Excursion #2

We shall meet the chiral anomalies as they appear already in simplest - MF or BCS types – models, particularly
at finite temperatures and
out-of-equilibrium when normal excitations are present.
The resulting effective Ginzburg-Landau theory will prove to be quite different from what is commonly expected –
non analytical with respect to the order parameter.

After collaboration with Natasha Kirova
Inspired by experimental studies in Grenoble, Cornell, etc.

Symmetry breaking and multiple fluids

Thermodynamics of classical phase transitions $F(\eta)$
Landau-Ginzburg free energy functional for the order parameter

Kinetics of classical phase transitions:
Landau-Khalatnikov, etc. phenomenology
PLUS ultimately (Halperin, Hohenberg, Ma)
hydrodynamic modes

$$\frac{\partial \eta}{\partial t} = -\frac{\delta F}{\delta \eta}$$

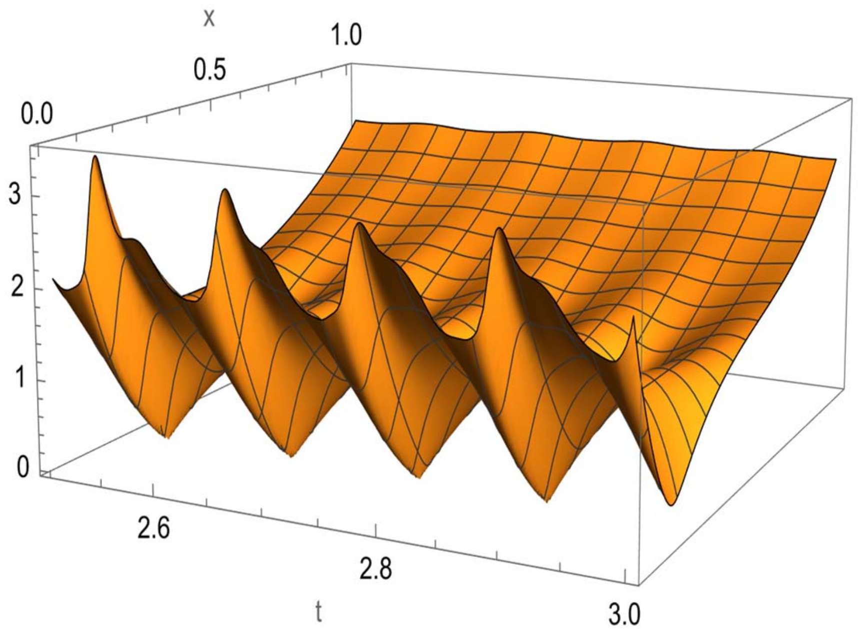
Kinetics of phase transitions in quantum systems:
Bose condensates, Superconductors, Charge Density Waves, etc.
for an order parameter $\boldsymbol{\eta}$ plus noncondensed particles

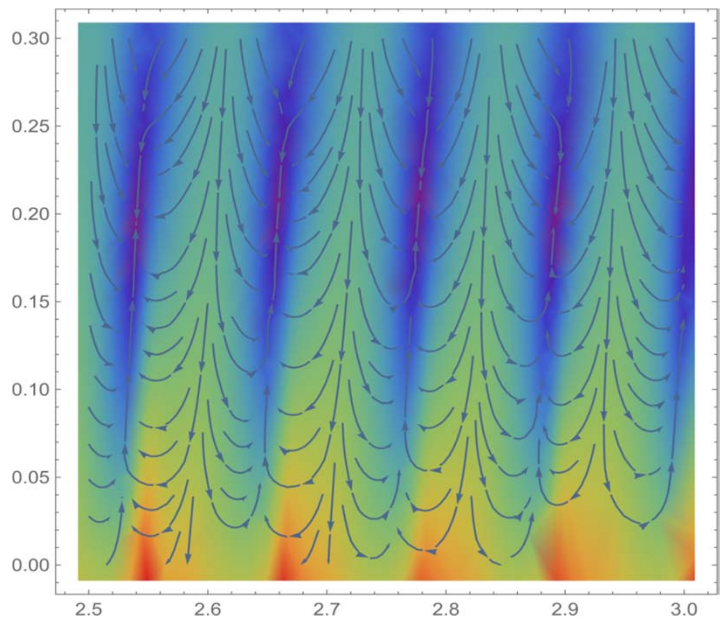
From microscopics (Green Functions, Gorkov and Keldysh technics, etc)
to Gross-Pitaevskii – for bosons or
TDGL (time dependent Ginzburg Landau) for fermions
phenomenology for $\boldsymbol{\eta}$ alone – the follow-up carriers are integrated.

