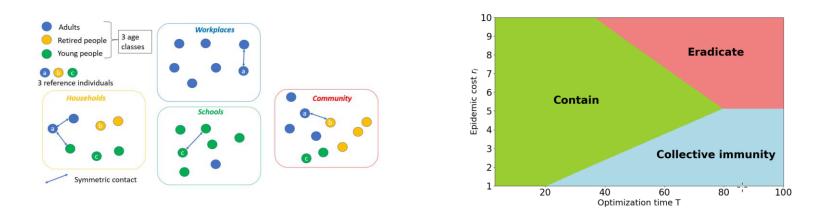






Mean Field Game Approach to Non-Pharmaceutical Interventions in a Social Structure model of Epidemics

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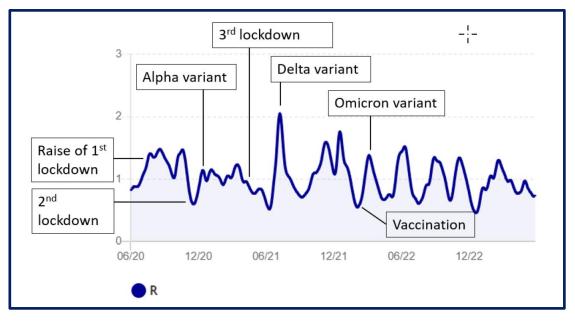


[Phys. Rev. E 106, L062301 (2022); arXiv:2404.08758]

Part I : Mean Field Game & Social Structure model of Epidemics

Evolution of the "effective reproduction number" during the Covid-19 pandemic between June 2020 and June 2023

 $R_{\rm eff}$ = average number of infected persons by a sick individual.



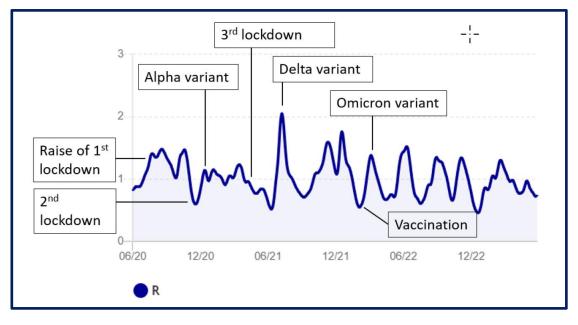
⇒ Significant variations

- Some with easily identified causes.
- Some of theses causes are biological in nature
- Some others are due to changes in behavior

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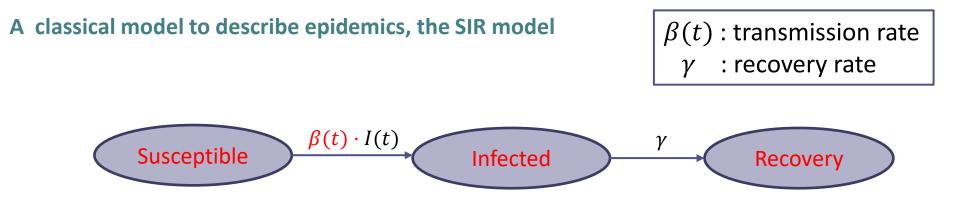
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our focus

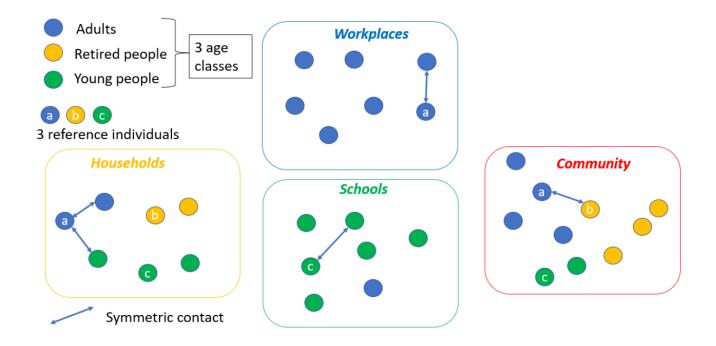


- $\beta(t)$: extrinsic time dependent functional parameter of the model.
- Hard to fit with experimental data (the dynamics of $\beta(t)$ is coupled to the one of the epidemic itself)

- We would like to **make this parameter intrinsic** (i.e. an output, rather than an input of the model)
 - ⇒ Mean Field Game description
- We also want a less homogenous description of the society
 ⇒ Social Structure model of Epidemics
- Eventually, we would like to be able to use our model to discuss "**non-pharmaceutical interventions**" on the epidemics (ie, from the point of view of the health authorities, ways to control the epidemics, other than vaccine or medical treatment)

The SIR model with Social Structure

[Fumanelli et al., PLoS Computational Biology 8 (2012).]



