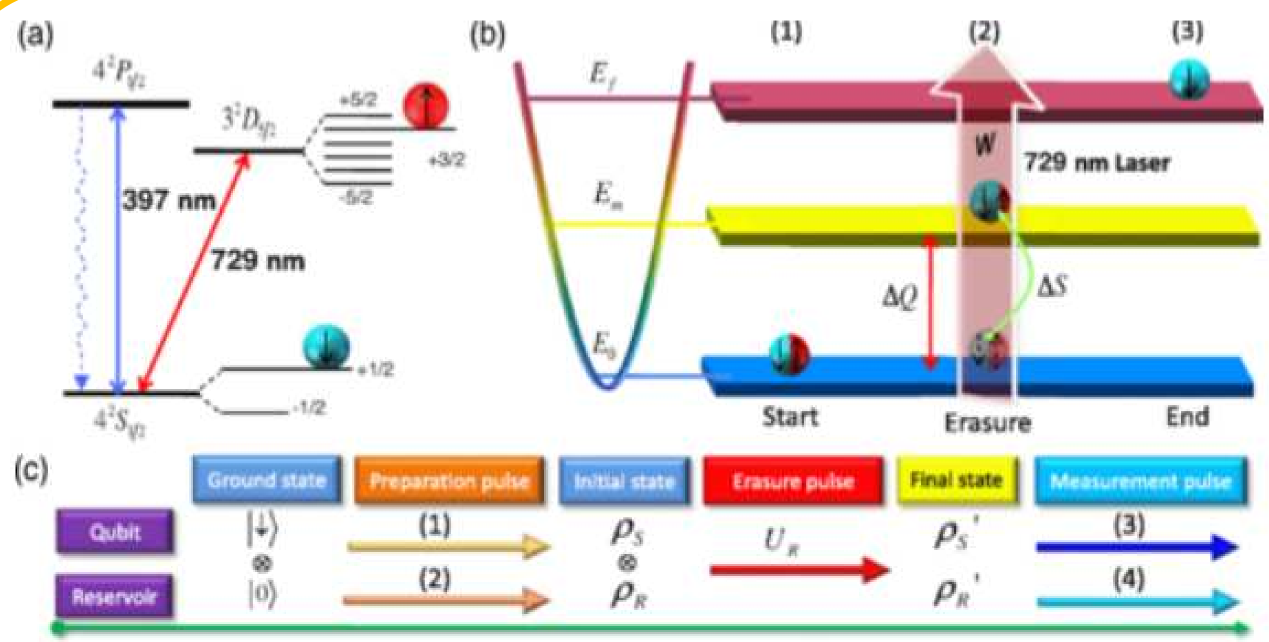
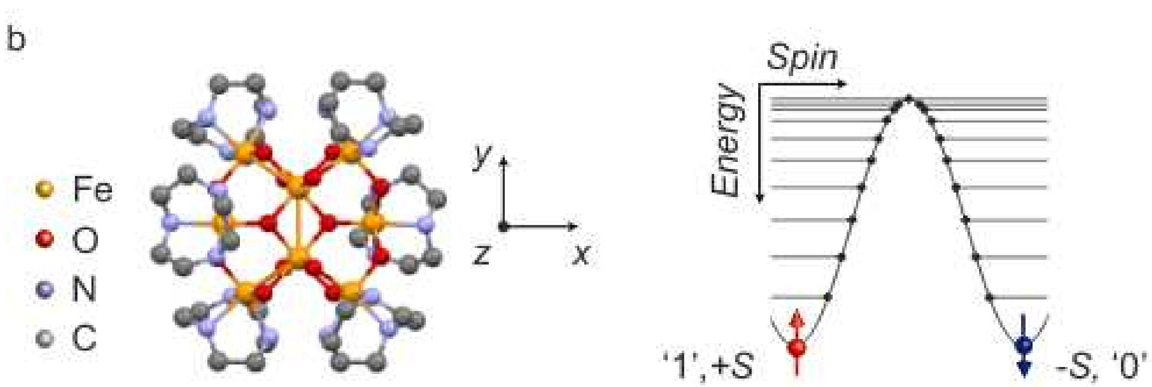
Experiments on Landauer’s bound in quantum systems



L. L. Yan, T. P. Xiong, K. Rehan, F. Zhou, D. F. Liang, L. Chen, J. Q.Zhang, W. L. Yang, Z. H. Ma, and M. Feng. Single-atom demonstration of the quantum Landauer principle. PRL., 120:210601 (2018).



R. Gaudenzi, E. Burzuri, S. Maegawa, H. S. J. van der Zant, and F. Luis. Quantum landauer erasure with a molecular nanomagnet. Nucl. Phys. 14 (6):565–568, JUN 2018.

