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INCLUSIVE OBSERVABLES AND HARD GLUON EMISSION IN NEUTRINO DEEP INELASTIC SCATTERING

C. BOUCHIAT, Ph. MEYER and M. MEZARD

Laboratoire de Physique Théorique de l'Ecole Normale Supérieure, Paris, France*

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We derive the predictions of perturbative QCD together with non-perturbative corrections for a set of inclusive observables connected with the angular distribution of light-cone energy in deep inelastic neutrino scattering. Our particular choice of observables has been made in order to meet important physical requirements besides the necessary condition of infrared regularity. Our inclusive observables receive their dominant contribution from the quark fragmentation region. The non-perturbative contribution is calculable in a rather model-independent way and stays at an acceptable level in realistic experimental conditions. The QCD perturbative contribution, which takes the simple form of a convolution product, exhibits a strongly decreasing behaviour as a function of the Bjorken scaling variable x, superimposed on a constant background associated with the non-perturbative terms, allowing a rather clean separation of the two effects. The perturbative term being dominated by the process of hard-gluon emission, an experimental investigation of the observables discussed here may be a good way to detect the effect of gluon emission in deep inelastic neutrino scattering.

1. Introduction

There is a widespread belief that quantum chromodynamics (QCD) is the field theoretical description of strong interactions. At the present stage of the development of the theory clear predictions are possible only within the context of the so-called "improved perturbation theory". The gap between perturbative QCD and observable physical processes is then bridged with the help of certain plausible assumptions concerning the long-distance behaviour of the theory, which is involved whenever real hadrons enter the picture. On this basis the "jet" structure of e^+e^- collisions and deep inelastic scattering of leptons has been extensively studied [1]. In this general context we have derived the predictions of perturbative QCD together with non-perturbative corrections for a set of inclusive observables which are connected to the angular distribution of the "light-cone" final hadronic energy in deep inelastic neutrino scattering [2]. The definition of our observables involves the choice of a jet axis. Unlike the case of thrust, spherocity [3], etc., where

* Laboratoire Propre du CNRS, associé à l'Ecole Normale Supérieure et à l'Université de Paris-Sud. Postal address: 24 rue Lhomond, 75231 Paris Cedex 05, France.

the jet axis is reconstructed from the observed data, our axis is taken once and for all to be the direction of the virtual intermediate boson.

In sect. 2 we define and partly justify our choice of inclusive observables. Besides the necessary condition of infrared regularity we have tried—and we believe with some success—to meet other important physical requirements. Our inclusive observables should receive their main contribution from the fragmentation of the quark having interacted directly with the virtual intermediate boson; the non-perturbative contribution associated with low transverse momentum processes should be kept at an acceptable level, computable in a model-independent way and be easily separable from the QCD contribution. In sect. 4 we show that the non-perturbative effects are completely described by a single physical parameter and can be written in such a way that they do not depend on the x Bjorken scaling variable.

The QCD perturbative computations are presented in sect. 3. For simplicity we consider only the parity-violating part of the neutrino cross section where the t-channel singlet contribution is absent. The results of the complete QCD calculation are given in sect. 5 and computational details can be found in the appendix.

The renormalization group equations can be used to justify the "improved perturbation" theory only if the infrared and mass singularities which one meets in the calculation have been eliminated. In sect. 3 we show explicitly, to order α_s, first that our observables are free of infrared singularities and secondly that the mass singularities can be factorized and reabsorbed in the experimental structure functions. Relying on the heuristic proofs of "factorization" we shall conjecture that these properties are valid to all orders in α_s [4]. With our particular choice of observables the QCD results can be written as a convolution product very similar in its mathematical form to the right-hand side of the Altarelli-Parisi equation [5]. As a consequence, consideration of the moments in the x Bjorken scaling variable gives QCD predictions which are normalized in the sense that the initial quark momentum distributions no longer appear in the result. However, since the whole range of x is difficult to explore experimentally, we have given, in sect. 5, a set of curves which allow the computation of our set of observables for any value of x. Some parametrization of the structure functions has to be chosen but the results, as they are given, are not very sensitive to small modifications of the parameters. We conclude the paper by comparing the perturbative and non-perturbative results under realistic experimental conditions. The discrimination between the two effects is easily done by looking at the x dependence. The QCD contribution will appear as a strongly decreasing function of x superimposed on a constant background of comparable magnitude associated with the non-perturbative effects. The QCD result being dominated by the process of hard gluon emission, an experimental study of the inclusive observables discussed in this paper may lead to a rather good identification of the gluon effects in neutrino-nucleon inelastic scattering.

2. The choice of observables

In order to use a QCD improved perturbation theory in the running coupling constant $\alpha(Q^2)$ we must deal with quantities which are free of infrared divergences [6]. Infrared free inclusive observables can be obtained by taking polynomials of quantities X having the general form [7]

$$X=\sum_{i=0}^{N} |\boldsymbol{p}_i| f(\hat{p}_i)\,, \tag{1}$$

where $\hat{p}_i$ is the unit vector along the momentum $\boldsymbol{p}_i$ of the ith particle (here assumed to be massless) in the final state. There is obviously a great arbitrariness in the definition of X, especially in the case of lepton-hadron inelastic scattering where there is no obvious choice for the reference frame in which the $\boldsymbol{p}_i$ are measured.

A very convenient variable to characterize the direction of the emission of a particle is the transverse light-cone velocity $\boldsymbol{v}$ defined by

$$\boldsymbol{v}=\boldsymbol{p}_{\mathrm{T}}/p_+\,, \tag{2}$$

where $\boldsymbol{p}_{\mathrm{T}}$ is the transverse momentum with respect to the direction of the virtual boson momentum $\boldsymbol{q}$ (the x_3 axis is taken along $\boldsymbol{q}$) and p_+ is the usual light-cone variable, $p_\pm = p_0 \pm p_3$.

The transformation properties of $\boldsymbol{v}$ with respect to Lorentz boosts are particularly simple: for boosts along any direction in the x_1-x_2 plane $\boldsymbol{v}=(v_1,\ v_2)$ transforms like a galilean velocity ($\boldsymbol{v}\to\boldsymbol{v}'=\boldsymbol{v}+\boldsymbol{a}$); whereas boosts along the x_3 axis reduce to a scale transformation ($\boldsymbol{v}\to\boldsymbol{v}'=\lambda\boldsymbol{v}$).

In the case of zero mass (or in the high-energy limit) the two-vector $\boldsymbol{v}$ has a specially simple expression:

$$\begin{aligned} v_1 &= \cos\varphi\tan\tfrac{1}{2}\theta,\\ v_2 &= \sin\varphi\tan\tfrac{1}{2}\theta, \end{aligned} \tag{3}$$

where θ and φ are the polar and azimuthal angles respectively of the momentum $\boldsymbol{p}$. This leads to a simple geometrical interpretation of the two-vector $\boldsymbol{v}$: the point of coordinates $(v_1, v_2, 0)$ is nothing but the stereographic projection on the $x_3=0$ plane of the point M of the unit sphere such that $\mathbf{OM}=\hat{p}$.

We shall choose, as reference frame where the observable X is defined, the so-called "Breit frame" of the lepton-nucleon system, i.e., the frame in which the momentum of the virtual vector boson has no time component: $q=(0, 0, 0, Q)$. In particular, the transverse velocity $\boldsymbol{v}$ defined above is assumed throughout the paper to be measured in the Breit frame. This frame is one of the favourite ones for parton model calculations but, as we shall see later, we have further arguments for such a choice.

In the case of lepton-hadron inelastic scattering the inclusive observable X should meet another important physical requirement: it should receive its dominant contribution from particles produced by the fragmentation of the struck quark. In the Breit frame the final momentum of the struck quark (in the parton model) has the components $(\frac{1}{2}Q,0,0,\frac{1}{2}Q)$ while those of the total momentum of the spectator quarks are $(Q(1-x)/2x,0,0,-Q(1-x)/2x)$ with x the usual scaling variable. The particles associated with the fragments of the struck quark (the target) have longitudinal momenta large and positive (negative). A convenient choice for the energy variable entering into X appears to be the light-cone variable z defined by

$$z_i = p_{+i}/\mathcal{P}_+ , \tag{4}$$

where $\mathcal{P}$ is the total energy-momentum of the final hadron system. In the Breit frame $\mathcal{P}_+ = Q \simeq p'_+$, where p' is the final momentum of the struck quark. For a particle coming from the fragmentation of the struck quark with $p_{3,i} = \eta_i \frac{1}{2} Q$ $(0 < \eta_i < 1)$ we have $z_i \simeq \eta_i \ (1 + \mathrm{O}(m_\mathrm{T}^2/Q^2))$ (where we have introduced the transverse square mass $m_\mathrm{T}^2 = p_\mathrm{T}^2 + m^2$). On the other hand the variable z_i associated with a target fragment of $p_{3,j} = -\zeta_j Q(1-x)/2x$ $(0 < \zeta_j < 1)$ is of the order of $(x/[\zeta_j(1-x)])m_\mathrm{T}^2/Q^2$. It is then clear that an inclusive observable of the form

$$X = \sum_{i=1}^{N} z_i f(\boldsymbol{v}_i) \tag{5}$$

should receive its main contribution from the fragmentation of the struck quark. A further enhancement of this effect will be obtained by requiring that the transverse velocities $\boldsymbol{v}_i$ measured in the Breit frame should lie inside the unit circle $0 < |\boldsymbol{v}_i| < 1$. In the zero-mass limit this cut simply means that we restrict ourselves to particles emitted in the forward hemisphere of the Breit frame: $0 \leqslant \theta_\mathrm{B} \leqslant \frac{1}{2}\pi$. We are then led to consider the infinite set of inclusive observables

$$X_n = \sum_{i=1}^{N} z_i |\boldsymbol{v}_i|^n , \tag{6}$$

where n is a positive integer and $0 \leqslant |\boldsymbol{v}_i| \leqslant 1$. As we shall see later this last condition on $|\boldsymbol{v}_i|$ ensures that the average value $\langle X_n \rangle$ exists for all $n \geqslant 1$, which would not be the case if no cut-off were imposed on the $|\boldsymbol{v}_i|$.

