# Topics in atom laser physics

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work in collaboration with:

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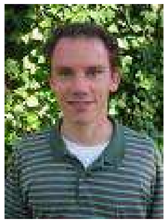
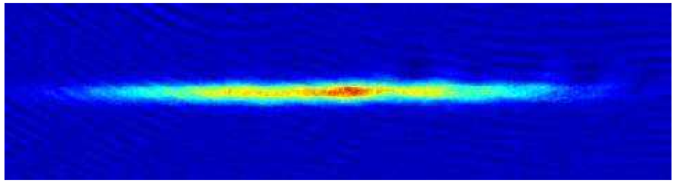
      

# ~~Topic~~ Top peak in Atom Laser PhysicS

l'Aiguille du Midi (3842 m)

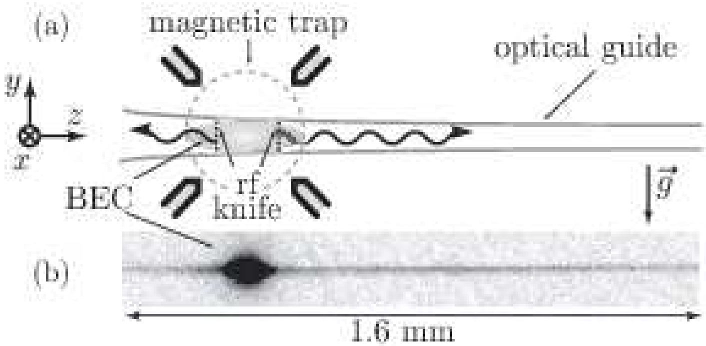
# quasi-1D condensates :



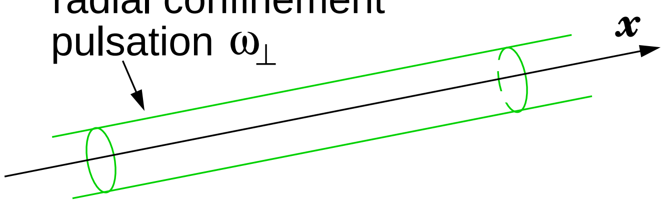
*quasi-1D condensate*

longitudinal size $\sim 10^2 \mu$m

transverse size $\sim 1 \mu$m




W. Guérin *et al.*, Phys. Rev. Lett. **97**, 200402 (2006)




harmonic radial confinement :

$$V_\perp(\vec{r}_\perp) = \frac{1}{2} m \omega_\perp^2 r_\perp^2 \; .$$

## Mesoscopic physics & BECs :

interaction in phase coherent systems,
non-linear transport.

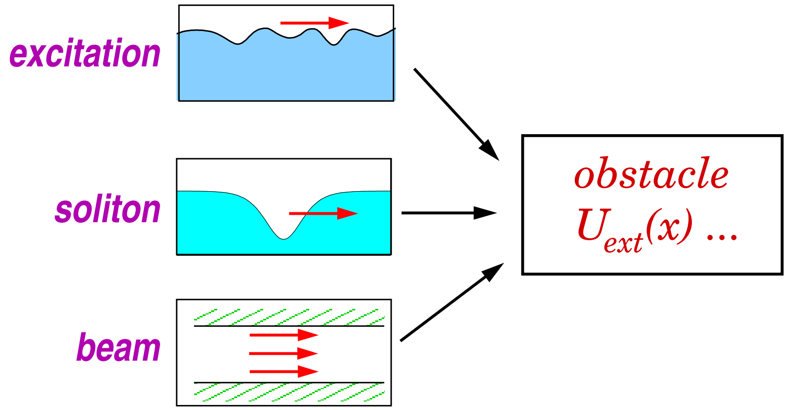
Large range of interaction regimes :

$\rightsquigarrow$ From "atom lasers" practicaly without interaction $\rightarrow$ strongly correlated 1D systems

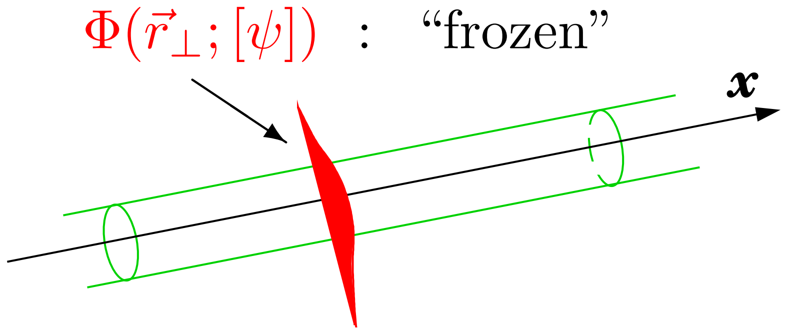
$\rightsquigarrow$ well defined theoretical framework (Bose-Hubbard/Gross-Pitaevskii)

Situations of 1D transport :

- Propagation of excitations, of (dark) solitons, of a beam ...
- In presence of localized or extended obtacles
- Effects of disorder
- Black-hole configuration
- Dispersive shock waves



# 1D mean field regime




Born-Oppenheimer :

$\Psi(\vec{r},t) = \psi(x,t) \times \Phi(\vec{r}_\perp;[\psi])$

1$D$ mean field regime with order parameter $\psi(x,t)$ verifying

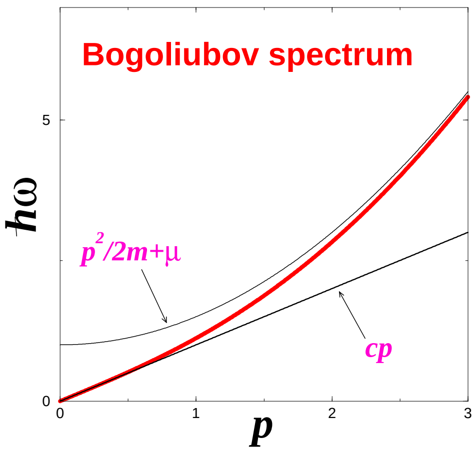
$$-\frac{\hbar^2}{2\,m}\partial_x^2\psi + \left(U_{\rm ext}(x) + g\,|\psi|^2\right)\psi = i\hbar\,\partial_t\psi \quad \text{or} \quad \mu\,\psi \qquad (1)$$

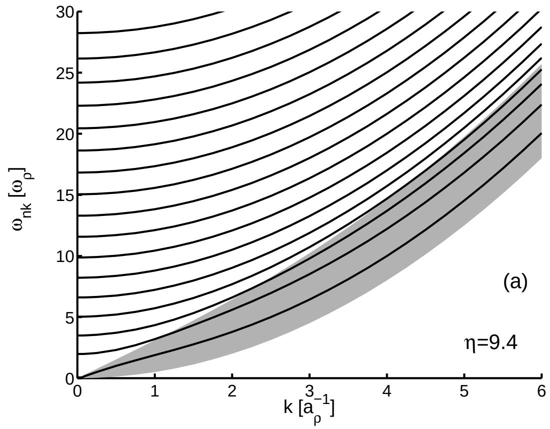
where $|\psi|^2 = n_1(x,t)$ is the longitudinal density of the condensate,
and $g = 2\,\hbar\,\omega_\perp\,a$, where $a$ : 3D s-wave scattering length $(a > 0)$

domain of validity :
$$\frac{\hbar\,\omega_\perp}{\hbar^2/ma^2} \ll n_1 a \sim \frac{\mu}{\hbar\omega_\perp} \ll 1$$

• The first inequality allows to avoid the Tonks-Girardeau regime and implies $E_{\rm int} \ll E_{\rm kin}$. Also $L_\phi \ggg \xi$ $\quad L_\phi = \xi \exp\left[\pi\sqrt{\frac{\hbar n_1}{2ma\omega_\perp}}\right]$

• the second inequality allows to avoid the 3D-like transverse Thomas-Fermi regime and implies that transverse motion is frozen

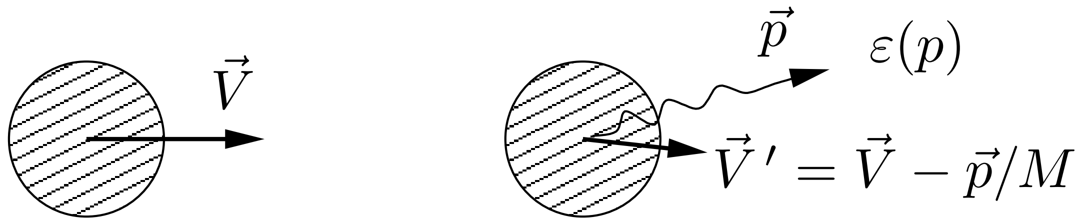


C. Tozzo & F. Dalfovo, Phys. Rev. A (2002)

← $\eta = \mu/\hbar\omega_\perp$ only axi-symmetric excitations included $(m = 0)$

## Landau criterion :




Energy and momentum conservation: $\frac{M}{2}V^2 = \frac{M}{2}\left(\vec{V} - \frac{\vec{p}}{M}\right)^2 + \varepsilon(p).$
for $M \gg m$ this reads $\varepsilon(p) = \vec{V}.\vec{p}$

emission of excitations possible only if $V > v_{\rm L} = \min\left[\frac{\varepsilon(p)}{p}\right].$