Landauer's bound and the Jarzynski equality

$$< \exp(-W_s) > = \exp(-\Delta F)$$

with $$W_s = -\int_o^{\tau_{cycle}} \dot{\lambda} \frac{\partial H(x,\lambda)}{\partial \lambda} dt$$

In our case this equality transforms

$$W_s = \int_o^{\tau} \dot{f}\ x\ dt = [f\ x]_o^{\tau} - \int_o^{\tau} f\ \dot{x}\ dt = -W_f$$

Since the height of the barrier is always finite there is no change in the ***equilibrium* F** of the system between **the beginning and the end of the procedure**.

$$< \exp(-W_s) > = \frac{\rho_{eq}(\tau)}{\rho(\tau)} \exp(-\Delta F)$$

S. Vaikuntanathan and C. Jarzynski, Euro. Phys. Lett. 87, 60005 (2009).

Generalized Jarzynski equality

Landauer's limit and the Jarzynski equality

We consider the erasure protocol $\begin{matrix} 0 \searrow \\ \quad\; 0 \\ 1 \nearrow \end{matrix}$

If the final state is 0 then $\rho = r \simeq 1, \quad \rho_{eq} = 1/2, \quad \Delta F = 0$

and the Generalized Jarzynski is : $< \exp(-W_s) >_{\to 0} = \frac{1/2}{r}$

from Jensen inequality $< W_s >_{\to 0} \geq (\ln 2 \; + \; \ln r)$

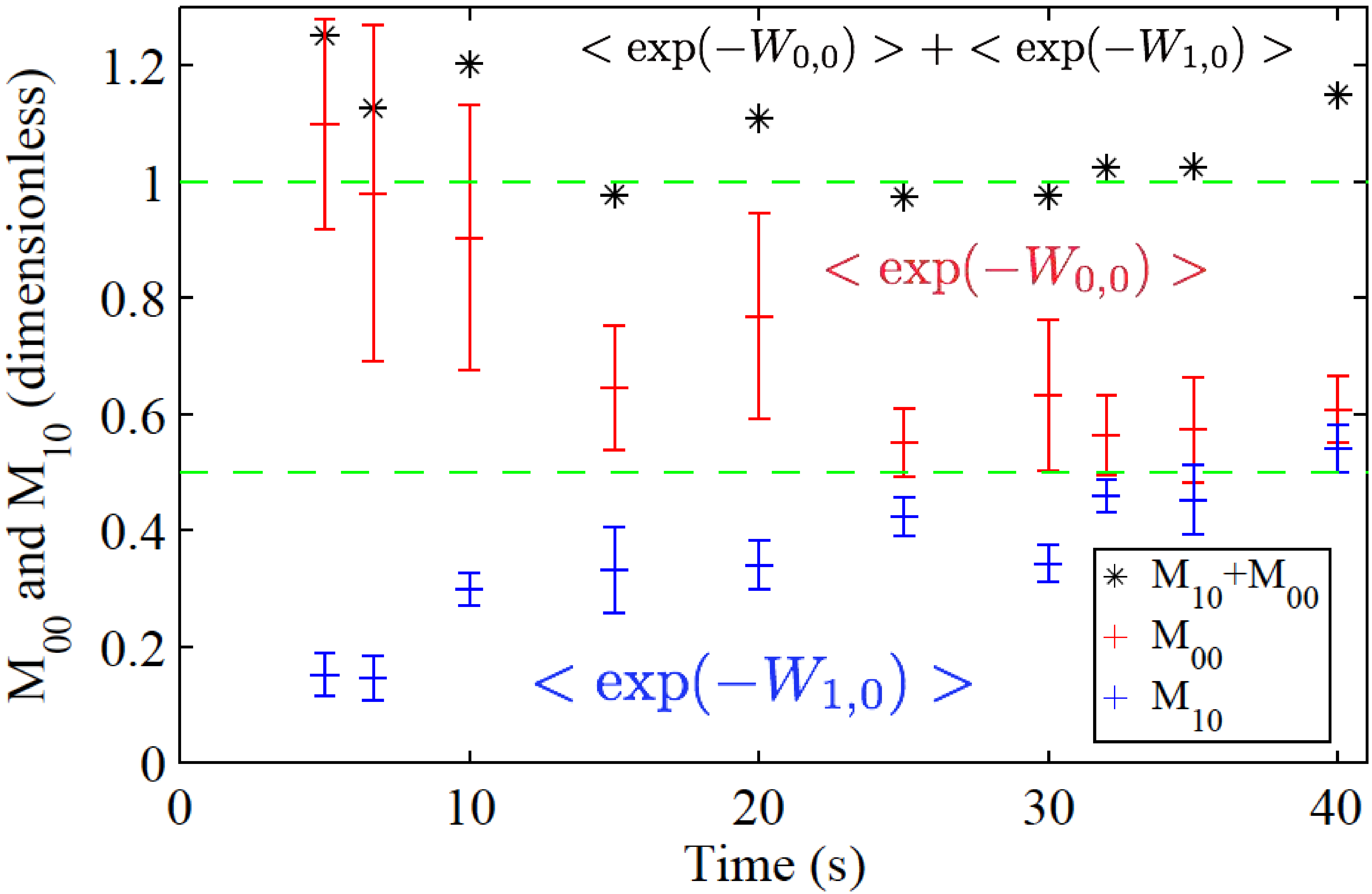
$$\frac{1}{2} < \exp(-W_{1,0}) > + \frac{1}{2} < \exp(-W_{0,0}) > = \frac{1}{2}$$

Work done if the particle makes the jump from 1 to 0

Work done when the particle starts in the final state

Landauer's limit and the Jarzynski equality

$$-\ln\left(\frac{<\exp(-W_{0,0})>+<\exp(-W_{1,0})>}{2}\right)=\Delta F_{eff}$$



Landauer's limit and the Jarzynski equality

We consider the erasure procedure $\begin{matrix} 0 \searrow \\ 1 \nearrow \end{matrix} 0$