- 3 age classes : Young, Adults, Retired
- 4 "settings" : Households, Workplaces, Schools, Community

Notations and Hypothesis

- N_{α} : proportion of agents in the age class α .
- $(S_{\alpha}, I_{\alpha}, R_{\alpha})$: proportions of (Susceptible, Infected, Recover) in the age class α $[S_{\alpha} + I_{\alpha} + R_{\alpha} = 1]$
- Probability that a pair of individuals (a, b) of age class (α, β) meet in the setting γ in the time interval $[t, t + dt] \rightarrow W^{\gamma}_{\alpha,\beta} dt$
- If they meet when a is infected and b susceptible → probability q of infection.

Dynamical equation for the epidemics

$$\begin{split} \dot{S}_{\alpha} &= -\lambda_{\alpha}(t) S_{\alpha}(t) \\ \dot{I}_{\alpha} &= \lambda_{\alpha}(t) S_{\alpha}(t) - \xi I_{\alpha}(t) \\ \dot{R}_{\alpha} &= \xi \ I_{\alpha}(t) \,. \end{split}$$

$$\lambda_{\alpha}(t) \equiv q \sum_{\beta=1}^{n_{\rm cl}} \sum_{\gamma=1}^{n_{\rm set}} W_{\alpha\beta}^{\gamma} N_{\beta} I_{\beta}(t)$$
$$\lambda_{\alpha}(t) : \text{force of infection}$$

Mean Field Game description

[Elie et al., Mathematical Modelling of Natural Phenomena 15 (2020)]

Optimization for a given representative agent $a \in \alpha$

<u>State variable</u> $x_a \in \{S, I, R\}$

Control variable

- $W_{\alpha,\beta}^{\gamma} dt$: pb that a pair of individuals meet in the time interval [t, t + dt]
- $W_{\alpha,\beta}^{\gamma} = w_{\alpha,\beta}^{\gamma} w_{\beta,\alpha}^{\gamma}$, with $w_{\alpha,\beta}^{\gamma} =$ "willingness of agents of class α to meet an agent of class β in setting γ " ($W_{\alpha,\beta}$ symmetric, $w_{\alpha,\beta}$ not necessarily)

$$w_{\alpha,\beta}^{\gamma}(t) = n_{\alpha}(t)w_{\alpha,\beta}^{\gamma(0)}$$

 $n_{\alpha}(t) \in [n_{\alpha,\min}, 1]$

hyp:

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:

$$w_{\alpha,\beta}^{\gamma}(t) = n_{\alpha}(t) v_{\alpha,\beta}^{\gamma(0)}$$
 $n_{\alpha}(t) \in [n_{\alpha,\min}, 1]$
Control variable

hyp:

Cost function

• Cost paid by individual a susceptible at time t if infected at time τ

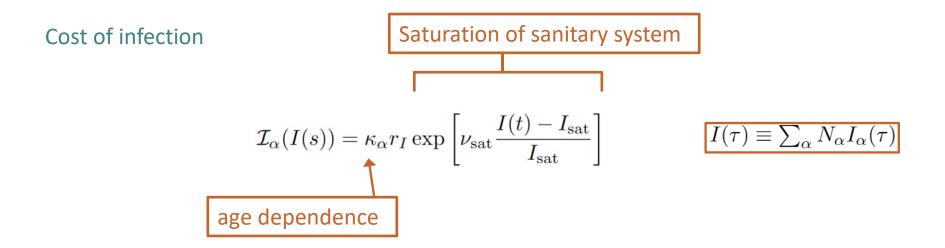
Strategy of
agent *a*
$$C_a\left(n_a^{\gamma}(\cdot), \{n_{\beta}^{\gamma}(.)\}, t, \tau\right) \equiv \mathcal{I}_{\alpha}(I(\tau))\mathbb{1}_{\tau < T} + \int_t^{\min(\tau, T)} f_{\alpha}\left(n_a^{\gamma}(s)\right) ds$$
(social) ds
(social) cost of effort

