Fluctuations of Internal Energy Flow in a Vibrated Granular Gas

Andrea Puglisi,[1] Paolo Visco,[1,2] Alain Barrat,[1] Emmanuel Trizac,[2] and Frédéric van Wijland[1,3]

[1]*Laboratoire de Physique Théorique (CNRS UMR8627), Bâtiment 210, Université Paris-Sud, 91405 Orsay cedex, France*

[2]*Laboratoire de Physique Théorique et Modèles Statistiques (CNRS UMR 8626), Bâtiment 100, Université Paris-Sud, 91405 Orsay cedex, France*

[3]*Pôle Matière et Systèmes Complexes (CNRS UMR 7057), Université Denis Diderot (Paris VII), 2 place Jussieu, 75251 Paris cedex 05, France*

(Received 3 February 2005; published 9 September 2005)

The nonequilibrium fluctuations of power flux in a fluidized granular media have been recently measured in an experiment [Phys. Rev. Lett. **92**, 164301 (2004)], which was announced to be a verification of the fluctuation relation (FR) by Gallavotti and Cohen. An effective temperature was also identified and proposed to be a useful probe for such nonequilibrium systems. We explain these results in terms of a two-temperature Poisson process. Within this model, supported by independent molecular dynamics simulations, power flux fluctuations do not satisfy the FR and the nature of the effective temperature is clarified. In the pursuit of a hypothetical global quantity fulfilling the FR, this points to the need of considering candidates other than the power flux.

DOI: 10.1103/PhysRevLett.95.110202 PACS numbers: 02.50.Ey, 05.20.Dd, 81.05.Rm

Granular gases, i.e., gases of macroscopic grains losing part of their kinetic energy during collisions, have been the subject of ardent investigation in the last decade [1]. They display, in experiments as well as in simulations, a rich and intriguing phenomenology: non-Gaussian statistics, breakdown of energy equipartition, spontaneous symmetry breaking (clustering, shear, convection, surface waves, shocks. etc.) are the most striking features of this surprising state of matter (see, e.g., [2] and references therein). In addition, a driven granular gas is an ideal testing ground for nonequilibrium statistical mechanics of dissipative stationary states. An inelastic gas can be kept in a state of constant average total kinetic energy thanks to an external driving mechanism, e.g., by rapidly vibrating its container. In such a system, energy is flowing from the external source/thermostat into the gas and then from the gas into an irreversibly draining sink, represented by inelastic collisions. As a consequence, the fluctuations of global or microscopic physical observables, such as the total energy or the velocity of particles or all internal currents, do not behave as expected in equilibrium statistical mechanics. It is therefore tempting to give an interpretation of the observed fluctuations in terms of the few known theoretical results in nonequilibrium statistical physics. In a recent experiment on vibrated granular gases [3] it has been argued that the statistics of the power flux fulfilled the fluctuation relation (FR) by Gallavotti and Cohen [4,5], even if—as emphasized in [3]—(a) the FR holds under conditions that are not met in the situation under study (in particular, microscopic reversibility is required) and (b) there is no proven relation between the measured power flux and the entropy production entering the original FR.

We show here that these experimental results can be explained in terms of a simple Poisson process. We argue that the measured "power flux" Q is well reproduced by a sum of random and independent energy amounts characterized by two different typical temperatures. Such a quantity does not verify the FR. We nevertheless demonstrate that a straight line with slope $\beta_{\rm eff}$ can be observed when plotting $\log[f_Q(Q)/f_Q(-Q)]$ vs Q, (f_Q being the probability density distribution of Q), in the range which is accessible in experiments and simulations. In [3] the predictability of $\beta_{\rm eff}$ remained an open problem and $1/\beta_{\rm eff}$ was interpreted as an effective temperature. Within the two-temperature model, the value of $\beta_{\rm eff}$ is obtained and it further appears that interpreting this quantity as an inverse effective temperature is problematic. We validate our analysis by a direct confrontation against molecular dynamics (MD) simulations, obtaining very good agreement.