---

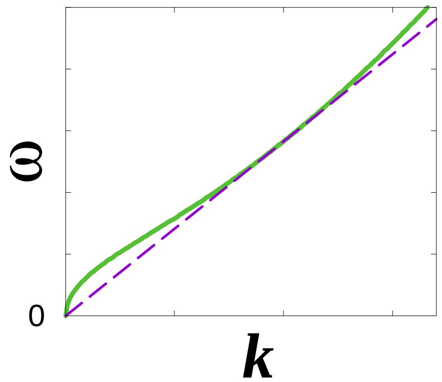
## Wave resistance

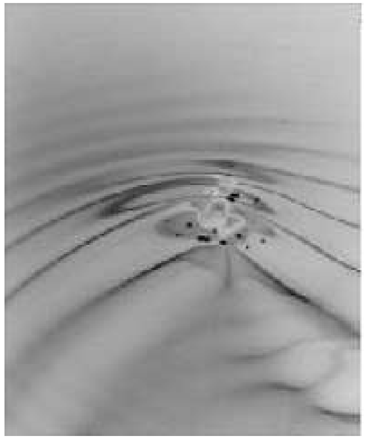
gravity-capillary waves at the surface of water:

$\omega^2 = k(g + \frac{\sigma}{\rho}k^2)$

$v_{\rm L} = \left(\frac{4\,g\,\sigma}{\rho}\right)^{1/4} = 23$ cm/s

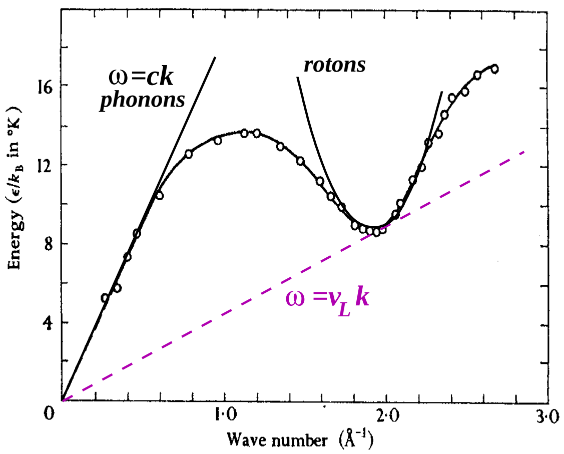
Kelvin (1871)



$V = 25.33$ cm/s

# Landau criterion




in $^4$He

$v_L = \min \frac{\varepsilon(k)}{k} \simeq 60$ m/s

due to vortex formation,

in most experiments :

1 mm/s $\lesssim v_{\text{crit,exp}} \lesssim 5$ m/s

in BEC

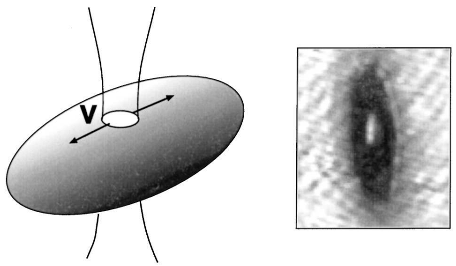
M.I.T. experiment

$\omega^2(k) = k^2\,(c^2 + k^2/4)$

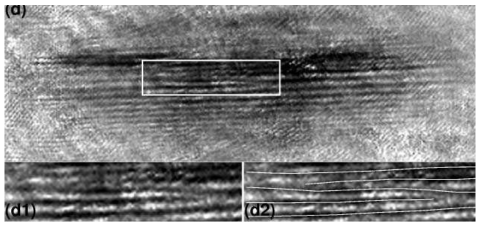
$\rightarrow v_L = c = 6.2$ mm/s

$v_{\text{crit,exp}} = 1.6$ mm/s

Phys. Rev. Lett. **83**, 2502 (1999)



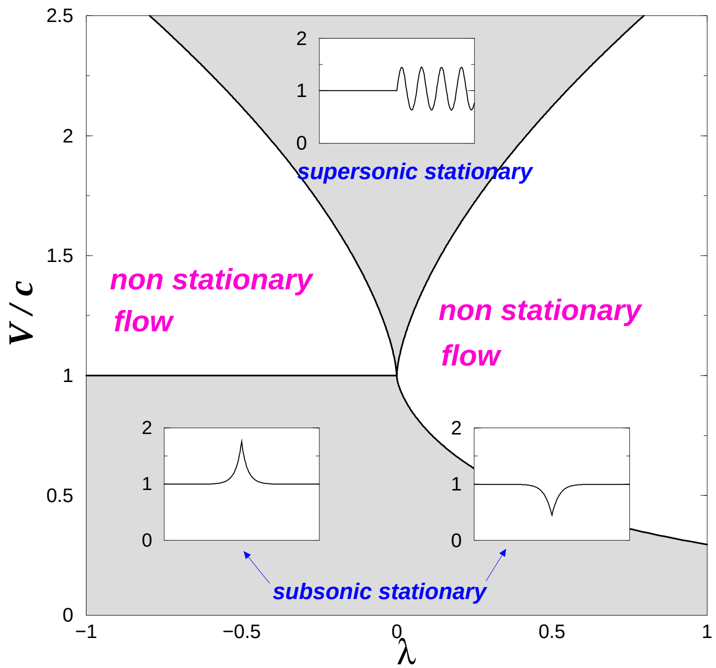

Phys. Rev. Lett. **87**, 080402 (2001)




Evidence of vortex formation

# Flow past an impurity

$$U_{\text{ext}}(x) = \lambda\, \mu\, \xi\, \delta(x)\ .$$




V. Hakim, Phys. Rev. E **55**, 2835 (1997)

P. Lebœuf & N. Pavloff, Phys. Rev. A **64**, 033602 (2001)

Perturbative treatment $(V > c)$ :

- in $1D$, $F \propto |\hat{U}(\kappa)|^2$

where $\kappa = \frac{m}{\hbar}|V^2 - c^2|^{1/2}$

- For a $\delta$ impurity :

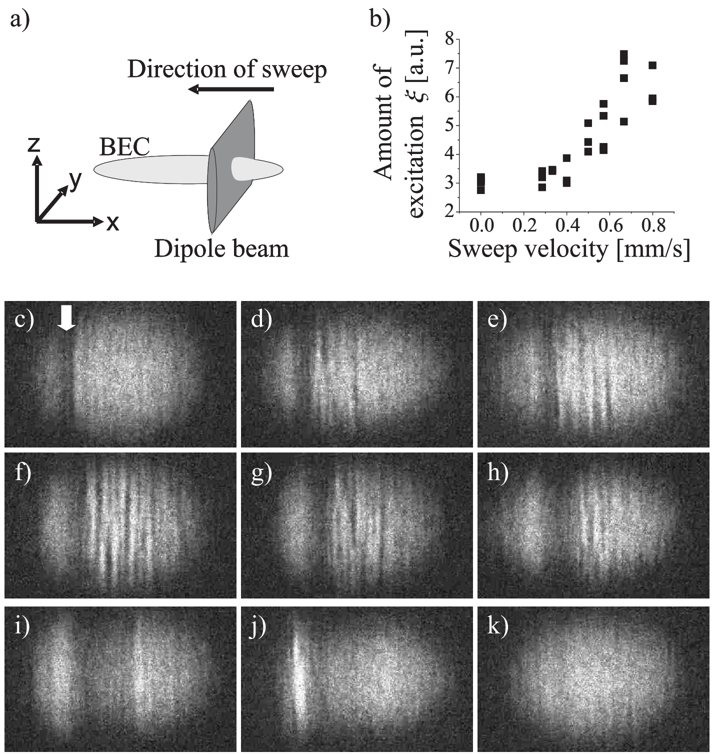
$$\begin{cases} F \propto C^{\text{st}} & 1D \\ F \propto (V^2 - c^2)/V & 2D \\ F \propto V^2\,(1 - c^2/V^2)^2 & 3D \end{cases}$$

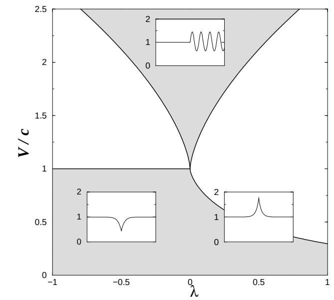
N. Pavloff, Phys. Rev. A **66**, 013610 (2002)

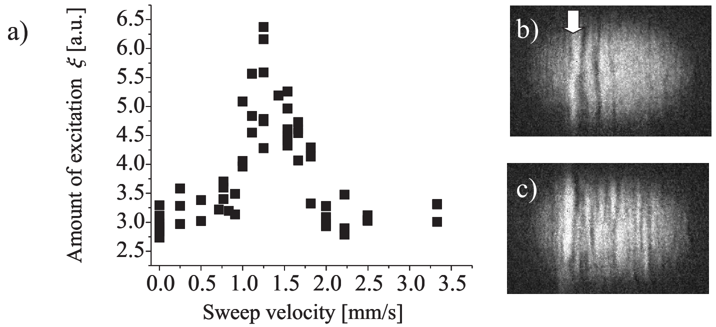
G. E. Astrakharchik & L. P. Pitaevskii, Phys. Rev. A **70**, 013608 (2004)