As an intermediate step in the evaluation of $\langle X_n \rangle$ it is convenient to introduce a quantity Σ, similar to the antenna pattern, which is the angular distribution of hadronic energy (more precisely of the light-cone energy $\mathcal{P}_+$). Focussing now on neutrino (antineutrino) reactions, we define:

$$\frac{d\Sigma^{\nu,\bar\nu}}{dx\,dy\,d^2v} = \left(\frac{d\sigma^{\nu,\bar\nu}}{dx\,dy}\right)\left\langle \sum_{i=1}^{N} z_i\delta(v - v_i)\right\rangle^{\nu,\bar\nu}$$

$$= \sum_{\{h\}}\int\left(\prod_{i=1}^{N} d^2p_{Ti}\,dp_i^+\right)\frac{d\sigma^{\nu,\bar\nu}}{dx\,dy\,d^2p_{Ti}\,dp_{Ti}^+\ldots}\sum_{i=1}^{N} z_i\delta(v-v_i), \quad (7)$$

where $\Sigma_{\{h\}}$ is extended over all possible final hadronic states, and $d\sigma^{\nu,\bar\nu}/dx\,dy$ is the total neutrino (antineutrino) cross section, x and y have their usual definitions $x = Q^2/(2P\cdot q)$; $y = (P\cdot q)/(P\cdot l)$. Once the antenna pattern Σ is calculated it is an easy matter to get the "average" of the X_n defined in (6):

$$\frac{d\sigma^{\nu,\bar\nu}}{dx\,dy}\langle X_n\rangle^{\nu,\bar\nu} = \int_{|v|<1}\frac{d\Sigma^{\nu,\bar\nu}}{dx\,dy\,d^2v}|v|^n\,d^2v\,. \quad (8)$$

In this paper we are not interested in the energy distribution in the azimuthal variable φ so it will be sufficient to calculate the angular distribution (7) averaged over φ. Using rotational invariance around the direction of the virtual intermediate boson (here the x_3 axis) it can be shown that the angular distribution of hadronic energy averaged over φ has the same tensorial decomposition and hence the same kinematic factors as the total cross section. To be specific we can write

$$\frac{d\Sigma^{\nu,\bar\nu}}{dx\,dy\,2\pi v\,dv} = G_F^2\frac{ME_\nu}{\pi}\left[xy^2\mathcal{F}_1^{\nu,\bar\nu}(x, Q^2, v) + (1-y)\mathcal{F}_2^{\nu,\bar\nu}(x, Q^2, v)\right.$$
$$\left.\pm xy\left(1-\tfrac{1}{2}y\right)\mathcal{F}_3^{\nu,\bar\nu}(x, Q^2, v)\right], \quad (9)$$

where the generalized structure functions $\mathcal{F}_i$ depend only on $v = |v|$ and are related to the ordinary structure functions by

$$F_i(x, Q^2) = \int d^2v\,\mathcal{F}_i(x, Q^2, v)\,, \quad (10)$$

Obviously the same tensorial properties are relevant in the computation of $\langle X_n\rangle$ and one can write, from eqs. (8) and (9),

$$\frac{d\sigma^{\nu,\bar\nu}}{dx\,dy}\langle X_n\rangle^{\nu,\bar\nu} = G_F^2\frac{ME_\nu}{\pi}\left[xy^2E_{n,1}^{\nu,\bar\nu}(x, Q^2) + (1-y)E_{n,2}^{\nu,\bar\nu}(x, Q^2)\right.$$
$$\left.\pm xy\left(1-\tfrac{1}{2}y\right)E_{n,3}^{\nu,\bar\nu}(x, Q^2)\right], \quad (11)$$

where

$$E_{n,i}^{\nu,\bar{\nu}}(x, Q^2) = \int_0^1 2\pi v \, \mathrm{d}v \, \mathcal{F}_i^{\nu,\bar{\nu}}(x, Q^2, v) v^n . \tag{12}$$

3. Perturbative QCD calculations of the generalized structure functions

As a first step in the perturbative calculation of the generalized form factors $\mathcal{F}_i(x, Q^2, v)$ we shall consider the problem of the scattering of a virtual vector boson of momentum q on a single quark of momentum p. In order to regularize the mass divergences the quark will be assumed to be slightly off-mass-shell with $p^2 = p_0^2 - \boldsymbol{p}^2 < 0$. It is convenient to introduce the quark Bjorken scaling variable x_p given by

$$x_p = \frac{Q^2}{2p\cdot q} , \qquad 0 \leqslant x_p \leqslant 1 . \tag{13}$$

The transverse momentum of the initial quark with respect to the direction of the virtual momentum is assumed to be zero. The effects of the transverse momentum distribution of the quark parton inside the nucleon, which are ignored here will be discussed later.

We shall here present the calculation for the parity-violating quark form factor $\mathcal{F}_{q3}(x,Q^2,v)$. The results relative to the other form factors are given in the appendix.

Let us begin with the lowest-order contribution to the generalized quark structure function. A straightforward calculation gives

$$\mathcal{F}_{q,3}^{(0)}(x_p,Q^2,v) = 2\delta(x_p - 1)\delta^2(v). \tag{14}$$

The details of the computation of the second-order inelastic contribution associated with the gluon emission are given in the appendix. One arrives at the relatively simple expression (in the Breit frame)

$$\mathcal{F}_{q3}^{\text{inel}}(x_p, Q^2, v) = \frac{4}{3}\frac{\alpha_s}{\pi^2}\left[\frac{1 + x_p^2}{x_p\left(v^2 + (1 - x_p)/x_p\right)\left[v^2 + x_p(1 - x_p)(-p^2/Q^2)\right]} - \frac{1 - x_p}{x_p^2}\frac{1 - 2x_p + 4x_p^2}{\left(v^2 + (1 - x_p)/x_p\right)^3}\right]. \tag{15}$$

If instead of the Breit frame one chooses another frame like the virtual boson-nucleon target center of mass, a more complicated expression is obtained, involving an explicit $x = Q^2/(2P\cdot q)$ dependence. If one goes to the limit $p^2/Q^2 \to 0$ in the above expression $\mathcal{F}_{q3}^{\rm ine}$ acquires a non-integrable singularity of $1/v^2$ type in the neighbourhood of $v = 0$.

In order to isolate the singularity we rewrite $\mathcal{F}_{q3}^{\rm ine}$ for a finite value of p^2/Q^2 as follows:

$$\mathcal{F}_{q3}^{\rm ine}(x_p, Q^2, \boldsymbol{v}) = \mathcal{F}_{q3}^{\rm ine}(x_p, Q^2, \boldsymbol{v})_\oplus + \delta^2(\boldsymbol{v}) \int d^2v \mathcal{F}_{q3}^{\rm ine}(x_p, Q^2, \boldsymbol{v}), \tag{16}$$

where the $\oplus$ operation associates, to a given function $f(\boldsymbol{v})$, the distribution $f(\boldsymbol{v})_\oplus$ defined by

$$\int f(\boldsymbol{v})_\oplus \varphi(\boldsymbol{v})\, d^2v = \int f(\boldsymbol{v})[\varphi(\boldsymbol{v}) - \varphi(0)]\, d^2v\,. \tag{17}$$

It is now easy to verify that $\mathcal{F}_{q3}^{\rm ine}(x_p, Q^2, \boldsymbol{v})_\oplus$ has a limit as a distribution in both variables $\boldsymbol{v}$ and x_p when p^2/Q^2 goes to zero. More precisely, the singularities which occur in $\mathcal{F}_{q3}^{\rm ine}(x_p, Q^2, \boldsymbol{v})_\oplus$ at $\boldsymbol{v} = 0$ and $x_p = +1$ are integrable. In this way the mass singularities have been transferred to the term proportional to $\delta^2(\boldsymbol{v})$:

$$\mathcal{F}_{q3}^{\rm ine}(x_p, Q^2, \boldsymbol{v}) = \mathcal{F}_{q3}^{\rm ine}(x_p, Q^2, \boldsymbol{v})_\oplus + \frac{4}{3}\frac{\alpha_s}{\pi}\left[-\frac{1 - 2x_p + 4x_p^2}{2(1-x_p)} + \frac{1+x_p^2}{1-x_p}\log\frac{Q^2}{x_p^2(-p^2)} \right]\delta^2(\boldsymbol{v})\,. \tag{18}$$

Beside the mass singularity $\log Q^2/(-p^2)$ which is associated to the emission of gluons with momentum collinear to the initial quark momentum, there is also an infrared singularity at $x_p = 1$. A simple way to regularize this singularity is to give a small mass $p'^2 > 0$ to the final quark. The pole term $1/(1 - x_p)$ is then replaced by $1/(1-x_p)_+ + \delta(1-x_p)\log(Q^2/p'^2)$, where the distribution $1/(1-x)_+$ is defined by

$$\int_0^1 \frac{1}{(1-x)_+}\varphi(x)\, dx = \int_0^1 \frac{1}{1-x}[\varphi(x) - \varphi(1)]\, dx\,.$$

The radiative corrections to the lowest-order elastic contribution contain terms of the form $\log(Q^2/p'^2)\delta^2(\boldsymbol{v})$ which, as expected, cancel exactly those occurring in $\mathcal{F}_{q3}^{\rm ine}$.