The event driven MD simulations have been performed for a system of N inelastic hard disks with restitution coefficient α, diameter d, and mass $m = 1$. The vertical 2D box of width $L_x = 48d$ and height $L_y = 32d$ is shaken by a sinusoidal vibration with frequency f (period $\tau_{\rm box} = 1/f$) and amplitude $2.6d$. In a collision, two particles lose a fraction $1 - \alpha^2$ of their relative kinetic energy while the total momentum is conserved. Collisions with the elastic walls inject energy and allow the system to reach a stationary state. We have checked that possible inelastic collisions with the walls hardly affect the results. Gravity—set to $g = -1.7df^2$ in order to be consistent with the experiment—has a negligible influence on the measured quantities. We have varied the restitution coefficient from 0.8 up to 0.99 (glass beads yield on average $\alpha \approx 0.9$) and the total area coverage from 0.138 (i.e., $N = 270$) up to 0.32 ($N = 620$). In Fig. 1 (left) a snapshot of the system is shown. During the simulations all the main physical observables are statistically stationary. The local area coverage field $\Phi(x, y)$ and the granular temperature field $T(x, y)$ (defined in $2D$ as the average kinetic energy

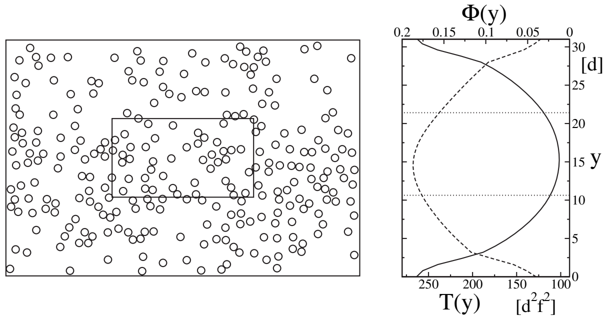

FIG. 1. Left: Snapshot of the system considered for MD simulations, with the inner region marked by the solid rectangle. Right: Corresponding vertical profiles of density [$\Phi(y)$, dashed line] and temperature [$T(y)$, solid line]. The dotted lines mark the bottom and top boundaries of the inner region. Here $N = 270$ and $\alpha = 0.9$. The mean free path is $\sim 5.7d$.

per particle) are almost uniform in the horizontal direction, apart from small layers near the side walls. In Fig. 1 (right) the profiles $\Phi(y) = (1/L_x)\int dx \Phi(x, y)$ and $T(y) = (1/L_x)\int dx T(x, y)$ are shown to be symmetric with respect to the bottom and the top of the box. Following the experimental procedure, we have focused our attention on a "window" in the center of the box, fixed in the laboratory frame, of width $2L_x/5$ and height $L_y/3$, marked in Fig. 1(left). Apart from the negligible change of potential energy due to gravity, the total kinetic energy of the particles inside the window changes during a time τ because of two contributions: $\Delta K_\tau = Q_\tau - W_\tau$, where Q_τ is the kinetic energy transported by particles through the boundary of the window (summed when going in and subtracted when going out), and W_τ is the kinetic energy dissipated in inelastic collisions during time τ. For several values of τ we have measured, as in the experiments, Q_τ, which is related to the kinetic contribution to the heat flux (we checked that inclusion of the collisional contribution, even if nonsmall [6], does not change the picture). With $N = 270$ and $\alpha = 0.9$ the characteristic times are the mean free time $\tau_{\text{col}} \approx 0.47\tau_{\text{box}}$, the diffusion time across the window $\tau_{\text{diff}} = 0.82\tau_{\text{box}}$, and the mean time between two subsequent crossings of particles (from outside to inside) $\tau_{\text{cross}} \approx 0.039\tau_{\text{box}}$.

We define the injected power as $q_\tau = Q_\tau/\tau$ and two relevant probability density functions (PDFs): $f_Q(Q_\tau)$ and $f_q(q_\tau)$. Figure 2(a) shows $f_q(q_\tau)$ for different values of τ. A direct comparison with Fig. 3 of Ref. [3] suggests a fair agreement between simulations of inelastic hard disks and the experiment. The PDFs are strongly non-Gaussian and asymmetric, becoming narrower as τ is increased. At small τ a strong peak in $q_\tau = 0$ is visible. More interestingly, $f_q(q_\tau)$ at small values of τ has two different exponential tails, i.e., $f_q(q_\tau) \sim \exp(\mp\beta_\pm \tau q_\tau)$ when $q_\tau \to \pm\infty$ with $\beta_- > \beta_+$. The peak and the exponential tails at small τ are observed also in the experiment (see Fig. 3 of [3]) and