# Recent experimental study

P. Engels & C. Atherton, Phys. Rev. Lett. **99**, 160405 (2007)




Repulsive potential

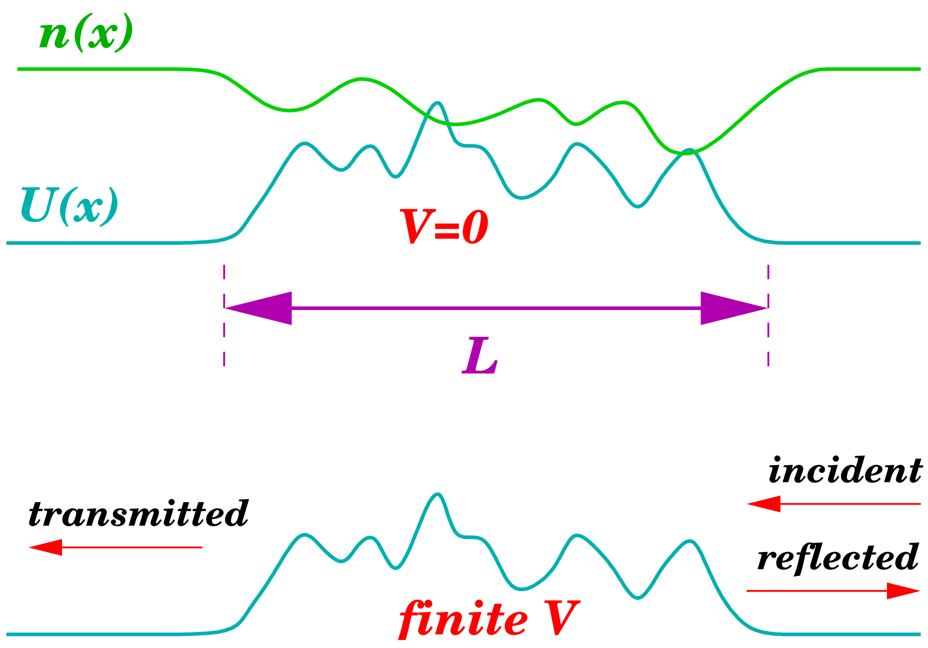
$U_{\max}/\mu \simeq 0.24$, $c = 2.1$ mm/s

$V$ =0.4 – 0.8, 1, 1.3, 2, 3.3 mm/s

Attractive potential $V$ =1.25 mm/s, $c = 2.1$ mm/s

$|U_{\min}|/\mu \sim 0.17, 0.32$

# A (nonlinear) beam incident on a disordered region of size $L$




What are the density profile, the transmission coefficient and the drag exerted on the obstacle when the velocity $V$ of the beam with respect to the obstacle is finite ?

How do these properties scale with $L$?

In the frame where the beam is at rest :

$$-\frac{\hbar^2}{2\,m}\partial_x^2\psi + \left[U(x - V\,t) + g\,|\psi|^2\right]\psi = i\hbar\,\partial_t\psi\ ,$$

## Different types of disorder

• model disordered potential : $U(x) = \lambda\,\mu\,\xi \sum_n \delta(x - x_n)$ ,

U. Gavish & Y. Castin, Phys. Rev. Lett. **95**, 020401 (2005)

$x_n$'s: uncorrelated random position of the impurities $0 = x_1 \leq x_2 \leq x_3 ...,$ with mean density $n_\mathrm{i}$

One has $\langle U(x) \rangle = \lambda\,\mu\,(n_\mathrm{i}\,\xi)$ and

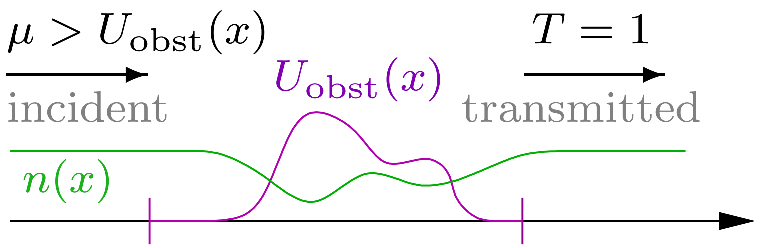
$\langle U(x) U(x') \rangle - \langle U \rangle^2 = \left(\frac{\hbar^2}{m}\right)^2 \sigma\,\delta(x - x')$

with $\sigma = n_\mathrm{i}\,\lambda^2/\xi^2$. $[\sigma] =$ length$^{-3}$.

• Other disordered potentials: Gaussian (white or correlated) noise, Speckle potential.

# Two contrasting phenomena

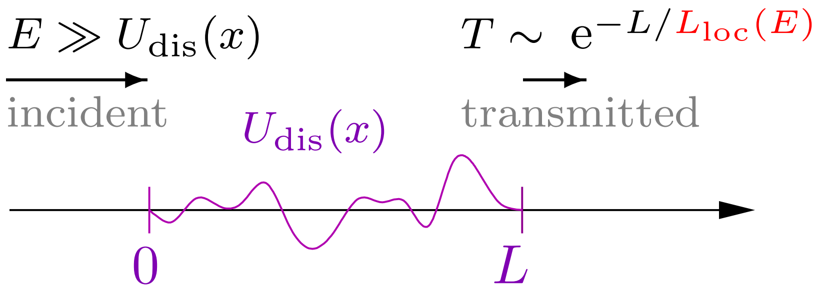
## Superfluidity




Perfect transmission

No drag, no dissipation

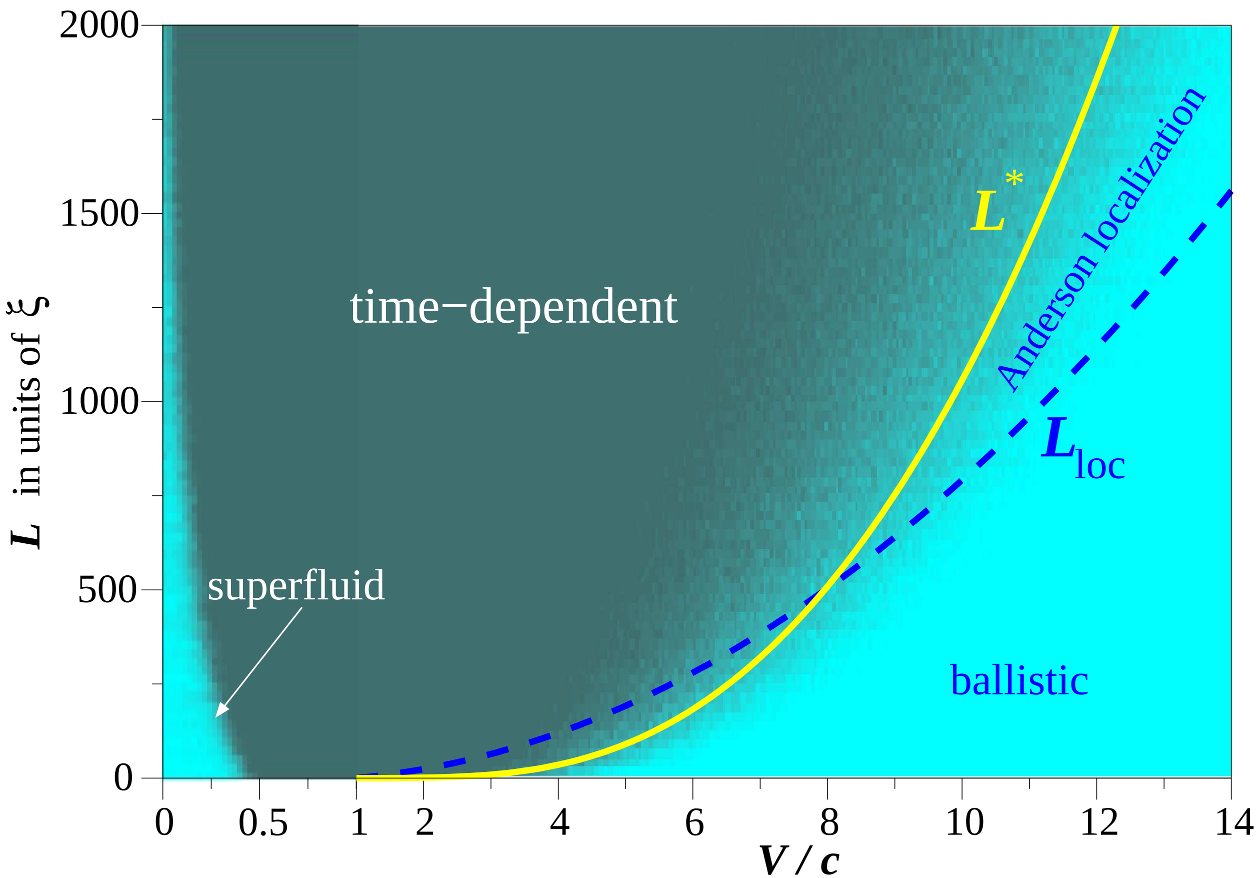
## Anderson localization




Large $L$ : no transmission

interaction $\Longleftrightarrow$ disorder

## Global Picture : conflict between superfluidity and localization




disordered delta peaks with $\lambda = 0.5$ and $n_i \xi = 0.5 \quad (\mu \gg \langle U \rangle)$.