One finally arrives at the second-order perturbative expression of the generalized quark form factor

$$\mathcal{F}_{q3}(x_p, Q^2, v) = \delta^2(v) F_{q3}(x_p, Q^2) + \mathcal{F}_{q3}(x_p, Q^2, v)_{\oplus}, \tag{19}$$

$$F_{q3}(x_p, Q^2) = 2\left[\delta(1-x_p) + \frac{4}{3}\frac{\alpha_s}{2\pi} P_{qq}(x_p) \log\frac{Q^2}{-p^2}\right],$$

$$P_{qq}(x_p) = \frac{1+x_p^2}{(1-x_p)_+} + \frac{3}{2}\delta(1-x_p); \tag{20}$$

$$\mathcal{F}_{q3}(x_p, Q^2, v)_{\oplus} = \frac{4}{3}\frac{\alpha_s}{2\pi} 2\left\{\frac{1+x_p^2}{\pi x_p}\left[\frac{1}{v^2\left(v^2 + (1-x_p)/x_p\right)}\right]_{\oplus} - \frac{(1-x_p)(1-2x_p+4x_p^2)}{\pi x_p^2} \times \left[\frac{1}{\left(v^2 + (1-x_p)/x_p\right)^3}\right]_{\oplus}\right\}. \tag{21}$$

In the expression for F_{q3} we have only kept the log dominant terms. In order to obtain the generalized form factor $\mathcal{F}_3(x, Q^2, v)$ relative to a nucleon target we have to introduce a quark (antiquark) momentum distribution $q(\bar{q})$ inside the nucleon

$$q(\xi, \boldsymbol{p}_T)\frac{d\xi}{\xi} d^2 p_T,$$

where $\xi = p_+/\mathcal{P}_+$. (In the Breit frame ξ can be identified to $p_3/\mathcal{P}_3$ up to a correction of the order of $1/Q^2$). If we neglect the transverse momentum distribution of the quark parton inside the nucleon, we may write

$$q(\xi, \boldsymbol{p}_T) = q(\xi)\delta^2(\boldsymbol{p}_T).$$

Using the relation $x_p = x/\xi$ and remembering that a $1/\xi$ factor has to be introduced in order to account for the different flux factors for a quark and nucleon target, we get

$$\mathcal{F}_3(x, Q^2, v) = \tfrac{1}{2}\int_x^1 \frac{d\xi}{\xi^2}[q(\xi) - \bar{q}(\xi)]\mathcal{F}_{q3}\left(\frac{x}{\xi}, Q^2, v\right), \tag{22}$$

where $\mathscr{F}_{q3}(x, Q^2, v)$ is given by (19)–(21). Note that, as in the case of the ordinary inelastic form factors $F_i(x, Q^2)$, we have a simple convolution product. The above formula is meant to give the average between neutrino and antineutrino for a target of arbitrary isospin with $q(x) = u(x) + d(x) + s(x) + c(x)$ and a corresponding expression for $\bar{q}(x)$.

Let us assume that the strong coupling constant α_s appearing in (20) has been defined at some value Q_0^2 such that $\alpha_s = \bar{\alpha}_s(Q_0^2)(\bar{\alpha}_s(Q^2)$ is the "running" coupling constant of the renormalization group equation). We now introduce a new quark distribution function:

$$\tilde{q}(\xi) = q(\xi) + \frac{4}{3}\frac{\alpha_s}{2\pi}\log\frac{Q_0^2}{-p^2}\int_\xi^1 \frac{d\xi'}{\xi'} q(\xi') P_{qq}\left(\frac{\xi}{\xi'}\right). \tag{23}$$

To the leading log approximation, the renormalization group equations for $F_3(x,Q^2)$ can be written as the following integral equation [5]:

$$F_3(x,Q^2) = F_3(x,Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{4}{3}\frac{\bar{\alpha}_s(t)}{2\pi}\frac{dt}{t}\int_x^1 \frac{d\xi}{\xi} F_3(\xi,t) P_{qq}\left(\frac{x}{\xi}\right). \tag{24}$$

The first-order expansion in α_s reads:

$$F_3(x,Q^2) = F_3(x,Q_0^2) + \frac{4}{3}\frac{\alpha_s}{2\pi}\log\frac{Q^2}{Q_0^2}\int_x^1 \frac{d\xi}{\xi} F_3(\xi,Q_0^2) P_{qq}\left(\frac{x}{\xi}\right) + O(\alpha_s^2). \tag{25}$$

This expression coincides with the coefficient of the term proportional to $\delta^2(v)$ in (23) provided $F_3(x, Q_0^2)$ is defined as

$$xF_3(x, Q_0^2) = \tilde{q}(x) - \tilde{\bar{q}}(x)\,.$$

We have just verified at the one-loop level that the mass singularities which are present in the generalized form factor $\mathscr{F}_3(x, Q^2, v)$ can be eliminated by a proper redefinition of the quark distribution function. We have here an example of the factorization properties of the mass singularities: heuristic arguments in favour of the validity of this result to all orders in α_s have been given by several authors [4] for very general situations which include the one considered in this paper. The final answer within the leading log approximation can be written as follows:

$$\mathscr{F}_3(x, Q^2, v) = \delta^2(v) F_3(x, Q^2) + \tfrac{1}{2}\int_x^1 \frac{d\xi}{\xi} F_3(\xi, Q^2)\mathscr{F}_{q3}\left(\frac{x}{\xi}, Q^2, v\right)_{\oplus}, \tag{26}$$

In this expression $\mathscr{F}_{q3}(x/\xi, Q^2, v)_{\oplus}$ is given by formula (21) with α_s replaced by the running coupling constant $\alpha_s(Q^2)$. In (26) the first term is the "one-jet" contribution, the second one is the "two-jet" contribution which has been separated by the $\oplus$ operation. Note that the choice of the Breit frame for the measurement of the transverse velocity v was essential in order to arrive at a final answer having the form of a convolution product.

As explained in sect. 1 the average value of the inclusive observable X_n is given in terms of the truncated moments of the generalized form factors $E_{n,i}(x, Q^2)$:

$$E_{n,i} = \int_0^1 2\pi v \, dv v^n \mathcal{F}_i(x, Q^2, v) . \tag{27}$$

Let us give here the explicit expression of $E_{n,3}$:

$$E_{n,3} = \frac{4}{3} \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{du}{u^2} F_3\left(\frac{x}{u}\right)\left[(1+u^2)I_n(u) - \frac{1-u}{u}(1-2u+4u^2)J_n(u)\right]. \tag{28}$$

The functions $I_n(u)$ and $J_n(u)$ are given by the integrals

$$I_n(u) = \int_0^1 \frac{2v^{n-1}\, dv}{v^2 + (1-u)/u} ,$$

$$J_n(u) = \int_0^1 \frac{2v^{n+1}\, dv}{\left[v^2 + (1-u)/u\right]^3} . \tag{29}$$

In the appendix simple recurrence laws are derived which allow a complete determination of $I_n(u)$ and $J_n(u)$ in terms of simple functions. We quote the results here for $n = 2$:

$$I_2(u) = \log \frac{1}{1-u} , \qquad J_2(u) = \frac{1}{2} \frac{u^3}{1-u} . \tag{30}$$

The complete expressions of the v moments $E_{n,i}(x)$ are given in the appendix in terms of the quark and gluon distribution functions.

4. Non-perturbative effects

In sect. 3 the generalized moments $\mathcal{F}_i(x, Q^2, v)$ were computed as if the quarks and gluons, and not the real hadrons, were the observed particles in the final state. In particular, even if the transverse momentum of the initial quark is neglected, the actual $\mathcal{F}_i(x, Q^2, v)$ should not contain a term proportional to $\delta^2(v)$. In order to make contact with the true hadronic world the high-p_T cross sections involving quarks and gluons obtained by the perturbative QCD calculations have to be folded with the corresponding fragmentation functions. More precisely, let

$$\frac{\mathrm{d}\sigma_{\mathrm{q}}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}^2p_{\mathrm{T}}\,\mathrm{d}p_+/p_+}=R_{\mathrm{q}}\left(x,\,y,\,Q^2,\,\frac{p_+}{\mathcal{P}_+},\,\boldsymbol{p}_{\mathrm{T}}\right)$$

be the differential cross section for producing a quark having a given value of p_+ and $\boldsymbol{p}_{\mathrm{T}}$ and R_{g} be the similar quantity for a gluon. The cross section for producing a hadron of given p_+ and $\boldsymbol{p}_{\mathrm{T}}$ can be written as

$$\frac{\mathrm{d}\sigma^{\mathrm{h}}}{\mathrm{d}x\mathrm{d}y\mathrm{d}^2p_{\mathrm{T}}\mathrm{d}p_+/p_+}\equiv R_{\mathrm{h}}\left(x,y,Q^2,\frac{p_+}{\mathcal{P}_+},\boldsymbol{p}_{\mathrm{T}}\right)$$

$$=\int\mathrm{d}^2p'_{\mathrm{T}}\frac{\mathrm{d}p'_+}{p'_+}\left[R_{\mathrm{q}}\left(x,y,Q^2,\frac{p'_+}{\mathcal{P}_+},\boldsymbol{p}'_{\mathrm{T}}\right)\right.$$

$$\left.\times F_{\mathrm{q}\to\mathrm{h}}(p'_+,\boldsymbol{p}'_{\mathrm{T}};p_+,\boldsymbol{p}_{\mathrm{T}})+\mathrm{q}\to\mathrm{g}\right],\qquad(31)$$

where $F_{\mathrm{q}\to\mathrm{h}}$ and $F_{\mathrm{g}\to\mathrm{h}}$ are the fragmentation functions of the quarks and the gluons respectively. Using the invariance under longitudinal and transverse Lorentz boosts they can be written in the following reduced forms:

$$F_{\substack{\mathrm{q}\to\mathrm{h}\\(\mathrm{g}\to\mathrm{h})}}(p'_+,\,\boldsymbol{p}'_{\mathrm{T}};\,p_+,\,\boldsymbol{p}_{\mathrm{T}})=f_{\substack{\mathrm{q}\to\mathrm{h}\\(\mathrm{g}\to\mathrm{h})}}\left(\frac{z}{z'},\,\boldsymbol{p}_{\mathrm{T}}-\frac{z}{z'}\boldsymbol{p}'_{\mathrm{T}}\right),\qquad(32)$$

where $z=p_+/\mathcal{P}_+$ and $z'=p'_+/\mathcal{P}_+$.