• Expectation value $(P_a(\tau) = d\phi_a/d\tau \equiv \text{proba to be infected at } \tau)$

$$C_a\left(n_a^{\gamma}(\cdot), \{n_{\beta}^{\gamma}(\cdot)\}, t\right) \equiv \int_t^{\infty} d\tau \ P_a(\tau) \ C_a\left(n_a^{\gamma}(\cdot), \{n_{\beta}^{\gamma}(\cdot)\}, t, \tau\right),$$
$$= \int_t^T \left[\lambda_a(s) \ \mathcal{I}_{\alpha}(I(s)) + f_{\alpha}\left(n_a^{\gamma}(s)\right)\right] (1 - \phi_a(s)) ds.$$

$$\phi_a(\tau) = 1 - \exp\left(-\int_t^\tau \lambda_a(s)ds\right)$$

 $\lambda_{\alpha}(t)$: force of infection



Social cost of effort (same form as Elie et al.)

$$f_{\alpha}(n_{a}^{\gamma}(t)) = \sum_{\gamma} \left[\left(\frac{1}{n_{a}^{\gamma}(t)} \right)^{\mu_{\gamma}} - 1 \right]$$

Bellman equation

<u>Bellman</u>

$$U_a(t) = \min_{n_a^\gamma(t)} \; \mathbb{E}_{x_a(t+dt)} \left[U_a(t+dt) + c_a(t)
ight]$$

$$c_a(t) = \begin{cases} f_\alpha(n_a^\gamma(t)) \, dt & a \text{ susceptible at } t + dt \\ \mathcal{I}_\alpha(I(t)) & a \text{ infected at } t + dt \end{cases}$$

<u>HJB</u>

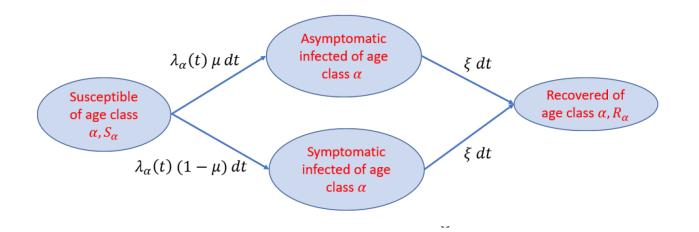
$$-\frac{dU_a(t)}{dt} = [\lambda_a(t) \left(\mathcal{I}_\alpha(I(t)) - U_a(t)\right) + f_\alpha(n_a^{\gamma*}(t))]$$
$$n_a^{\gamma*}(t) = \operatorname*{argmin}_{n_a^{\gamma}(t)} [\lambda_a(t) \left(\mathcal{I}_\alpha(I(t)) - U_a(t)\right) + f_\alpha(n_a^{\gamma}(t))]$$
Optimization at *t* only

What about infected agent behavior ?

The force of infection $\lambda_a(t)$ depend on the strategies (i.e. the $\{n_\beta\}$) of infected agents.

Possible assumptions :

- Infected agents stay at home $\rightarrow \lambda_a(t) = 0$, no propagation
- Infected agents do not care $\rightarrow n_{eta}(t) = 1$
- **Our choice** : propagation of the epidemics is due to a small number of **asymptomatic agents** (who behave as susceptible ones).



Mean Field Game equations

<u>Dynamics</u> ("Kolmogorov")

$$\dot{S}_{\alpha} = -\lambda_{\alpha}(t)S_{\alpha}(t)$$
$$\dot{I}_{\alpha} = \lambda_{\alpha}(t)S_{\alpha}(t) - \xi I_{\alpha}(t)$$
$$\dot{R}_{\alpha} = \xi I_{\alpha}(t).$$

$$\lambda_{\alpha}(t) \equiv \mu q \sum_{\beta=1}^{n_{\rm cl}} \sum_{\gamma=1}^{n_{\rm set}} n_{\alpha}^{\gamma}(t) n_{\beta}^{\gamma}(t) W_{\alpha\beta}^{\gamma(0)} N_{\beta} I_{\beta}(t)$$

Biology
$$(\xi = 1.2, q = 0.2, \mu = 0.1)$$

$$(\varsigma = 1.2, q = 0.2, \mu = 0.1)$$

<u>Social Structure</u> $(M^{\gamma}_{\alpha\beta} \equiv W^{\gamma}_{\alpha\beta}N_{\beta})$

M^S	M^W	M^C	M^H	N_{lpha}
$(100 \ 0 \ 0)$	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	$(12.5 \ 25 \ 12.5)$	$(15 \ 25 \ 10)$	
0 0 0	0 75 0	$12.5\ \ 25\ \ 12.5$	$12.5 \ \ 32.5 \ \ 5$	$\left(0.25, 0.5, 0.25\right)$
$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	$(12.5 \ 25 \ 12.5)$	10 10 30	

[Fumanelli et al.]

