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1997 Europhys. Lett. 37 591

(http://iopscience.iop.org/0295-5075/37/9/591)

EUROPHYSICS LETTERS 20 March 1997

Europhys. Lett., **37** (9), pp. 591-596 (1997)

Exact low-density free energy and algebraic tails of static correlations in quantum plasmas in a uniform magnetic field

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(received 20 January 1997; accepted 17 February 1997)

PACS. 05.30–d – Quantum statistical mechanics.
PACS. 05.70Ce – Thermodynamic functions and equations of state.
PACS. 71.45Gm– Exchange, correlation, dielectric and magnetic functions, plasmons.

Abstract. – In nonrelativistic Coulomb plasmas with quantum dynamics and statistics, the effect of an external uniform magnetic field upon thermodynamics and the algebraic tails of position correlations is discussed. The results are obtained in the framework of a path integral formalism. The exact virial expansion of the free-energy density at finite temperature is calculated up to order $\rho^{5/2}$ in the density ρ. It exhibits the interplay of Coulomb interactions and quantum statistics with bare diamagnetism and paramagnetism. Moreover, the static particle correlations are shown to decay as $1/r^5$ at large distances r, and the exact analytical coefficients of these tails are given at first order in ρ.

From the point of view of statistical mechanics and fundamental interactions, matter in many situations has to be considered as a quantum nonrelativistic plasma of point charges with Coulomb interactions. Recently, the exact free-energy density [1] up to order $\rho^{5/2}$ (where ρ is a generic notation for the particle densities) was produced, and the limiting values of the large-distance behaviours of static correlations [2] were also calculated at first order in ρ. These exact analytical results, which contain all quantum effects at any order in $\hbar$, have been obtained at low density and finite temperature, which corresponds to a regime of low degeneracy and weak Coulomb coupling. (For instance, they are valid for the nucleus-electron plasma in the core of the sun or for the electron-hole gas in intrinsic semiconductors). In the present letter, we study the changes that arise in the presence of a uniform external magnetic field $\mathbf{B}_0$, which is coupled both to position and spin variables. The difficulty which was eventually overcome in the above references is the exact treatment of Coulomb interaction at any distance (without any regularization) in the quantum many-body problem. In the presence of $\mathbf{B}_0$, we generalize the formalism of ref. [3], which takes exchange effects systematically into account and which uses the path integral representation of the quantum Gibbs factor in order to exactly resum the long-ranged Coulomb divergences directly in position space. Thus, the large-distance decay of position correlations can be investigated at finite density most adequately. Moreover, we can devise systematic low-density expansions. The whole formalism is valid for any strength of $\mathbf{B}_0$.

A detailed account of this work will be published elsewhere.

The results are the following. The expression of the free-energy density up to order $\rho^{5/2}$, which is derived from the microscopic insight of quantum statistical mechanics, exhibits how the orbital diamagnetism arising from quantum dynamics and the Pauli paramagnetism due to the coupling between $\mathbf{B}_0$ and the spin quantum degree of freedom are renormalized and coupled by interactions and quantum statistics. In agreement with Bohr-van Leeuwen theorem, the purely classical terms in the expansion of the free energy are unchanged when $\mathbf{B}_0$ is switched on. On the contrary, the contribution from bound and diffusion states as well as a quantum "diffraction" term, specific to the long range of Coulomb potential, depend on $\mathbf{B}_0$. Moreover, the root of ferromagnetism appears in the effective coupling between spins caused by the combination of Coulomb interactions and quantum statistics (though the fundamental magnetic dipolar interactions between spins are neglected). By a limiting process, we also consider the One-Component Plasma (OCP), which is a fluid of one species of charges moving in a rigid background that ensures global neutrality, and we compare our results with the semiclassical expansion of ref. [4]. On the other hand, quantum Coulomb screening is such that the monopole-monopole and monopole-multipole interactions between charges surrounded by their polarization clouds are exponentially screened at large distances r, whereas the multipole-multipole interactions are partially screened. When $\mathbf{B}_0 = \mathbf{0}$, rotational-invariance arguments together with the harmonicity of the $1/r$ potential imply that the large-distance behaviours of the static particle correlations are controlled by the quantum fluctuations of dipolar-like interactions and decay algebraically as $1/r^6$ [3], [2]. When $\mathbf{B}_0 \neq \mathbf{0}$, the invariance under rotations is broken in one space direction, so that quadrupole-quadrupole interactions survive partially after statistical averaging, and the particle correlations fall off only as $1/r^5$. Here, we exhibit the exact analytical coefficients of the $1/r^5$ tails in the low-density limit mentioned above.