The light-cone antenna pattern $\mathrm{d}\Sigma/\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}^2v$ defined in eq. (7) is given by:

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}^2v}=\sum_{\mathrm{h}}\int R_{\mathrm{h}}(x,\,y,\,Q^2,\,z,\,\boldsymbol{p}_{\mathrm{T}})\delta\left(v-\frac{\boldsymbol{p}_{\mathrm{T}}}{p_+}\right)\mathrm{d}z\,\mathrm{d}^2p_{\mathrm{T}}\,.$$

Using (31) and (32) and performing a change of variables one gets the following expression:

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}x\mathrm{d}y\mathrm{d}^2v}=\sum_{\mathrm{h}}\int\mathrm{d}^2\pi_{\mathrm{T}}\mathrm{d}\eta f_{\mathrm{q}\to\mathrm{h}}(\eta,\boldsymbol{\pi}_{\mathrm{T}})$$

$$\times\int\mathrm{d}^2p'_{\mathrm{T}}\mathrm{d}z'\delta\left(v-\frac{\boldsymbol{p}'_{\mathrm{T}}+(1/\eta)\boldsymbol{\pi}_{\mathrm{T}}}{p'_+}\right)R_{\mathrm{q}}(x,y,Q^2,z',\boldsymbol{p}'_{\mathrm{T}})$$

$$+\,\mathrm{q}\to\mathrm{g}.\qquad(33)$$

In the above expression the fragmentation function is supposed to describe, in a light-cone quantized version of QCD, all "soft" processes where a quark is converted into a given hadron h plus an arbitrary number of hadrons. In a typical "hard" process described by R_q, the transverse momentum $|\boldsymbol{p}'_T|$ is of the order Q while $|\boldsymbol{\pi}_T|$ is of the order of the transverse momentum p_T^{NP} characteristic of non-perturbative "soft" processes. In the δ function we can set $\boldsymbol{\pi}_T = 0$ except for the small values of η such that $0 < \eta \lesssim p_T^{NP}/Q$. With our normalization the functions $f_{q\to h}(\eta, \boldsymbol{\pi}_T)$ are well behaved near $\eta = 0$ and obey the p_+ momentum conservation sum rule:

$$\int \sum_h f_{q\to h}(\eta, \boldsymbol{\pi}_T) d\eta d^2\pi_T = 1. \tag{34}$$

If in the quark production cross section R_q we subtract the naive quark model contribution [first term in the right-hand side of (26)], dropping the $\boldsymbol{\pi}_T/\eta$ term will introduce an error of the order of $\alpha_s(Q^2)p_T^{NP}/Q$. To $\alpha_s(Q^2)$ order, expression (33) then reduces to

$$\frac{d\Sigma}{dx dy d^2 v} = \int d^2 p'_T dz' \delta\left(v - \frac{\boldsymbol{p}'_T}{p'_+} \right) R_q(x, y, Q^2, z', p'_T) + q \to g, \tag{35}$$

which is nothing but the light-cone antenna pattern for quark and gluon final states. The dominant non-perturbative contribution—of the order of p_T^{NP}/Q—will be obtained when R_q is taken to be given by the naive parton model; if for a moment we ignore the transverse momentum of the initial quark, R_q is proportional to $\delta(z'-1)\delta^2(p'_T)$. The dominant non-perturbative contribution to the generalized form factor $\mathcal{F}_i^{NP}(x, Q^2, \boldsymbol{v})$ is then given by

$$\mathcal{F}_i^{NP}(x, Q^2, \boldsymbol{v}) = F_i(x, Q^2) \sum_h \int f_{q\to h}(z, |\boldsymbol{p}_T|) \delta\left(\boldsymbol{v} - \frac{\boldsymbol{p}_T}{zQ} \right) dz d^2 p_T. \tag{36}$$

We recall that $\boldsymbol{v}$ has to be measured in the Breit frame where $p'_+ = z'\mathcal{P}_+ = \mathcal{P}_+ = Q$.

The non-perturbative contribution to the mean value of the inclusive observable X_n [see formula (11)] can be expressed in terms of the truncated v-moments $E_{ni}^{NP}(x)$:

$$\frac{E_{n,i}^{NP}}{F_i(x)} = \int_0^1 \frac{2\pi v^{n+1} dv \mathcal{F}_i^{NP}(x, Q^2, v)}{F_i(x)}$$

$$= \int d^2 p_T \left(\frac{p_T}{Q} \right)^n \int_{p_T/Q}^1 z^{-n} f(z, p_T) dz. \tag{37}$$

We have introduced the total quark fragmentation function $f(z, p_T)$ defined by

$$f(z, p_T) = \sum_h f_{q\to h}(z, p_T) . \tag{38}$$

It is important to notice that without the condition $|v| \leqslant 1$, the lower limit of the z integration would have been zero instead of p_T/Q, leading to a badly divergent expression.

It is convenient to introduce the average transverse momentum distribution at a given z

$$\langle p_T(z)\rangle = \frac{\int f(z, p_T) p_T \, d^2 p_T}{\int f(z, p_T) \, d^2 p_T} . \tag{39}$$

Experiments seem to indicate that $\langle p_T(z)\rangle$ is a slowly varying function of z. As a definition of p_T^{NP} we shall take

$$p_T^{NP} = \int f(z, p_T) p_T \, d^2 p_T \, dz . \tag{40}$$

(Remember the sum rule: $\int f(z, p_T) \, d^2 p_T \, dz = 1$.)

Performing integration by parts in the right-hand side of (37) one easily gets the following asymptotic formulas valid in the limit $Q \to \infty$:

$$n \geqslant 2: \frac{E_{n,i}^{NP}}{F_i(x)} = \frac{1}{(n-1)Q} \lim_{z\to 0} (\langle p_T(z)\rangle z D_q(z)) \left[1 + O\left(\frac{p_T^{NP}}{Q} \log \frac{p_T^{NP}}{Q}\right)\right], \tag{41}$$

$$n = 1: \frac{E_{1,i}^{NP}}{F_i(x)} = \frac{1}{Q} \lim_{z\to 0} \left\{ \log\left(\frac{Q}{\langle p_T(z)\rangle}\right) \langle p_T(z)\rangle z D_q(z) \right\}$$

$$\times \left[1 + O\left(\frac{1}{\log(p_T^{NP}/Q)}\right)\right]. \tag{42}$$

In the above expression $D_q(z)$ is the p_T-integrated total fragmentation function:

$$z D_q(z) = \sum_h z D_q^h(z) = \int f(z, p_T) d^2 p_T .$$

An evaluation of $zD_q(z)|_{z=0}$ has been obtained in two ways.

(a) We have used the new quark-jet parametrization of Field and Feynman [8] which is adjusted to fit the experimental results on the total charged hadron fragmentation function: $\sum_{h^\pm} z D_q^h(z)$.

(b) The charged hadron part of $zD_q(z)|_{z=0}$ is taken directly from experiment [9]. The neutral contribution is deduced from the charged one using the $z=0$ limit of the $K^{\pm}/\pi^{\pm}$ ratio together with the "plateau universality" hypothesis:

$$\lim_{z\to 0}\left[\frac{D_q^{K^+}(z)}{D_q^{K^0}(z)}=\frac{D_q^{\bar{K}^0}(z)}{D_q^{K^-}(z)}\right]=1\,.$$

By these methods one gets values for $zD_q(z)|_{z=0}$, which are equal for u and d quarks and scatter around 3 with a dispersion of 10% depending on the adopted values of the $K^{\pm}/\pi^{\pm}$ ratio.

The Field and Feynman model predicts—in agreement with experimental observation [10]—that $\langle p_T(z)\rangle$ is a decreasing function of z with $\langle p_T(z)\rangle|_{z=0}\approx$ 0.3 GeV/c.

We finally arrive at the following numerical results for the non-perturbative contribution to the truncated v-moments $E_{n,i}^{NP}(x)$:

$$n\geqslant 2:\ \frac{E_{n,i}^{NP}(x)}{F_i(x)}\simeq\frac{1}{n-1}\,\frac{0.9}{Q(\mathrm{GeV}/c)}\,,$$

$$n=1:\ \frac{E_{1,i}^{NP}(x)}{F_i(x)}\simeq\frac{0.9}{Q(\mathrm{GeV}/c)}\log\frac{Q(\mathrm{GeV}/c)}{0.3}\,. \tag{43}$$

Let us mention that it can be easily proved that for $n\geqslant 2$ the above asymptotic limits are reached from below.

