

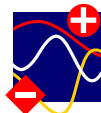
Numerical aspects of large deviations

Motivation

Alexander K. Hartmann

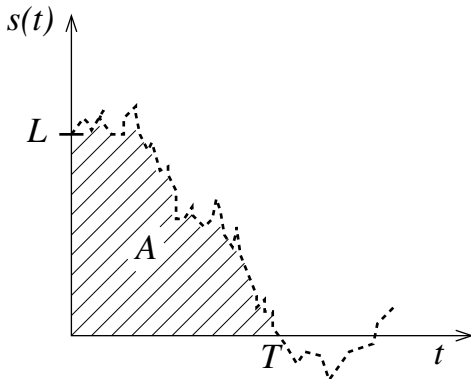
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Les Houches Summer School, 11. July 2024



Area under walk

- Walks $s(t)$ start at distance L
- First passage to $s = 0$, A : area under curve



- Wanted: distribution $P(A)$ even for very small probabilities

Results

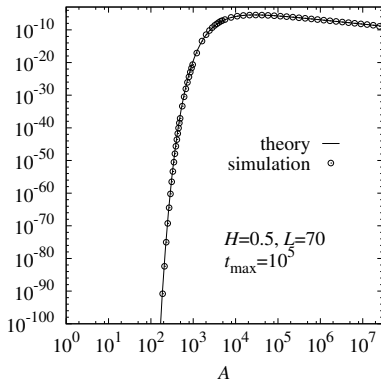
Exact result known for standard RW case ($H = 1/2$)

$$P(A) = \frac{3^{-2/3} L}{\Gamma\left(\frac{1}{3}\right) (DA^4)^{1/3}} e^{-\frac{L^3}{9DA}}$$

[M.J. Kearney & S.N. Majumdar (2005)] $P(A)$

[S.N. Majumdar & B. Meerson (2020)]

→ perfect agreement

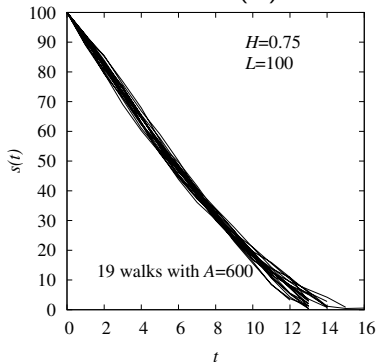


[AKH & B. Meerson, Phys. Rev. E 2024]

Paths

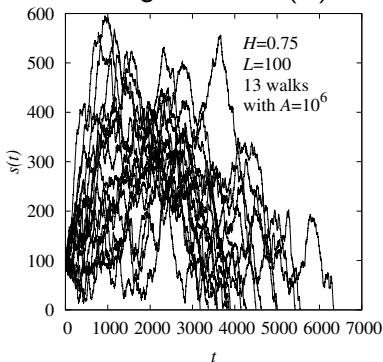
How do the rare paths look like?

left tail of $P(A)$



only “optimal” paths

right tail of $P(A)$



many different paths

Ising model

Ising spins $S_i = \pm 1$ on (here square $L \times L$) lattice

B : external field (=parameter). Energy:

$$H = - \sum_{\langle i,j \rangle} S_i S_j - B \sum_i S_i$$

Work when

$T = T_c \approx 2.269$; B :

$0 \rightarrow B_{\max} / B_{\max} \rightarrow 0$

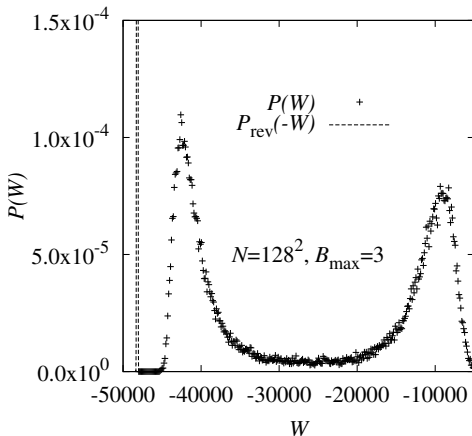
$W = -\Delta B \sum_i S_i$

Crooks (1998):

$W = \Delta F$ at intersection

NOT visible!