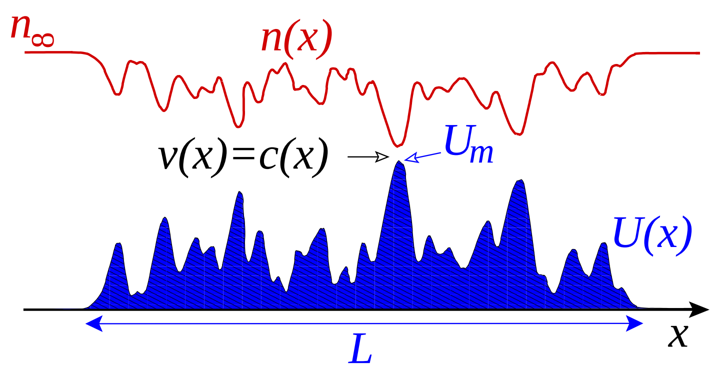
T. Paul, P. Schlagheck, P. Leboeuf & N. Pavloff, Phys. Rev. Lett. **98**, 210602 (2007)

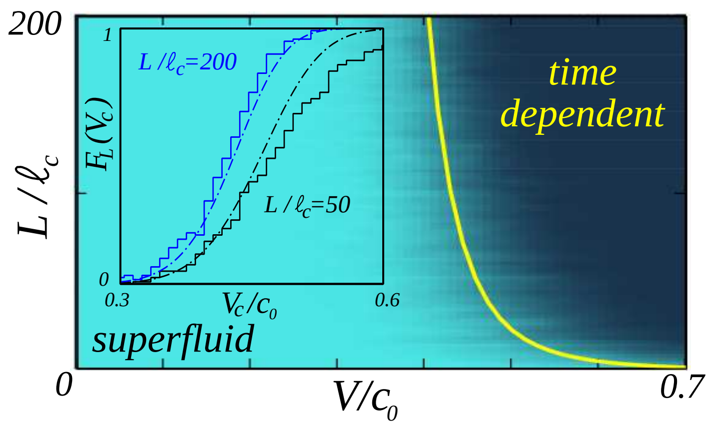
# Breakdown of superfluidity

Similar to the non-disordered case.
V. Hakim, Phys. Rev. E **55**, 2835 (1997)

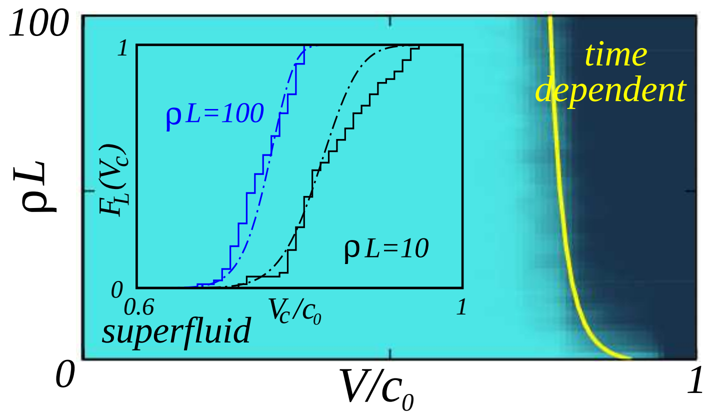
Linked to statistics of extremes of the random potential.

One obtains analytical results in two limiting cases:




Smooth disorder $L_{\text{typ}} \gg \xi$



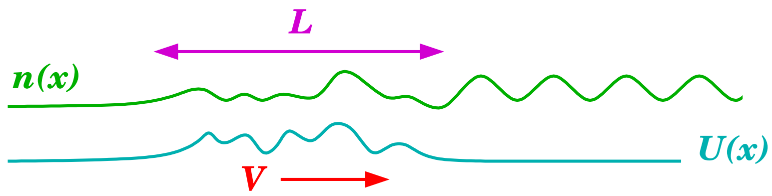

random delta peaks

M. Albert, T. Paul, N. Pavloff & P. Leboeuf, Phys. Rev. A **82**, 011602(R) (2010)

# Supersonic stationary regime

## Ballistic ($\equiv$ perturbative) region

$$\delta n(\zeta) \simeq \frac{2mn_0}{\hbar^2\kappa}\int_{-\infty}^{\zeta} \mathrm{d}y\, U(y)\, \sin[2\kappa(\zeta - y)]$$



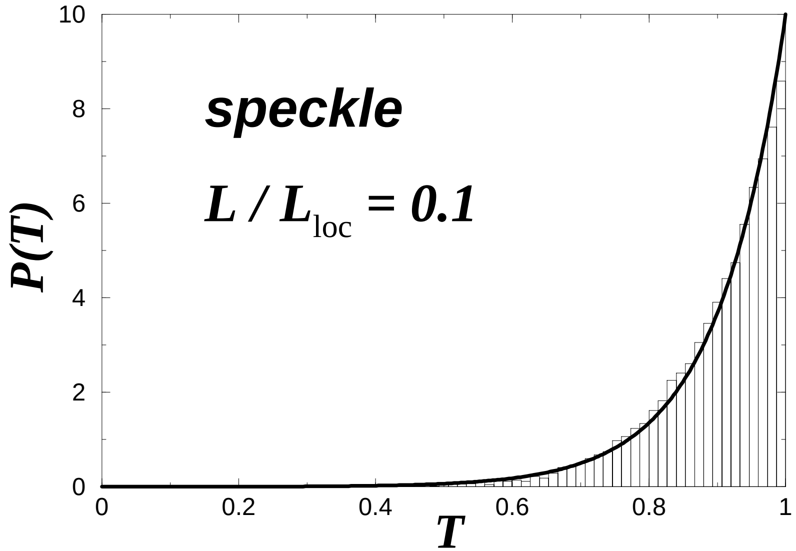

where $\zeta = x - Vt$. This yields $\langle T\rangle \simeq 1 - L/L_{\rm loc}$ where

$$L_{\rm loc}(\kappa) = \frac{\kappa^2}{\sigma} \,. \qquad (2)$$

and $$\kappa = \frac{m}{\hbar}\left|V^2 - c^2\right|^{1/2} \,. \qquad (3)$$

probability distribution of $T$ :

$$P(T) = \frac{L_{\rm loc}}{L}\exp\left\{-(1-T)\frac{L_{\rm loc}}{L}\right\} \,.$$



# Anderson localization

$L > L_{\rm loc}$ : non perturbative. $P(\lambda, t)$ ($\lambda = T^{-1} - 1$, $t = L/L_{\rm loc}$) is solution of the DMPK equation: $\partial_t P = \partial_\lambda \left[\lambda(\lambda+1)\partial_\lambda P\right]$

This implies that

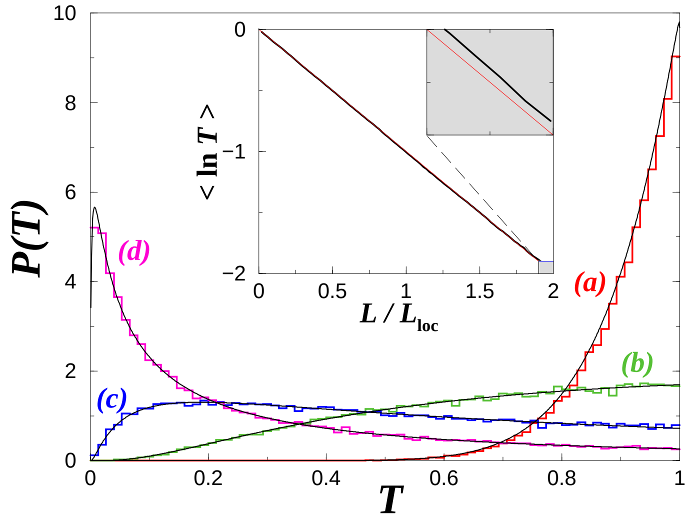
$$\langle \ln T \rangle = -L/L_{\rm loc}(\kappa) \ ,$$

where $L_{\rm loc}(\kappa)$ is given by Eqs. (2,3).