More precisely, we consider a multicomponent plasma of several species α of point particles which obey quantum nonrelativistic dynamics and quantum statistics. Each species is characterized by its mass m_α, its spin $\hbar S_\alpha$, its charge e_α and its Landé factor g_α. The dynamical variables of a particle with index i are its position $\mathbf{r}_i$, with conjugate momentum $\mathbf{p}_i$, and its spin $\hbar\mathbf{S}_i$. In the gauge where the potential vector is $(1/2)\mathbf{B}_0 \wedge \mathbf{r}$, the Hamiltonian of the system reads

$$H(\mathbf{B}_0) = \sum_i \frac{1}{2m_{\alpha_i}} \left(\mathbf{p}_i - \frac{e_{\alpha_i}}{2c}\mathbf{B}_0 \wedge \mathbf{r}_i\right)^2 - \sum_i g_{\alpha_i}\mu_{B\alpha_i}\mathbf{S}_i \cdot \mathbf{B}_0 + \frac{1}{2}\sum_{i\neq j} \frac{e_{\alpha_i}e_{\alpha_j}}{|\mathbf{r}_i - \mathbf{r}_j|}, \tag{1}$$

where c is the light velocity and $\mu_{\mathrm{B}\alpha} = e_\alpha\hbar/2m_\alpha c$ is the Bohr magneton. The first two terms are the Hamiltonian of the ideal gas in Pauli's theory. At thermal equilibrium characterized by the inverse temperature β and a set of densities $\{\rho_\alpha\}$, the latter system is well-behaved in the Maxwell-Boltzmann (MB) approximation. However, when Coulomb interactions are involved in the description, quantum statistics must be taken into account in order to prevent the macroscopic collapse of charges with opposite signs [5]. In a path integral formalism, the grand partition function of a system of quantum particles with quantum statistics, and which interact through a two-body potential, can be written as the grand partition function of a system of classical loops with random shapes and MB statistics and which also interact via some two-body potential [6], [7]. In the case of Coulomb interactions, it was shown in ref. [3] that the long-ranged $1/r$ tail of the potential between loops can be exactly resummed in some Mayer-like expansions. In fact, the latter formalism can be generalized to the fundamental Hamiltonian (1) in the presence of $\mathbf{B}_0$, for the following reasons. First, the spin variables are coupled only with the external field $\mathbf{B}_0$, and not with the position variables. Second, in the Feynman-Kac-Itô formula [8] for the density-matrix element associated with a given permutation of positions, a loop is associated with each of the cycles in which the permutation of exchanged particles can be decomposed, and the coupling of the position variables with $\mathbf{B}_0$ reduces to the appearance of a product of one-loop phase factors. Moreover, every one-loop phase factor involves only the shape of the loop. Subsequently, the presence of $\mathbf{B}_0$ just renormalizes the statistical weight of loops in the above identity between grand partition functions, and it does not modify the exact resummation process for large-distance Coulomb divergences.

Systematic low-density expansions at finite temperature are devised from the above general loop formalism. The method is different from that of ref. [1], but it uses a similar scaling analysis. (In the method of ref. [1], the exchange effects are not taken into account systematically from the start and appear only as perturbative corrections, while the free energy is derived from other basic formulae.) When $\mathbf{B}_0 = \mathbf{0}$, our method allows to retrieve the exact free-energy density up to order $\rho^{5/2}$ [1]. (We notice that a multiplicative factor 1/2 was omitted in the expressions (4.2) and (4.3) in the third paper of ref. [1].) When $\mathbf{B}_0 \neq \mathbf{0}$, analytical results can still be obtained, because the covariance of independent Brownian paths can be computed explicitly even in the presence of $\mathbf{B}_0$. Let λ_α be the thermal de Broglie wavelength, $\lambda_\alpha \equiv \sqrt{\beta\hbar^2/m_\alpha}$. The dimensionless parameters $u_{C\alpha} \equiv \beta\mu_{B\alpha}B_0$ and $u_{S\alpha} \equiv (g_\alpha/2)u_{C\alpha}$ (with $B_0 \equiv |\mathbf{B}_0|$) are equal to $\beta/2$ times the energies associated with the cyclotronic orbital motion and the spin precession, respectively: $u_{C\alpha} = \beta\hbar\omega_{C\alpha}/2$, where $\omega_{C\alpha} = e_\alpha B_0/m_\alpha c$ is the cyclotronic frequency. For sets of densities that satisfy the local neutrality relation $\sum_\alpha e_\alpha\rho_\alpha = 0$, the difference, up to order $\rho^{5/2}$, between the exact volumic densities f of free energies with or without $\mathbf{B}_0$ reads

