Phase dislocations in the 2D scattering of microcavity polaritons

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work in collaboration with
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Institute of Spectroscopy
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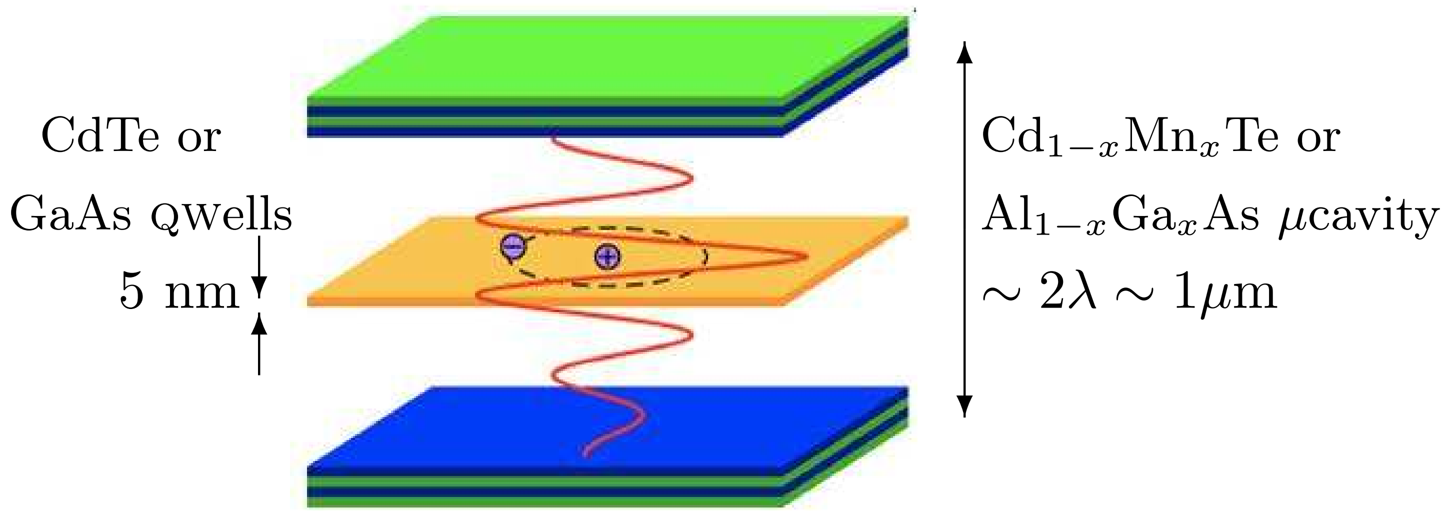
EPJ. D 69, 32 (2015)

also A. Amo, J. Bloch, A. Bramati, I. Carusotto, C. Ciuti, B. Deveaud-Plédran, E. Giacobino, G. Grosso, A. Kamchatnov, G. Malpuech, N. Pavloff, S. Pigeon, D. Sanvitto & D. D. Solnyshkov, arXiv:1401.7347

cavity polaritons



interacting bosons

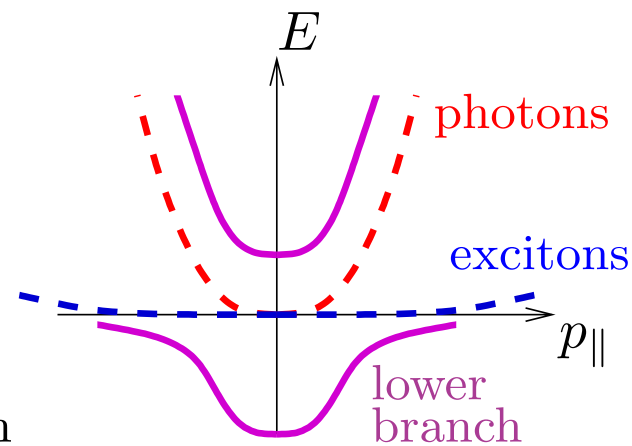
$m_{\rm eff} \lesssim 10^{-4}\, m_{\rm e}$