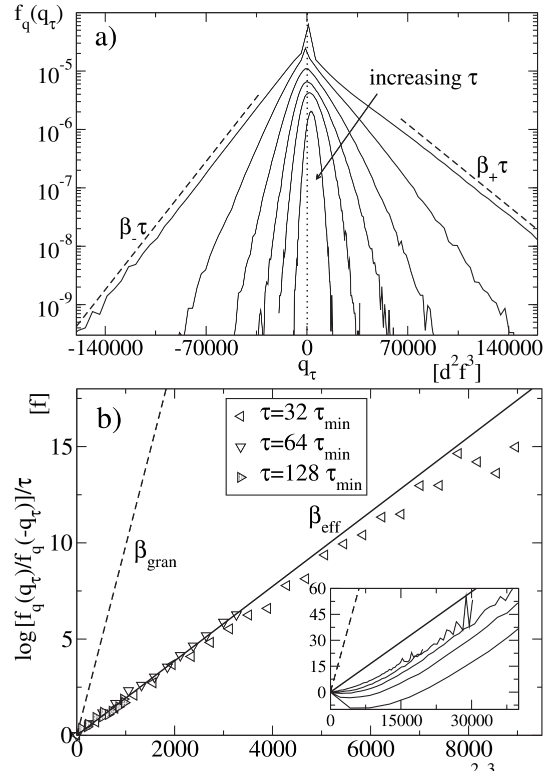

FIG. 2. (a) PDFs of injected power $f_q(q_\tau)$ from MD simulations for different values of $\tau = (1, 2, 4, 8, 16, 32) \times \tau_{\min}$ with $\tau_{\min} = 0.015\tau_{\text{box}}$. Here $N = 270$ and $\alpha = 0.9$. The distributions are shifted vertically for clarity. The dashed lines put in evidence the exponential tails of the PDF at $\tau = \tau_{\min}$. (b) plot of $(1/\tau) \times \log[f_q(q_\tau)/f_q(-q_\tau)]$ vs q_τ from MD simulations (same parameters as above) at large values of τ. The solid curve is a linear fit (with slope β_{eff}) of the data at $\tau = 128\tau_{\min}$. The dashed line has a slope $\beta_{\text{gran}} = 1/T_{\text{gran}}$. In the inset, the same graph is shown for small values of $\tau = (1, 2, 4, 8) \times \tau_{\min}$ (from bottom to top).

in similar simulations [7]. In Fig. 2(b) we display $\log[f_q(q_\tau)/f_q(-q_\tau)]/\tau$ vs q_τ, which is equivalent to the graph of $\pi(q_\tau) - \pi(-q_\tau)$ vs q_τ where $\pi(q_\tau) = \log[f_Q(\tau q_\tau)]/\tau$. When $\tau \to \infty$, $\pi(q_\tau) \to \Pi(q)$, i.e., the large deviation function associated to $f_Q(Q_\tau)$. Under a number of hypothesis, the Gallavotti-Cohen fluctuation theorem [4] states that for the entropy production σ (defined in a dynamical system as the phase space contraction rate) $\Pi(\sigma) - \Pi(-\sigma) = \sigma$. From Fig. 2 it appears that at large values of τ, $\pi(q_\tau) - \pi(-q_\tau)$ is linear with a τ-independent slope $\beta_{\text{eff}} \neq 1$. We have measured β_{eff} with various choices of the restitution coefficient α and of the covered area fraction finding similar results. Reference [3] reports $\beta_{\text{eff}}T_{\text{gran}} \sim 0.25$ where T_{gran} is the mean granular temperature in the observation window. Similar values are measured in our MD simulations. At area fraction 13.8% and $\alpha = 0.9$ we have $\beta_{\text{eff}}T_{\text{gran}} \approx 0.23$. At fixed α and increasing area fraction, $\beta_{\text{eff}}T_{\text{gran}}$ slightly increases, as in the experiment. As $\alpha \to 1$ the slope $\beta_{\rm eff}$ decreases. At $\alpha = 1$ (without gravity and external driving) the distribution of Q_τ is symmetrical and $\beta_{\rm eff} = 0$, indicating that $1/\beta_{\rm eff}$ is not a physically relevant temperature concept. Interestingly, it appears that $\beta_{\rm eff}$ is a nonhydrodynamic quantity: different systems may show the *same* density and temperature profiles, with very different values of $\beta_{\rm eff}$ [8].