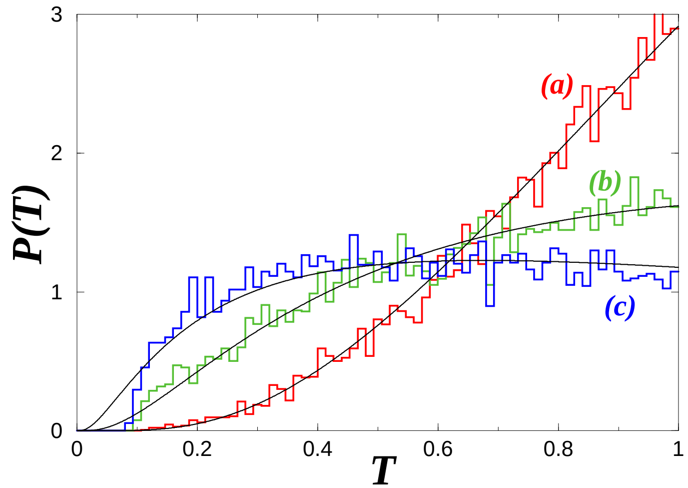
and that the asymptotic probability distribution is log-normal

$$P(\ln T, t = \frac{L}{L_{\rm loc}}) = \frac{1}{\sqrt{4\pi t}} \, {\rm e}^{-\frac{1}{4t}\,(t+\ln T)^2}$$

delta peaks




speckle potential



# Diffusion equation for the transmission

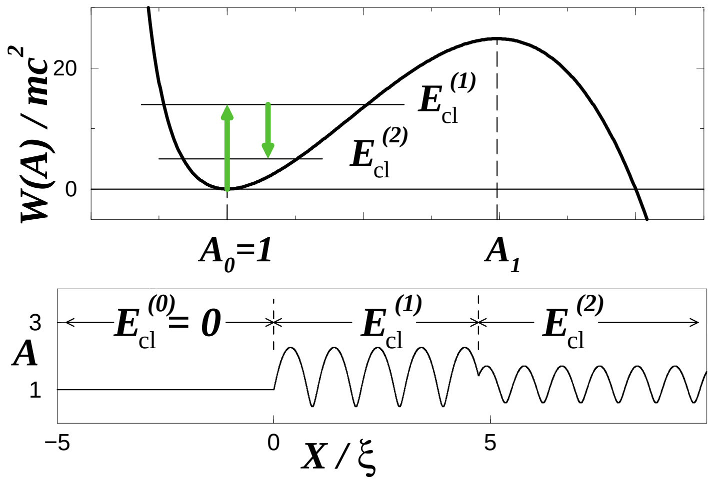
First integral in regions where $U(x) \equiv 0$ (between $x_n$ and $x_{n+1}$ say)

$$\frac{\xi^2}{2}\left(\frac{\mathrm{d}A}{\mathrm{d}X}\right)^2 + W[A(X)] = E_{\rm cl}^{(n)} \, ,$$

where $A = |\psi|/\sqrt{n_0}$, $E^n_{\rm cl}$ is a constant and $W(A) = \frac{1}{2}(A^2-1)(1+v^2-A^2-v^2/A^2)$. From the final $E_{\rm cl}^{(N_{\rm i})}$ one computes the transmission[a]

$$T = \frac{1}{1+(2\kappa^2\,\xi^2)^{-1} E_{\rm cl}^{(N_{\rm i})}} \, .$$

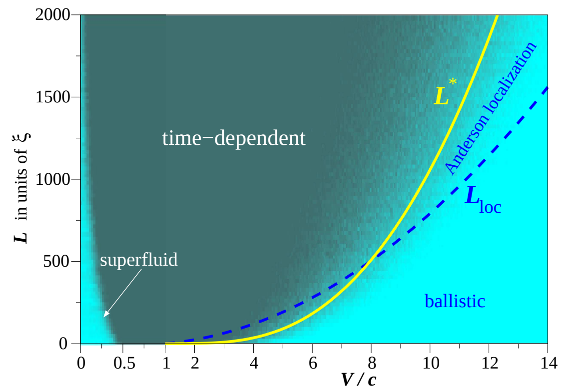
[a] P. Lebœuf, N. Pavloff & S. Sinha, PRA **68**, 063608 (2003)




*Upper panel: $W(A)$ (drawn for $v = V/c = 4$). $A_0(=1)$ and $A_1$ are the zeros of $\mathrm{d}W/\mathrm{d}A$. The fictitious particle is initially at rest with $E_{\rm cl}^{(0)} = 0$. The value of $E_{\rm cl}$ changes at each impurity. The lower panel displays the corresponding oscillations of $A(X)$, with two impurities (vertical dashed lines) at $x_1 = 0$ and $x_2 = 4.7\,\xi$.*

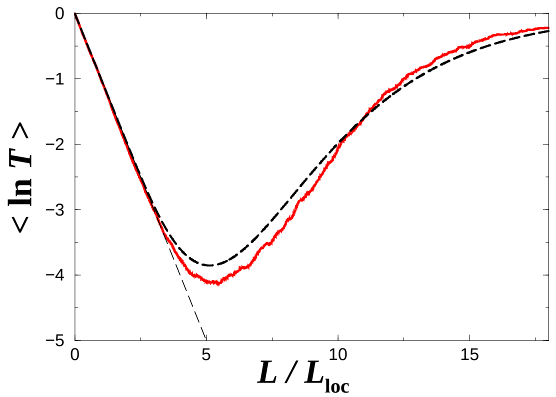
## Upper threshold

(for the supersonic stationary regime)

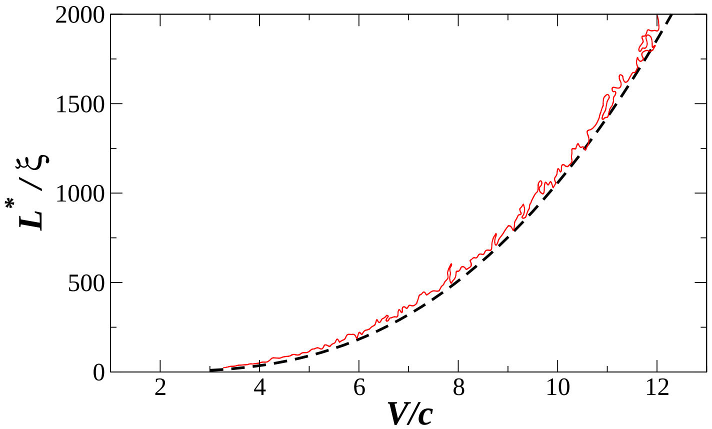



One solves the DMPK equation with the boundary condition that there exists a $\lambda_{\max}$ at which $P(\lambda_{\max}, t) = 0$.

fixed $V$ ($V/c = 450$)

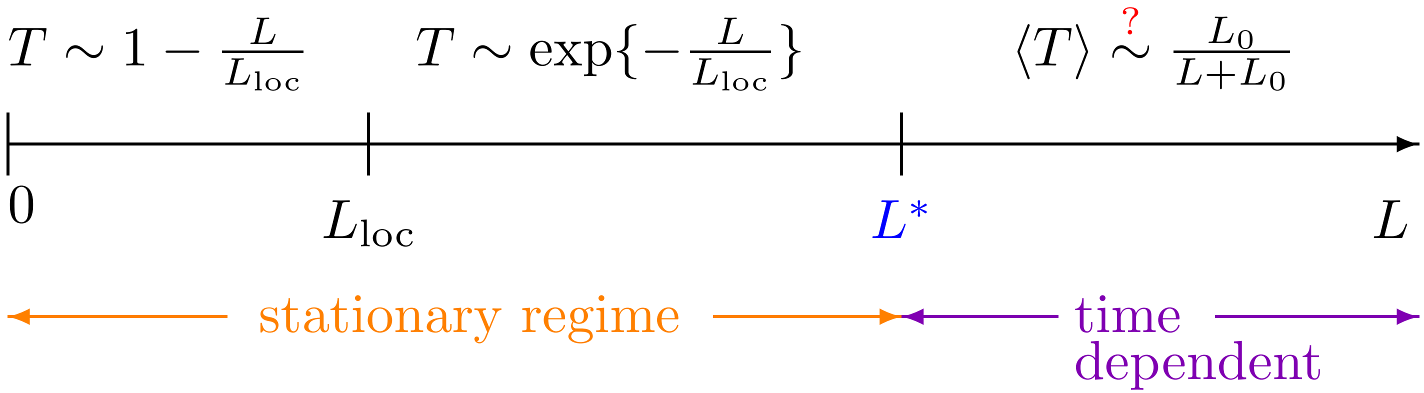



phase diagram




T. Paul, M. Albert, P. Schlagheck, P. Leboeuf & N. Pavloff, PRA **80**, 033615 (2009)

## Picture in the supersonic regime :




- $L_{\rm loc}$ has the same expression as for non-interacting particles with

$$k = \frac{m\,V}{\hbar} \quad \text{replaced by} \quad \kappa = \frac{m}{\hbar}\sqrt{V^2 - c^2} = \sqrt{k^2 - \frac{1}{\xi^2}}\ .$$

- $$L^* \sim L_{\rm loc}(\kappa)\,\ln\left(\frac{V^2}{c^2}\right)\ .$$

# Partial Conclusion

Different types of set-ups lead to a large variety of phenomena :

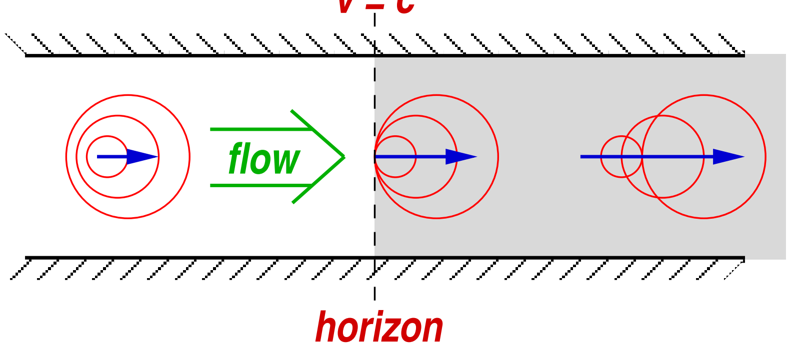
→ Algebraic decay of a dark soliton.