$T_{\rm BEC} \sim 10$ K

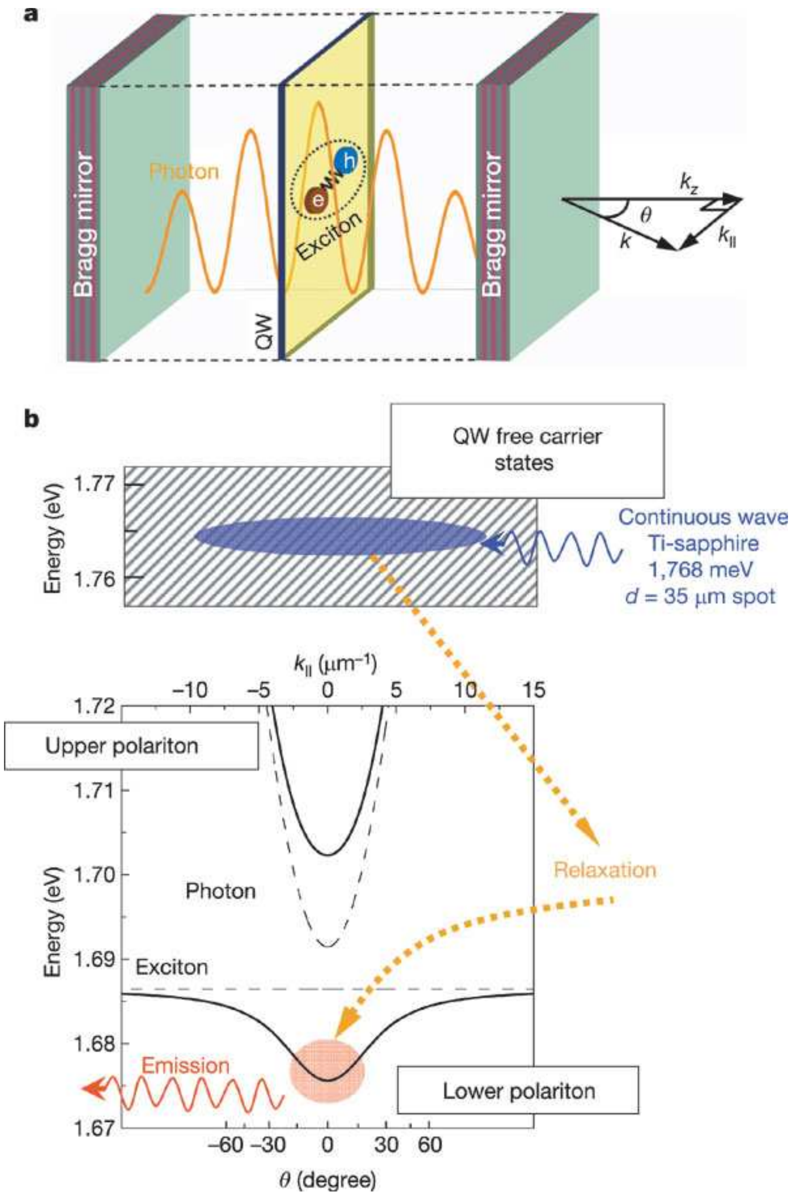
lifetime $\lesssim 50$ ps

optical detection

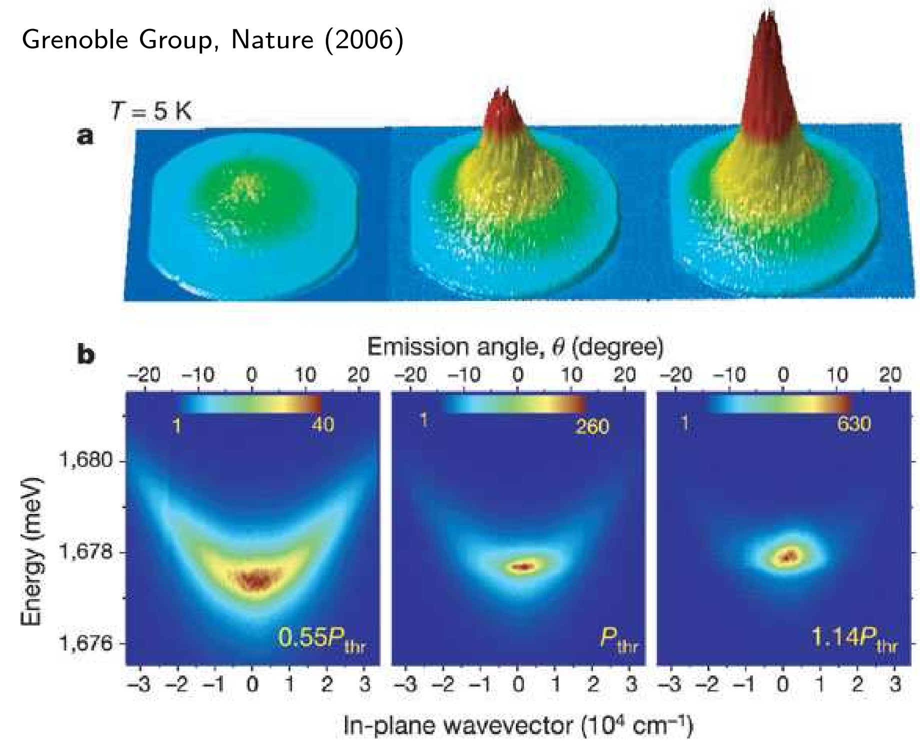
polarization degree of freedom

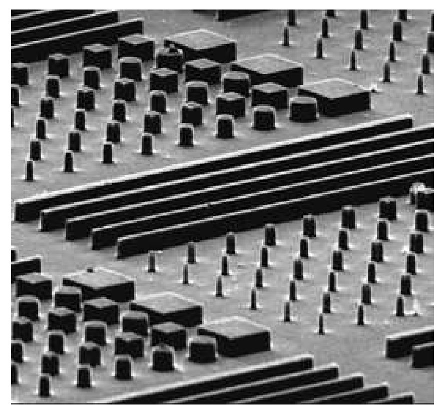


BEC of polaritons (2006)



Grenoble Group, Nature (2006)



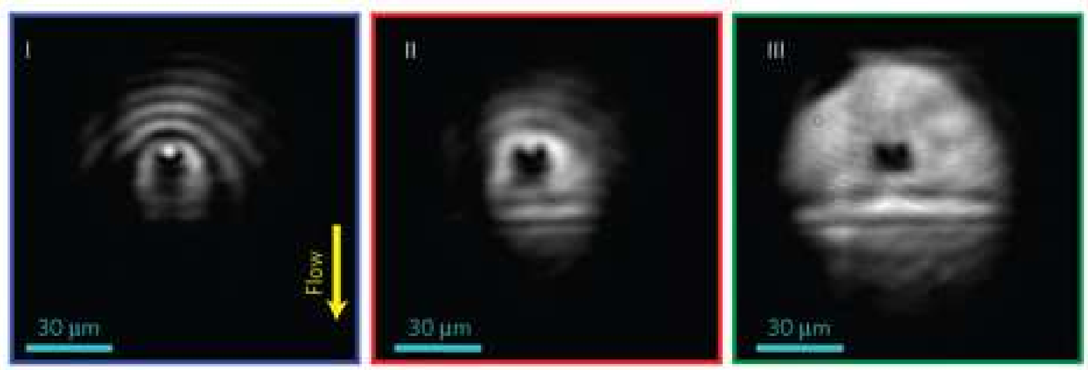


Grenoble : Institut Néel

Marcoussis : LPN

Paris : LKB

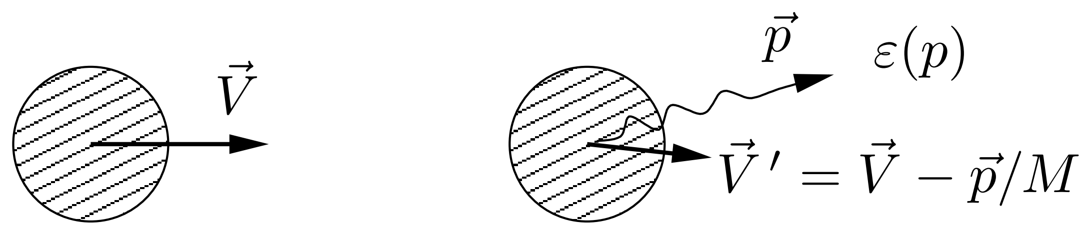
Superfluidity - hydrodynamics



LKB group, Nat. Phys. (2009)

Landau criterion	
left :	$V_{\text{flow}} > V_{\text{crit}}$
right :	$V_{\text{flow}} < V_{\text{crit}}$

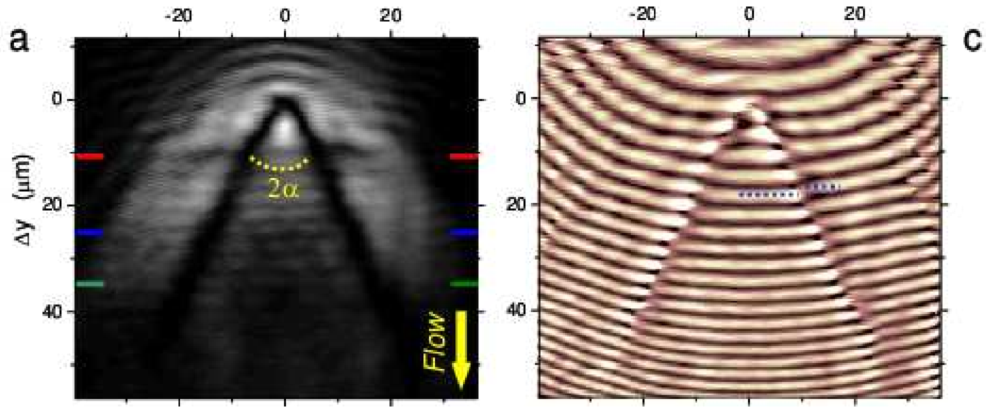
Landau criterion :



Energy and momentum conservation: $\frac{M}{2}V^2 = \frac{M}{2}\left(\vec{V} - \frac{\vec{p}}{M}\right)^2 + \varepsilon(p)$.
for $M \gg m$ this reads $\varepsilon(p) = \vec{V}.\vec{p}$

emission of excitations possible only if $\boldsymbol{V > v_{\mathrm{L}} = \min\left[\frac{\varepsilon(p)}{p}\right]}$

