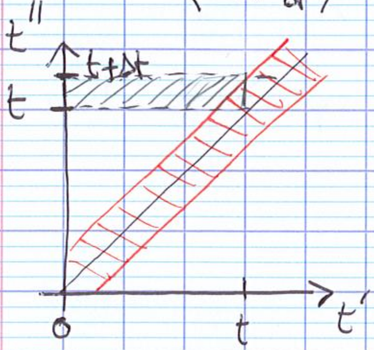
27/11/2022

In Stratonovich discretization calculus, we find quantities of type
$\langle x(t) B_{\Delta t} \rangle$, put to 0 (see below definition of $B_{\Delta t}$)
Why? Take Wiener process for simplicity: $\dot{x}(t) = \sqrt{2D}\, \eta(t)$
with $\langle \eta(t) \eta(t') \rangle = \mathcal{C}(t-t')$ and $\int_{-\infty}^{+\infty} \mathcal{C}(\tau)\, d\tau = 1$
and $\mathcal{C}(\tau) = \mathcal{C}(-\tau)$, τ_c being the correlation time
(ie $\mathcal{C}(\tau) \simeq 0$ for $|\tau| > \tau_c$) $\rightarrow$ thus $\mathcal{C}(0)\tau_c$ is of order 1.
Take $t \gg \Delta t$ and choose x_0 s.t.

$$x(t) = x_0 + \sqrt{2D} \int_0^t \eta(t')\, dt'$$

$$B_{\Delta t} = \int_t^{t+\Delta t} \eta(t'')\, dt'' \sqrt{2D}$$

$$\langle x(t) B_{\Delta t} \rangle = 0 + 2D \int_0^t dt' \int_t^{t+\Delta t} dt'' \langle \eta(t') \eta(t'') \rangle$$

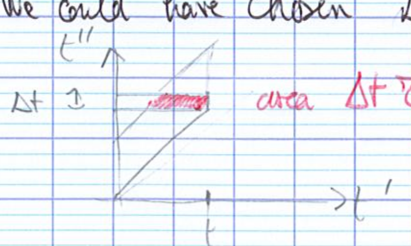


The integral is over //// region and $\langle \eta(t') \eta(t'') \rangle$ is non 0 only on the diagonal and on a region width τ_c around it (in red)
↳ the integral is of order $\mathcal{C}(0)\, \tau_c^2 \simeq \tau_c$

Hence $\langle x(t) B_{\Delta t} \rangle \propto D \tau_c$. We have taken here $\boxed{\Delta t \gg \tau_c}$.

The next step is to consider $\tau_c \rightarrow 0$, to get rid of this non zero correlation.

We could have chosen $\Delta t \ll \tau_c$: $\langle x(t) B_{\Delta t} \rangle \propto \tau_c \Delta t\, \mathcal{C}(0)\, D$
$\propto D \Delta t \neq 0$



area $\Delta t\, \tau_c$

Besides, $B_{\Delta t}$ is no longer decoupled from $\int_{t+\Delta t}^{t+2\Delta t} \eta(t')\, dt'$ $\rightarrow$ complicated.

$\rightarrow$ When computing quantities following Stratonovich discretization, one implicitly has $\Delta t \gg \tau_c$ and even $\tau_c \rightarrow 0$.