Paris-Sud / Paris-Saclay University
M1 General Physics

Year 2017-2018
STATISTICAL PHYSICS
Partial exam 2019

2 HOUR 30 MINUTES
DOCUMENTS, POCKET CALCULATORS AND ANY ELECTRONIC DEVICE NOT ALLOWED
Concise but explicative answers expected throughout. No bonus for verboseness

1 Basic questions

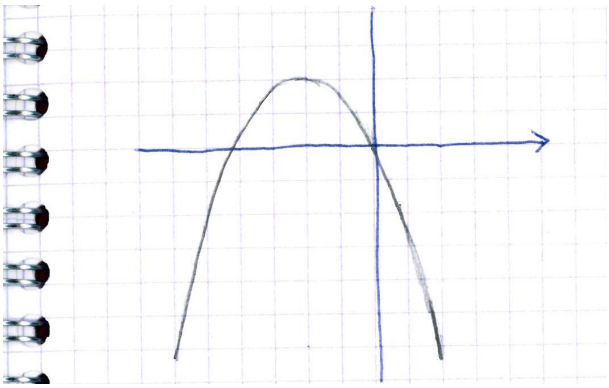
1) Starting from the microcanonical description of a fluid through the function $S(U, V, N)$, express the differential forms of F, G, H and U. Name the various quantities.

2) What is (arguably) the simplest treatment one can think of to describe ferromagnetism ?

3) Explain what the Gibbs-Bogoliubov inequality is about. What is it interesting for in the realm of phase transitions?

4) In a binary mixture, how many intensive variables have to be fixed to specify fully the state of the system? Same question with a ternary mixture.

5) Sketch graphically the Legendre transform of the function in the graph below



6) How is Maxwell plateau argument related to the double-tangent construction?

7) In a fluid, how is c_v, the specific heat at fixed volume, related to the free energy F and the temperature T ? Derive a similar relation between c_p, the specific heat at constant pressure, and some relevant thermodynamic potential to be given.

8) In the theory of continuous phase transitions, how many critical exponents refer to macroscopic properties, and how many do pertain to microscopics? Are these exponents related?

9) We consider a Gaussian random variable X with mean 0 and standard deviation 3. Compute $\langle e^X \rangle$, $\langle e^{-X} \rangle$, $\langle X^2 \rangle$, $\langle X^3 \rangle$ and $\langle X^4 \rangle$. Same question for $\langle e^X \rangle$ and $\langle e^{-X} \rangle$ if the mean is changed to unity, leaving the standard deviation unaffected.

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2 Fluctuation-response connection for ideal magnets

We aim at verifying the fluctuation-response connection for ideal magnets. We consider to this end an ensemble of N non interacting spins taking value ± 1, in an external magnetic field B.

1) What is the Hamiltonian of the system ?

2) Give the corresponding partition function.

3) What is the mean magnetization $\langle M \rangle$?

4) Define and compute the susceptibility, as a function of B (*i.e.* not restricted to small fields).

5) Compute the second moment $\langle M^2 \rangle$? How does it behave in the thermodynamic limit?

6) Relate the variance of M to the susceptibility (for $B \neq 0$).

7) Can ferromagnetism be observed here?

8) What could we call "the ideal magnet equation of state"?

9) Show the fluctuation-response connection on general grounds (for an arbitrary Hamiltonian of an N-spin system).

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3 Landau vs Bragg-Williams approaches

We consider a simplified free energy $F(m)$, where m is the order parameter of some transition, in the form

$$F(m) \,=\, \frac{1}{2} a_2(T)\, m^2 \,+\, a_4\, m^4 \,-\, B\, m \tag{1}$$

where B is some external field. The coefficient a_4 is a positive constant, while a_2 is temperature dependent, taken linear of the form $a_2(T) = \widetilde{a}_2(T - T^*)$, where $\widetilde{a}_2$ is a positive constant.

1) For $B = 0$, provide a sketch of the free energy profiles, identifying the different temperature ranges to be distinguished.

2) To which type of phase transition does this behaviour correspond to?

3) Compute the critical exponent β characterizing the behaviour of m vs T at $B = 0$.

4) Compute the critical exponent γ.

5) Same question for the critical exponent δ, related to the B dependence of the order parameter at the critical temperature.

6) Write Bragg-Williams free energy for the ferromagnetic Ising model.

7) How is it possible to recast Bragg-Williams free energy in the form of Eq. (1)?

8) How do exponents compare to their van der Waals counterpart?

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