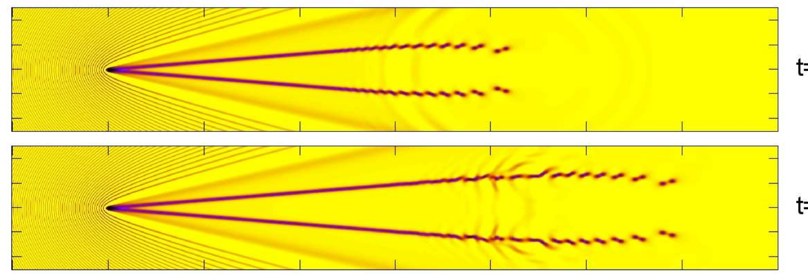
supersonic flow



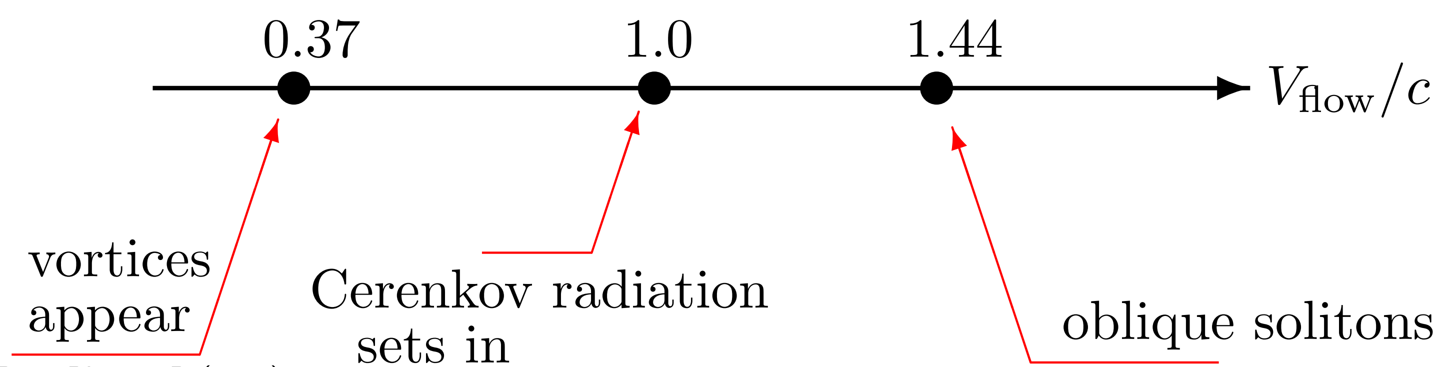
LKB group, Science (2011)

convective instability of oblique dark solitons

El,Gammal,Kamchatnov PRL (2006)



Expected scenario in 2D: (neglecting damping and polarization effects)

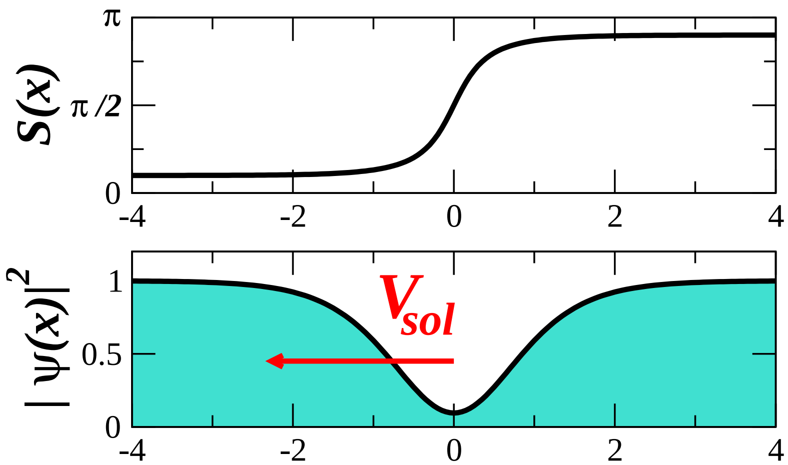


Rica, Physica D (2001)
Berloff, Roberts, J Phys A (2001)
Stießberger, Zwerger, PRA (2000)

Kamchatnov,Pitaevskii PRL (2008)

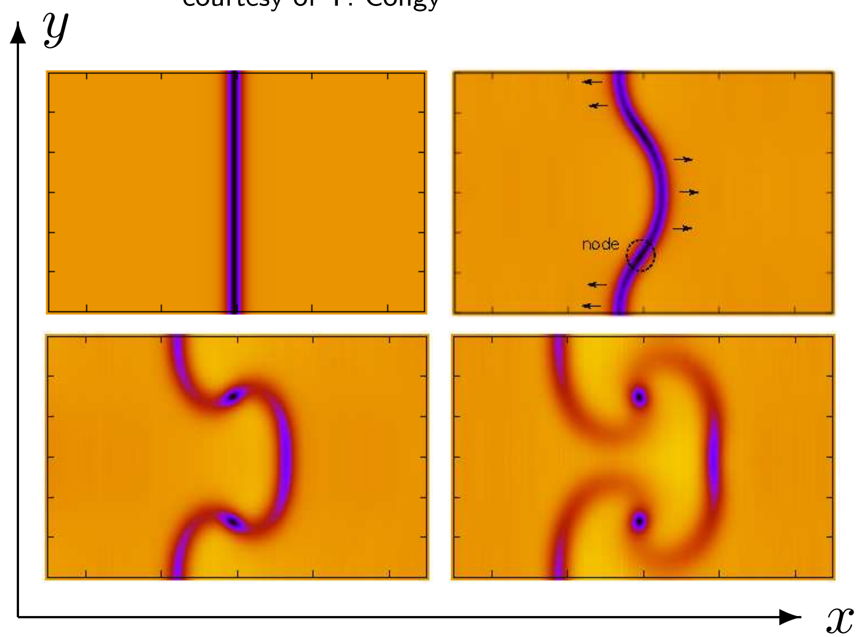
Dark solitons : 1D objects:

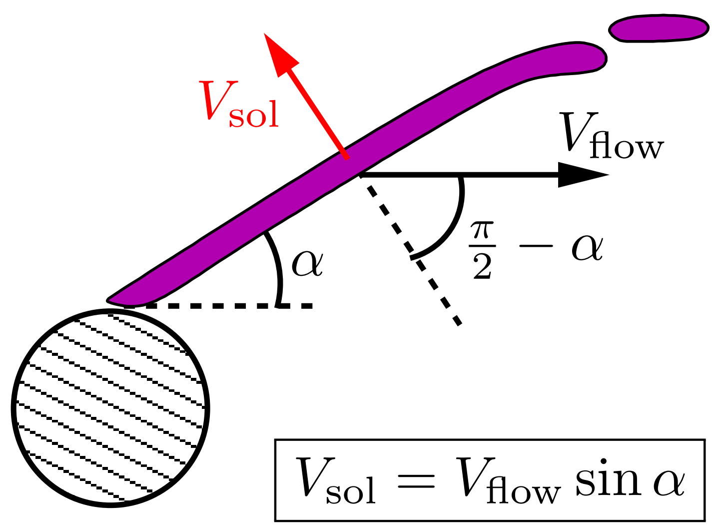
$V_{sol} < c$



in 2D, snake instability:

courtesy of T. Congy

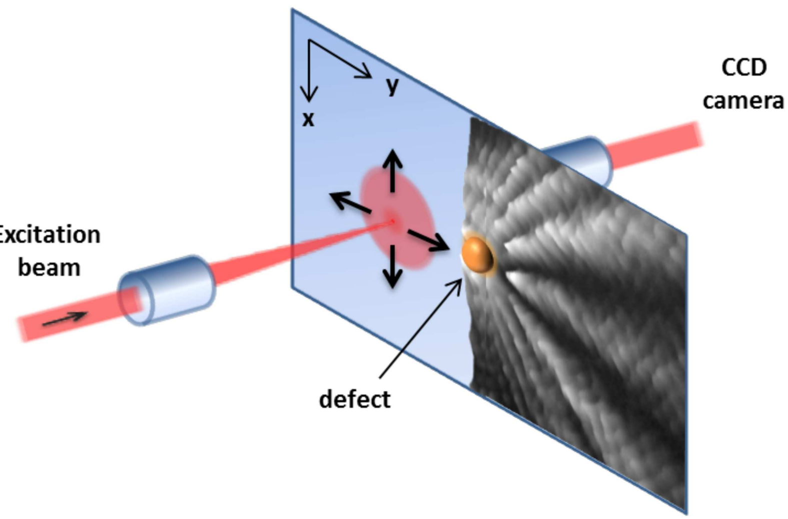


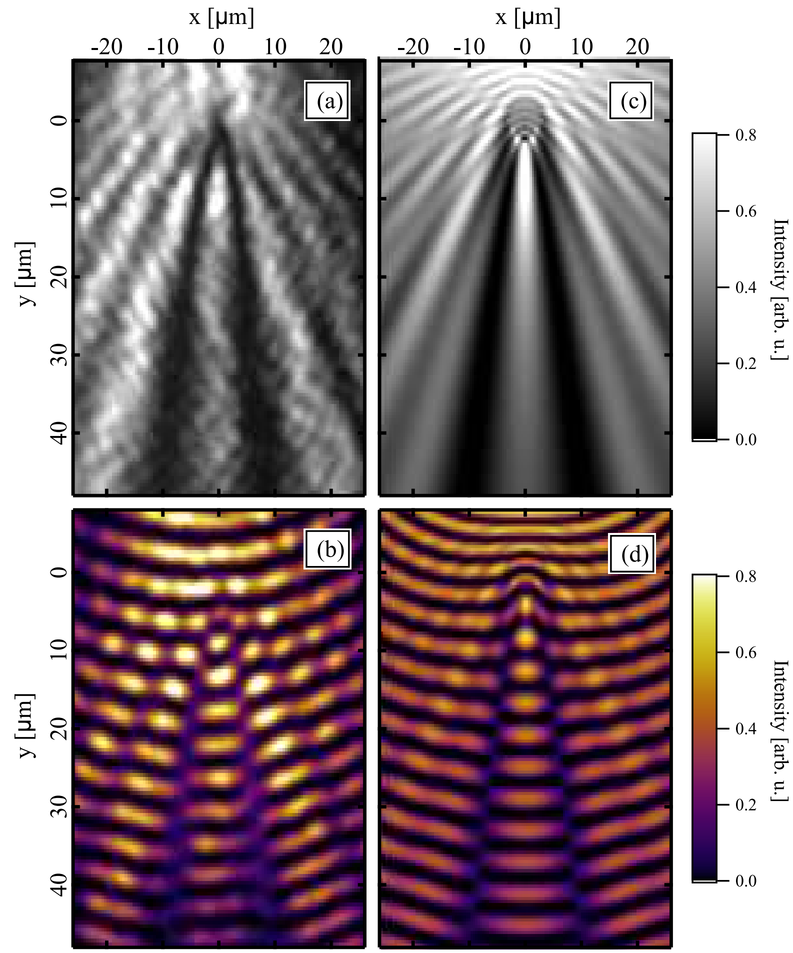


A controversy :

Lagoudakis' group
at Southampton

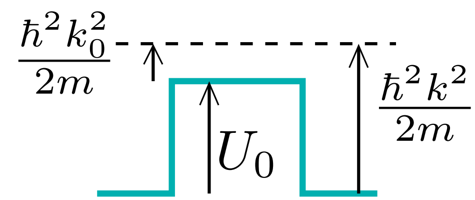
Cilibrizzi *et al.* Phys. Rev. Lett. (2014)





A simple model :

Lord Rayleigh, Phil. Mag. (1918)



$$\mathrm{i}\,\hbar\,\psi_t = -\frac{\hbar^2}{2m}\vec{\nabla}^2\psi + U(\vec{r})\,\psi\;, \quad U(\vec{r}) = \begin{cases} U_0\ (\lessgtr 0 \text{ or } \infty) & \text{if} \quad r < a\;, \\ 0 & \text{if} \quad r > a\;. \end{cases}$$

$$\psi(\vec{r}) = e^{\mathrm{i}kx} + \psi_{\mathrm{scat}}(\vec{r})\;, \quad \psi_{\mathrm{scat}}(\vec{r}) = \begin{cases} \sum_{n=-\infty}^{\infty} \mathrm{i}^n \tilde{A}_n J_n(k_0 r)\, e^{\mathrm{i}n\varphi} & r < a\;, \\ \sum_{n=-\infty}^{\infty} \mathrm{i}^n \tilde{B}_n H_n^{(1)}(kr)\, e^{\mathrm{i}n\varphi} & r > a\,. \end{cases}$$

$$k_0 = \sqrt{k^2 - 2\,m\,U_0/\hbar^2} \quad \text{(possibly complex)}\;, \quad \vec{r} = (r,\varphi) = (x,y)\;.$$

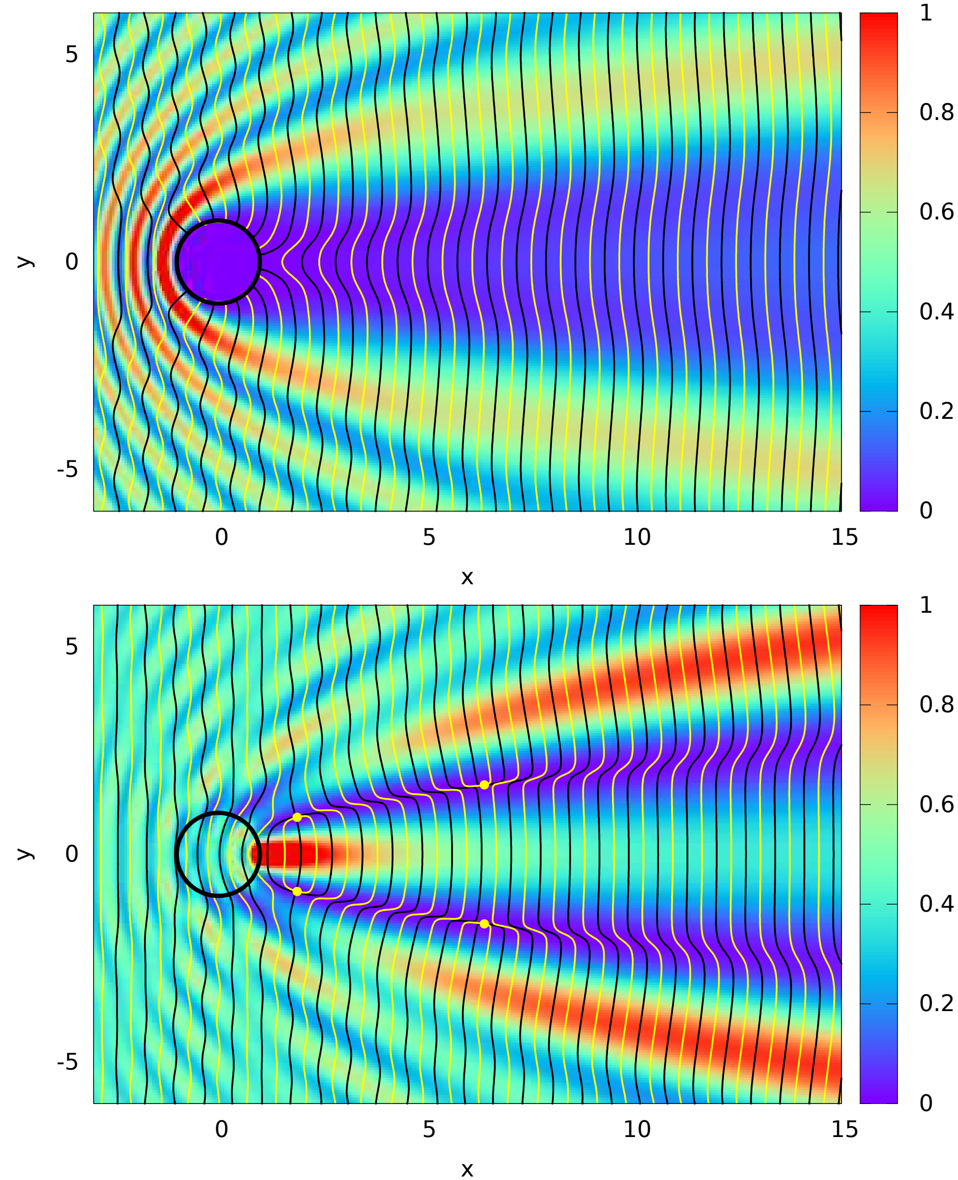
$$\tilde{B}_n = \frac{-k_0 J_n'(k_0 a) J_n(ka) + k J_n(k_0 a) J_n'(ka)}{k_0 J_n'(k_0 a) H_n^{(1)}(ka) - k J_n(k_0 a) H_n^{(1)\prime}(ka)} \xrightarrow{U_0 \to \infty} -\frac{J_n(ka)}{H_n^{(1)}(ka)}\;,$$

simple numerics ($n_{\max} \sim ka$)
$ka = 4.5$

> top figure: hard disk
>
> below : attractive disk
>
> with $2ma^2U_0/\hbar^2 = -15.$