$$
\begin{aligned}
&\beta f(\beta,\{\rho_\alpha\},B_0) - \beta f(\beta,\{\rho_\alpha\},B_0{=}0) = \sum_\alpha \rho_\alpha \ln\left(\frac{\sinh u_{C\alpha}}{u_{C\alpha}}\right) + \\
&\quad + \sum_\alpha \rho_\alpha \ln\left(\frac{(2S_\alpha+1)\sinh u_{S\alpha}}{\sinh[(2S_\alpha+1)u_{S\alpha}]}\right) - \frac{1}{2}\sum_\alpha \frac{(-1)^{2S_\alpha}}{2S_\alpha+1}[1+\beta\kappa_D e_\alpha^2]\,\rho_\alpha^2\,(4\pi\lambda_\alpha^2)^{3/2} \times \\
&\qquad \times \int \mathrm{d}\mathbf{r}\left[\frac{(2S_\alpha+1)\tanh u_{S\alpha}}{\tanh[(2S_\alpha+1)u_{S\alpha}]}\,\frac{\sinh u_{C\alpha}}{u_{C\alpha}}\langle -\mathbf{r}|e^{-\beta h_{\mathrm{rel},\alpha}(\mathbf{B}_0)}|\mathbf{r}\rangle - \right.\\
&\quad \left. - \langle -\mathbf{r}|e^{-\beta h_{\mathrm{rel},\alpha}(\mathbf{B}_0=\mathbf{0})}|\mathbf{r}\rangle\right] - \frac{1}{2}\sum_{\alpha,\gamma}[1+\beta\kappa_D e_\alpha e_\gamma]\,\rho_\alpha\rho_\gamma\,(2\pi\lambda_\alpha\lambda_\gamma)^3 \times \\
&\qquad \times \int \mathrm{d}\mathbf{r}\left[\frac{\sinh u_{C\alpha}}{u_{C\alpha}}\frac{\sinh u_{C\gamma}}{u_{C\gamma}}\langle \mathbf{0},\mathbf{r}|e^{-\beta H_{\alpha\gamma}(\mathbf{B}_0)}|\mathbf{0},\mathbf{r}\rangle - \langle \mathbf{0},\mathbf{r}|e^{-\beta H_{\alpha\gamma}(\mathbf{B}_0=\mathbf{0})}|\mathbf{0},\mathbf{r}\rangle\right] + \\
&\quad + \frac{1}{6}\frac{\beta\hbar c}{B_0}\kappa_D^3\sum_\alpha \rho_\alpha e_\alpha L^{[3]}(\beta\mu_{B\alpha}B_0) + o(\rho^{5/2}) \,.
\end{aligned} \tag{2}
$$

At order ρ, all effects are contained in the MB contribution from the gas of independent particles described by Pauli's Hamiltonian. This contribution is the sum of the standard diamagnetic and paramagnetic terms. Two-body exchange effects, which are short-ranged whether there are interactions or not, arise only at the order ρ^2. The long-ranged Coulomb potential is partially screened by collective effects over a length scale κ^{-1} which depends on the density and tends to the Debye value $\kappa_{\mathrm{D}}^{-1} = [4\pi\beta\sum_\alpha \rho_\alpha e_\alpha^2]^{-1/2}$ when exchange effects vanish [3]. Thus, half-integer powers of the density appear in the free-energy density from order $\rho^{3/2}$ on. However, the $\rho^{3/2}$ "Debye" contribution is purely classical and does not involve $\mathbf{B}_0$, in agreement with Bohr-van Leeuwen theorem. $\mathbf{B}_0$ appears only from order ρ^2 on, in contributions from quantum dynamics and statistics, through normalization factors involving $u_{C\alpha}$ and $u_{S\alpha}$ and through Hamiltonian operators. $H_{\alpha\gamma}$ is the two-body Hamiltonian without the spin contribution and