PLUS kinetics or simpler hydrodynamics of normal fermions.
- Beyond the hardly treatable microscopics, all steps are badly questionable !

Dynamics of topological defects in charge and spin density waves and the role of the chiral anomaly.

REVISION OF THE TIME DEPENDENT GINZBURG-LANDAU APPROACH TO EVOLUTION OF INHOMOGENEOUS STATES IN SLIDING CHARGE DENSITY WAVES.

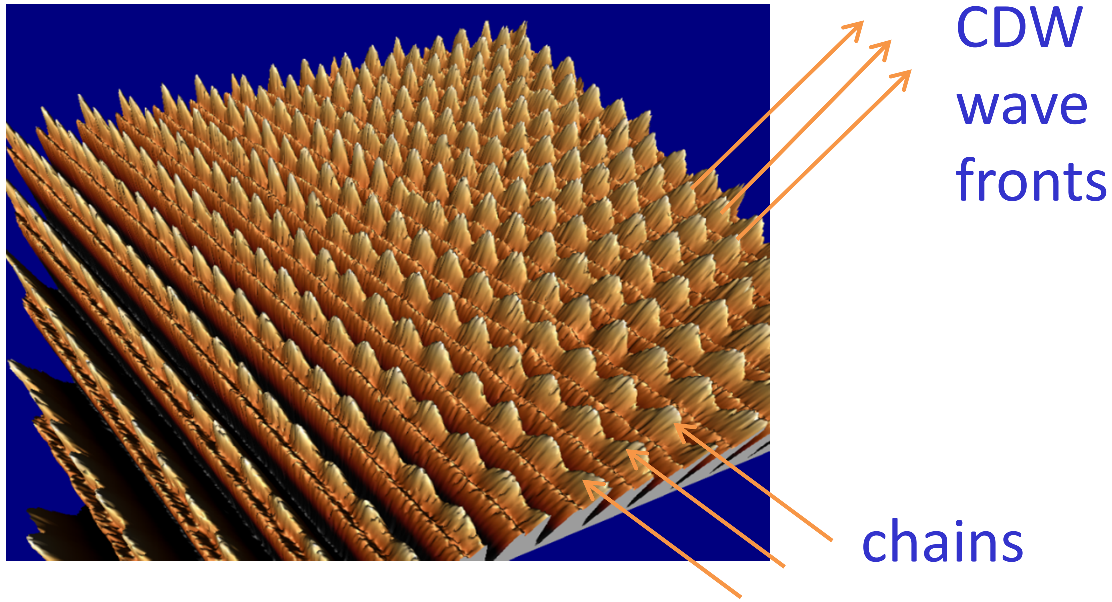




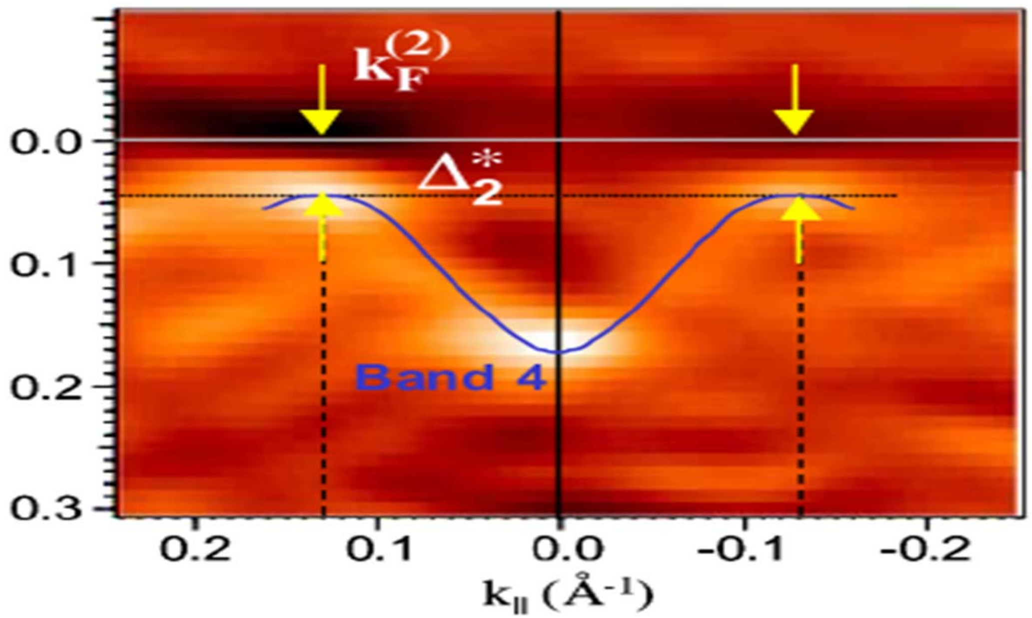
Graphical abstract: modeling of a sequence of phase slips – space-time vortices. Left – amplitude nodes, right – phase vorticity

Incommensurate, 3D ordered Charge Density Waves - CDW

Scanning Tunneling Microscopy STM shows the CDW modulation in atomic displacements and the electronic density.
C. Brun et al



ARPES gives electrons' spectral density, hence E(k) with the gap Δ formed by the CDW



Common features of incommensurate CDWs and the superconductors

CDW: $<\psi^*_+\psi_- > \neq 0$ SC: $<\psi_+\psi_- > \neq 0$

→ complex order parameters $O_{cdw,sc} \sim A \exp[i\varphi_{cdw,sc}]$

hence vortices ←→ dislocations, phase slips ←→ phase solitons

Similar microscopic theories: Peierls-Frohlich vs BCS

Pair-breaking gaps **2Δ** - hence tunneling, FFLO ←→ solitonic lattices

Tighter links at **D=1**:

phases φ_{cdw} , φ_{sc} are in conjugation: $[\varphi_i, \partial_x\varphi_j] \sim \delta(x)\, \delta_{ij}$

pairbreaking goes via common spinons as amplitude solitons

2Δ becomes the common spin gap; broken pair becomes 2 free spinons

A universal link: the same current is $j = -\partial_t\varphi_{cdw}/\pi \sim \partial_x\varphi_{sc}$

Differences in currents and densities of condensates

$$\psi_{CDW} = A\exp(i\varphi)$$

$$n_{CDW} = A^2 \frac{e}{\pi}\frac{\partial\varphi}{\partial x}$$

$$j_{CDW} = -A^2 \frac{e}{\pi}\frac{\partial\varphi}{\partial t}$$

$$\psi_{SC} = C\exp(i\theta)$$

$$j_{SC} \propto C^2 e v_F \frac{\partial\theta}{\partial x}$$

$$n_{SC} \propto -C^2 \frac{e}{v_F}\frac{\partial\theta}{\partial t}$$

SC, explicit gradient invariance:
conservation of the condensate is contained in the eq. for the SC phase.

CDW, chiral = translational invariance: conservation of the condensate is preserved as for conventional crystal displacements.

$$\partial t(\partial x)\varphi + \partial x(-\partial t\varphi)\equiv 0$$

Collective motion of CDWs (**Frohlich 1950’s; Lee, Rice, PWA 1970’e**):

- nonlinear conduction by the collective sliding.
- Topological defects: solitons, dislocations (electronic vortices).
- Phase slips as space-time vorticity.
- Current conversion among normal and condensed electrons.