(units: $a \equiv 1$)

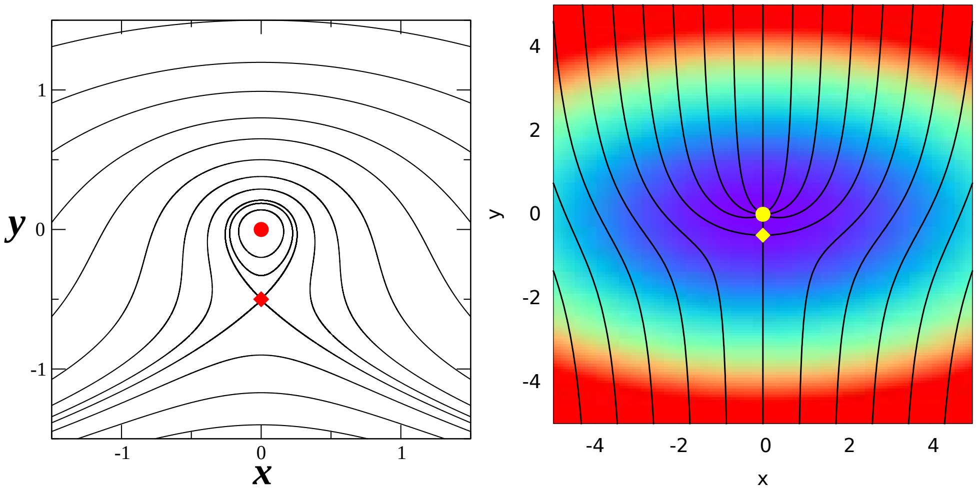
black lines: Im $\psi = 0$
yellow lines: Re $\psi = 0$



Wave singularity

model case:

$$\psi \cong (\alpha\, x - \mathrm{i}y)\, e^{\mathrm{i}kx}$$

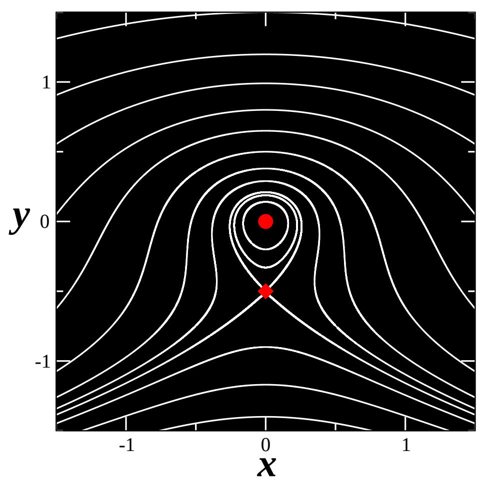


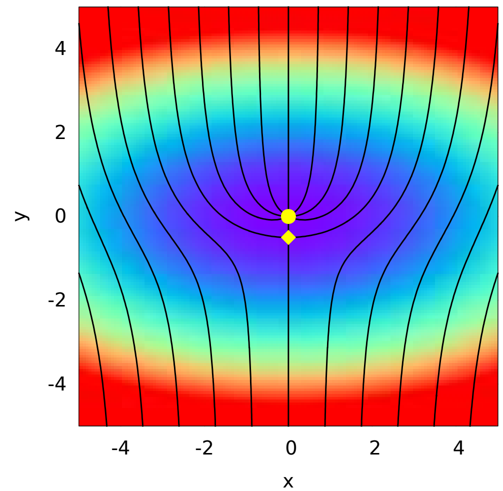
Wave singularity

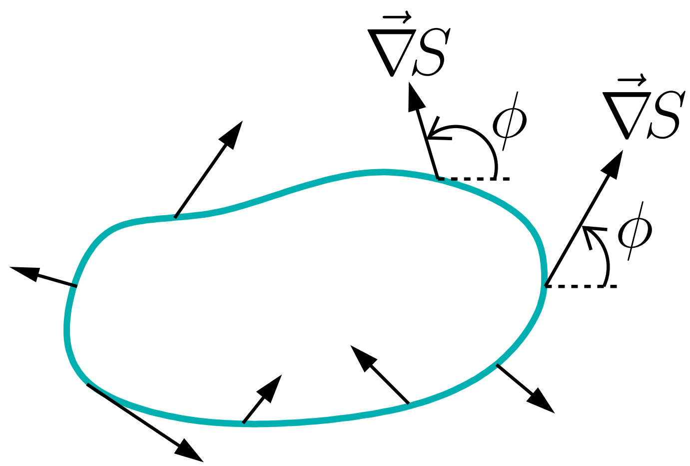
model case:

$$\psi \cong (\alpha\, x - \mathrm{i}y)\, e^{\mathrm{i}kx}$$

$$S(\vec{r}) = \arg\psi$$

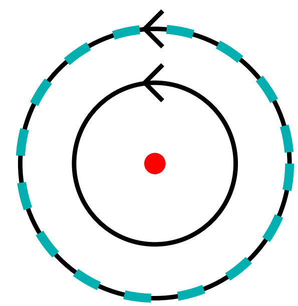






- $I_{\mathrm{V}} = \frac{1}{2\pi} \oint \vec{\nabla}S \cdot \mathrm{d}\vec{\ell} = \oint \frac{\mathrm{d}S}{2\pi}$
- $I_{\mathrm{P}} = \frac{1}{2\pi} \oint \vec{\nabla}\phi \cdot \mathrm{d}\vec{\ell} = \oint \frac{\mathrm{d}\phi}{2\pi}$

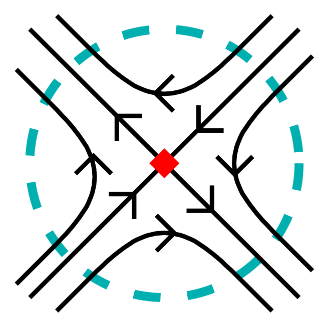
vortex



$I_{\mathrm{V}} = \pm 1$

$I_{\mathrm{P}} = +1$

saddle



$I_{\mathrm{V}} = 0$

$I_{\mathrm{P}} = -1$

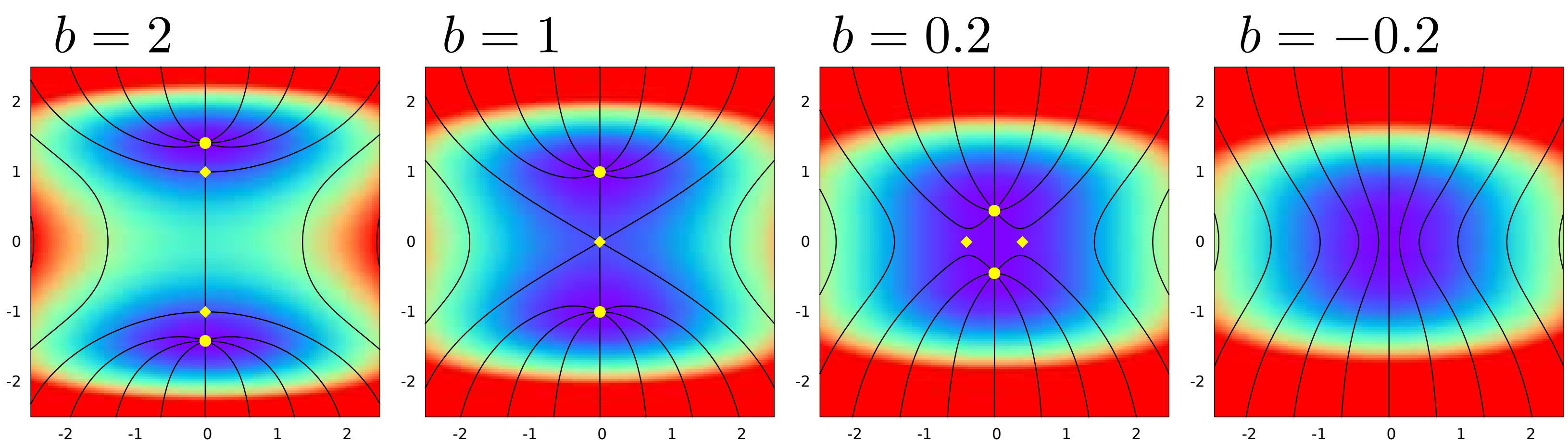
Wave singularity

model case bis:

$$\psi = [x - \mathrm{i}k(y^2 - b)]\, e^{\mathrm{i}kx} \ , \quad b \in \mathbb{R}$$

vortices: $(0, \pm\sqrt{b})$, saddles: $\begin{cases} (0, \pm\sqrt{b - k^{-2}}) & \text{for} \quad b > k^{-2}, \\ (\pm\sqrt{b - k^2 b^2}, 0) & \text{for} \quad 0 \le b < k^{-2}. \end{cases}$

