A time to cast away stones, and a time to gather stones;
A time to search and a time to give up,
A time to keep and a time to throw away

Is there anything of which one can say,  "Look! This is something new"?
It was here already, long ago; it was here before our time.
*Ecclesiastes 3:4-3:6*

# WHEN THE INTUITION BETRAYS OR SAVES YOU: VOICES FROM THE 1D WORLD

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Two-lectures, four-stories course given on occasion of the half-a-century anniversary
June - September 1969
of the author's first public presentation of his first research (DEA defense),
first journal club report (Landau seminar),
and the first PhD duty: a month of collecting potato at the state farm

*Today's stories will be old but still not so antic*

*Acknowledging collaboration:*

| | |
|---|---|
| Natasha Kirova | story 2 |
| Sergei Matveenko | story 3 |
| Philippe Nozieres | story 3 |
| Victor Yakovenko | story 4 |

## Excursion #1:
## Meandering river of the Luttinger liquid: from Felix Bloch source through Tomonaga lake

For whosover hath, to him shall be given,
and he shall have more abundance.
But whosoever hath not,
from him shall be taken away even what he hath.

*Matthew, 13:12*

*In translation from the antic English to the most modern one:*
The first takes everything!

Luttinger liquid: the most popular concept in physics of 1D electrons (+ fermionic cold atoms nowadays).

**Basic statements:** the background of free sound-like Bose excitations provides complete degrees of freedom to describe all fermionic Green functions and related observables.

The nonlinear exponential relations between Bose and Fermi operators makes the fermionic response to follow power laws with indices dependent on interactions.

Different perturbations, both irrelavant ones and ones destroying the LL (e.g. giving rise to spin (SC, CDW) or charge (Mott state) gaps) can be studied by an RG within the bosonisation scheme.

The bosonisation was derived for weak interactions but is believed to be asymptotically valid well beyond inwards strong coupling regimes with phenomenological powers related to observable macroscopic properties (compressibility and sound velocity).

## Warnings of semantic curiosities – back in time:

Titles of Luttinger and rarely Tomonaga recently (thanks cold atoms).
Mostly Luttinger for QH edge states (while a purely Tomonaga case).
Overwhelmingly LL in meso and nano science of quantum wires of 90's.
Explosion of the LL concept since the high-Tc epoch as a crusade announce by Ph. Anderson against the Fermi liquid.
Dark ages with the actually unattended suggestion by D.Haldain in 1981.
Resurrections of the concept in the epoch (late 70s to late 70s) of quasi-1D conductors but exclusively under the title, if any, of the Tomonaga model. Examples:

*I.E. Dzyaloshinskii and A. I. Larkin (1973)*
Correlation functions for a one dimensional fermi system with long-range interaction **(Tomonaga model)**
*R. A. Klemm, A. I. Larkin (1978)*
$4k_F$ response function in the **Tomonaga model**

Digging deeper, but still above the original publication
*J. M. Luttinger, J. Math. Phys. 4, 1154 (1963)*
we find a "skeleton in the closet":
*D. Mattis and E. Lieb, J. Math. Phys. 6, 304 (1965)*
*Exact Solution of a Many-Fermion System and Its Associated Boson Field*

"… We show that [Luttinger] did not solve his model properly because of the paradoxical fact that the density operator commutators, which always vanish for any finite number of particles, no longer vanish in the field-theoretic limit of a filled Dirac sea.
In fact the [density] operators $\rho(\mathbf{p})$ define a boson field which is *ipso facto* associated with the Fermi-Dirac field.
We then use this observation to solve the model, and obtain the exact (*and now nontrivial*) spectrum, free energy, and dielectric constant. …"

Time to recall a recognized talent:
*S. Tomonaga, (1950) "Remark on Bloch's method of sound waves applied to many fermion problems" .*
Referring to the absolutely abandoned foundation:
*F. Bloch, Z.Physik 81, 363 (1933); Helv. Phys. Acta 7, 385 (1934).*

Felix Bloch observations: The conventional two-parametric band of two-fermionic (e-h) excitations of the Fermi gas, in 1D is reduced to a single-parametric imitation of the sound spectrum.