We now adopt a coarse-grained description of the experiment which is able to entirely capture the observed phenomenology. The measured flow of energy is given by

$$Q_\tau = \frac{1}{2}\left(\sum_{i=1}^{n_+} v_{i+}^2 - \sum_{i=1}^{n_-} v_{i-}^2\right), \qquad (1)$$

where n_- (n_+) is the number of particles leaving (entering) the window during the time τ, and v_{i-}^2 (v_{i+}^2) are the squared moduli of their velocities. In order to analyze the statistics of Q_τ we take n_- and n_+ as Poisson-distributed random variables with average $\omega\tau$, where ω corresponds to the inverse of the crossing time $\tau_{\rm cross}$. In doing so we neglect correlations among particles entering or leaving successively the central region. A key point, supported by direct observation in the numerical experiment, lies in the assumption that the velocities $\mathbf{v}_{i+}$ and $\mathbf{v}_{i-}$ come from populations with different temperatures T_+ and T_-, respectively. Indeed, compared with the population entering the central region, those particles that leave it have suffered on average more inelastic collisions, so that $T_- < T_+$. Finally we assume Gaussian velocity PDFs [9]. Within such a framework, the distribution $f_Q(Q_\tau)$ of Q_τ can be studied analytically. Here it is enough to recall that $\frac{1}{2} \times \sum_{i=1}^n v_i^2$, in D dimensions, if each component of $\mathbf{v}_i$ is independently Gaussian distributed with zero mean and variance T, is a stochastic variable x with a distribution $\chi_{n,T}(x) = f_{1/T, Dn/2}(x)$, where $f_{\alpha,\nu}(x)$ is the Gamma distribution, and whose generating function reads $\tilde{\chi}_{n,T}(z) = (1 - Tz)^{-Dn/2}$ [10]. It is then straightforward to obtain the generating function of Q_τ in the form $\tilde{f}_Q(z) = \exp[\tau\mu(z)]$ with

$$\mu(z) = \omega[-2 + (1 - T_+ z)^{-D/2} + (1 + T_- z)^{-D/2}]. \quad (2)$$

We observe that $\tilde{f}_Q(z)$ has two poles in $z = \pm 1/T_\pm$ and two branch cuts on the real axis for $z > 1/T_+$ and $z < -1/T_-$. From $\mu(z)$ we immediately obtain the cumulants of $f_Q(Q_\tau)$ through the formula $\langle Q^n \rangle_c = \tau \frac{d^n}{dz^n}\mu(0)$.

For $\tau \to \infty$ the large deviation theory states that $f_Q(Q_\tau) \sim \exp[\tau \Pi(Q_\tau/\tau)]$ and $\Pi(q)$ can be obtained from $\mu(z)$ through a Legendre transform, i.e., $\Pi(q) = \max_z[\mu(z) - qz]$. The study of the singularities of $\mu(z)$ reveals the behavior of the large deviation function $\Pi(q)$ for $q \to \pm\infty$. It can be seen that

$$\Pi(q) \sim -\frac{q}{T_+}(q \to \infty), \qquad \Pi(q) \sim \frac{q}{T_-}(q \to -\infty). \quad (3)$$

We emphasize, however, that it is almost impossible to appreciate these tails in simulations and in experiments, since the statistics for large values of q and τ is very poor.

A Gallavotti-Cohen-type relation [4,5], e.g., $\Pi(q) - \Pi(-q) = \beta q$ for any q and an arbitrary value of β would imply $\mu(z) = \mu(\beta - z)$. One can see that such a β does not exist, i.e., the fluctuations of Q_τ do not satisfy a Gallavotti-Cohen-like relation. The observed linearity of the graph $\log[f_q(q_\tau)/f_q(-q_\tau)]/\tau = \pi(q_\tau) - \pi(-q_\tau)$ vs q_τ can be explained by the following observation [7]: at large values of τ it is extremely difficult, in simulations as well as in experiments, to reach large values of q, while for small q, $\pi(q) - \pi(-q) \approx 2\pi'(0)q + o(q^3)$, i.e., a straight line with a slope $\beta_{\rm eff} = 2\pi'(0)$ is likely to be observed. It has been already shown [11] that in dissipative systems deviations from the FR can be hidden by insufficient statistics at high values of q. The knowledge of $\mu(z)$ is useful to predict this slope. At large τ, $\pi'(0) \approx \Pi'(0) = -z^*(0)$ where $z^*(q)$ is the value of z for which $\mu(z) - qz$ is extremal. This gives

$$\beta_{\rm eff} = 2\frac{\gamma^\delta - 1}{\gamma + \gamma^\delta}\frac{1}{T_-} \text{ with } \gamma = \frac{T_+}{T_-}; \quad \delta = \frac{2}{2 + D}. \qquad (4)$$