While the phase evolution or deformation are the principle ingredients, no collective effects can be set in without perturbations of the amplitude **A(x,t)**, particularly with **A** passing though zero.

Intuitive Ginzburg-Landau – like model for CDW

$$\psi = A\exp(i\varphi)$$

A is normalized to its equilibrium value A_{eq} at a given *T*

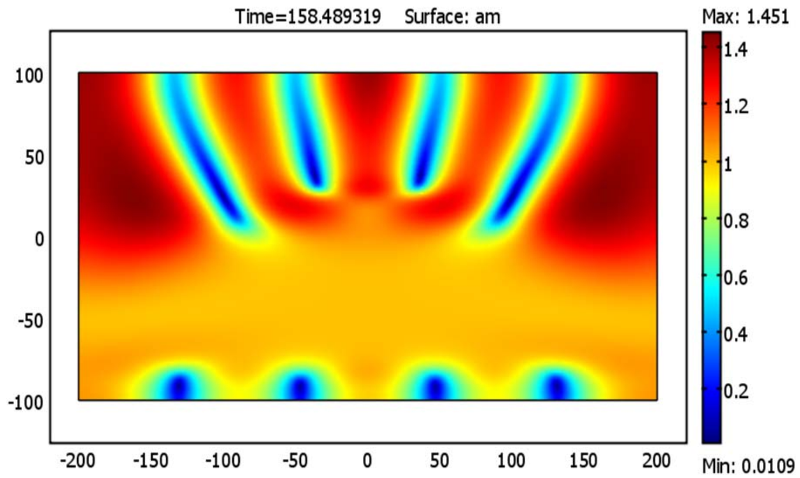
$$H_{CDW} = \int d^3r \left\{ \left[\left| \frac{\partial \psi}{\partial x} \right|^2 + \alpha \left| \frac{\partial \psi}{\partial y} \right|^2 \right] + |\psi|^2 \ln \frac{|\psi|^2}{e} \right\}$$

$$H_{\text{int}} = \int d^3r \left[\Phi A^2 \partial_x \varphi / \pi + \Phi n(\varsigma) + F(n) - |\nabla \Phi|^2 \varepsilon / 8\pi \right]$$

Only extrinsic (other bands) carriers *n* are taken explicitly.
In the GL spirit, the intrinsic carriers (in the gap region) are integrated out, their effect is hidden in A_{eq} and coefficients (not all are shown).

Nonstationary processes in CDWs within the TDGL phenomenology. Numerical solution of coupled PDE for the order parameter phase and amplitude, the electric potential and carriers concentration.

T. Yi, A. Rojo, N. Kirova and S.B.



traces of A=0 nodes at cores of nucleating and moving vortices

Rectangle.avi

Many vortices appear temporarily in the course of the evolution. For that run, only one will be left in the steady state.

The result is as spectacular as it is wrong!
The TDGL approach is principally deficient here.

Deadly problems with the TDGL model for CDW

Well established and works for stationary state and as a tool to reach it. Takes explicitly the extrinsic fermions (not interacting with the CDW)

Restrictions, inherent to the GL spirit:

Intrinsic fermions have been integrated out and come into the model only via the equilibrium value of the amplitude A and related parameters.

Major problem: Violation of the local charge conservation for the condensate density if the amplitude is variable.

$$n_c = \frac{A^2}{\pi}\frac{\partial\varphi}{\partial x} \qquad j_c = -\frac{A^2}{\pi}\frac{\partial\varphi}{\partial t} \quad \Longrightarrow \quad \frac{dn}{dt} = \frac{\partial n_c}{\partial t} + \frac{\partial j_c}{\partial x} = 0$$

automatically if A = const

But with
$A(x,y,t) \neq \mathrm{cnst}$

$$\pi\frac{dn}{dt} = \frac{\partial A^2}{\partial x}\frac{\partial\varphi}{\partial t} - \frac{\partial\varphi}{\partial x}\frac{\partial A^2}{\partial t} \neq 0$$

WRONG

Way of resolution: keep carriers in hand and decompose n,j

More general scheme :