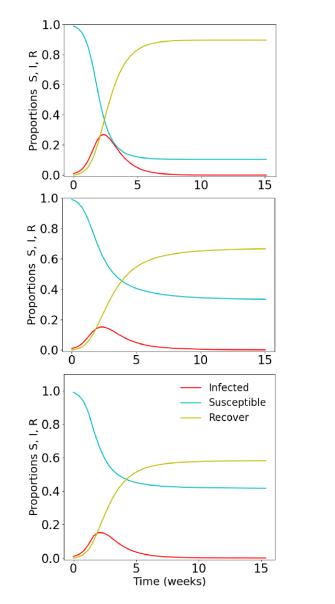
Cost of infection

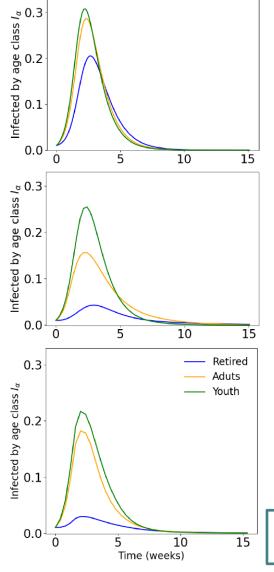
Social cost

$$\frac{\hline r_I \quad \kappa_{\alpha} \quad (I_{\rm sat}, \nu_{\rm sat})}{1 \quad (1, 10, 100) \quad (0.1, 0.1)}$$

$$\frac{n_{\min}^{\gamma} \quad \mu_{\gamma}}{\left(\frac{1}{3}, \frac{1}{5}, \frac{1}{5}, \frac{1}{2}\right) \ (2, 2, 1, 3)}$$

Epidemic dynamics





$$\frac{\text{Business as usual}}{(n_{\alpha}^{\gamma}(t) \equiv 1)}$$

 $\frac{\text{(free) Nash}}{(n_{\alpha}^{\gamma}(t) \leftarrow \text{MFG})}$

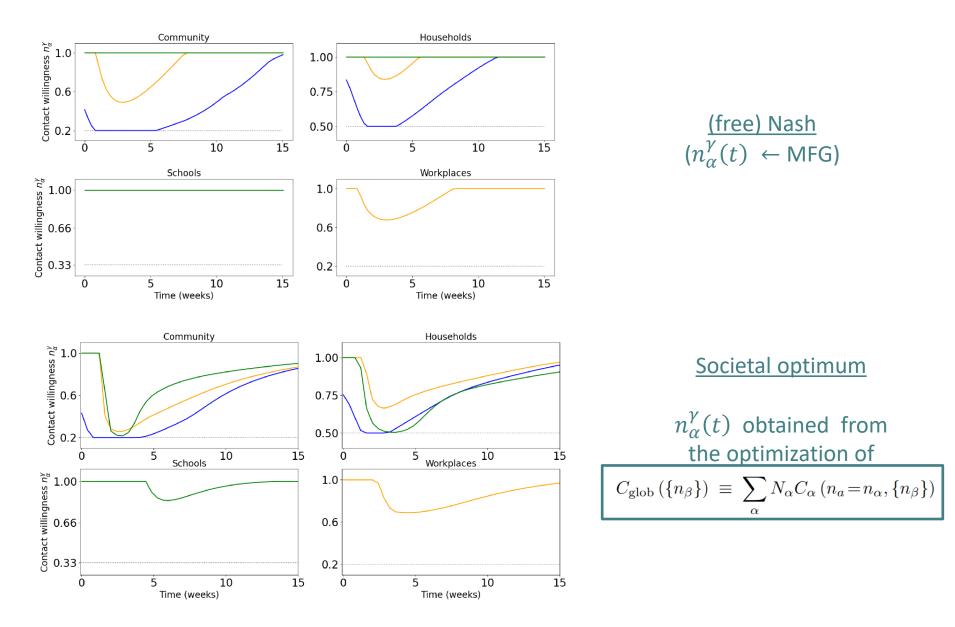


 $n_{\alpha}^{\gamma}(t)$ obtained from the optimization of

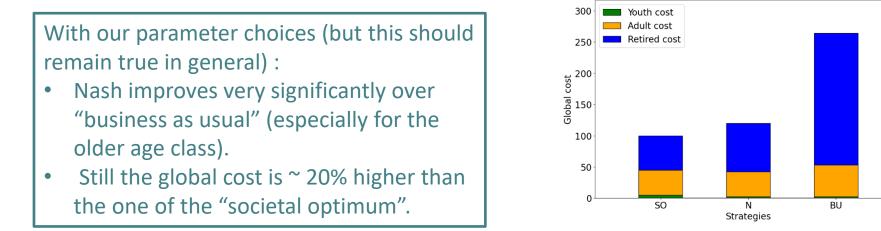
$$C_{\text{glob}}\left(\{n_{\beta}\}\right) \equiv \sum_{\alpha} N_{\alpha} C_{\alpha}\left(n_{a} = n_{\alpha}, \{n_{\beta}\}\right)$$