We would now like to discuss the effect of the transverse momentum distribution of the initial quark. Let $\boldsymbol{p}_{T_0}$ be the transverse momentum of the initial quark, the invariance under transverse Lorentz boosts [see eq. (32)] implies that in formula (37) $f(z,p_T)$ has to be replaced by $f(z,|\boldsymbol{p}_T-z\boldsymbol{p}_{T_0}|)$. From the fact that $E_{n,i}^{NP}(x)/F_i(x)$ is of the order of $[1/(n-1)]\langle p_T\rangle/Q$ for any $n\geqslant 2$, one deduces that the dominant contribution is coming from values of z of the order of p_T^{NP}/Q. Since $|\boldsymbol{p}_T-z\boldsymbol{p}_{T_0}|$ and $|\boldsymbol{p}_{T_0}|$ are both of the order of p_T^{NP}, the correction to $E_{n,i}^{NP}(x)$ associated with the initial quark transverse momentum is at most of the order of $(p_T^{NP}/Q)^2$ and can then be neglected.

To end up this section we would like to point out a possible source of ambiguity connected with finite mass effects. In the QCD perturbative computation the masses of all the final-state particles are taken to be zero. The cut-off condition $|v|\leqslant 1$ is equivalent in the Breit frame to the angular condition $|\tan\frac{1}{2}\theta_B|\leqslant 1$. For massive particles the two conditions are not equivalent. The condition $|\tan\frac{1}{2}\theta_B|\leqslant$ 1 implies that the lower limit on the z integration in formula (37) is replaced by m_T/Q with $m_T^2=p_T^2+m^2$ where m^2 is the hadronic mass. In formulae (41) and (42) $\langle p_T(z)\rangle$ has to be replaced by: $\langle m_T(p_T/m_T)^n\rangle$, the variation with n is no longer given by $1/(n-1)$ and depends on the shape of the p_T (or m_T) distribution near $z=0$. Assuming a distribution of the form $\exp(-bm_T)$, we find that the correction factor

$$K_n = \frac{1}{\langle p_T \rangle} \left\langle m_T \left(\frac{p_T}{m_T} \right)^n \right\rangle ,$$

which is always smaller than one, does not deviate from unity by more than 15%.

5. Results and conclusions

In this section we shall give the results of the improved perturbative QCD calculation of the quantities $E_{n,i}^{\nu,\bar{\nu}}(x, Q^2)$ which allow the computation of the average values of the inclusive observables $\langle X_n \rangle$ defined by formulae (7) and (8). In sect. 2 only the parity-violating generalized structure function $\mathcal{F}_3(x, Q^2, v)$ was considered; the calculation of the other two ($i=1, 2$) is sketched in the appendix.

Although the explicit result for $i=3$ can be found in sect. 2, we give here, for completeness, the final expressions of $E_{n,i}^{\nu,\bar{\nu}}(x)$ for $i=1, 2, 3$.

Let

$$\hat{E}_{n,1}^{\nu,\bar{\nu}} = 2xE_{n,1}^{\nu,\bar{\nu}}(x) ,$$

$$\hat{E}_{n,2}^{\nu,\bar{\nu}} = E_{n,2}^{\nu,\bar{\nu}}(x) ,$$

$$\hat{E}_{n,3}^{\nu,\bar{\nu}} = xE_{n,3}^{\nu,\bar{\nu}}(x) .$$

Then

$$\hat{E}_{n,i}^{\nu,\bar{\nu}}(x) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \left\{ \left[q\left(\frac{x}{u}, Q^2\right) + \bar{q}\left(\frac{x}{u}, Q^2\right) \right] Q_{n,i}(u) + g\left(\frac{x}{u}, Q^2\right) G_{n,i}(u) \right\} \mathrm{d}u , \tag{44}$$

for $i=1, 2$,

$$\hat{E}_{n,3}^{\nu,\bar{\nu}}(x) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \left\{ \left[q\left(\frac{x}{u}, Q^2\right) - \bar{q}\left(\frac{x}{u}, Q^2\right) \right] Q_{n,3}(u) \right\} \mathrm{d}u . \tag{45}$$

$q(\xi, Q^2)$, $\bar{q}(\xi, Q^2)$, $g(\xi, Q^2)$ are respectively the total quark, antiquark, and gluon momentum distributions inside the nucleon.

The functions $Q_{n,i}(x)$ and $G_{n,i}(x)$ are simple linear combinations of the integrals $I_n(x)$ and $J_n(x)$ defined by formula (29):

$$Q_{n,1}(x)=\frac{4}{3}\left[\frac{1+x^2}{x}I_n(x)+\frac{(1-x)(1-2x-2x^2)}{x^2}J_n(x)\right], \tag{46}$$

$$Q_{n,2}(x)=\frac{4}{3}\left[\frac{1+x^2}{x}I_n(x)+\frac{(1-x)(1+2x-6x^2)}{x^2}J_n(x)\right], \tag{47}$$

$$Q_{n,3}(x)=\frac{4}{3}\left[\frac{1+x^2}{x}I_n(x)-\frac{(1-x)(1-2x+4x^2)}{x^2}J_n(x)\right], \tag{48}$$

$$G_{n,1}(x)=2\frac{1-x}{x}\left[x^2+(1-x)^2\right]\left[I_n(x)-2\frac{1-x}{x}J_n(x)\right], \tag{49}$$

$$G_{n,2}(x)=2\frac{1-x}{x}\left\{\left[x^2+(1-x)^2\right]I_n(x)-2\frac{1-x}{x}(1-6x+6x^2)J_n(x)\right\}. \tag{50}$$

Explicit expressions of the integral $I_n(x)$ and $J_n(x)$ in terms of elementary functions are given in the appendix.

Formulas (44) and (45) clearly show that the v moments are convolution products involving quark and gluon momentum distributions. By considering double moments both in v and x variables it is possible to factorize the quark and gluon distributions. The most interesting case is the parity-violating term ($i=3$). We define the double x-v moments $\mathcal{M}_{n,3}(N, Q^2)$ as follows:

$$\mathcal{M}_{n,3}(N, Q^2)=\frac{\int_0^1 x^{N-1}E_{n,3}(x, Q^2)\,\mathrm{d}x}{\int_0^1 x^{N-1}F_3(x, Q^2)\,\mathrm{d}x}$$

$$=\frac{\alpha_s(Q^2)}{2\pi}\int_0^1 x^{N-1}Q_{n,3}(x, Q^2)\,\mathrm{d}x\,. \tag{51}$$

From (48) one readily gets

$$\mathcal{M}_{n,3}(N,Q^2)=\frac{4}{3}\frac{\alpha_s(Q^2)}{2\pi}\int_0^1 \mathrm{d}x\,x^{N-1}$$

$$\times\left[\frac{1+x^2}{x}I_n(x)-\frac{(1-x)(1-2x+4x^2)}{x^2}J_n(x)\right]. \tag{52}$$

Using the explicit expressions of $I_n(x)$ and $J_n(x)$ given in the appendix, a straightforward calculation leads to the formulas

$$\mathcal{M}_{2,3}^{\rm QCD}(N,Q^2)=\frac{4}{3}\frac{\alpha_s(Q^2)}{2\pi}\left[\frac{1}{N-1}\sum_{q=1}^{q=N-1}\frac{1}{q}+\frac{1}{N+1}\sum_{q=1}^{q=N+1}\frac{1}{q}-\frac{1}{2}\left(\frac{1}{N+1}-\frac{2}{N+2}+\frac{4}{N+3}\right)\right], \qquad (53)$$

$N>1$,

$$\mathcal{M}_{4,3}^{\rm QCD}(N,Q^2)=\frac{4}{3}\frac{\alpha_s(Q^2)}{2\pi}\left[\frac{2}{N-1}-\frac{5}{2}\frac{1}{N}+\frac{11}{2}\frac{1}{N+1}-\frac{1}{N+2}-\frac{2}{N+3}-\frac{2}{N-2}\sum_{q=1}^{q=N-2}\frac{1}{q}+\frac{4}{N-1}\sum_{q=1}^{q=N-1}\frac{1}{q}-\frac{7}{N}\sum_{q=1}^{q=N}\frac{1}{q}+\frac{5}{N+1}\sum_{q=1}^{q=N+1}\frac{1}{q}\right]. \qquad (54)$$

The $N=1$ moments also exist and can be computed directly. Similar expressions could be obtained for $n=1$ and $n=3$. From the results of sect. 3 one immediately deduces that the non-perturbative contributions to the double x-v moments are independent of N:

$$\mathcal{M}_{n,3}^{\rm NP}(N,\,Q^2)\simeq\frac{1}{n-1}\,\frac{0.9}{Q({\rm GeV}/c)}\,, \qquad (55)$$

$n\geqslant 2$, although the double x-v moments $\mathcal{M}_{n,i}(N,\,Q^2)$ are quite interesting from a theoretical point of view since the QCD predictions are "normalized" in the sense that they do not depend on the initial quark and gluon distributions, their practical interest is limited by the difficulty of exploring the whole range of x experimentally.