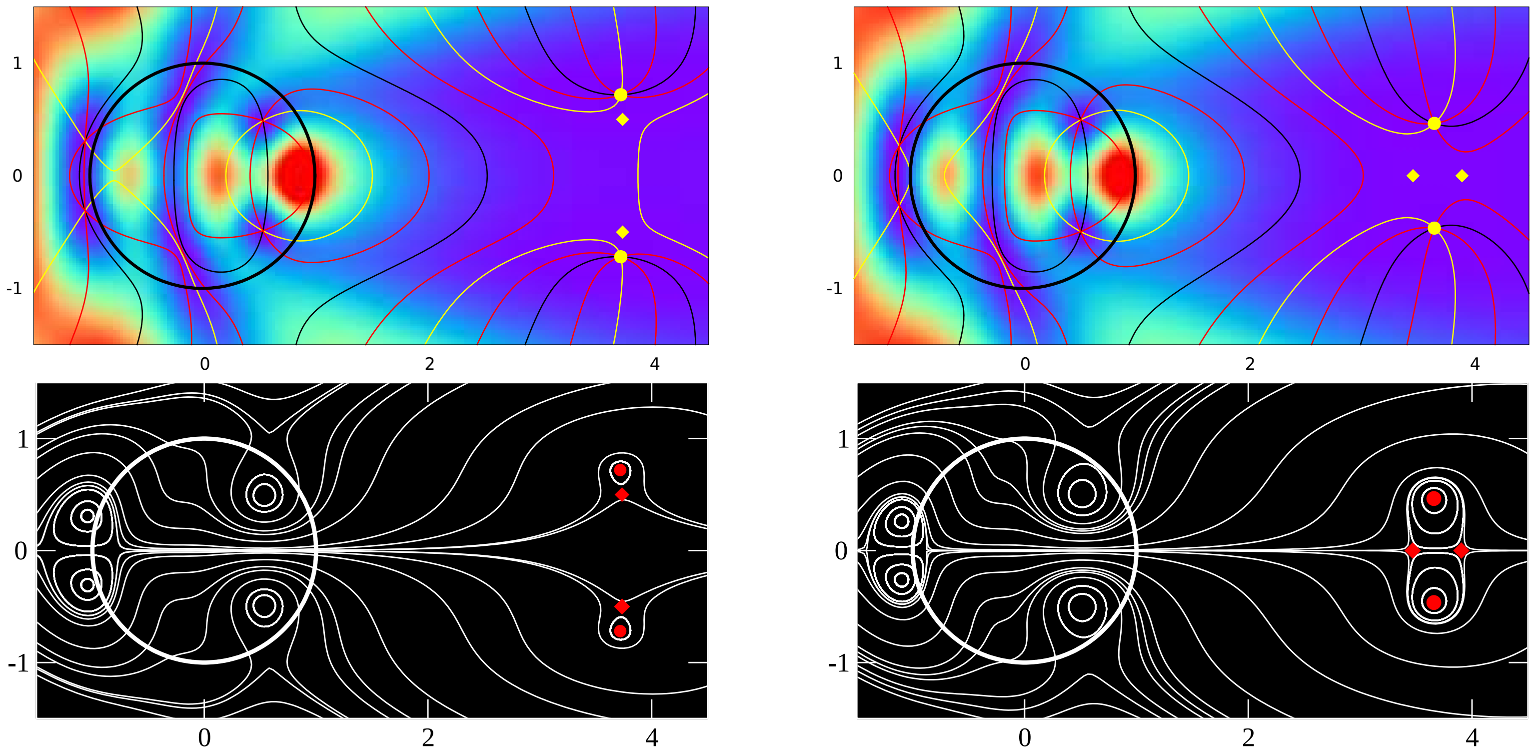
Scenario of Nye, Hajnal and Hannay (1988):



Wave singularity

2D scattering $2ma^2U_0/\hbar^2 = -15.$

$ka = 2.0$ $ka = 1.9$



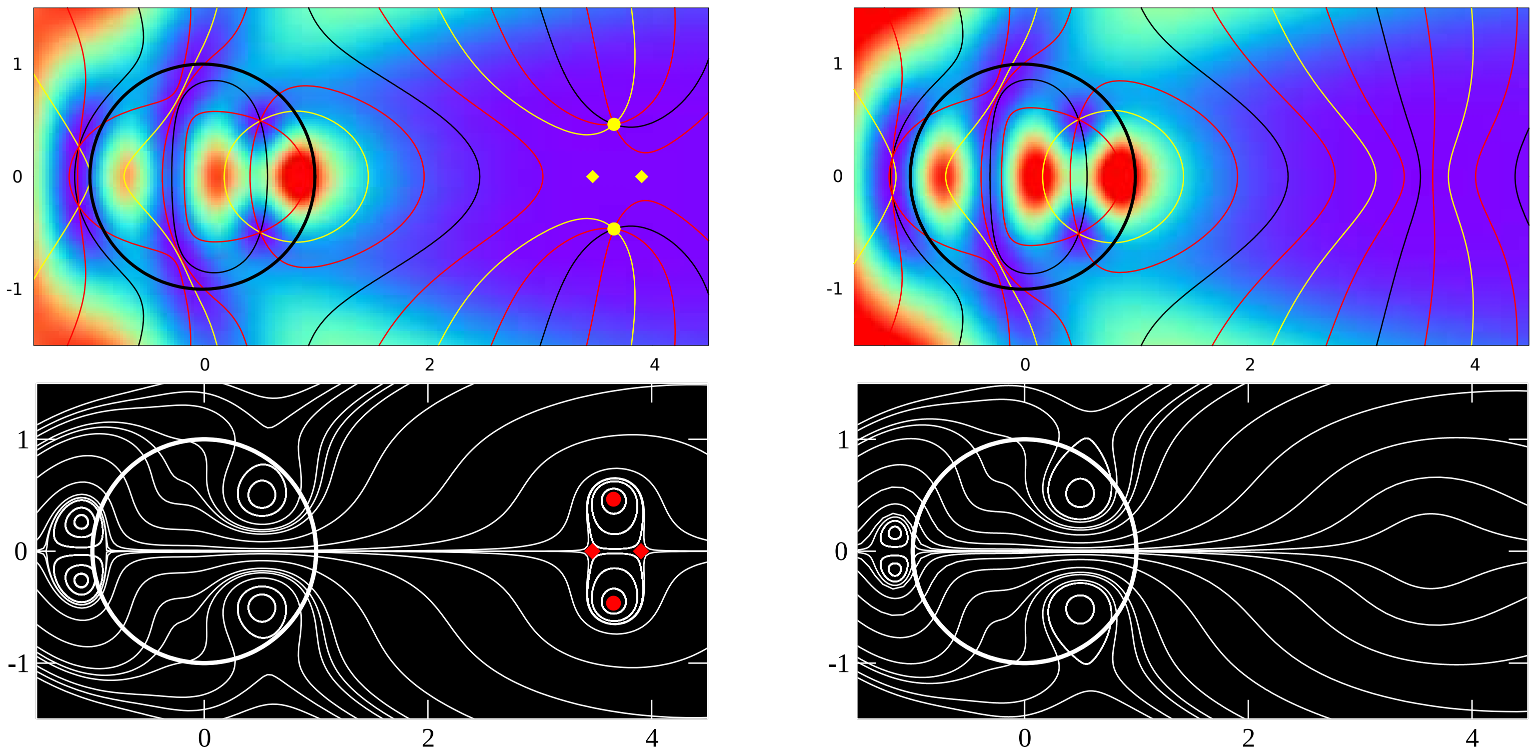
wavefronts:

0 and π : yellow ; $\pm\pi/2$: black : $\pm\pi/4$ and $\pm 3\pi/4$: purple.