Corresponding strategies (i.e. $n_{\alpha}^{\gamma}(t)$, $\alpha = (young, adults, retired)$,

 γ = (community, housholds, schools, workplace))



Constrained Nash



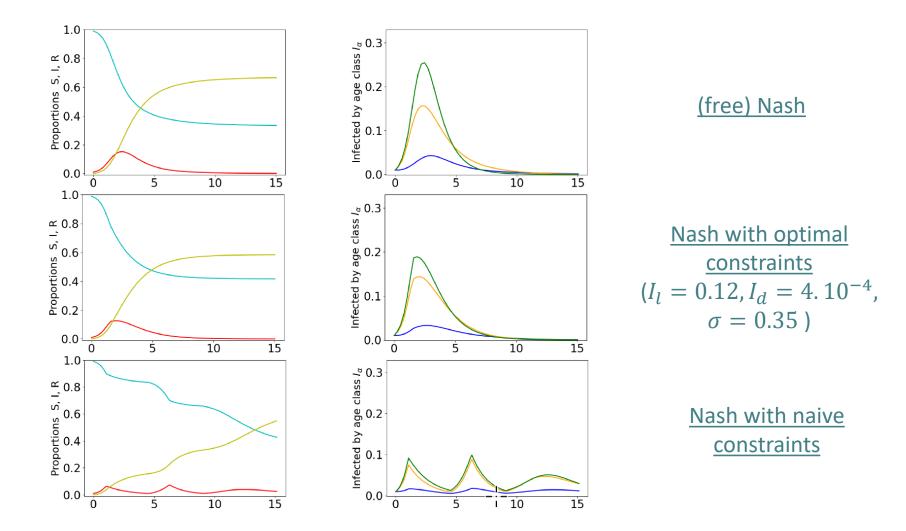
Can we bridge the gap (at least partially) by imposing **local constraints** similar to **lockdowns** ?

Constrained Nash (still a MFG)

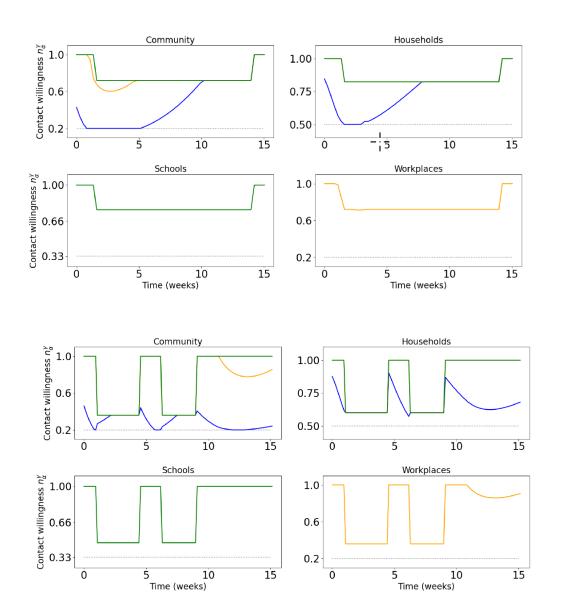
- Two thresholds :
 - \circ when $I(t) > I_l$: lockdown imposed
 - when $I(t) < I_d$: lockdown lifted
- Lockdown $\implies n_{\alpha}^{\gamma}(t) \in [n_{\alpha,\min}^{\gamma}, n_{\alpha,l}^{\gamma}]$

 $n_{\alpha,l}^{\gamma} = \sigma n_{\alpha,\min}^{\gamma} + (1-\sigma)$

Dynamics for constrained Nash



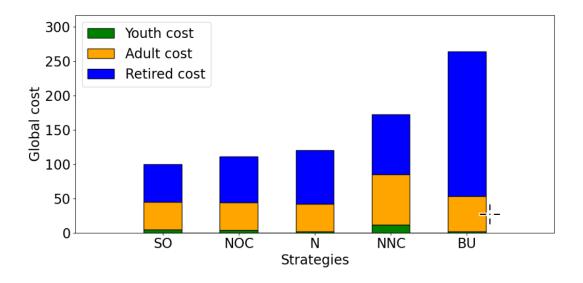
Strategies for constrained Nash



Nash with optimal constraints

Nash with naive constraints

Local conclusion



- Nash improves very significantly over "business as usual" (especially for the older age class).
- The gap between free Nash and Societal Optimum can be partially bridged by imposing well chosen constraints.
- Naïve constraints on the other hand can result in a cost significantly worse than free Nash, by degrading the situation for both adults and young without improving it for retired people.