Decomposition in right and left moving fermions with the spectrum linearization – most fruitful and exploited tool in theory of 1D fermions

$\psi=(\psi_+,\psi_-)$ - electronic wave function components near $\pm p_F$

$\Delta e^{i\varphi}$ – order parameter

Φ and $\boldsymbol{A_x}$ - scalar and vector potential,

$\boldsymbol{v_F}$ – Fermi velocity

The fermionic Lagrangian:

$$H_{el} = \begin{pmatrix} i\frac{\partial}{\partial t} - i\hbar v_F \frac{\partial}{\partial x} + e\Phi + \frac{e}{c}A_x & \Delta e^{i\varphi} \\ \Delta e^{-i\varphi} & i\frac{\partial}{\partial t} + i\hbar v_F \frac{\partial}{\partial x} + e\Phi - \frac{e}{c}A_x \end{pmatrix}$$

Inconvenience: the gap **Δ** is loaded with the essentially variable **x,t** dependent factor exp(±iφ)

Chiral transformation: $\psi_{\pm} \rightarrow \psi_{\pm} e^{\pm i\phi/2}$

actually puts the electrons to the breathing frame of shifted Fermi momentum and Fermi energy: **$\delta P_F = \partial_x \phi/2 \rightarrow \delta E_F = \hbar v_F/2\partial_x \phi$**

The the phase factor is unloaded from the gap **Δ**,
we arrive at a semiconductor model,
but in expense of elongating the applied potentials:

$$e\Phi \rightarrow e\Phi + \hbar v_F / 2\partial_x \phi \qquad e/cA_x \rightarrow e/cA_x + \hbar v_F / 2\partial_t \phi$$

$$eE = -\partial_x \Phi + \partial_t A_x \rightarrow F = eE + (\partial_t^2 - \partial_x^2)\phi/2$$

F – chiral invariant effective electric field
experienced by the floating fermions.
Resulting energy, collective charge and current, etc.
Seem (sic !) to be functions of entire ***F*** only.

PROBLEM 1 Action = free energy **W** after integration over fermions:

$$W = W(e\Phi + \hbar v_F / 2\partial_x \phi)$$ can choose $$e\Phi + \hbar v_F / 2\partial_x \phi = 0$$

False local "gauge" invariance – the potential can be excluded at no cost

In absence of normal carriers we definitely expect:

$$W = \frac{\rho^2}{2N_F} + e\rho\Phi \Rightarrow \frac{\hbar v_F}{4\pi}\left(\frac{\partial \phi}{\partial x}\right)^2 + \frac{e\Phi}{\pi}\frac{\partial \phi}{\partial x}$$

- Not a full square of the chiral-invariant combination:

$\sim\Phi^2$ looks to be an anomaly - and not only

PROBLEM 2

By definition (in lowest quadratic approximation) :

$$W_e = -\frac{\varepsilon(k,\omega)F^2}{8\pi} = -\frac{\varepsilon(k,\omega)}{8\pi e^2}\left(-\frac{\partial e\Phi}{\partial x} - \frac{\hbar v_F}{2}\frac{\partial^2\varphi}{\partial x^2}\right)^2$$

Where the electrical permittivity: $\epsilon = \epsilon_\Delta + \epsilon_e$

$$\varepsilon_\Delta \propto \left(\frac{\omega_p}{\Delta}\right)^2 \Rightarrow W_e \propto \left(\frac{\partial^2\phi}{\partial x^2}\right)^2 = k^4\phi_k^2$$

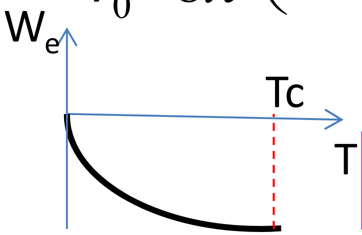
Not important, higher order in k

$$\varepsilon_e = \frac{1}{\lambda^2 k^2} = \frac{\rho_n}{r_0^2 k^2} \qquad \frac{1}{\lambda^2} = 4\pi e^2\frac{dn}{d\mu} = 4\pi e^2 N_F\rho_n$$