If the final state is 0 then $\rho = r \simeq 1, \quad \rho_{eq} = 1/2, \quad \Delta F = 0$

$$< \exp(-W_s) >_{\rightarrow 0} = \frac{1/2}{r} \quad \text{and} \quad < W_s >_{\rightarrow 0} \geq (\ln 2 + \ln r)$$

If the final state is 1 then $\rho = (1-r) \simeq 0, \quad \rho_{eq} = 1/2, \quad \Delta F = 0$

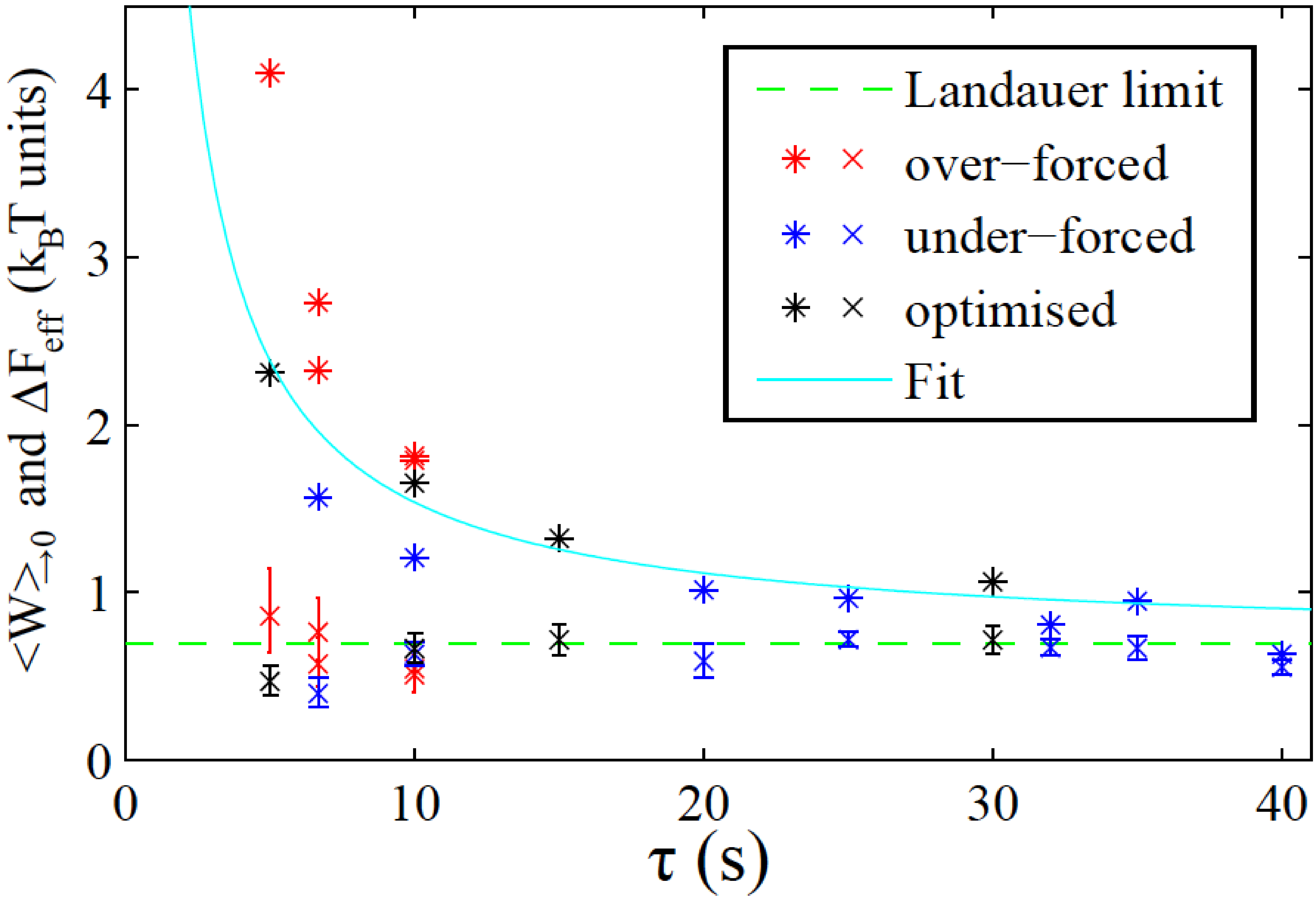
$$< \exp(-W_s) >_{\rightarrow 1} = \frac{1/2}{1-r} \quad \text{and} \quad < W_s >_{\rightarrow 1} \geq \ln 2 + \ln(1-r)$$