Part II : phase transition for optimal strategies

- Until now we have assumed :
 - *T* (total optimization time) $\rightarrow \infty$
 - *N* (total number of agents) $\rightarrow \infty$

 \Rightarrow The only way out of the epidemic was "herd immunity".

- However,
 - If *T* finite (anticipation of a vaccine, seasonality of the disease, etc.) ⇒ one may try to "contain" the epidemic.
 - If *N* finite (Island, small country with tight borders)

 \Rightarrow one may even try to "eradicate" the epidemic.



What kind of change would this imply ?

Herd immunity

Herd immunity for the basic SIR

In our case

We say we have herd immunity at t, if, without effort $\dot{S}(t') < 0$, $\forall t' > t$

Reproduction number of age class α :

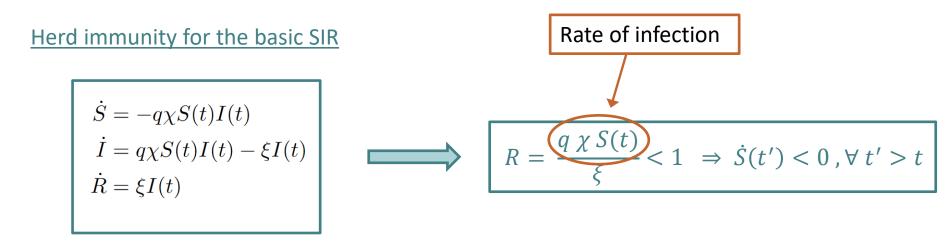
$$R_{\alpha}(t) = \frac{\mu q}{\xi} \sum_{\beta,\gamma} n_{\alpha}^{\gamma}(t) n_{\beta}^{\gamma}(t) W_{\alpha\beta}^{\gamma} N_{\beta} S_{\beta}(t)$$

$$\dot{I} = \xi \sum_{\alpha} N_{\alpha} I_{\alpha} (R_{\alpha} - 1) \quad \Longrightarrow \qquad \text{``Sufficient'' criterion} : R_{\alpha} < 1, \ \forall \alpha$$

Effective criterion :

$$R^{(0)}\equiv\sum_lpha N_lpha R^{(0)}_lpha < 1$$

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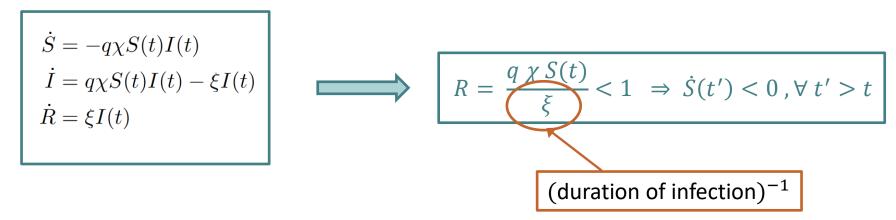
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Scenario classification

- Herd immunity : $R^{(0)}(T) \equiv N_{\alpha}R_{\alpha}^{(0)} < 1$
- Eradication : $I(t < T) = I_{thr} = O(\frac{1}{N})$
- Containment : $R^{(0)}(T) > 1 \&\& I(T) > I_{thr}$

Can we do better ?

For each scenario : **template** = approximation for the **optimal strategies** $n(.) \equiv \{n_{\alpha}^{\gamma}(.)\}$

• Template for herd immunity :

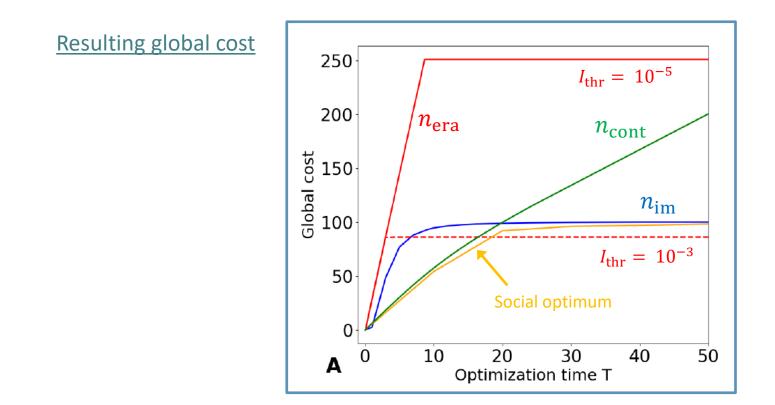
$$\mathbf{n}_{ ext{im}}(.\,) = rgmin_{n_eta^\gamma(.)} \Big[\, C_{ ext{glob}} \left(\{ n_eta^\gamma(.\,) \}, T \longrightarrow \infty
ight) \Big]$$

