Chaos gives quantum tunnelling a hand

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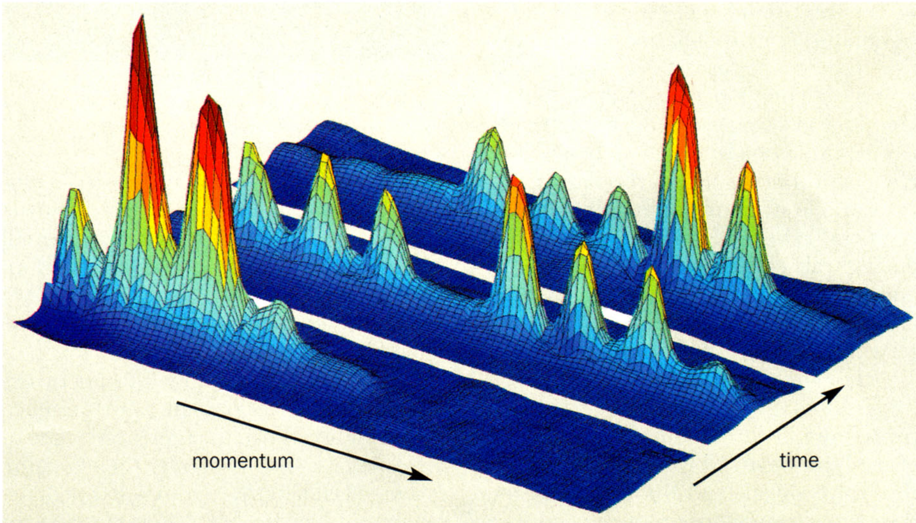
Everyday macroscopic objects follow the laws of classical mechanics. For microscopic objects, such as atoms or nuclei, the wave character of their dynamics has to be taken into account within the framework of quantum mechanics. While wave-like behaviour can show up in various ways, one of the most striking is the tunnelling effect: some events that are classically forbidden for energetic reasons happen because the quantum particles can tunnel under a potential barrier. Similarly, events that are prohibited due to some other dynamical constraints can occur via a process known as dynamical tunnelling. Indeed, tunnelling plays a major role in a variety of physical phenomena, ranging from α-particle radioactivity to the current–voltage characteristics of transistors.



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From the front. The evolution of the momentum distribution of a Bose–Einstein condensate as it oscillates within the wells of an optical lattice. The diffraction peaks at each stage are due to the interference between atoms in different wells. The atoms on the left have negative momentum, while those on the right have positive momentum. Classically we would expect most of the atoms to remain in a state with negative momentum (i.e. remain on the left). At some later time, however, most of the atoms have positive momentum (i.e. they have moved to the right), which is due to the occurrence of dynamical tunnelling.

Being genuinely quantal in origin, one might think that the tunnelling of an atom is unrelated to the motion of a classical particle subject to the same forces. In particular, the classical phenomenon of chaos seems irrelevant, at first sight, in understanding tunnelling because it deals with the extreme sensitivity of classical trajectories to initial conditions. However, theoretical analyses and numerical studies of systems with a few degrees of freedom have shown that this is actually not the case, and that the nature of the underlying classical dynamics can alter drastically the way tunnelling takes place.

In the early 1990s Oriol Bohigas and one of the current authors (DU) at Orsay, together with Steven Tomsovic at Washington State University, demonstrated that the presence of chaotic trajectories in classical systems can increase the rate of quantum tunnelling by several orders of magnitude, and make the trajectories highly fluctuating functions of external parameters. Such effects were first studied experimentally in microwave and optical resonators that can be described by wave equations similar to the Schrödinger equation, rather than with quantum-mechanical objects.

Now two independent teams at the University of Texas, and at the University of Queensland in Australia together with the National Institute of Standards and Technology in Gaithersburg, US, have studied the tunnelling of atoms in the presence of chaos. The work was made possible by the sophisticated tools that have been recently developed to study cold atoms and Bose–Einstein condensates (D Steck *et al.* 2001 *Science* **293** 274; W Hensinger *et al.* 2001 *Nature* **412** 52).