When $\gamma = 1$ (i.e., if $\alpha = 1$) $\beta_{\rm eff} = 0$. As α decreases, γ increases, since the collisions dissipate more energy, and $\beta_{\rm eff} T_-$ grows, reaches a maximum, and subsequently decreases asymptotically toward 0 as $\sim\gamma^{\delta-1}$. We emphasize that $\beta_{\rm eff}$ does not depend upon ω. We have compared these predictions with the numerical and experimental results, measuring the temperatures T_+ and T_- in the simulation. To bypass the problem of temperature anisotropy (discussed below), we have used the horizontal component of the temperature, obtaining (with $\alpha = 0.9$ and area fraction 13.8%) $T_+ \approx 141 d^2 f^2$ and $T_- \approx 91 d^2 f^2$, i.e., $\gamma = 1.55$ and, from Eq. (4) in $D = 2$, $\beta_{\rm eff} = 0.00193$ in very good agreement with the measured value 0.0022. It should be noted that the temperature ratio can also be approximated by $(\alpha^2)^{-m}$ where m is the average number of collisions undergone by a particle between the moments of entering and leaving the observation region. In our numerical simulations (as well as in the experiment) $m \approx 2$, which corresponds to $\gamma \approx 1.5$.

What happens for small values of τ? We note that $\tilde{f}_Q(z)$ has the form $\exp[\tau\mu(z)]$ for *any* value of τ and not only for large τ. Therefore at small τ one can expand the exponential, obtaining $\tilde{f}_Q(z) \sim 1 + \omega\tau[-2 + (1 - T_+ z)^{-D/2} + (1 + T_- z)^{-D/2}]$. This immediately leads to an analytical expression for $f_Q(Q_\tau) = \text{const} \times \delta(Q_\tau) + \chi_{1,T_+}(Q_\tau) + \chi_{1,T_-}(-Q_\tau)$, which fairly accounts for the strong peak which is observed in the experiment and in the simulations, and predicts exponential tails for $f_Q(Q_\tau)$: $\chi_{1,T}(x) \propto x^{D/2-1}\exp(-x/T)$ so that $\beta_+ = 1/T_+$ and $\beta_- = 1/T_-$. This suggests an experimental test of this theoretical approach: the measure at small values of τ of the slopes of the

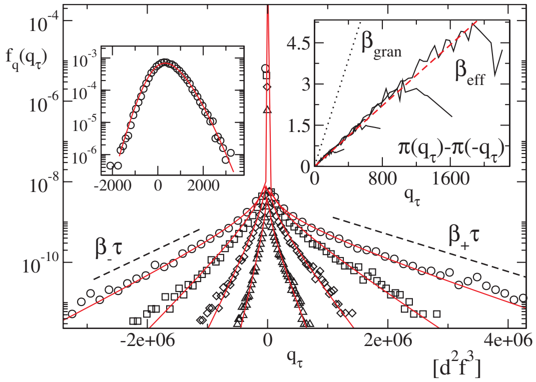

FIG. 3 (color online). PDF of transverse (xy) injected power $f_q(q_\tau)$ for low values of $\tau = (1, 2, 4, 8) \times \tau_{\min}$ with $\tau_{\min} = 0.00015\tau_{\mathrm{box}}$ (resp., circle, squares, diamond, and triangles) from MD simulations with $N = 270$ and $\alpha = 0.9$. The distributions are shifted vertically for clarity. The (red) solid lines show the solution of the "two-temperatures" model. The dashed lines indicate exponential tails $\sim \exp(\mp\beta_\pm \tau q_\tau)$. Left inset: same plot for a large value $\tau = 6400\tau_{\min}$. Right inset: plot of $(1/\tau) \times \log[f_q(Q_\tau)/f_q(-Q_\tau)]$ vs $q = Q/\tau$ from MD simulations (same parameters as above) at large values of $\tau = (1, 2, 4, 8) \times \tau'_{\min}$ with $\tau'_{\min} = 3200\tau_{\min}$, together with a dashed line of slope β_{eff} predicted by Eq. (4) and the dotted line of slope $\beta_{\mathrm{gran}} = 1/T_{\mathrm{gran}}$.