• Template for containment

$$\mathbf{n}_{ ext{cont}}(\mathbf{1}) = rgmin_{\{n_eta^\gamma\}} \left[\sum_lpha f_lpha(n_lpha^\gamma) \ / \ R(\{n_eta^\gamma\},\{S_eta\!=\!1\}) = 1
ight]$$

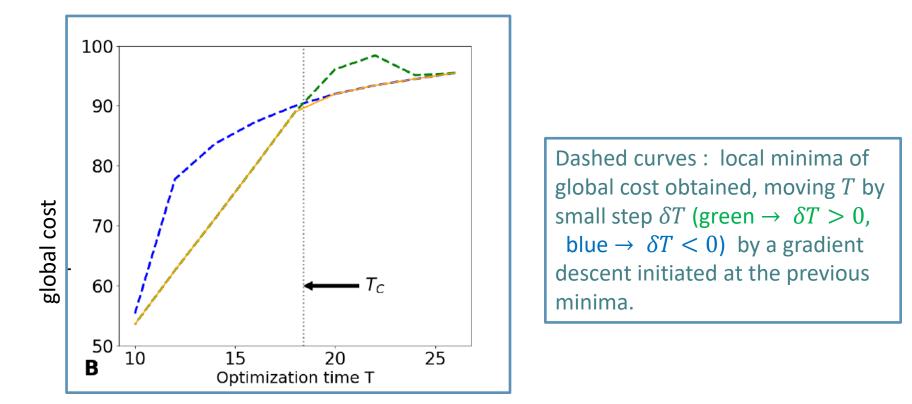
• Template for eradication

Comparison between template and social optimum strategies



Template strategies appear as good proxies for the optimal one

Phase transition between "herd immunity" and "containment



- Existence of local minima of the global cost in a relatively large region around T_C .
- Local minima ~ template strategies \Rightarrow we expect discontinuous change of the optimal strategy at T_C .
- Also discontinuous derivative of C_{glob} at T_C .

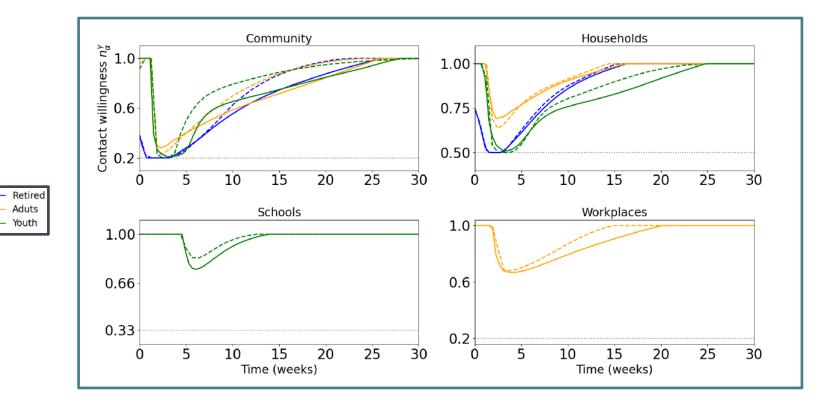


What we have here is a first order phase transition

Comparison between template strategies (dash) and true optimal ones (solid)

Herd immunity regime

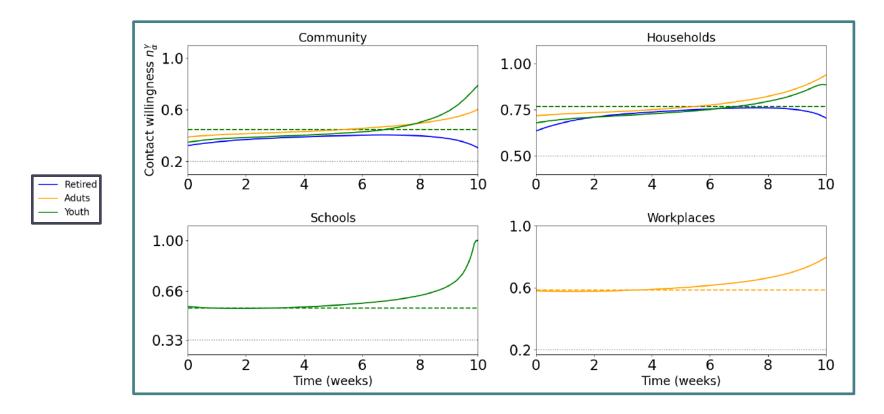
 $(T = 30, I_{\rm thr} = 0)$



Extra effort, especially from the youth, appears beneficial to limit the number of infected.