Wave singularity

2D scattering $\qquad 2ma^2U_0/\hbar^2 = -15.$

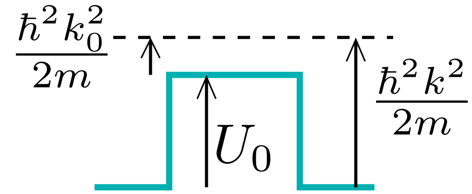
$ka = 1.9 \qquad\qquad ka = 1.8$



wavefronts:

0 and π : yellow ; $\pm\pi/2$: black : $\pm\pi/4$ and $\pm 3\pi/4$: purple.

Resonant scattering $(U_0 > 0)$



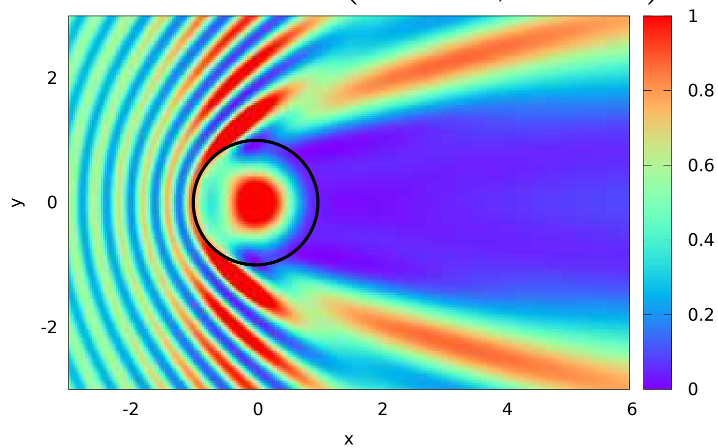
$$U_0 \gg \frac{\hbar^2}{ma^2}\ , \quad \text{and} \quad \frac{\hbar^2 k^2}{2m} - U_0 \ll U_0\ :$$

pole of the scattering amplitude:

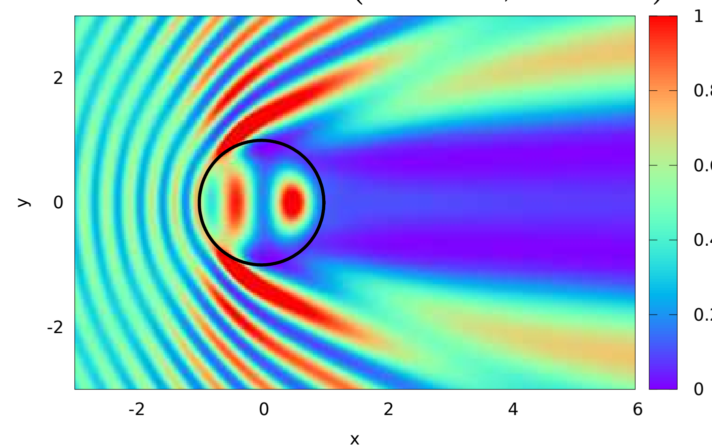
$k_0 J_n'(k_0 a) H_n^{(1)}(ka) = k J_n(k_0 a) H_n^{(1)\prime}(ka)$. Hence $k_0 a \simeq$ zero of J_n.

$$ka \approx Q_0 a + \frac{1}{2}\frac{j_{n,s}^2}{Q_0 a}\ , \quad \text{where} \quad \begin{cases} Q_0^2 = 2mU_0\hbar^2\ , \quad \text{here} \quad (Q_0 a)^2 = 60\ , \\ j_{n,s} : s^{\text{th}} \text{ zero of } J_n \end{cases}$$