Total work $\quad < W_s > = r \; < W_s >_{\rightarrow 0} + (1-r) \; < W_s >_{\rightarrow 1}$

using the inequalities $\quad < W_s > \geq \ln 2 + r \ln r + (1-r) \ln(1-r)$

The generalized Landauer's Bound

Landauer's limit and the Jarzynski equality

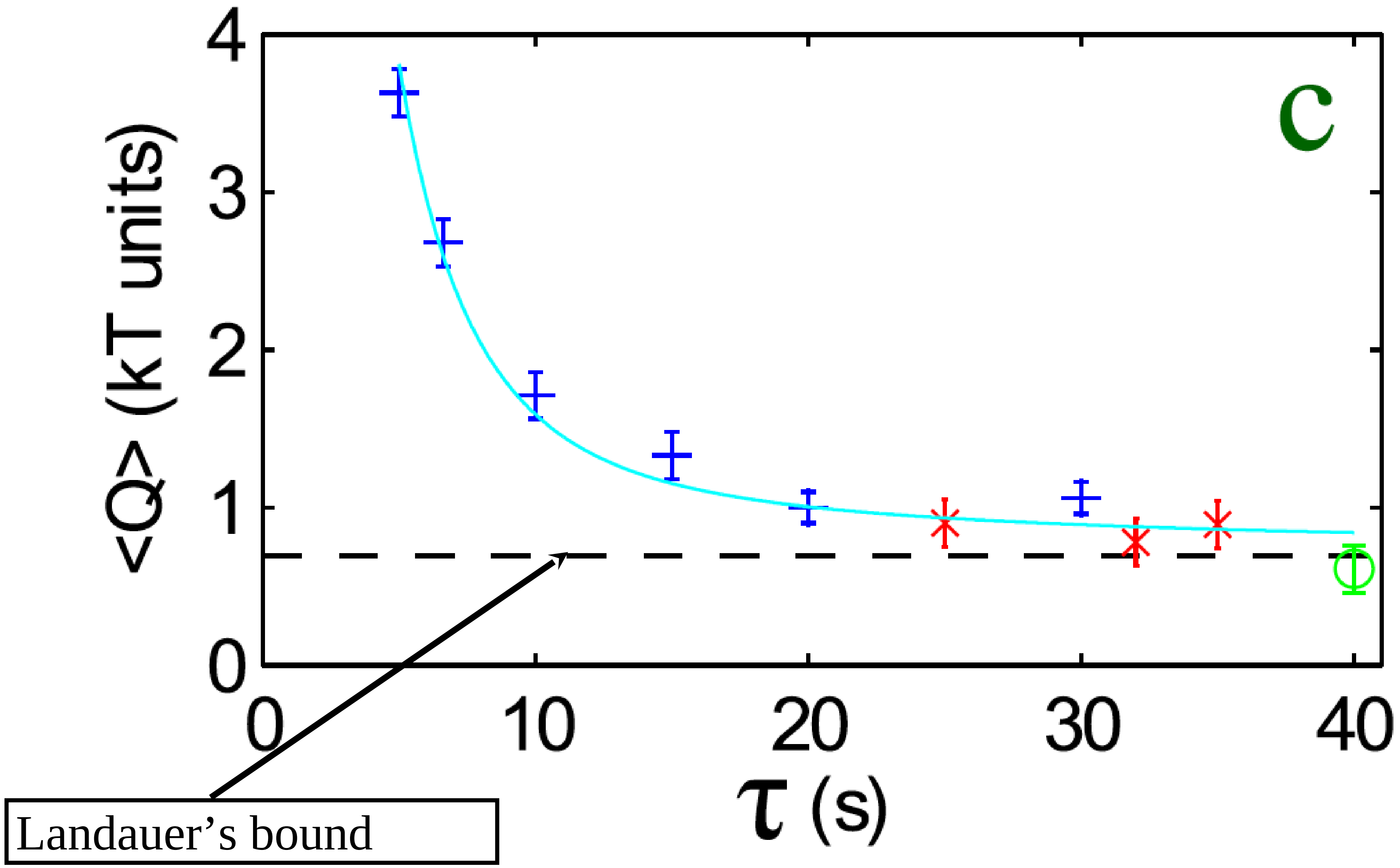


Landauer's bound

Question : can Landauer's bound be reached in short times ?

Optimal protocol :

E. Aurell, et al. J. Stat. Phys.147, 487-505 (2012).



Nature 483, 187-189 (2012)