The contact with experiment will be easier if we give the numerical values of the v-moments $E_{n,i}^{\nu,\bar\nu}(x)$ as a function of x. In order to do so we have to choose some analytic forms for the quark and gluon distribution functions. We shall limit ourselves to the case of an isoscalar target. Furthermore we shall neglect the effect of scaling violations on the structure functions $F_i(x)$ and use the CDHS [11] parton distributions which are averaged over the neutrino energy range: $30 < E_\nu <$ 200 GeV:

$$q(x) - \bar{q}(x) \propto \sqrt{x}\,(1-x)^{3.5}\,,$$

$$\bar{q}(x) \propto (1-x)^{6.5}\,,$$

$$g(x) \propto (1-x)^{5}\,.$$

We have fixed the normalization of the three distributions by the conditions

$$\frac{\int_0^1 \bar{q}(x)\,\mathrm{d}x}{\int_0^1 [q(x)+\bar{q}(x)]\,\mathrm{d}x} = 0.16\,, \tag{56}$$

$$\int_0^1 [q(x)+\bar{q}(x)]\,\mathrm{d}x = 1 - \int_0^1 g(x)\,\mathrm{d}x = 0.48\,. \tag{57}$$

A convenient way to present the results is to write, for $n \geqslant 1$,

$$\frac{E_{n,i}^{\mathrm{QCD}}(x)}{F_i(x)} = \frac{\alpha_s(Q^2)}{2\pi} f_{n,i}(x)\,. \tag{58}$$

We have verified that the functions $f_{n,i}(x)$ are not very sensitive to the choice of the parameters used in the CDHS fit. For instance, changing the exponent of $(1-x)$ in $q(x)-\bar{q}(x)$ from the value 3.5 to 3 modifies the $f_{n,i}(x)$ by less than 10%.

The three functions $f_{2,i}(x), f_{3,i}(x), f_{4,i}(x)$ are plotted together in figs. 1, 2, 3 for $i = 1,2,3$ respectively. It is interesting to note that the functions $(n-1)f_{n,i}(x)$, for a given i and $n = 2,3,4$, look very much alike. This suggests that for $n \geqslant 2$ everything happens as if the v probability distribution were of the form d^2v/v^3. For a given n the functions $f_{n,i}(x)$ almost coincide in the large x region where the antiquark and gluon distributions are negligible. A very important feature of all the functions $f_{n,i}(x)$ is their rapid decrease with x; they vanish in the limit $x \to 1$ where there is no phase space left for "hard" gluon emission. The values taken by the functions $f_{n,i}(x)$ for $n \geqslant 2$ look "normal" in the sense that they are of the order of unity. The case $n = 1$ which is displayed in fig. 4 looks more peculiar specially in the small x region where the $f_{1,i}(x)$ reach values of the order of ten. This suggests that, although no divergences are present, higher-order contributions in $\alpha_s(Q^2)$ are probably not negligible.

In order to get a feeling about what is to be expected in actual experimental conditions we have plotted, in fig. 5, the quantities

$$g_{n,i}(x, Q^2) = \frac{E_{n,i}^{\mathrm{QCD}}(x, Q^2) + E_{n,i}^{\mathrm{NP}}(x, Q^2)}{F_i(x, Q^2)}\,,$$

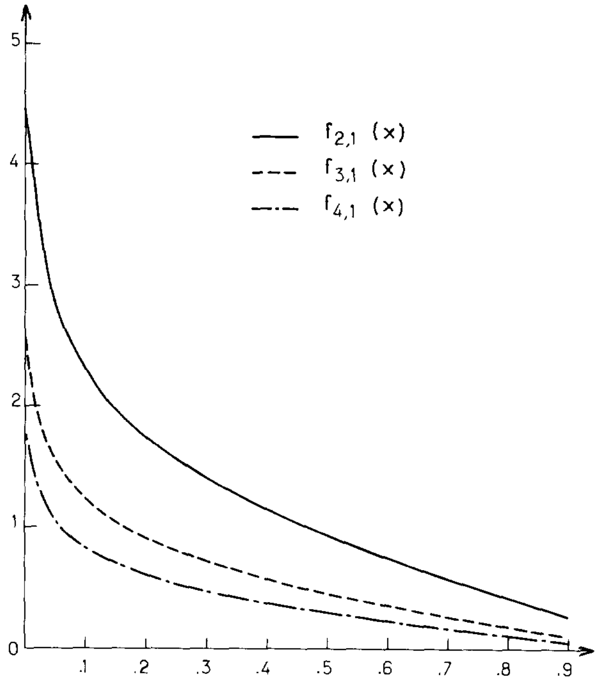

Fig. 1. QCD predictions for the $n = 2, 3, 4$ v moments of the generalized form factors $\mathcal{F}_i(x, Q^2, v)$

$$\frac{\int_0^1 \mathcal{F}_i(x, Q^2, v) 2\pi v^{n+1}\, \mathrm{d}v}{F_i(x, Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} f_{ni}(x)\,.$$

for $n = 2$ and $i = 1, 2, 3$. The value chosen for Q^2 is 30 $(\mathrm{GeV}/c)^2$. The energy scale parameter Λ which appears in the running coupling constant $\alpha_s(Q^2) = 12\pi/25 \log(Q^2/\Lambda^2)$ has been taken equal to 0.5 GeV/c. As already noted, the QCD effects, which are dominated by the gluon emission, exhibit a strong x-dependence in striking contrast with the constant background associated with the non-perturbative effects.

An experimental determination of the average values $\langle X_n \rangle$ looks feasible using neutrino events produced in a bubble chamber filled with neon or equipped with a track sensitive target (hydrogen target immersed in a neon bath allowing an identification of neutral particles). If a reasonable accuracy can be achieved, the x-dependence of the v moments will constitute a good signature for the production of gluons in neutrino-hadron inelastic scattering.

Appendix

We describe, here, the important steps in the derivation of the perturbative QCD results given in sect. 4. As explained before, we first calculate the antenna pattern

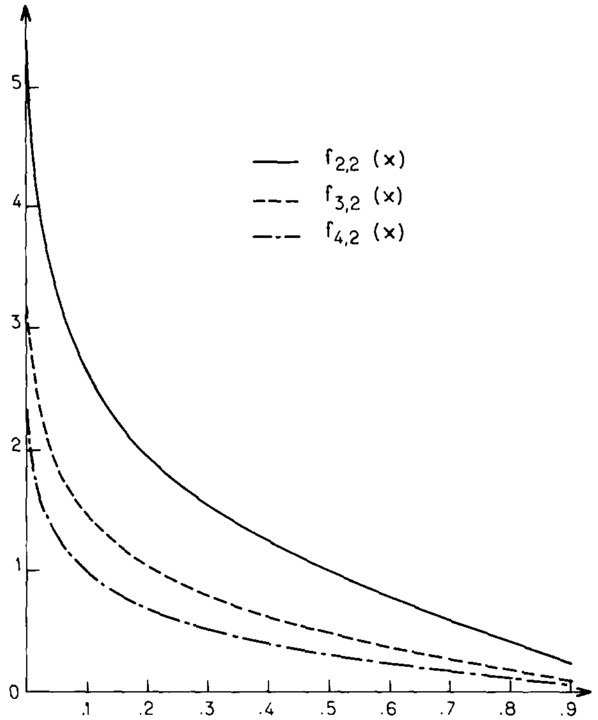

Fig. 2. See caption to fig. 1.

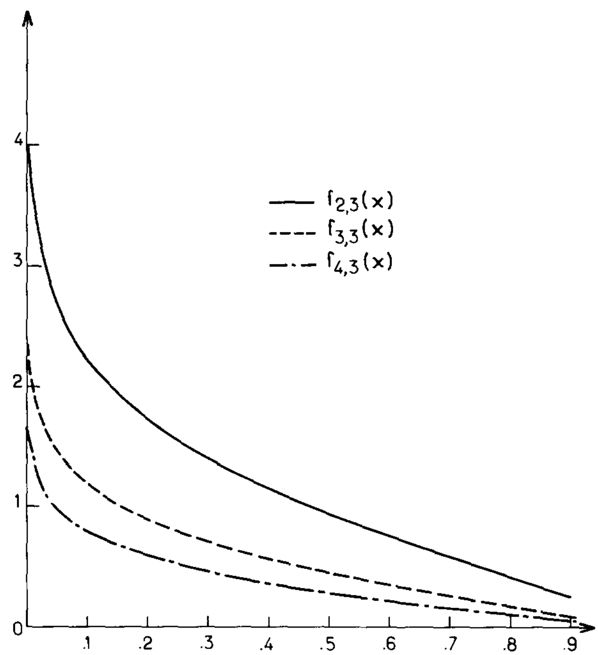

Fig. 3. See caption to fig. 1.

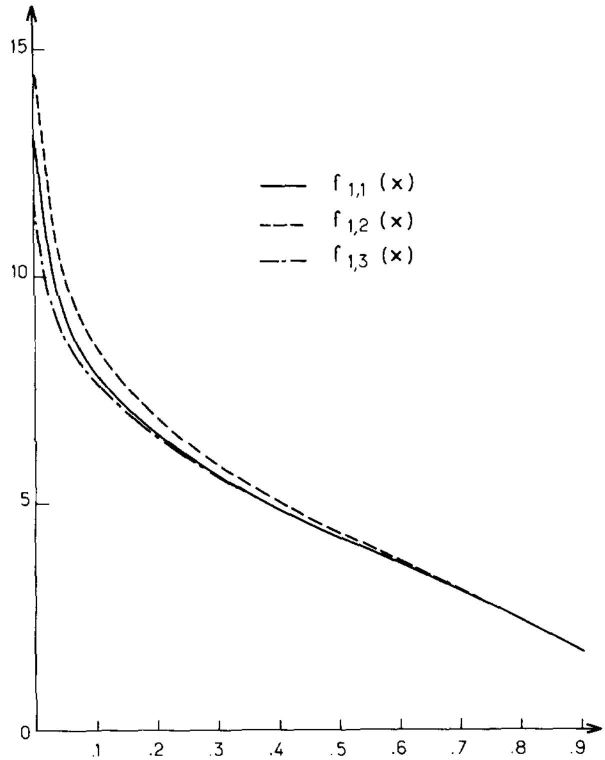

Fig. 4. First v moment ($n = 1$) of the three generalized form factors $\mathcal{F}_i(x, Q^2, v)$.