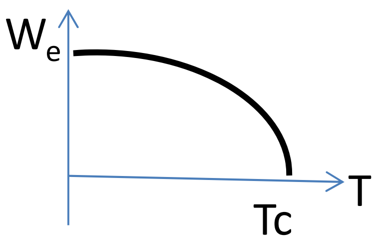
Right order in k but ridiculous

λ - length of screening by unpaired fermions, $\mathbf{r_0}$ = microscopic TF scale

$$W_e^* = -\frac{\rho_n}{r_0^2}\frac{1}{8\pi}\left(e\Phi + \frac{\hbar v_F}{2}\frac{\partial\varphi}{\partial x}\right)^2 \qquad W_e = \frac{\rho_c}{r_0^2}\frac{1}{8\pi}\left(e\Phi + \frac{\hbar v_F}{2}\frac{\partial\varphi}{\partial x}\right)^2$$



Obtained:



Expected:

Resolution: the whole expression has been lost

Chiral anomaly

$$\delta W = \frac{\hbar v_F}{4\pi}\left(\frac{\partial \varphi}{\partial x}\right)^2 + \frac{e\Phi}{\pi}\frac{\partial \varphi}{\partial x}$$

On top of perturbational part

$$W_e^* = -\frac{\rho_n}{r_0^2}\frac{1}{8\pi}\left(e\Phi + \frac{\hbar v_F}{2}\frac{\partial \varphi}{\partial x}\right)^2$$

$$W_e = W_e^* + \delta W = \rho_c\frac{\hbar v_F}{4\pi}\left(\frac{\partial \phi}{\partial x}\right)^2 + \frac{e}{\pi}\rho_c\Phi\frac{\partial \phi}{\partial x} - \frac{\rho_n (e\Phi)^2}{\pi\hbar v_F} - \frac{(\nabla\Phi)^2}{8\pi}$$

The contribution of normal carriers bites from the T=0 anomalous action erasing it down to zero at $\mathbf{T_c}$ when A~Δ→**0**, $\boldsymbol{\rho_c \sim A^2}$

Local energy functional

$$\psi = A\exp(i\varphi);$$

$$W\{\phi,\Phi,n_{in},A\} = \frac{\hbar v_F}{4\pi}\left[\phi_x^2 + \alpha A^2\phi_y^2\right] + C\frac{\hbar v_F}{4\pi}\left[\left|\frac{\partial A}{\partial x}\right|^2 + \beta^2\left|\frac{\partial A}{\partial y}\right|^2\right] +$$

Expect A^2 – actually 1.
Non analytic in Ψ

$$\frac{\phi_x}{\pi}\Phi + \left(\Phi + \frac{\hbar v_F}{2}\phi_x\right)n_{in} - \frac{\varepsilon_h}{8\pi}(\nabla\Phi)^2 + F(A,n_{in})$$

Both terms come from the chiral anomaly.

$$\phi_i = \partial\phi/\partial x_i$$

Expressions for total density and current conserve number of particles

$$n = \frac{1}{\pi}\partial_x\varphi + n_{in}\ , \qquad j = -\frac{1}{\pi}\partial_t\varphi + j_{in} \Rightarrow \frac{dn}{dt} = 0$$

+ need a mechanism for n_{in} , n_{in} to compensate $\partial\varphi$ at $A\rightarrow 0$

$$n_c = \frac{A^2}{\pi}\frac{\partial\varphi}{\partial x} \qquad j_c = -\frac{A^2}{\pi}\frac{\partial\varphi}{\partial t}$$ for A=cnst

That will come implicitly from counter-charges, counter-currents which react to CDW bringing compensating contributions $-\rho_n\,\partial\varphi/\pi$