$h_{\mathrm{rel},\alpha}(\mathbf{B}_0) \equiv (1/m_\alpha)\,(\mathbf{p} - (e_\alpha \mathbf{B}_0/4c) \wedge \mathbf{r})^2 + e_\alpha^2/r$ is the Hamiltonian of a relative particle. (Indeed, for two particles of the same species, the position of the center of mass, with mass $2m_\alpha$ and charge $2e_\alpha$, and that of the relative particle, with mass $m_\alpha/2$ and charge $e_\alpha/2$, are separable variables even when $\mathbf{B}_0 \neq \mathbf{0}$.) The bound and diffusion states are contained in the quantum density-matrix elements. The two-body exchange effects, which are short-ranged, are not altered by any collective phenomenon at order ρ^2, while, at order $\rho^{5/2}$, the bare contribution is only renormalized by a multiplicative factor arising from classical Debye screening. In the direct terms, the difference between the quantities with $\mathbf{B}_0 \neq \mathbf{0}$ and $\mathbf{B}_0 = \mathbf{0}$ automatically performs the truncations needed for extended states, and the corresponding contribution at order ρ^2 is the same as in the case of a short-ranged interaction. However, the long range of Coulomb potential is responsible for an extra quantum diffraction term, which arises from the partial quantum screening at large distances. This term vanishes at order ρ^2 because of the local neutrality relation. It can be decomposed into a part independent of $\mathbf{B}_0$ plus a correction which involves a generalization $L^{[3]}(u_{C\alpha})$ of the Langevin function that appears in the orbital magnetization of a gas of independent charges: $L^{[3]}(u_{C\alpha}) \equiv \coth u_{C\alpha} - 1/u_{C\alpha} - u_{C\alpha}/3$. Thus, the diffraction contribution at order $\rho^{5/2}$ is proportional to B_0^2 when $\mathbf{B}_0$ is weak.

The free energy of the OCP is derived from the formulae valid for a two-component plasma (TCP) by sending the mass of one species to infinity, then its charge to zero while keeping charge neutrality. Up to order $\rho^{5/2}$, the result is similar to (2), apart from the diffraction term, which does not vanish at order ρ^2 and reads $(4\pi/3)(\beta\hbar ec/B_0)\rho^2 L^{[3]}(\beta\mu_{\mathrm{B}}B_0)$. Besides, in regimes of low degeneracy and weak quantum dynamical effects at $u_C \equiv \beta\mu_{\mathrm{B}}B_0$ fixed, the expression of the OCP free energy can be expanded with respect to $\hbar$, because the exchange density-matrix element in position space vanishes exponentially fast when $\hbar$ goes to zero [4] and because the OCP has a well-defined thermodynamic limit even with MB statistics. In this semiclassical limit, valid for any strength of Coulomb and magnetic couplings, the quantum term of lowest order in $\hbar$ in the free-energy density is the contribution from the MB gas of independent charges, which is of order ρ. The interactions are involved only in the term of next order in $\hbar$, which is exactly proportional to ρ^2 [4]. We have checked that the semiclassical and low-density expansions are coherent. (In particular, the $\rho^{5/2}$ terms in the low-density expression cancel up to second order in $\hbar$, as they should.)

All thermodynamic quantities can be obtained from the free-energy density. For instance, the pressure $P = -f + \sum_\alpha \rho_\alpha \partial f/\partial\rho_\alpha$ has an expression similar to f up to order $\rho^{5/2}$. On the contrary, the expression of the volumic magnetization $M = -\partial f/\partial B_0$ requires a detailed spectral analysis, which is far beyond the scope of the present paper. The diamagnetic and paramagnetic magnetizations of the MB quantum ideal gas are renormalized and coupled by interactions and quantum statistics. In (2) the term $\rho_\alpha^2 \tanh u_{S\alpha}/\tanh[(2S_\alpha+1)u_{S\alpha}]$ is the sum of the squared densities of particles α in the $2S_\alpha+1$ spin states in the absence of Coulomb interactions, and the combination of the exchange and direct density-matrix elements in position space is linked to the origin of ferromagnetism.

The analysis of the large-distance behaviours of static position correlations in quantum plasmas in a uniform magnetic field is performed along the same lines as in ref. [3]. From the study of the nonanalyticities at small wave numbers in Fourier space, it can be shown that the truncated two-body distribution function $\rho^{\mathrm{T}}_{\alpha\gamma}(r)$ (called particle-particle correlation in the following) decays more slowly than when $\mathbf{B}_0 = \mathbf{0}$, namely as $1/r^5$ instead of $1/r^6$, at any density and finite temperature. After integration over the orientation of the relative position of the particles, the $1/r^5$ tail disappears, while the subleading $1/r^6$ term survives. These algebraic tails are compatible with the screening rule for an external infinitesimal charge [9]. Moreover, the particle-charge and charge-charge correlations, $\sum_\gamma e_\gamma \rho^{\mathrm{T}}_{\alpha\gamma}(r)$ and $\sum_{\alpha,\gamma} e_\alpha e_\gamma \rho^{\mathrm{T}}_{\alpha\gamma}(r)$, also behave as $1/r^5$, whereas, when $\mathbf{B}_0 = \mathbf{0}$, they fall off as $1/r^8$ and $1/r^{10}$, respectively, because of some interplay with the partially exponential screening created by other quantum charges combined with rotational invariance and harmonicity arguments [2], [3]. As expected, the induced charge density given by the linear response theory decays with the same inverse power law as the particle-charge correlation.