Comparison between template strategies (dash) and true optimal ones (solid)

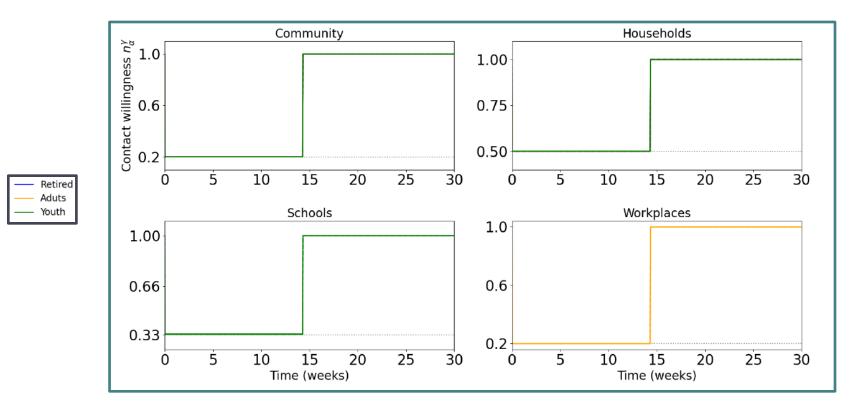
$\frac{\text{Containment regime}}{(T = 10, I_{\text{thr}} = 0)}$



Slightly more effort at the beginning, and quite a bit less in near the end (except for the retired who have to compensate for it), appears beneficial.

Comparison between template strategies (dash) and true optimal ones (solid)

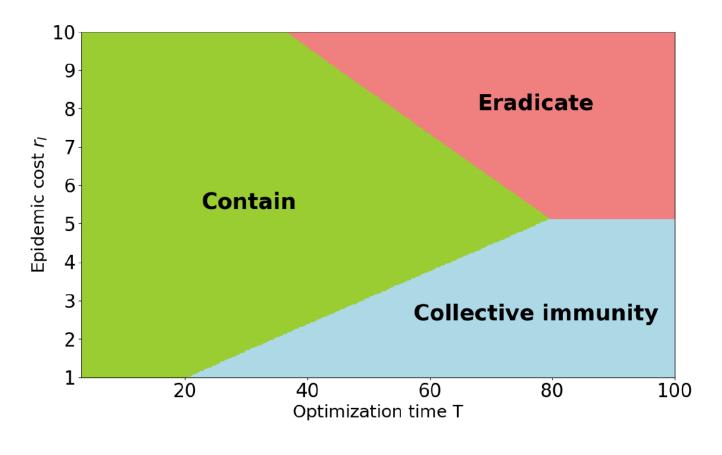
Eradication regime ($T = 30, I_{thr} = 10^{-5}$)



Here the template is the optimal strategy

Phase diagram (T, r_I)

$$\mathcal{I}_{\alpha}(I(s)) = \kappa_{\alpha} r_{I} \exp\left[\nu_{\text{sat}} \frac{I(t) - I_{\text{sat}}}{I_{\text{sat}}}\right]$$



Phase diagram derived from template strategies

Previous phase diagram obtained from (the proxies of) optimal strategies : what about Nash / MFG ?

- Herd immunity : Nash, or even better, the optimized constrained Nash, provides a good approximation of the societal optimal.
- However the **Containment** and **Eradication** strategies are characterized by very low I_{α} (thus small force of infection λ_{α})

 \Rightarrow for an individual *a* optimizing for himself, the best strategy is always to do no effort at all $(n_a^{\gamma} \equiv 1)$.

 \Rightarrow for these two strategies, the societal optimum cannot be approached by a Nash / MFG approach.

In the MFG context, phase diagram becomes trivial (only one phase = "collective-immunity-like"). The green and red regions of the previous figure now correspond to parameter values for which the "cost of anarchy" is large.

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 \Rightarrow but one may argue that a rough approximation can be better than just guesswork as a negotiation basis.

In particular, even at a qualitative level, the role of Nash equilibrium is presumably not well thought.