→ Anderson localization : non-interacting elementary excitations or supersonic beams in presence of interaction.

→ For a beam :
- $L_{\rm loc}$ is renormalized in presence of interaction
- time dependent regime (for $L \geq L^*$ )
- different regimes ⇒ different heating rates

→ Dipolar oscillations : negative answer, albeit possible indirect evidences...

# Sonic black holes : "dumb holes"




W. G. Unruh, Phys. Rev. Lett. (1981)

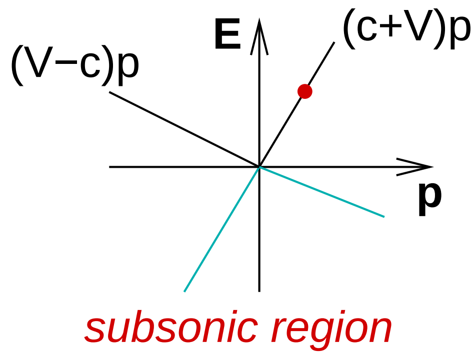
even without a source,
vacuum fluctuations $\rightsquigarrow$
Hawking radiation

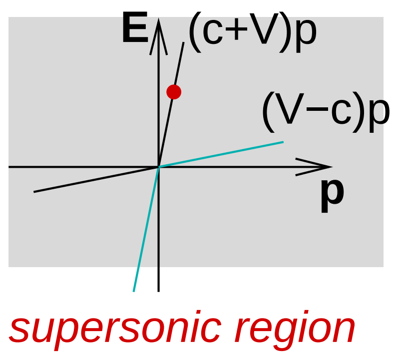
in the laboratory :

$$E(p) = c\,|p| + V\,p$$

$\swarrow$ $\downarrow$

comoving Doppler

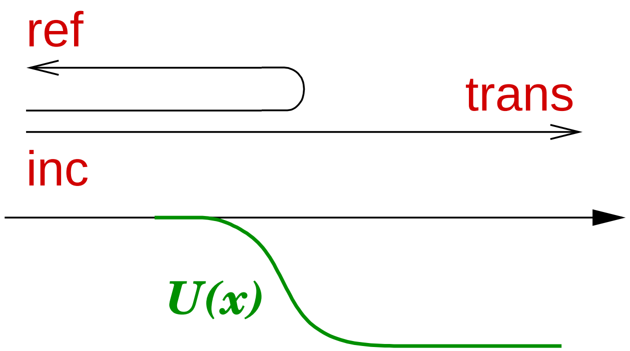


## Analogous to tunnel effect : (quantum reflexion)

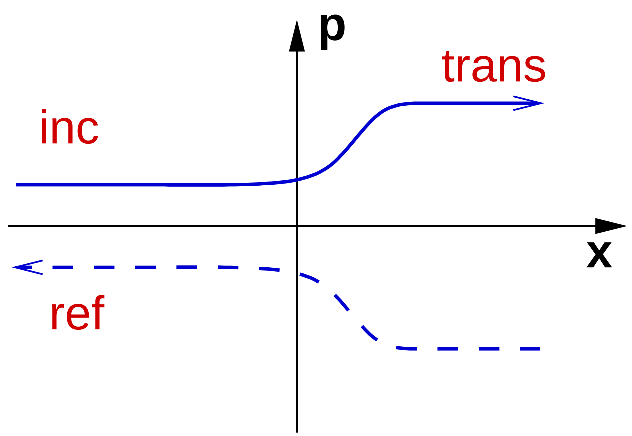
real space

particle incoming from the left with $E > U_{\max}$



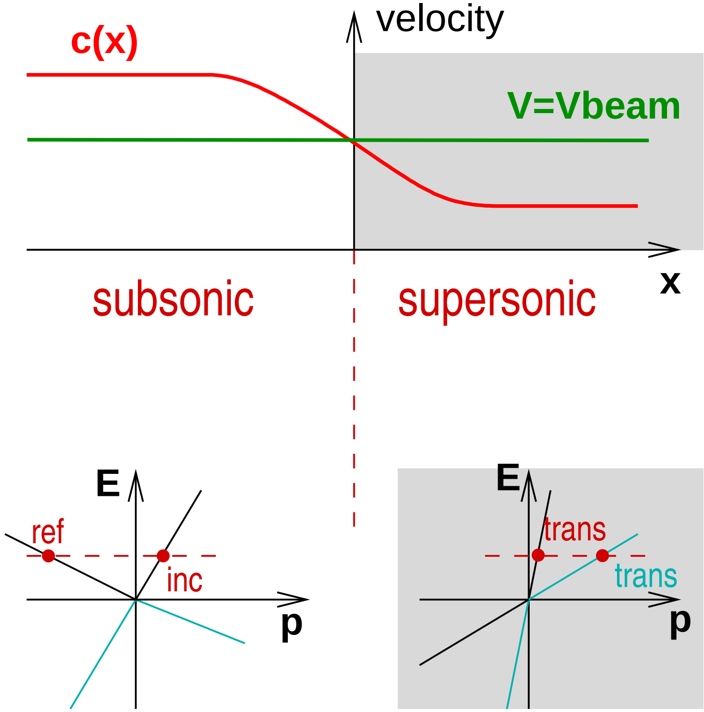

phase space trajectory

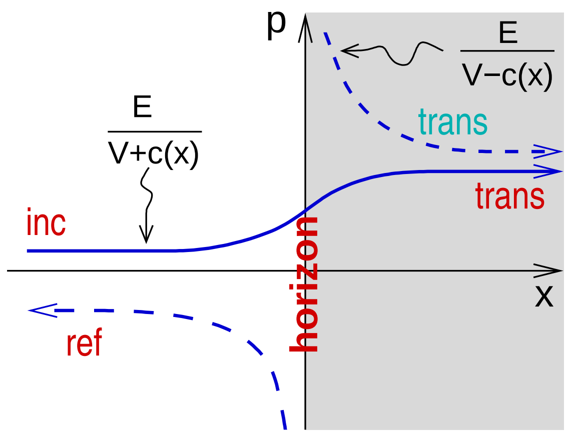
$E = p^2/2m + U(x)$



A model configuration :

$$H(x,p) = c(x)\,|p| + V\,p$$



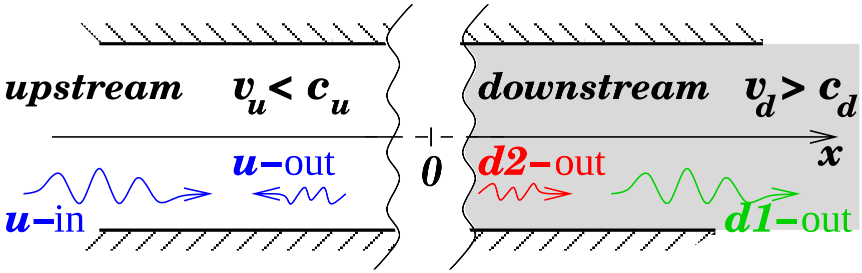



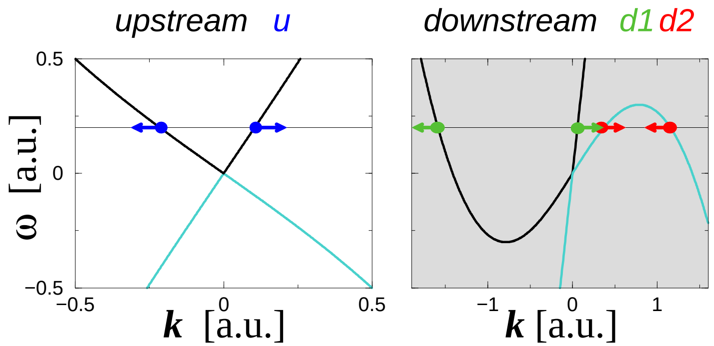

tunnel proba $R \propto \exp\left\{-\frac{2\,S}{\hbar}\right\}$

$$S = \left|\mathrm{Im} \int p(x)\mathrm{d}x\right| \simeq \frac{\pi\,E}{c'(0)}$$

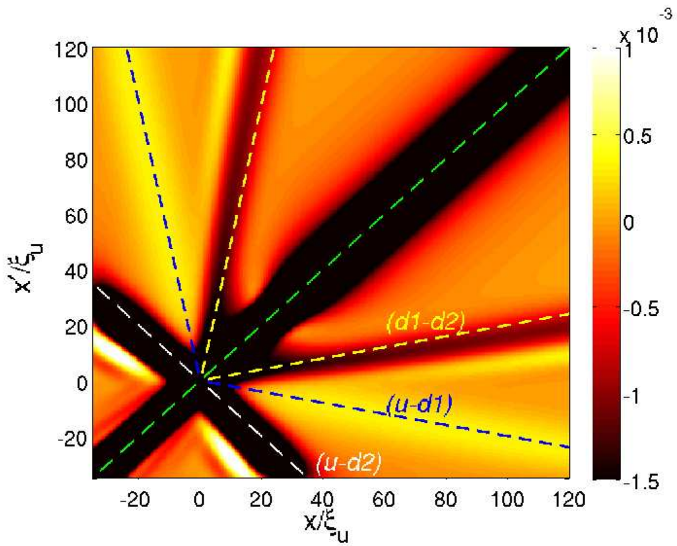
of the form $R \propto \exp\{-E/k_B T_H\}$

with $T_H \sim 10$ nK very weak ...