In the low-density limit, we have calculated the exact coefficient of the $1/r^5$ tail of the particle-particle correlation at first order in ρ. Its value turns out to coincide with the limit of weak Coulomb coupling for the coefficient found in a simple solvable model. For the sake of pedagogy, instead of giving technical details about the derivation for the fully quantum many-body problem, we present this model. It was first introduced by Alastuey and Martin in ref. [10] in order to exemplify the origin of the algebraic decays generated by quantum fluctuations when $\mathbf{B}_0 = \mathbf{0}$. The system is made of two quantum particles immersed in a classical plasma and whose relative position is $\mathbf{r}$. By using the Feynman-Kac representation of the diagonal matrix-element of the quantum Gibbs factor in position space, it can be shown that this problem is equivalent to that of two loops with random shapes $\lambda_1\boldsymbol{\xi}_1$ and $\lambda_2\boldsymbol{\xi}_2$, with typical extent λ_j and a Gaussian measure $\mathcal{D}(\boldsymbol{\xi}_j)$ [8]. (The $\boldsymbol{\xi}_j(s)$'s are dimensionless Brownian bridges which vanish at $s=0$ and $s=1$.) As shown in sect. VII of ref. [10], the fundamental interaction energy between the loops can be written as the electrostatic potential between charged wires with the same shapes, plus a purely quantum term W,

$$W(\mathbf{r},\boldsymbol{\xi}_1,\boldsymbol{\xi}_2) = e_1 e_2 \int_0^1 \mathrm{d}s \int_0^1 \mathrm{d}s'[\delta(s-s') - 1]\frac{1}{|\mathbf{r} + \lambda_1\boldsymbol{\xi}_1(s) - \lambda_2\boldsymbol{\xi}_2(s')|}. \tag{3}$$

W can be written as a series of terms W_n, each of which decays as $1/r^n$, with $n \geq 3$. After averaging over microscopic configurations of the classical plasma, the correlation between the two quantum particles contains algebraic tails at large distances. These tails are given by the integration of $\exp[-\beta W] - 1$ over the shapes $\boldsymbol{\xi}_j$'s of the loops with a measure $\overline{\mathcal{D}}(\boldsymbol{\xi}_j)$ which involves the excess free energy $F(\boldsymbol{\xi}_j)$ associated with the immersion of a single loop in the classical bath. When $\mathbf{B}_0 \neq \mathbf{0}$, the only difference is that $\overline{\mathcal{D}}(\boldsymbol{\xi}_j)$ contains an extra phase factor $\exp[i(e_j\lambda_j^2/2\hbar c)\mathbf{B}_0 \cdot \int \boldsymbol{\xi}_j \wedge \mathrm{d}\boldsymbol{\xi}_j]$, where $\int \mathrm{d}\boldsymbol{\xi}_j$ is Itô's integral [8]. When $\mathbf{B}_0 = \mathbf{0}$, the measure $\overline{\mathcal{D}}(\boldsymbol{\xi}_j)$ is invariant under rotations, so that $\int \overline{\mathcal{D}}(\boldsymbol{\xi}_1) \int \overline{\mathcal{D}}(\boldsymbol{\xi}_2)[-\beta W]$ contains only powers of the Laplacian of $1/r$, which is short-ranged: $\Delta(1/r) = -4\pi\delta(\mathbf{r})$. Thus, the slowest algebraic tail is given by $\int \overline{\mathcal{D}}(\boldsymbol{\xi}_1) \int \overline{\mathcal{D}}(\boldsymbol{\xi}_2)[\beta W_3]^2/2$ and behaves as $1/r^6$. On the contrary, when $\mathbf{B}_0 \neq \mathbf{0}$, the measure $\overline{\mathcal{D}}(\boldsymbol{\xi}_j)$ is invariant only under rotations in the plane perpendicular to $\mathbf{B}_0$ and under the inversion $\boldsymbol{\xi}_j \to -\boldsymbol{\xi}_j$. Then, if $\mathbf{B}_0$ is parallel to the z-axis, the slowest algebraic tail is given by a $\partial_{zzzz}(1/r)$ term which survives in $\int \overline{\mathcal{D}}(\boldsymbol{\xi}_1) \int \overline{\mathcal{D}}(\boldsymbol{\xi}_2)[-\beta W_5]$ apart from short-ranged terms. Its value involves the differences between the covariances along the z- and x-axes when $\mathbf{B}_0 \neq \mathbf{0}$ for both Brownian bridges $\boldsymbol{\xi}_j$'s. In the limit of weak Coulomb coupling, $F(\boldsymbol{\xi}_j)$ becomes independent of the shape $\boldsymbol{\xi}_j$, the normalized measure $\overline{\mathcal{D}}(\boldsymbol{\xi}_j)$ reduces to its bare value $\mathcal{D}(\boldsymbol{\xi}_j)$ and the $1/r^5$ tail of the correlation in the model happens to be identical to the low-density limit of the exact $1/r^5$ tail in the quantum plasma. (This coincidence does not hold when $\mathbf{B}_0 = \mathbf{0}$ [2].) The result is