Landauer’s bound in an underdamped micromechanical oscillator

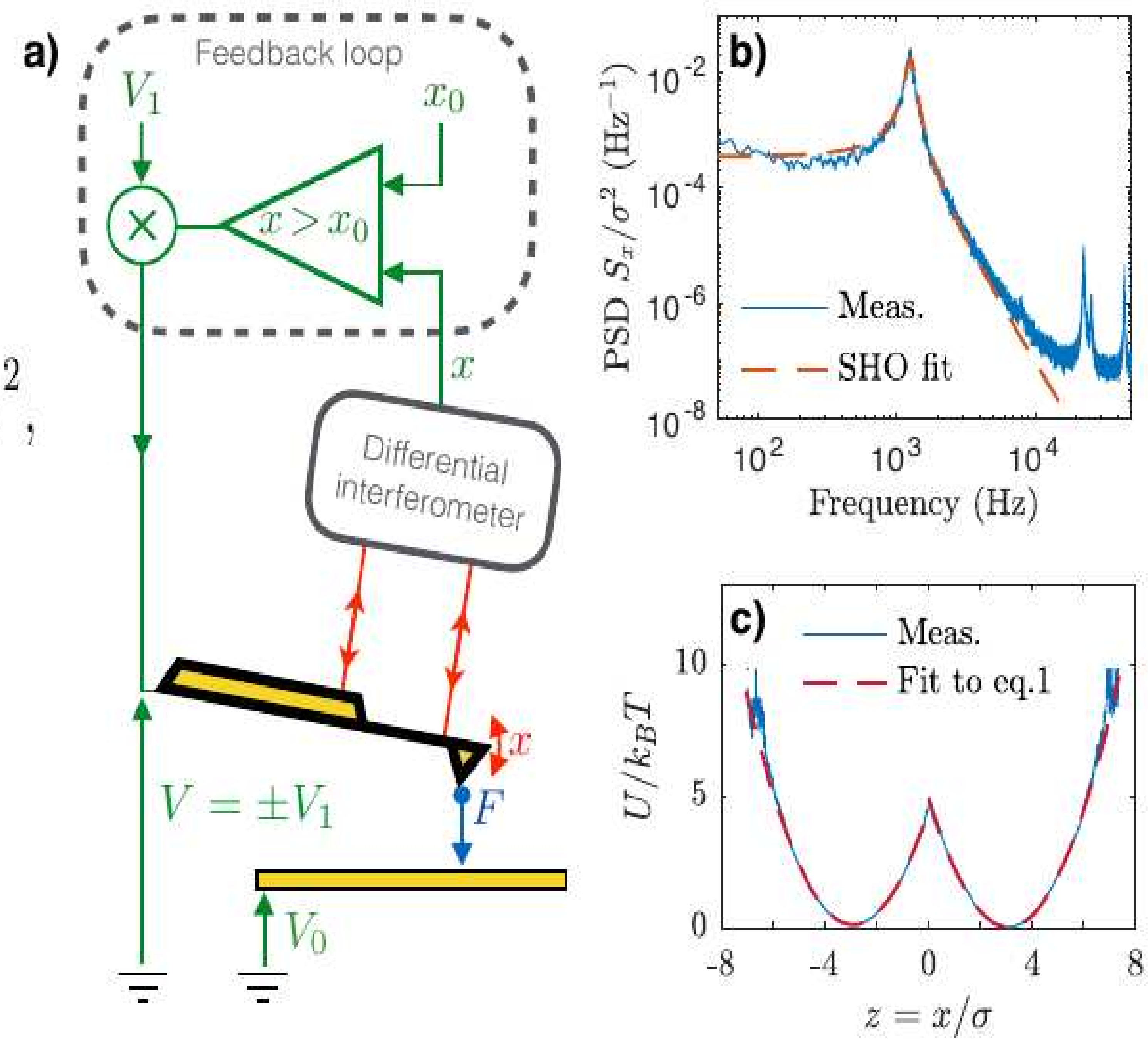
In this system, Landauer’s bound is reached with a 1 % uncertainty, with protocols as short as 100 ms.

S Dago, J. Pereda, N. Barros, S. Ciliberto,L. Bellon, PRL 126, 170601 (2021)