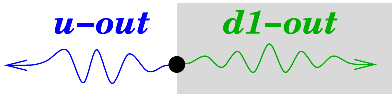

new theoretical and experimental interest : study of density correlation on each side of the horizon




$$G^{(2)}(x, x') = \frac{\langle : n(x) n(x') : \rangle}{\langle n(x') \rangle \langle n(x) \rangle} - 1$$

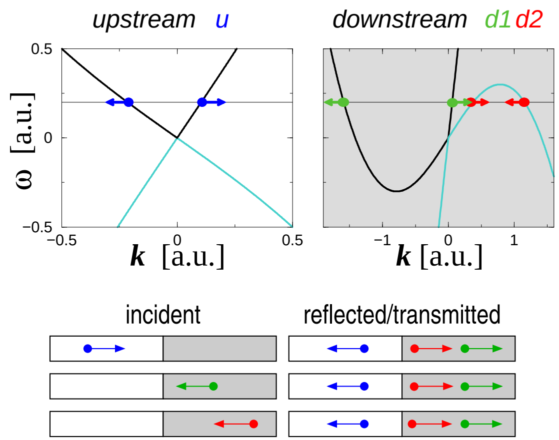
Balbinot, Carusotto, Fabbri, Fagnocchi, Recati
Phys. Rev. A (2008) & New J. Phys. (2008)

example :




$x = (v_d + c_d)t$ correlates with $x' = (v_u - c_u)t$

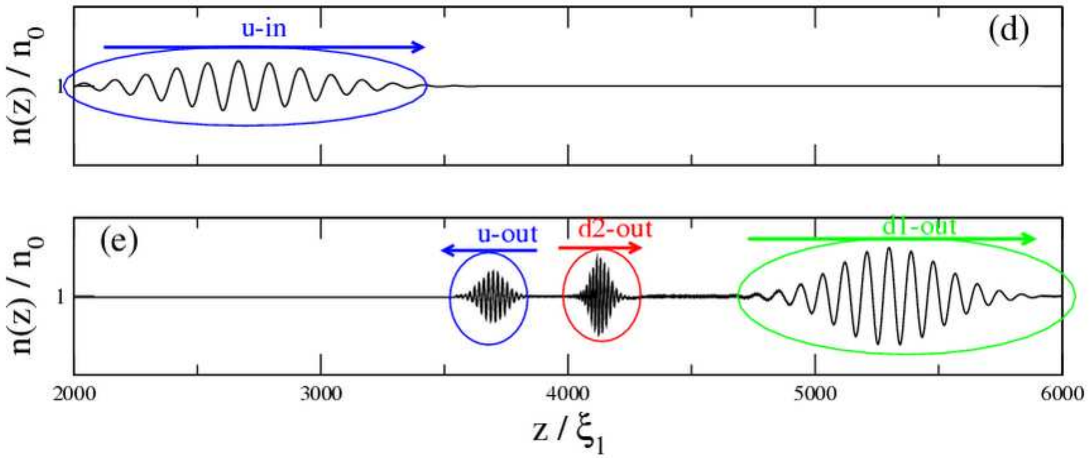
## In practice :




$$(\omega - k\,V_{\rm beam})^2 = \omega_{\rm B}^2(k)$$

$$\hat{\psi} = \psi_0 + \delta\hat{\psi} \text{ with } \delta\hat{\psi}(x) = \sum(\,3 \text{ modes}\,)$$

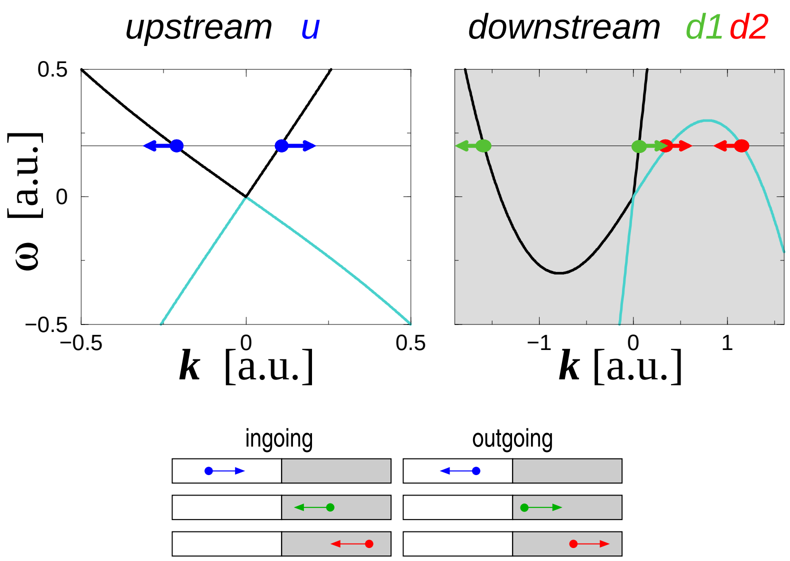
Model configuration : $U(x)$ and $g(x)$ step like with
$U(x) + g(x)n_0 = C^{\rm st}$ such that $\psi_0(x) = \sqrt{n_0}\exp\{ik_0 x\}$, $\forall x$.



# One-body Hawking signal

linear relation connecting the operators of the out-going modes $\hat{b}_{u,d1,d2}$ to the in-going $\hat{a}_{u,d1,d2}$ ones

$$\begin{pmatrix} \hat{b}_u(\omega) \\ \hat{b}_{d1}(\omega) \\ \hat{b}^\dagger_{d2}(\omega) \end{pmatrix} = \mathbf{S}(\omega) \begin{pmatrix} \hat{a}_u(\omega) \\ \hat{a}_{d1}(\omega) \\ \hat{a}^\dagger_{d2}(\omega) \end{pmatrix} .$$



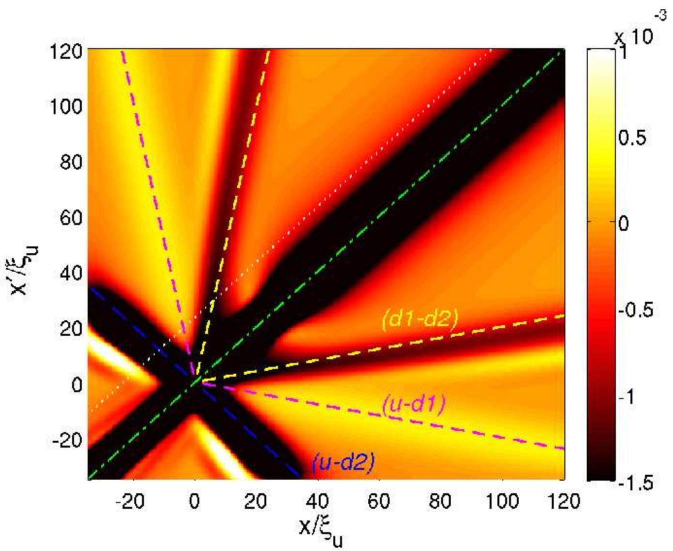

Radiation in the subsonic region occurs in the $u$-outgoing mode with

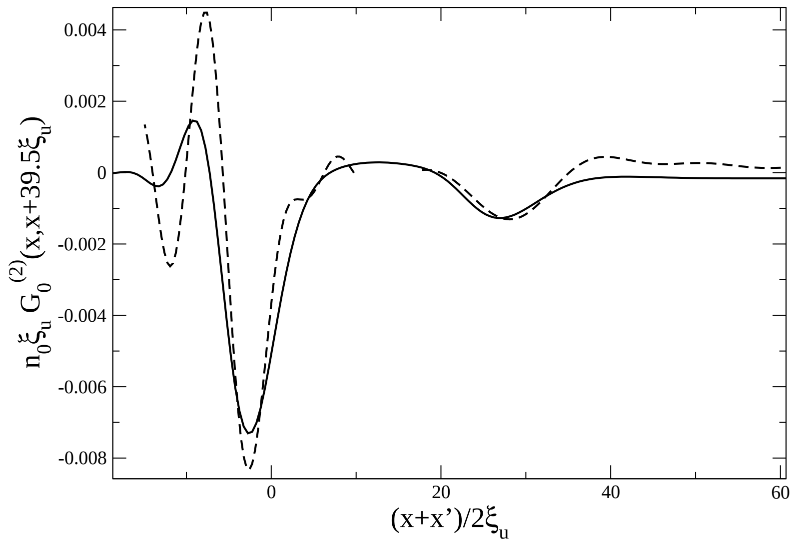
$$\frac{\mathrm{d}I_u^{\mathrm{out}}}{\mathrm{d}t\,\mathrm{d}\omega} = \langle \hat{b}^\dagger_u(\omega)\,\hat{b}_u(\omega)\rangle = |\mathbf{S}_{uu}|^2\, I_u^{\mathrm{in}} + |\mathbf{S}_{ud1}|^2\, I_{d1}^{\mathrm{in}} + |\mathbf{S}_{ud2}|^2\,(I_{d2}^{\mathrm{in}} + 1)\,.$$

at $T = 0$ : $\frac{\mathrm{d}I_u^{\mathrm{out}}}{\mathrm{d}t\,\mathrm{d}\omega} = |\mathbf{S}_{ud2}|^2$ needs $\left\{ \begin{array}{l} u \rightleftharpoons d2 \text{ mode conversion} \\ d2\text{-ingoing mode !} \end{array} \right.$