$$\varepsilon(\vec{p}+\vec{k})-\varepsilon(\vec{p}) \to \vec{\mathrm{v}}_{\mathrm{F}}(\mathrm{p})\vec{\mathrm{k}} \to \mathrm{v}_{\mathrm{F}}\,\vec{\mathrm{n}}(\vec{\mathrm{p}})\vec{\mathrm{k}} \to \mathrm{v}_{\mathrm{F}}\,k$$

*Remind A. Luther*

Tomanaga genuine step to materialize Bloch hypothesis:
Rise the bilinear, in fermions, kinetic energy to the quadratic, in fermions, density-density interaction

$$\Psi^{+}\widehat{\varepsilon}\Psi \rightarrow \rho_{+}^{2}+\rho_{-}^{2} \quad \rho_{\pm}=\Psi_{\pm}^{+}\Psi_{\pm}$$

Just opposite to what we usually do: reducing the order of interactions by a kind of Hubbard-Stratonovich decoupling.

Here was a tremendously useful and potentially harmful step:
division of the fermionic Hilbert space into its
right (-) and left (+) moving components.

Simplifying limitations of the TM: only long range interactions and moreover – only among fermions of the same direction

$$H_{int} = g_4(\rho_+^2 + \rho_-^2) \neq g(\rho_+ + \rho_-)^2$$

Luttinger addendum: towards reality of common LRI add the interaction

$$g_2 \rho_+ \rho_-$$

A new missed agenda: entanglement of L and R densities
which was not an issue of the TM.
Luttinger suggestion: they just commute.
Lieb and Mattice correct that referring to already known paradox of the Schwinger anomaly in particle physics and even the earlier Jordan work from late 30's. For us, in condensed matter, the answer is just the basic foundation of the quantum hydrodynamics, model independently:

$$\rho_+ + \rho_- = \rho \, , \rho_+ - \rho_- = j \; (v_F = 1)$$

$$[\rho \, , \mathrm{j}] = -i\nabla \, \delta(r - r') \neq 0$$

For a universal long-wave Hamiltonian of fermions in the potential $\Phi$

$$H_{LR}=K\rho^2/2+\rho\Phi$$

The commutator $$[\rho, \mathrm{j}] = -i\nabla\delta(r-r')$$

leads to the "chiral anomaly" of the "axial current"
of the 1+1 quantum field theory:

$$\partial_t j + K\partial_x\rho = -\partial_x\Phi$$

The "anomalous" – missed in the perturbation theory – RHS actually restores the most basic property of the ideal metal: **j=Et**
and of any conductor: $\boldsymbol{\rho=-\Phi/K= =-\Phi N_F}$ -origin of screening

Why has it been missed ?

The most principle property of a conductor: screening of the electric field at the length $\mathbf{r_0}$

$$\frac{1}{r_0^2} = 4\pi e^2 \frac{dn(\zeta)}{d\zeta}$$

For metal at T=0 $\quad \dfrac{dn(\zeta)}{d\zeta} = \dfrac{dn}{d\varepsilon_F} = N_F$ $\qquad \zeta$ is chemical potential

The nonlinearized Schroedinger eq. would give us, in the adequate here WKB approximation

$$\psi(x) = \frac{C}{\sqrt[4]{E - e\Phi}} \exp(\pm i \int \sqrt{E - e\Phi(y)} dy)$$

$$n = \sum |\psi|^2 = \sum_E \frac{1}{\sqrt{E - e\Phi}} \sim \sqrt{\varepsilon_F - e\Phi}$$

- the Thomas-Fermi theory $\quad \delta n \propto -\Phi$ - correct linear response

When we linearize the spectrum

$$\frac{p^2}{2m}-\frac{p_F{}^2}{2m}\cong \pm\hbar v_F k$$

$P=\pm p_F+k$

and separate L&R states $\psi=\psi_+ e^{ik_F x}+\psi_- e^{-ik_F x}$

$$\mp i\partial_x \psi_\pm + e\Phi\,\psi_\pm = E\psi_\pm \qquad \psi_\pm = C\exp\left(\pm i\int dx(E-e\Phi(x))\right)$$

$$n=\sum|\psi_\pm|^2=cnst$$

No density response to the potential hence **no screening**

By necessary but premature linearization we loose the possibility to change the density $|\psi|^2$ already for any wave function, hence for their ensemble.