$ka = 8.12\ (n = 0, s = 1)$



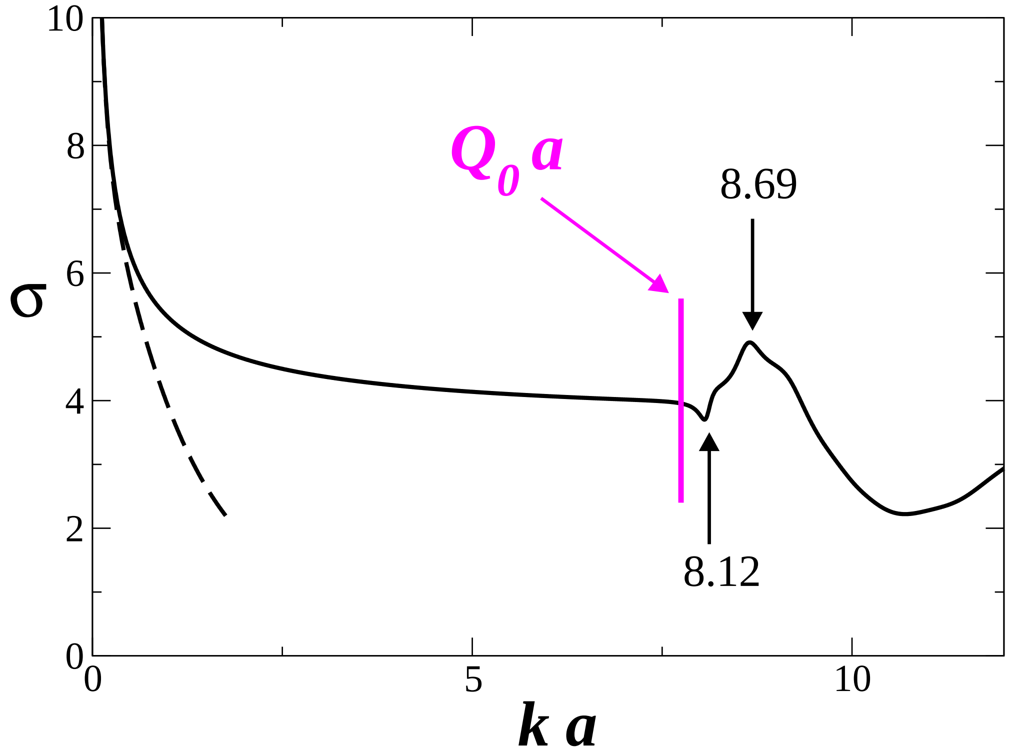
$ka = 8.69\ (n = 1, s = 1)$



Cross section

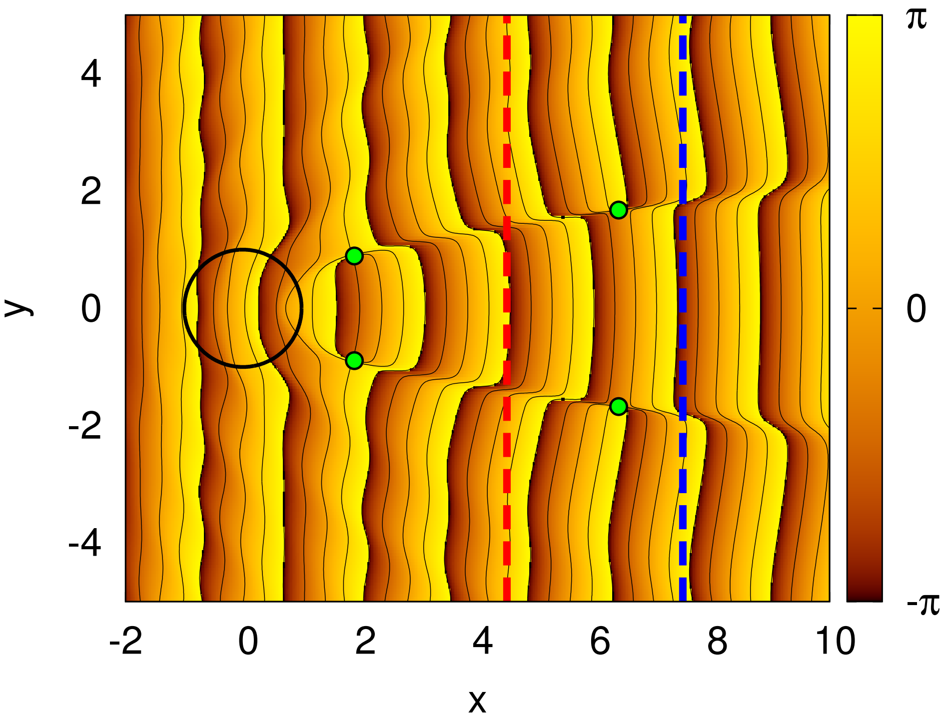
$\left[U_0 > 0\,, (Q_0 a)^2 = 60\right]$

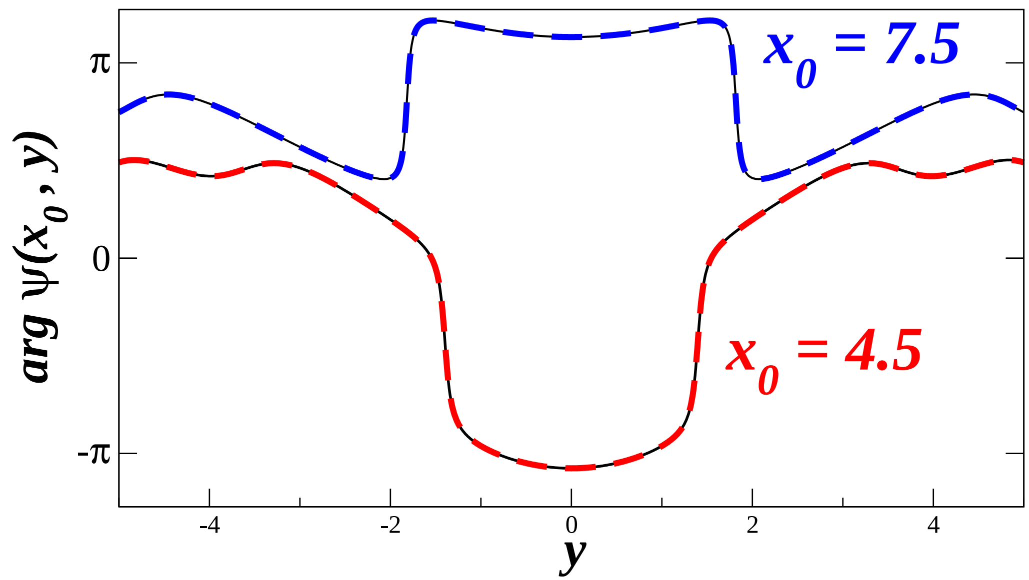
$$\psi \simeq e^{\mathrm{i}kx} + \frac{f(\varphi)}{\sqrt{r}} e^{\mathrm{i}kr} \to \sigma = \int_0^{2\pi} |f(\varphi)|^2 = \frac{4}{k} \sum_{n\in\mathbb{Z}} |\tilde{B}_n|^2 \,.$$



Low energy: $\tilde{B}_0 \simeq \frac{-1}{1+\frac{2\mathrm{i}}{\pi}\ln(k/\boldsymbol{\kappa})}$ where $\ln\left(\frac{\boldsymbol{\kappa}\, a\, e^{\gamma}}{2}\right) = \frac{1}{Q_0 a}\,\frac{I_0(Q_0 a)}{I_0'(Q_0 a)}\,.$
(only s-wave)

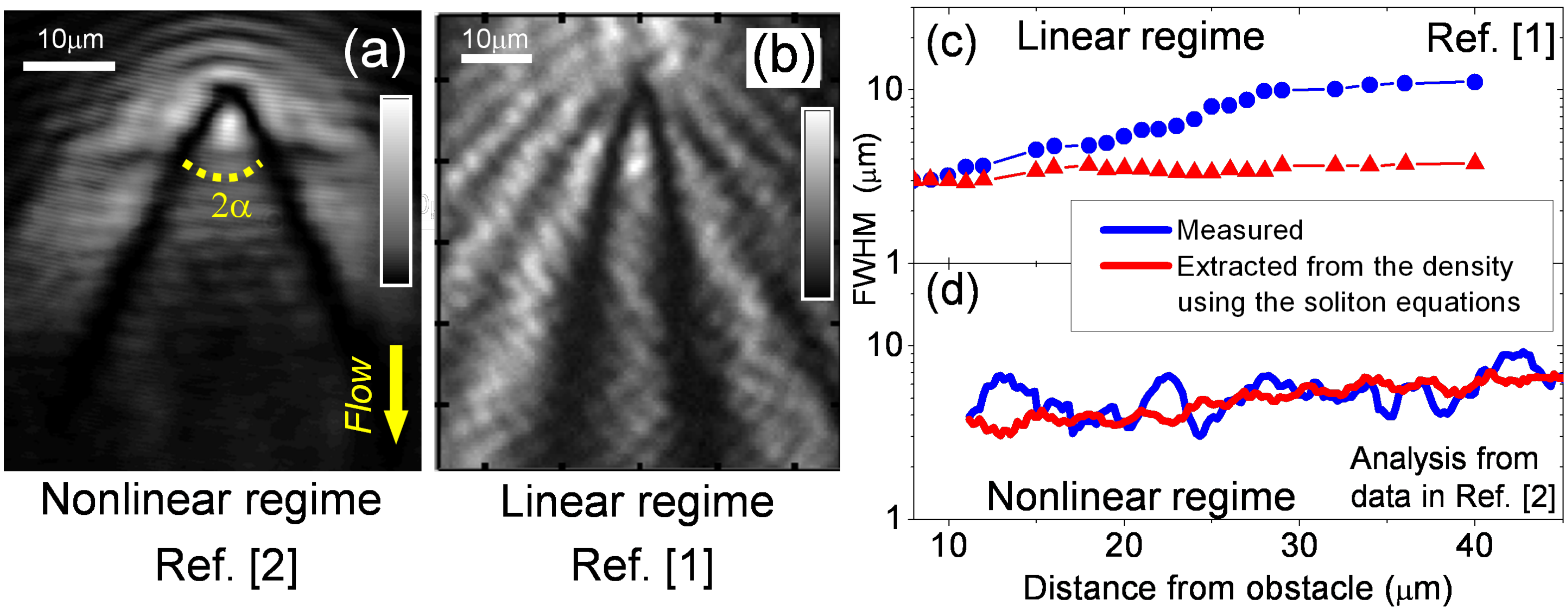
Qualitative analysis of the linear regime





Quantitative analysis

A. Amo *et al.* arXiv:1401.7347



[1] Cilibrizzi *et al.* PRL **113** (2014)

[2] Amo *et al.* Science **332** (2011)

Summary :

	$V_{\text{flow}} \to 0$	elongated regions of low density	phase pattern
linear	σ diverges	around nodal points / sensible to details	dislocations
non-linear	superfluid	around oblique solitons / robust	smooth phase jumps

CONCLUSION :

linear physics: interesting *per se* (motion of dislocation, resonances), effects within experimental reach.

What about the linear $\leftrightarrow$ non-linear transition ?

Still unclear :

A. Amo *et al.* Science **332** (2011)