At the energy scale of visible light, an isolated motionless atom can be thought of as an object with a rather simple internal structure that obeys the laws of quantum physics. Of course, this is an oversimplified description because under ordinary physical conditions atoms are neither isolated nor motionless. At room temperature, the speed of atoms is so high (about 100 m s^{-1} in a gas) that they usually interact many times with their environment.

To recover the richness of quantum dynamics, one must "domesticate" the atoms by cooling them down. At microkelvin temperatures, the atoms can be trapped and manipulated with lasers and electrostatic or magnetostatic fields. These atomic waves offer an alternative to light waves and become an extremely accurate and sensitive tool for applications in metrology, interferometry and lithography. Meanwhile, recent work on Bose–Einstein condensates has demonstrated the feasibility of a laser-like atomic beam (see *Physics World* August 1999 pp31–35).

Moreover, model systems can also be built from atoms and tuned to study phenomena in many other fields of physics. For example, neutral atoms exposed to monochromatic light feel a dipolar force that is proportional to the variations in the intensity of the light. Such systems have made it possible to mimic electronic waves in a crystalline potential, and thus provide a fuller understanding of electric conductivity at the quantum level. In this spirit, the Texas and NIST–Queensland teams placed atoms within two standing waves with slightly different frequencies. In this way they produced a time-dependent one-dimensional potential – the minimum condition for chaos to appear. Such a configuration for cold atoms was proposed in the mid-1990s and studied theoretically in great detail by Dominique Delande, one of the present authors (AM) and collaborators in Paris.

Imagine a surfer in the sea where the winds produce a superposition of counter-propagating waves of speed v and $-v$. In such circumstances, the surfer will usually feel the random effect of the waves, being shaken rather than pushed in any definite direction. On the other hand, if the surfer has an initial velocity close to v (or $-v$) just before a wave moving to the right (or left) reaches her, then she will be able to ride the wave.

The forces on the atoms in the above experiments produce a very similar situation. In general, the atoms essentially undergo chaotic motion, but if their initial speed and position are chosen carefully then the atoms will be pushed either to the right or to the left by the electromagnetic wave created by the lasers, while the classical laws of motion forbid any U-turn.

In principle, cold atoms can be prepared with sufficiently small uncertainty in their initial velocity, but this is rather difficult to achieve. Bill Phillips and co-workers at NIST and Queensland succeeded by first producing a Bose–Einstein condensate in which all the atoms are in the same quantum ground state. The condensate was released while the standing-wave potential was turned on in such a way that an atomic wavepacket with zero momentum was obtained. The researchers then shifted the position of the standing wave to give the atoms the proper initial momentum. The approach used by Mark Raizen and co-workers at Texas was similar, except that they used a cloud of cold atoms without actually forming a Bose–Einstein condensate. In both cases, the technology required to produce ultracold atoms was a prerequisite. Furthermore it was necessary to manipulate the resulting cold atoms in a very precise way.

It then became possible to follow the quantum-mechanical evolution of the wavepacket (see figure). At first the atoms followed a classical-like evolution, appearing to be trapped by a wave moving to the right, for example. But after some time, some atoms seemed to surf a wave moving in the *opposite* direction, although they were never observed to stop or change direction. After more time had elapsed, a coherent oscillation was found between the left and right motion. This classically forbidden back and forth "Houdinization" between two regular motions with different velocities was measured in both experiments and corresponds to the first observation of dynamical tunnelling for atoms with "non-integrable" dynamics.

In addition, Raizen and co-workers identified a significant increase in the tunnelling rate due to the presence of a third, presumably chaotic, delocalized state. This behaviour is typical of chaos-assisted tunnelling. Such experiments clearly offer the possibility of studying in depth the effect of chaos on quantum-mechanical tunnelling.

Some of the most exotic and fascinating concepts developed in the context of quantum chaos are now within range of experimental study, thanks to the remarkable progress that has been made in the production and manipulation of cold atoms. This should lead to better understanding and perhaps also to new applications in the context of quantum information.