$$\rho^{\mathrm{T}}_{\alpha\gamma}(\mathbf{r}) \underset{r\to\infty}{\sim} -\rho_\alpha\rho_\gamma\,\beta^3\hbar^4 \frac{e_\alpha}{m_\alpha}\frac{e_\gamma}{m_\gamma} A(u_{C\alpha}, u_{C\gamma})\frac{P_4(\cos\theta)}{r^5}, \tag{4}$$

where $P_4(x)$ is a Legendre polynomial, θ is the angle between $\mathbf{B}_0$ and $\mathbf{r}$, and $A(u_{C\alpha}, u_{C\gamma})$ is determined by the dynamics of independent charges in a magnetic field,

$$A(u_{C\alpha}, u_{C\gamma}) = \frac{3}{2}\left\{-\frac{1}{u_{C\alpha}^2 - u_{C\gamma}^2}\left[\frac{u_{C\gamma}^2}{u_{C\alpha}^3}\coth u_{C\alpha} - \frac{u_{C\alpha}^2}{u_{C\gamma}^3}\coth u_{C\gamma}\right] + \right.$$
$$\left. + \frac{1}{45} - \frac{1}{3u_{C\alpha}^2} - \frac{1}{3u_{C\gamma}^2} - \frac{1}{u_{C\alpha}^4} - \frac{1}{u_{C\gamma}^4} - \frac{1}{u_{C\alpha}^2 u_{C\gamma}^2}\right\}. \tag{5}$$

The $C_{\alpha\gamma}/r^5$ tail in (4) does disappear after integration over $\cos\theta$, as at any finite density. When $\mathbf{B}_0$ is weak, $C_{\alpha\gamma}$ is proportional to $\hbar^8 B_0^4$. In the strong-field limit, $C_{\alpha\gamma}$ becomes independent of B_0 and is only of order $\hbar^4$ —as well as the coefficient of the $1/r^6$ tail when $\mathbf{B}_0 = \mathbf{0}$ [2]— because the localization enforced by $\mathbf{B}_0$ makes the system less quantum. The low-density algebraic decays of the particle-charge and charge-charge correlations are given by (4) with adequate summation over charges. The tail of the induced charge density, derived from the linear response theory in the loop formalism [3], vanishes at order ρ and appears only at higher orders in density. Indeed, this infinitesimal induced charge is generically given by the linear term in e_α in the particle-charge correlation $\sum_\gamma e_\gamma \rho_{\alpha\gamma}^{\mathrm{T}}(r)$ [2], and $A(u_{C\alpha}, u_{C\gamma})$ is nonlinear in e_α when e_α tends to zero. Eventually, we mention that, in situations where the conditions of weak Coulomb coupling and low degeneracy are met, the algebraic tails dominate the Debye exponential decay only at distances of about ten Debye lengths, as in the absence of $\mathbf{B}_0$ [2].

We notice that all results about correlations also hold for the OCP. We have checked that the result for the OCP that is deduced from eqs. (4) and (5) for a TCP when one mass becomes infinite coincides with the low-density limit of the exact coefficient that is derived from the nonanalytic term in the ρ-expansion of the exact sum rule (5.63) in ref. [9].

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This work was partially supported by the Région Rhône-Alpes. I thank A. Alastuey and B. Jancovici for a careful reading of the manuscript.

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