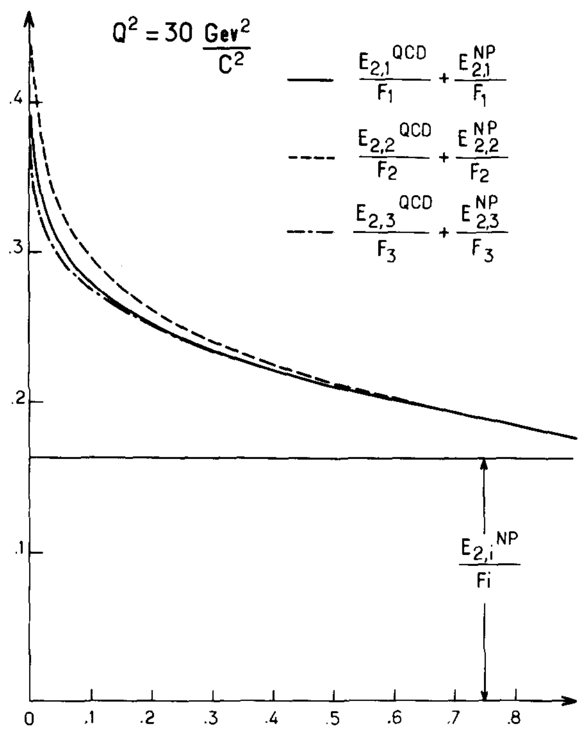

Fig. 5. QCD and non-perturbative results for the $n = 2$ v moments when $Q^2 = 30(\mathrm{GeV}/c)^2$ and $\Lambda = 0.5$ GeV/c.

$d\Sigma^{\nu,\bar{\nu}}/dx\,dy\,d^2v$ defined in (7) and, more precisely, its different tensorial components $\mathcal{F}_i^{\nu,\bar{\nu}}(x,Q^2,v)$, $1 \leqslant i \leqslant 3$.

We first consider the scattering of a virtual vector boson on a single parton. To zeroth order in α_s, the gluons are not coupled to the vector boson, and the scattering on a quark is described by a simple Born-type Feynman diagram; averaging over the initial quark color and spin, one immediately finds

$$\begin{aligned} \mathcal{F}_{q1}^{(0)}(x_p, Q^2, v) &= \delta(1-x_p)\delta^2(v)\,, \\ \mathcal{F}_{q2}^{(0)}(x_p, Q^2, v) &= 2x\delta(1-x_p)\delta^2(v)\,. \end{aligned} \tag{A.1}$$

The first-order contribution in α_s to the scattering on a quark comes from gluon emission; we calculate separately the inelastic term due to the emission of a real gluon, $\mathcal{F}_{qi}^{\text{ine}}$, and the radiative corrections to the lowest-order process, $\mathcal{F}_{qi}^{\text{el}}$.

We shall study the scattering on a gluon target later on.

The inelastic gluon emission is calculated most easily in the quark-virtual boson c.m. frame, in terms of the polar angles θ^*, φ^* defining the direction of the struck quark. One then goes to the Breit frame with a Lorentz boost along the x_3 axis. The last step of the derivation is the change of variables from the polar angles θ_b, φ_b in the Breit frame to the transverse speed v:

$$v_1 = \tan\tfrac{1}{2}\theta_b \cos\varphi_b\,,$$

$$v_2 = \tan\tfrac{1}{2}\theta_b \sin\varphi_b\,.$$

The results are

$$\begin{aligned} \mathcal{F}_{q1}^{\text{ine}} &= \frac{4}{3}\frac{\alpha_s}{2\pi^2}\left\{\frac{1+x_p^2}{x_p}\frac{1}{\left(v^2+(1-x_p)/x_p\right)\left[v^2+x_p(1-x_p)(-p^2/Q^2)\right]}\right. \\ &\qquad \left. + \frac{(1-x_p)(1-2x_p-2x_p^2)}{x_p^2\left(v^2+(1-x_p)/x_p\right)^3}\right\}, \\ \mathcal{F}_{q2}^{\text{ine}} &= \frac{4}{3}\frac{\alpha_s}{2\pi^2}2x\left\{\frac{1+x_p^2}{x_p}\frac{1}{\left(v^2+(1-x_p)/x_p\right)\left[v^2+x_p(1-x_p)(-p^2/Q^2)\right]}\right. \\ &\qquad \left. + \frac{(1-x_p)(1+2x_p-6x_p^2)}{x_p^2\left(v^2+(1-x_p)/x_p\right)^3}\right\}. \end{aligned} \tag{A.2}$$

$\mathcal{F}_{q3}^{\text{ine}}$ is given in (15).

We have calculated the radiative corrections due to virtual gluon emission using dimensional regularization and the minimal subtraction scheme. We regularize the infrared singularities by keeping the incoming and outgoing quarks off shell, space-like ($p^2 < 0$) and time-like ($p'^2 > 0$), respectively. We find

$$\begin{aligned}\mathcal{F}^{\mathrm{el}}_{\mathrm{q}1} &= A\delta(1-x_p)\delta^2(\boldsymbol{v}),\\ \mathcal{F}^{\mathrm{el}}_{\mathrm{q}2} &= 2xA\delta(1-x_p)\delta^2(\boldsymbol{v}),\\ \mathcal{F}^{\mathrm{el}}_{\mathrm{q}3} &= 2A\delta(1-x_p)\delta^2(\boldsymbol{v}),\end{aligned} \tag{A.3}$$

where A is given by

$$A = -\frac{2\alpha_s}{3\pi}\left[2\log\frac{Q^2}{-p^2}\log\frac{Q^2}{p'^2} - \frac{3}{2}\log\frac{Q^2}{-p^2} - \frac{3}{2}\log\frac{Q^2}{p'^2}\right]. \tag{A.4}$$

We have kept in A only the terms that are singular when p^2 or p'^2 go to zero, the other terms would appear as corrections in the coefficient of $\delta^2(\boldsymbol{v})$ for the $\mathcal{F}_i$, so they are irrelevant for the computation of the moments in $\boldsymbol{v}$ of the $\mathcal{F}_i$.

Adding (A.1), (A.2) and (A.3), one gets the total up-to-first order contribution to the scattering on a quark target. The singularities in $\boldsymbol{v} \to 0$ and $x_p \to 1$ are regularized using the method explained in sect. 2, and one gets for $\mathcal{F}_{\mathrm{q}1}$ and $\mathcal{F}_{\mathrm{q}2}$ the analogue of formula (19) for $\mathcal{F}_{\mathrm{q}3}$:

$$\mathcal{F}_{\mathrm{q}1,2}(x_p, Q^2, \boldsymbol{v}) = F_{\mathrm{q}1,2}(x_p, Q^2)\delta^2(\boldsymbol{v}) + \mathcal{F}_{\mathrm{q}1,2}(x_p, Q^2, \boldsymbol{v})_{\oplus}, \tag{A.5}$$

$$F_{\mathrm{q}1}(x_p, Q^2) = \delta(1-x_p) + \frac{4}{3}\frac{\alpha_s}{2\pi}P_{\mathrm{qq}}(x_p)\log\frac{Q^2}{(-p^2)},$$

$$F_{\mathrm{q}2}(x_p, Q^2) = 2xF_{\mathrm{q}1}(x_p, Q^2), \tag{A.6}$$

$$\begin{aligned}\mathcal{F}_{\mathrm{q}1}(x_p, Q^2, \boldsymbol{v})_{\oplus} &= \frac{4}{3}\frac{\alpha_s}{2\pi}\left\{\frac{1+x_p^2}{\pi x_p}\left[\frac{1}{\boldsymbol{v}^2(\boldsymbol{v}^2+(1-x_p)/x_p)}\right]_{\oplus}\right.\\ &\quad\left.+\frac{(1-x_p)(1-2x_p-2x_p^2)}{\pi x_p^2}\left[\frac{1}{(\boldsymbol{v}^2+(1-x_p)/x_p)^3}\right]_{\oplus}\right\},\\ \mathcal{F}_{\mathrm{q}2}(x_p, Q^2, \boldsymbol{v})_{\oplus} &= \frac{4}{3}\frac{\alpha_s}{2\pi}2x\left\{\frac{1+x_p^2}{\pi x_p}\left[\frac{1}{\boldsymbol{v}^2(\boldsymbol{v}^2+(1-x_p)/x_p)}\right]_{\oplus}\right.\\ &\quad\left.+\frac{(1-x_p)(1+2x_p-6x_p^2)}{\pi x_p^2}\left[\frac{1}{(\boldsymbol{v}^2+(1-x_p)/x_p)^3}\right]_{\oplus}\right\}.\end{aligned} \tag{A.7}$$