True equations are not analytical in Ψ: phase gradients are not multiplied by A^2

$$\frac{\partial}{\partial x}\left(\frac{\partial\varphi}{\partial x}+2\Phi+\pi(n_e-n_h)\right)+\alpha\frac{\partial}{\partial y}\left(A^2\frac{\partial\varphi}{\partial y}\right)=\gamma_\varphi A^2\frac{\partial\varphi}{\partial t}$$

$$\nabla^2 A+\alpha A\left(\frac{\partial\varphi}{\partial y}\right)^2+\frac{\partial F}{\partial A}=-\gamma_A\frac{\partial A}{\partial t}$$

$$-r_0^2\nabla^2\Phi=\frac{\partial\varphi}{\partial x}+\pi(n_e-n_h+n_{ex})$$

$$\nabla\hat{\sigma}\nabla\mu=\partial_t n \quad \mu=\zeta+\Phi+\partial_x\varphi/2$$

F=F(A,n) in principle, the minimal form:

Former G-L like equations

$$\nabla A^2\nabla\varphi+\frac{\partial}{\partial x}A^2\Phi=\gamma_\varphi A^2\frac{\partial\varphi}{\partial t}$$

$$\nabla^2 A+A(\nabla\varphi)^2+\frac{\partial F}{\partial A}=-\gamma_A\frac{\partial A}{\partial t}$$

$$-r_0^2\nabla^2\Phi=A^2\frac{\partial\varphi}{\partial x}+n_{ex}$$

$$-\nabla[\sigma\nabla(\varsigma+\Phi)]+\frac{\partial n}{\partial t}=0$$

F=F(A) only

$$F(n,A)=n^2/(2N_F)+(-\tau+(n/n_{cr})^2)(A\Delta_0)^2N_F/2+bA^4\Delta_0^2N_F/4$$

Possible simplifications and explicitness

Infinite conductivity: **-** a bridge to the naive GL eqs. $\mu=\zeta+\Phi+\partial_x\varphi/2\equiv 0$

$$\zeta=\frac{\partial F}{\partial n},\ \rho_n=N_F^{-1}\frac{\partial n}{\partial \zeta};\ \rho_c=1-\rho_n$$

$$\rho_c\partial_x^2\varphi = r_0^2\nabla^2 E_x - \rho_n E_x$$

Poisson eq.

LHS resembles the static effective charge

$n_c=A^2\partial_x\varphi/\pi$ - identifying ρ_c and A^2

But instead: $\partial_x n_c=\rho_c\partial_x^2\varphi/\pi$

Never a closed expression for **j**

screening of E_x with a standard local screening length $l^2=r_0^2/\rho_n$

$$-\rho_c E_x+\left(\rho_c\partial_x^2+\partial_y(\alpha A^2\partial_y)-\gamma_\varphi\partial_t\right)\varphi=0$$

Phase eq.

Resembles GL with ρ_c as A^2 but
with no differentiation of the amplitude :

$\rho_c\partial_x\Phi$ instead of $\partial_x(A^2\Phi)$

$\rho_c\partial_x^2\varphi$ instead of $\partial_x(A^2\partial_x\varphi)$

Not like variational eqs.

The limit of the local electro-neutrality $r_0 \to 0$ together with the infinite normal conductivity.

$$\partial_x \Phi + (\pi N_F)^{-1} \nabla_\perp A^2 \nabla_\perp \varphi - \gamma_\varphi \partial_t \varphi = 0 \ , \ \Phi = (n/N_F - \zeta)$$

$$\partial_x \varphi + \pi n = 0$$

Curiously, no commonly assumed longitudinal phase rigidity $\propto \partial_x^2 \varphi$

It is hidden in the term $\partial_x \Phi$ implicitly, via relations

$$\frac{\rho_c}{\rho_n} \partial_x^2 \varphi + \kappa_\perp \nabla_\perp \left(A^2 \nabla_\perp \varphi \right) - \gamma_\varphi \partial_t \varphi = \pi N_F \frac{\partial \zeta}{\partial A} \partial_x A$$

Coulomb hardening looks intuitive, but where is the driving force?

The drive comes only from the boundary conditions for Φ transferred to the phase via the local relations of Φ and $\partial \varphi$ mediated by n.

Examples of numerical solution of the anomolouse TDGL