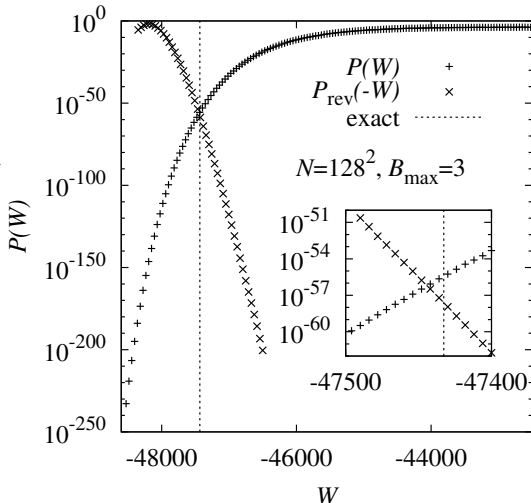
→ use large-deviation approach



Results

Ising model
 $L \times L = 128^2$
 $B : 0 \leftrightarrow 3$

Distribution of work
and reverse work



Curves intersect at $W^* \approx -47443$

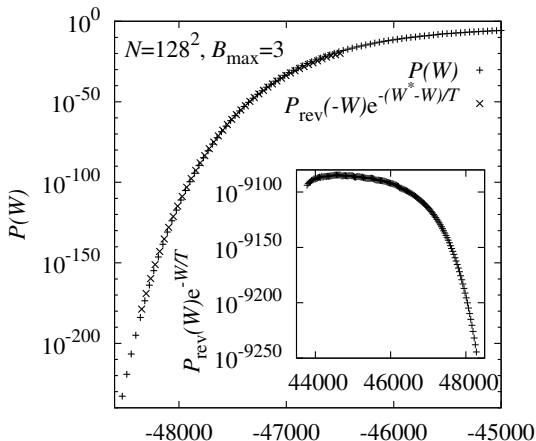
Exact result: $\Delta F = -47433$

[AKH, PRE 2014]

Test of Crooks theorem

$$\frac{P(W)}{P_{\text{rev}}(-W)} = e^{(W-\Delta F)/T}$$

Fits well!



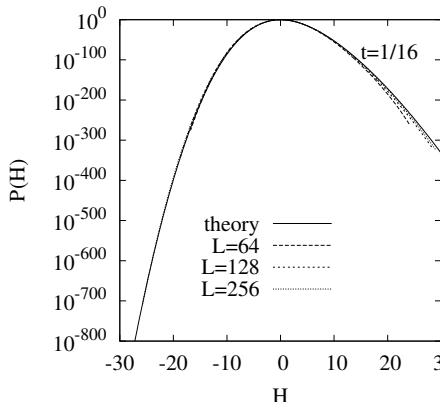
Inset: integrand $e^{-W/T}P(W) \rightarrow \Delta F \approx -47438$

Further results: $\Delta F(B_{\text{max}})$

[AKH, PRE 2014]

$P(H)$ for $d = 1 + 1$ KPZ equation

Distr. $P(H)$ of heights ($t \ll 1$), mapping to directed polymers



“small” prob’s reachable

$H < 0$: very good
agreement $\forall L$
(800 decades!)

$H > 0$:
convergence to analytic
result

[AKH, Le Doussal, Majumdar, Rosso,
& Schehr, Europhys. Lett. 2018]

Other initial conditions [AKH, Krajenbrink, & Le Doussal, PRE 2020]

Shape of “optimal” (dominating) height profiles

[AKH, Meerson, Sasorov, Phys. Rev. Res. 2019] [AKH, Meerson, Sasorov, PRE 2021]