Let us now turn to the quark pair production process on a gluon target. The calculation of this contribution is done in exactly the same way as that of $\mathcal{F}_{qi}^{\text{ine}}$ and leads to the following results:

$$\mathcal{F}_{g1}(x_p,Q^2,v)=\left\{\frac{\alpha_s}{2\pi}\left[x_p^2+(1-x_p)\right]\log\frac{Q^2}{(-p^2)}\right\}\delta^2(v)$$

$$+\frac{\alpha_s(Q^2)}{\pi}\left\{\frac{1-x_p}{2\pi x_p}\frac{x_p^2+(1-x_p)^2}{\left[v^2\left(v^2+(1-x_p)/x_p\right)\right]_\oplus}\right.$$

$$\left.-\left(\frac{1-x_p}{\pi x_p}\right)^2\frac{x_p^2+(1-x_p)^2}{\left[\left(v^2+(1-x_p)/x_p\right)^3\right]_\oplus}\right\} \tag{A.8}$$

$$\mathcal{F}_{g2}(x_p,Q^2,v)=2x\left\{\frac{\alpha_s}{2\pi}\left[x_p^2+(1-x_p)^2\right]\log\frac{Q^2}{(-p^2)}\right\}\delta^2(v)$$

$$+\frac{\alpha_s(Q^2)}{\pi}2x\left\{\frac{1-x_p}{2\pi x_p}\frac{x_p^2+(1-x_p)^2}{\left[v^2\left(v^2+(1-x_p)/x_p\right)\right]_\oplus}\right.$$

$$\left.-\left(\frac{1-x_p}{\pi x_p}\right)^2\frac{1-6x_p+6x_p^2}{\left[\left(v^2+(1-x_p)/x_p\right)^3\right]_\oplus}\right\}.$$

In order to obtain the form factors $\mathcal{F}_{1,2}(x,Q^2,v)$ for the scattering on a real nucleon, we have now to do the convolution of the functions $\mathcal{F}_{qi}$ and $\mathcal{F}_{gi}$ with the quark and gluon distributions inside the nucleon. Following the discussion of sect. 2, one can factorize the mass singularities in these distributions in order to obtain:

$$\mathcal{F}_i^{\nu,\bar\nu}(x,Q^2,v)=\int_x^1\frac{d\xi}{\xi^2}\left\{\tfrac{1}{2}\left[q(\xi,Q^2)+\bar q(\xi,Q^2)\right]\mathcal{F}_{qi}\left(\frac{x}{\xi},Q^2,v\right)\right.$$

$$\left.+g(\xi,Q^2)\mathcal{F}_{gi}\left(\frac{x}{\xi},Q^2,v\right)\right\}, \tag{A.9}$$

$i=1,2$. Here q, $\bar q$ and g are the total quark, antiquark and gluon momentum distributions.

There is at this stage an ambiguity we would like to point out: concerning the scattering on a gluon we have to sum over the flavours of the quark-antiquark pair produced by the gluon. For instance, for an incoming neutrino the W^+ can scatter either on a d (or ū), or on an s (or c̄) quark, so one should multiply the contribution of the gluons in the above formula by a factor 2, in a world of four flavors. This is true asymptotically, but at the present available Q^2 our computation does not apply to the process where the scattering of the virtual vector boson $W^{\pm}$ on a gluon produces a sc̄ or s̄c pair since the charm quark mass cannot be neglected. So we do not consider this process here, and our results are valid only for non-charmed final states. Well above the charm threshold the gluon target contribution (which is rather small except in the low x region) should be multiplied by 2.

We then calculate the moments in v of $\mathcal{F}_i^{\nu,\bar{\nu}}$, i.e., the functions $\hat{E}_{n,i}^{\nu,\bar{\nu}}(x)$ defined in (44) [$E_{n,3}^{\nu,\bar{\nu}}$ has been given in formula (28)]. Due to the contribution of the gluons, the expressions of $\hat{E}_{n,1}$ and $\hat{E}_{n,2}$ look slightly more complicated; they are given in (45) in terms of the following functions:

$$Q_{n1}(x)=\frac{4}{3}\left[\frac{1+x^2}{x}I_n(x)+\frac{(1-x)(1-2x-2x^2)}{x^2}J_n(x)\right],$$

$$G_{n1}(x)=2\frac{1-x}{x}\left[x^2+(1-x)^2\right]\left[I_n(x)-2\frac{1-x}{x}J_n(x)\right]; \tag{A.10}$$

$$Q_{n2}(x)=\frac{4}{3}\left[\frac{1+x^2}{x}I_n(x)+\frac{(1-x)(1+2x-6x^2)}{x^2}J_n(x)\right],$$

$$G_{n2}(x)=2\frac{1-x}{x}\left\{\left[x^2+(1-x)^2\right]I_n(x)-2\frac{1-x}{x}(1-6x+6x^2)J_n(x)\right\}. \tag{A.11}$$

The integrals $I_n(x)$ and $J_n(x)$ defined in (29) satisfy the following recurrence laws:

$$\text{for } n\geqslant 3:\ I_n(x)=\frac{2}{n-2}-\frac{1-x}{x}I_{n-2}(x); \tag{A.12}$$

$$\text{for } n\neq 4:\ J_n(x)=\frac{1}{4-n}\left[\frac{1-x}{x}nJ_{n-2}(x)-2x^2\right]. \tag{A.13}$$

I_0, I_1, I_2, J_0, J_1 and J_4 can easily be calculated directly, and one finds, for the first terms:

$$I_1(x)=2\sqrt{\frac{x}{1-x}}\,\tan^{-1}\sqrt{\frac{x}{1-x}},$$

$$I_2(x)=\log\frac{1}{1-x}, \tag{A.14}$$

$$I_3(x)=2\left[1-\sqrt{\frac{1-x}{x}}\,\tan^{-1}\sqrt{\frac{x}{1-x}}\right],$$

$$I_4(x) = 1 - \frac{1-x}{x} \log \frac{1}{1-x};$$

$$J_1(x) = \frac{1}{4} \frac{x}{1-x} \left[\sqrt{\frac{x}{1-x}} \tan^{-1} \sqrt{\frac{x}{1-x}} + x(2x-1) \right],$$

$$J_2(x) = \frac{1}{2} \frac{x^3}{1-x}, \tag{A.15}$$

$$J_3(x) = \frac{1}{4} \left[3\sqrt{\frac{x}{1-x}} \tan^{-1} \sqrt{\frac{x}{1-x}} - 2x^2 - 3x \right],$$

$$J_4(x) = \log \frac{1}{1-x} - x - \frac{1}{2} x^2.$$

We have used a simple parametrization given in sect. 4 for $q(x,Q^2)$, $\bar{q}(x,Q^2)$ and $g(x,Q^2)$ to compute the functions $E_{n,i}^{\nu,\bar{\nu}}(x)$ given by the relations (45) and (A.10) to (A.15).

References

[1] H. Georgi and H.D. Politzer, Phys. Rev. Lett. 40 (1978) 3;
G. Altarelli and G. Martinelli, Phys. Lett. 76B (1978) 89;
G. Ranft and J. Ranft, Phys. Lett. 82B (1979) 129;
A. Mendez, Nucl. Phys. B145 (1978) 199;
A. Mendez, A. Raychauduri and V.J. Stenger, Nucl. Phys. B148 (1979) 499;
J. Cleymans, Phys. Rev. D18 (1978) 954;
G. Köpp, R. Maciejko and P.M. Zerwas, Nucl. Phys. B144 (1978) 123;
P. Mazzanti, R. Odorico and V. Roberto, Phys. Lett. 80B (1978) 111;
F. Hayot and A. Morel, Z. Phys. C2 (1979) 243;
K.H. Streng, T.F. Walsh and P.M. Zerwas, Z. Phys. C2 (1979) 237;
P. Binetruy and G. Girardi, Nucl. Phys. B155 (1979) 150;
P.M. Stevenson, Nucl. Phys. B150 (1979) 357

[2] C.L. Basham, L.S. Brown, S.D. Ellis and S.T. Love, Phys. Rev. D17 (1978) 2298; D19 (1979) 2018; Phys. Rev. Lett. 41 (1978) 1585;
R.D. Peccei and R. Rückl, Phys. Lett. 84B (1979) 95; Nucl. Phys. B162 (1980) 125

[3] H. Georgi and M. Machacek, Phys. Rev. Lett. 39 (1977) 1237;
E. Fahri, Phys. Rev. Lett. 39 (1977) 1587;
A. De Rújula, J. Ellis, E.G. Floratos and M.K. Gaillard, Nucl. Phys. B138 (1978) 387;
J.D. Bjorken and S.J. Brodsky, Phys. Rev. Lett. D1 (1977) 1416;
G. Parisi, Phys. Lett. 74B (1978) 65

[4] A.H. Mueller, Phys. Rev. D18 (1978) 3705;
C.H. Llewellyn Smith, Proc. 17th Int. Universitätswochen f. Kernphysik, Schladming, 1978, Acta Phys. Austriaca Suppl. XIX (1978) 331;
K.H. Craig and C.H. Llewellyn-Smith, Phys. Lett. 72B (1978) 349;
Yu. L. Dokshitzer, D.I. D'Yakanov and S.I. Troyan, 13th Leningrad Winter School; SLAC TRANS-183 (1978);
D. Amati, R. Petronzio and G. Veneziano, Nucl. Phys. B140 (1978) 54; B146 (1978) 29;

R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer and C.G. Ross, Phys. Lett. 78B (1978) 281; Nucl. Phys. B152 (1979) 285;
S.B. Libby and G. Sterman, Phys. Rev. D18 (1978) 4737
[5] G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298
[6] G. Sterman and S. Weinberg, Phys. Rev. Lett. 39 (1977) 1436
[7] G. Tiktopoulos, Nucl. Phys. B147 (1979) 371;
L. Clavelli, Phys. Lett. 85B (1979) 111
[8] R.D. Field and R.P. Feynman, Nucl. Phys. B136 (1978) 1
[9] J. Bell et al., Phys. Rev. D19 (1979) 1
[10] L. Hand, Proc. 1977 Int. Symp. on Lepton and photon interactions at high energy, ed. F. Gutbrod (Hamburg, 1977)
[11] J.G.H. de Groot et al., Z. Phys. C1 (1979) 143