exponential tails of $f_Q(Q_\tau)$ should coincide with a direct measure of T_+ and T_-. However, we point out that the values of β_+ and β_- obtained by fitting the tails in the hard disks simulation, using values as small as $\tau = 0.00015\tau_{\mathrm{box}}$, yield estimates of T_+ and T_- which are larger (by a factor $\sim$1.6) than those found by a direct measure. This disagreement brings the limits of such a simple two-temperature picture to the fore. In the simulation and in the original experiment the measured injected energy is indeed the sum of several different contributions, namely $Q_\tau \approx Q_\tau^{xx} + Q_\tau^{xy} + Q_\tau^{yx} + Q_\tau^{yy}$ where Q_τ^{ij} is the kinetic energy transported by the i component of the velocity by particles crossing the boundary through a wall perpendicular to direction j. Two main differences with the simplified interpretation given above arise: (a) there are *two* couples of temperatures, i.e., T_+^x, T_-^x as well as T_+^y, T_-^y [12]; (b) the diagonal contributions Q_τ^{jj} are sums of squares of velocities whose distribution is not a Gaussian but is $\sim v \exp(-v^2/T)$, since the probability of crossing is biased by the velocity itself. The calculation of $f_Q(Q_\tau)$ is still feasible, with qualitatively similar results [8].

Here we focus on the proposed two-temperature model, showing its ability to explain the statistics of internal energy currents. To this extent we have repeated the MD simulations discussed above, but measuring only the transversal kinetic energy current Q_τ^{xy} through the bottom boundary of the central region. For new measurements, shown in Fig. 3, we have also improved the time resolution of the measure, using a minimum $\tau = 0.00015\tau_{\mathrm{cross}}$. The coefficients of the exponential tails of $f_q(q_\tau)$ at low τ are in perfect agreement with the predicted $\tau/T_\pm$ and the slope of the graph $\log[f_q(q_\tau)/f_q(-q_\tau)]/\tau$ vs q_τ, shown in the inset, is accurately recovered by Eq. (4) with $D = 1$. More remarkably, the two-temperature model (solved by numerical inversion of $\tilde{f}$) fully accounts for the PDF $f_q(q_\tau)$ at *any* value of τ, as evidenced in the same figure.

In conclusion, we have implemented MD simulations of inelastic hard disks fluidized in a vibrated box. We have studied the statistics of the power flux through a closed perimeter—which effectively separates a high temperature region from a low temperature one—thereby accurately reproducing the experimental measures of [3]. Inspired by these results, we have put forward a simple two-temperature Poissonian model, which entirely reproduces the phenomenology of the system but for which the fluctuations of the energy flow do not satisfy a Gallavotti-Cohen relation. Moreover, our approach puts forward a kinetic interpretation for the measured slope β_{eff}, a question previously left open. We conclude here that there is serious evidence against the power flux as a potential candidate for extending the FR to a granular gas, a system for which the validity of such a relation is therefore still open.

We would like to thank B. Meerson for useful discussions. A. P. acknowledges the Marie Curie Grant No. MEIF-CT-2003-500944.

[1] *Granular Gases*, edited by T. Pöschel and S. Luding, Lecture Notes in Physics Vol. 564 (Springer, Berlin, 2001).

[2] A. Barrat, E. Trizac, and M. H. Ernst, J. Phys. Condens. Matter **17**, S2429, (2005).

[3] K. Feitosa and N. Menon, Phys. Rev. Lett. **92**, 164301 (2004).

[4] G. Gallavotti and E. G. D. Cohen, Phys. Rev. Lett. **74**, 2694 (1995).

[5] J. L. Lebowitz and H. Spohn, J. Stat. Phys. **95**, 333 (1999); J. Kurchan, J. Phys. A **31**, 3719 (1998).

[6] O. Herbst, P. Müller, and A. Zippelius, cond-mat/0412334.

[7] S. Aumaitre, S. Fauve, S. McNamara, and P. Poggi, Eur. Phys. J. B **19**, 449 (2001); S. Aumaitre, J. Farago, S. Fauve, and S. McNamara, Eur. Phys. J. B **42**, 255 (2004).

[8] A. Puglisi and P. Visco (to be published).

[9] While non-Gaussian tails are quite common in granular gases [see T. P. C. van Noije and M. H. Ernst, Granular Matter **1**, 57 (1998)], we have checked that they play a negligible role here.

[10] W. Feller, *Probability Theory and its Applications* (John Wiley & Sons, New York, 1971).

[11] J. Farago, J. Stat. Phys. **107**, 781 (2002).

[12] J. Brey and D. Cubero, Phys. Rev. E **57**, 2019 (1998); S. McNamara and S. Luding, Phys. Rev. E **58**, 813 (1998); D. L. Blair and A. Kudrolli, Phys. Rev. E **67**, 041301 (2003).