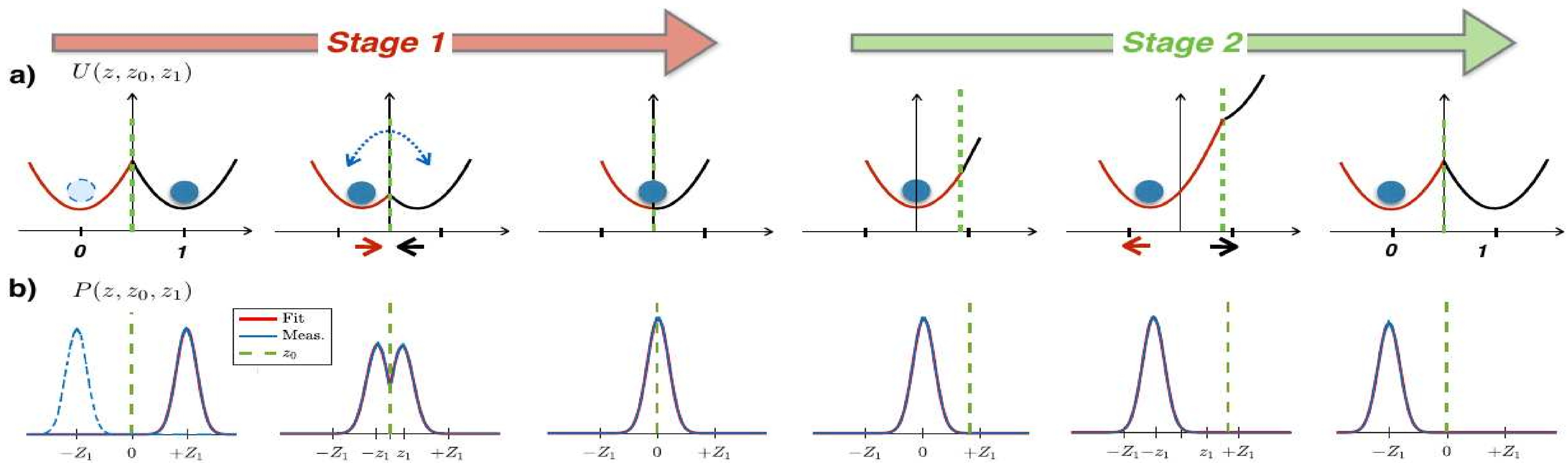
Experimental set-up

$$U(z, z_0, z_1) = \frac{1}{2}\left(z - S(z - z_0)z_1\right)^2,$$

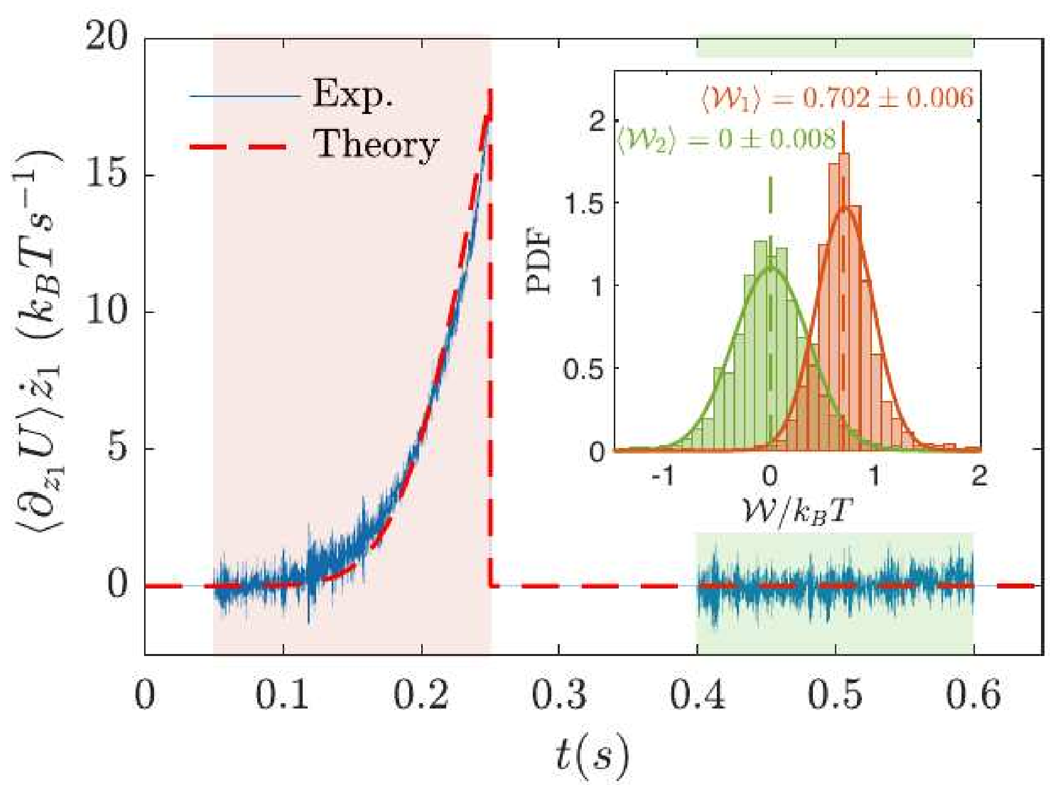
With S(u)=sign(u) is the sign function



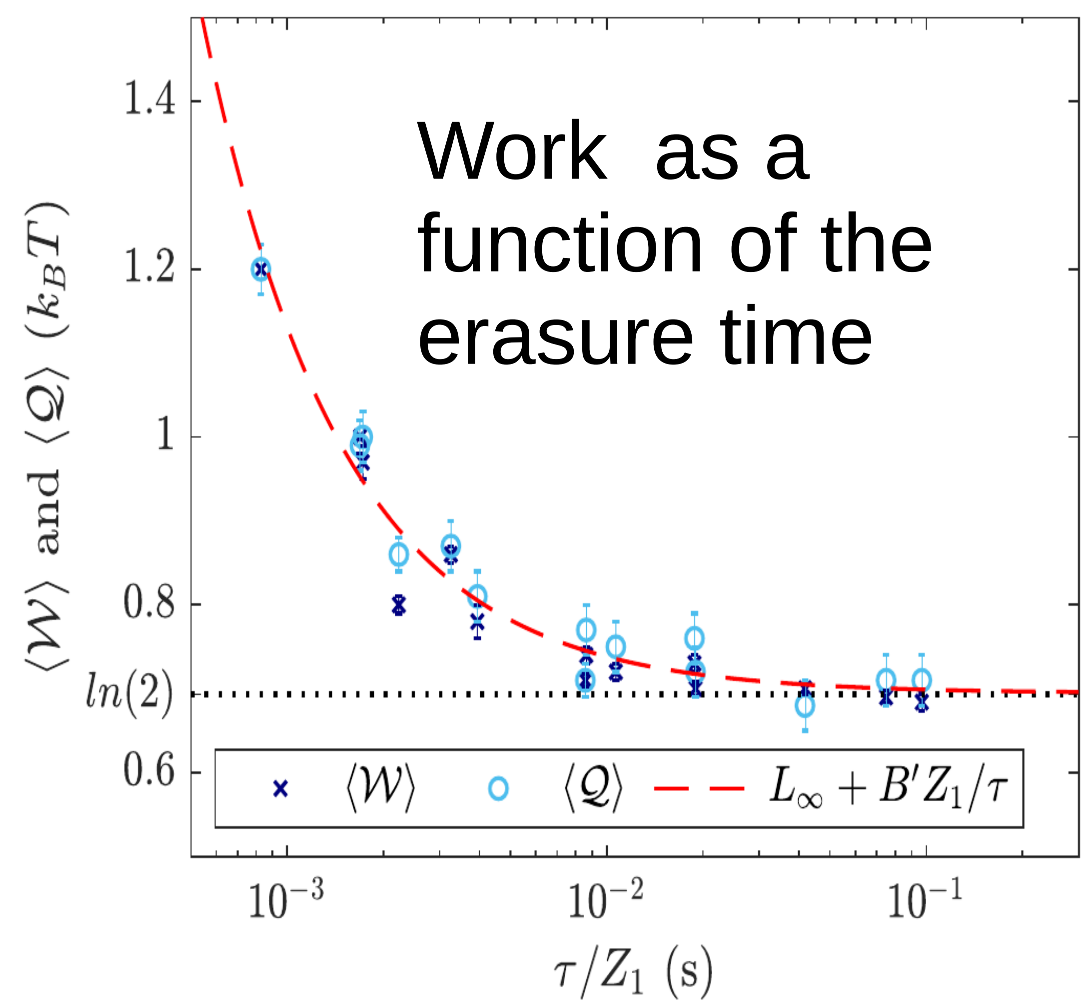
Landauer's bound in an underdamped micromechanical oscillator



The erasure protocol

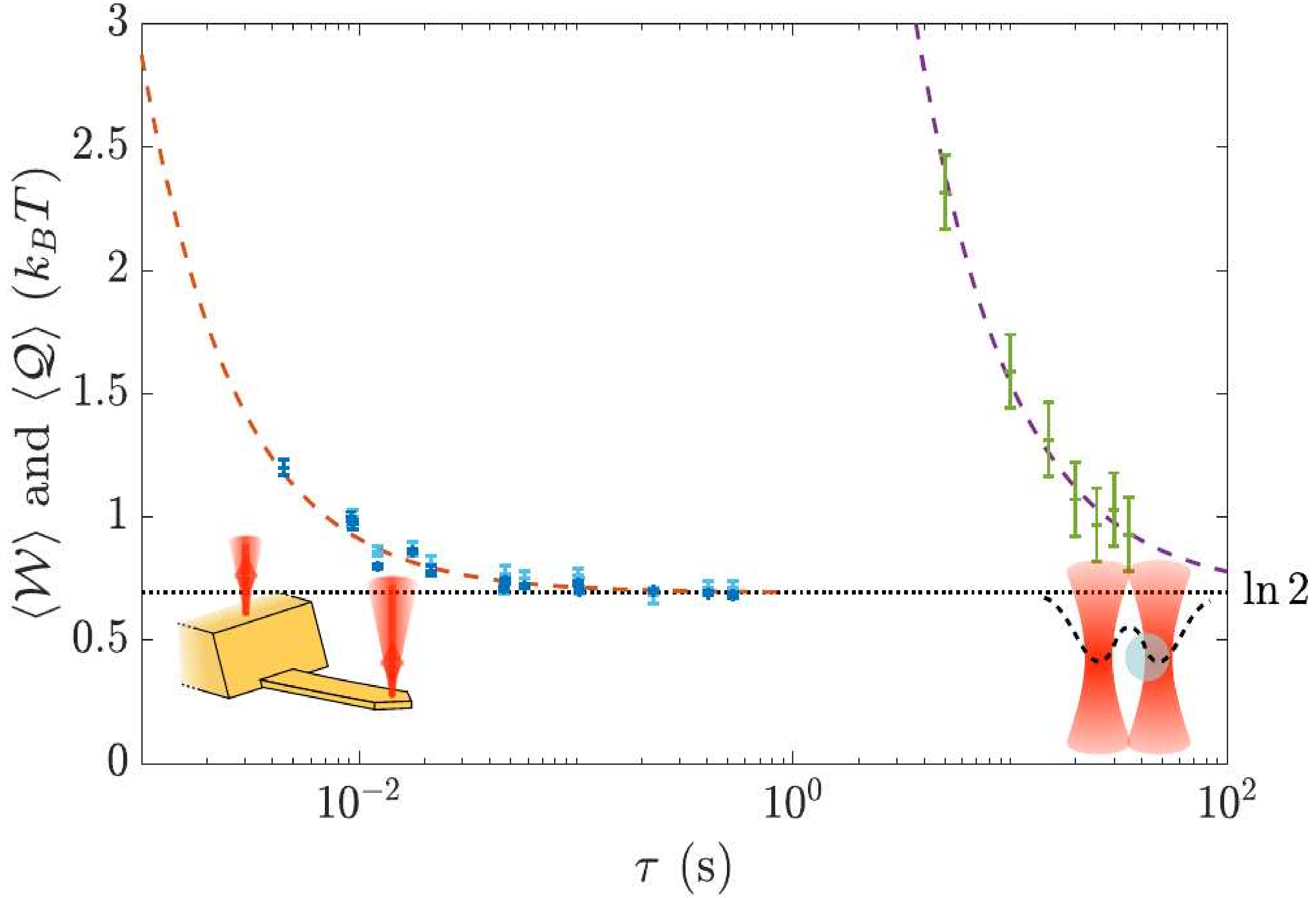


$$U(z, z_0, z_1) = \frac{1}{2}\big(z - S(z - z_0)z_1\big)^2,$$



Comparison of the mechanical oscillator with optical trap

$$\langle \mathcal{Q} \rangle = \langle \mathcal{W} \rangle = k_B T \left(\ln 2 + \frac{B}{\tau} \right)$$



Question : can Landauer's bound be reached in short times ?

This is possible using underdamped systems.

ENS DE LYON

Main messages

a) Maxwell's demon allows us to produce energy from information
b) Information needs a minimum energy in order to be processed

The energy gain from information is smaller/equal than the processing energy. The demon cannot overcome the second principle.

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2013 *EPL* **103** 60002 ; arXiv:1302.4417 ;
Detailed Jarzynski Equality applied on a Logically Irreversible Procedure

JSTAT, 2015, P06015
Information and thermodynamics: experimental verification of Landauer's Erasure principle

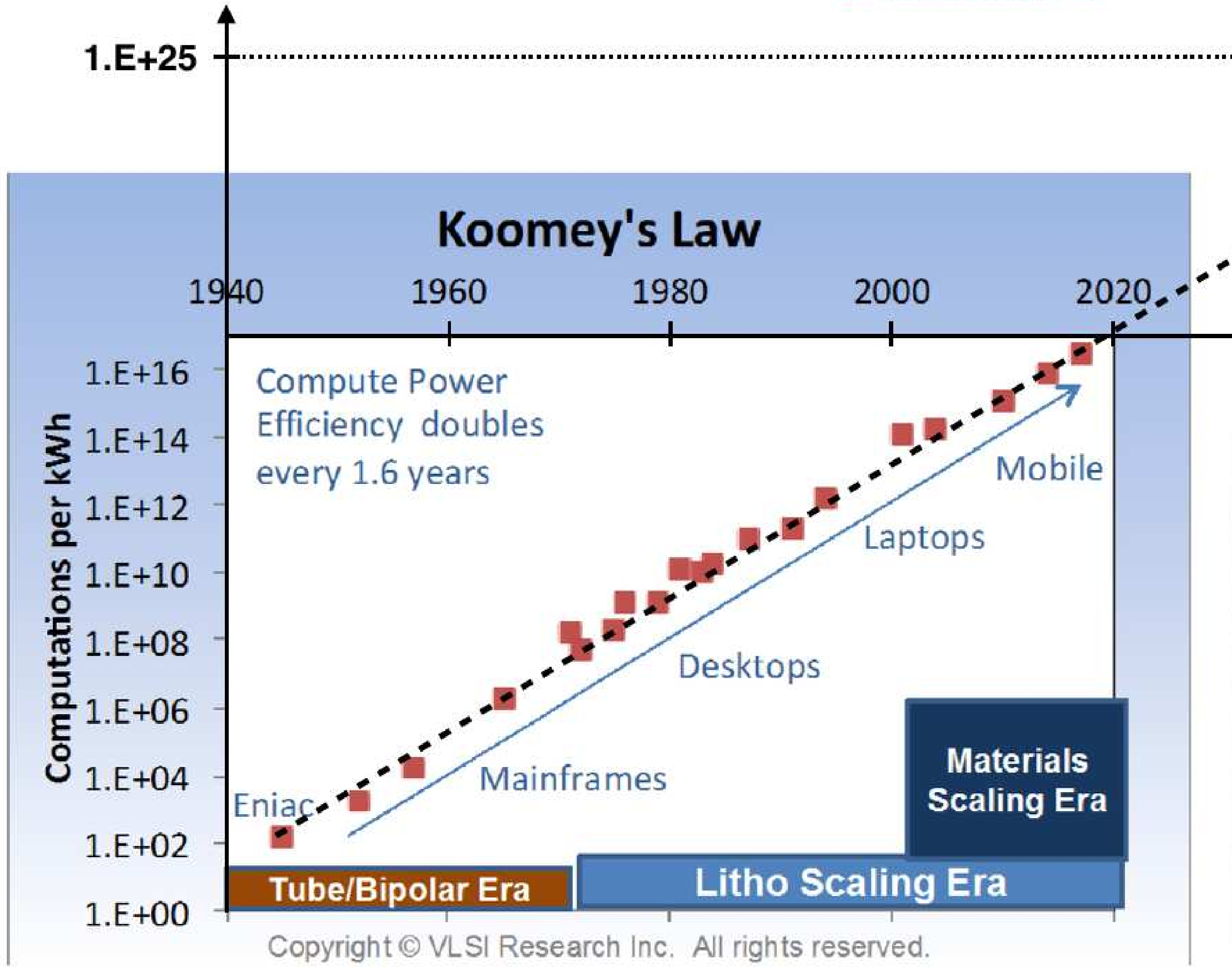
Physics Today, September 2015;
Information: From Maxwell's demon to Landauer's eraser

Information and Thermodynamics: Fast and Precise Approach to Landauer's Bound in an Underdamped Micromechanical Oscillator PRL 126, 170601 (2021)

The physics of information: From Maxwell to Landauer.
In ''Energy Limits in Computation'', Springer, 2019.

The energetic cost of information

1-bit erasure costs at least $\underline{k_B T \ln 2} = 3 \times 10^{-21}$ J



Computers costs are driven by Flops/KWh

THANK YOU !!

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A. Petrosyan

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erc

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