# Two-body Hawking signal

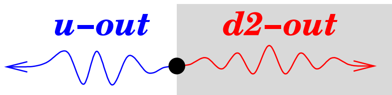
Comparison of numerical and analytic results (stationary phase neglecting interferences between the correlation signals) :



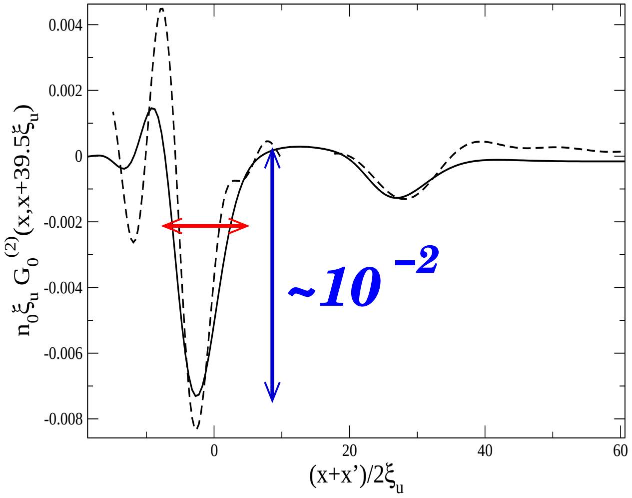

A. Recati, N. Pavloff & I. Carusotto, Phys. Rev. A **80**, 043603 (2009)

main correlation signal :




$x = V_{d2-out}\, t$ correlates with $x' = V_{u-out}\, t$

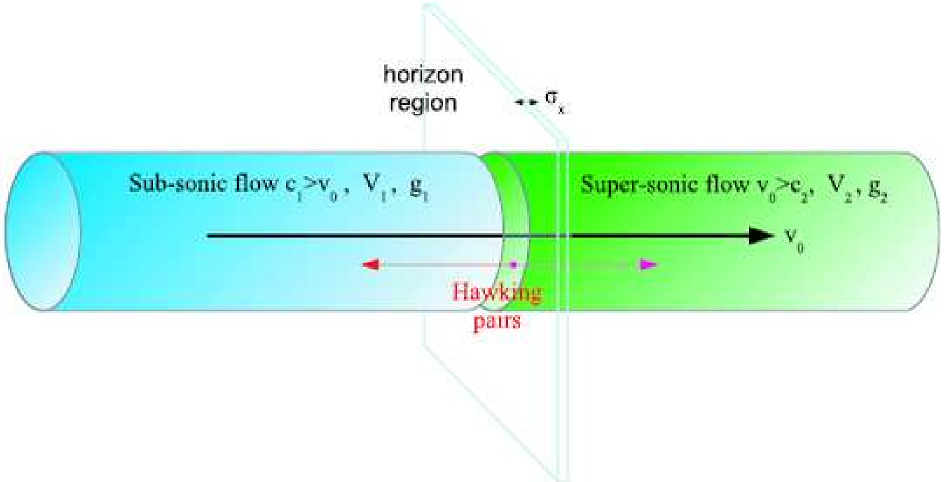
## orders of magnitude :




$^{87}$Rb

| | laser | cigar |
|---|---|---|
| | $n_0 \sim 50\ \mu\text{m}^{-1}$ | $100\ \mu\text{m}^{-1}$ |
| | $c \sim 1$ mm/s | 3 mm/s |
| | $\xi \sim 1\ \mu$m | 0.1 $\mu$m |

$$\left|G^{(2)}\right|_{\max} \sim 2 \times 10^{-4} \leftrightarrow 10^{-3}$$




need for new dumb-hole configurations !

## Partial Conclusion

Density correlations appear as promising tools for identifying Hawking radiation ...with some unessential limitations.

→ Clear signal , well understood. One knows where to look, and at which quantity.

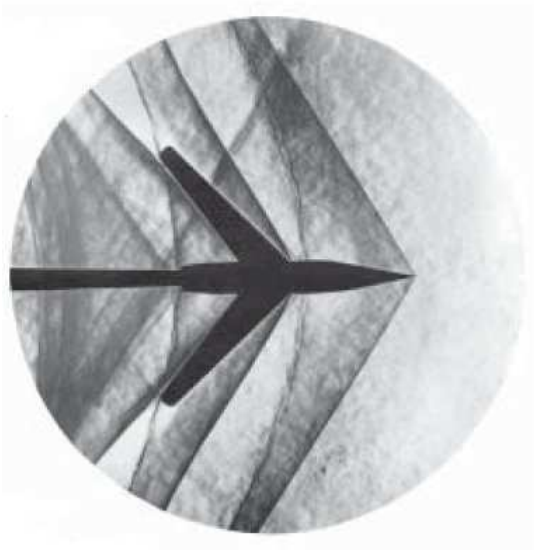
→ Poorly affected by noise and finite $T$ .

→ Drawbacks : 
- weak signal intensity.
- awkward configuration.
- what about transverse degrees of freedom ?

→ What comes next ? 
- more realistic dumb hole configurations,
- white hole stability ...

# Shock Waves

## Dissipative shock



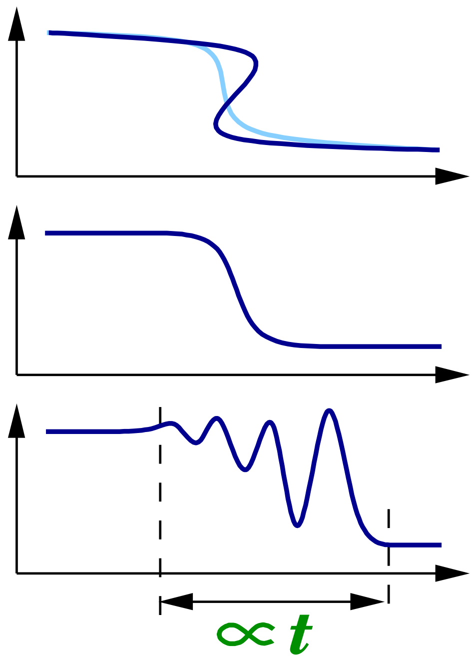
Schlieren photograph of a shock attached on a supersonic body

## Dispersive shock

Tidal bore on river Severn

# Shock Waves


$$u_t + u u_x = \begin{cases} 0 & \text{Hopf} \\ \epsilon\, u_{xx} & \text{Burgers} \\ \epsilon\, u_{xxx} & \text{KdV} \end{cases}$$




steepening then breaking

Taylor profile

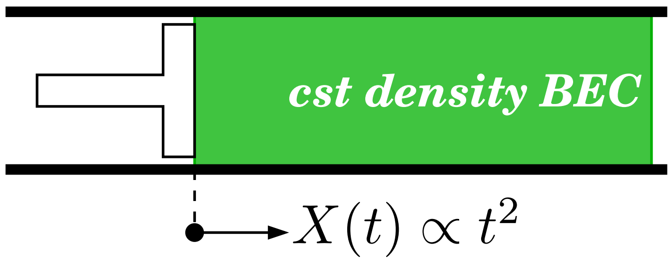
Gurevich Pitaevskii

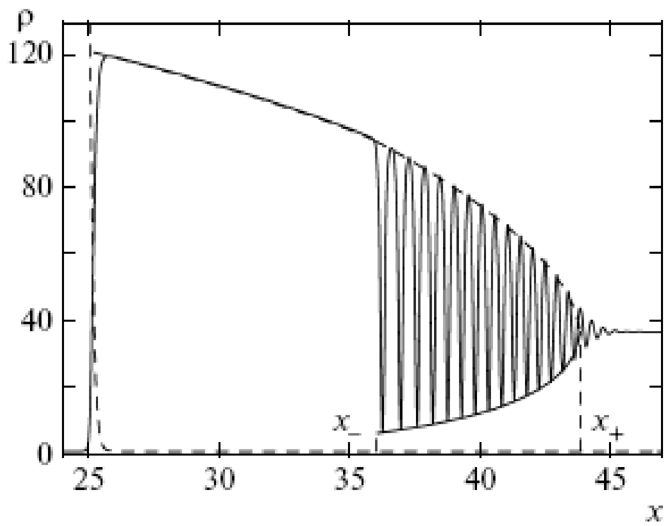
# Atom laser shock wave

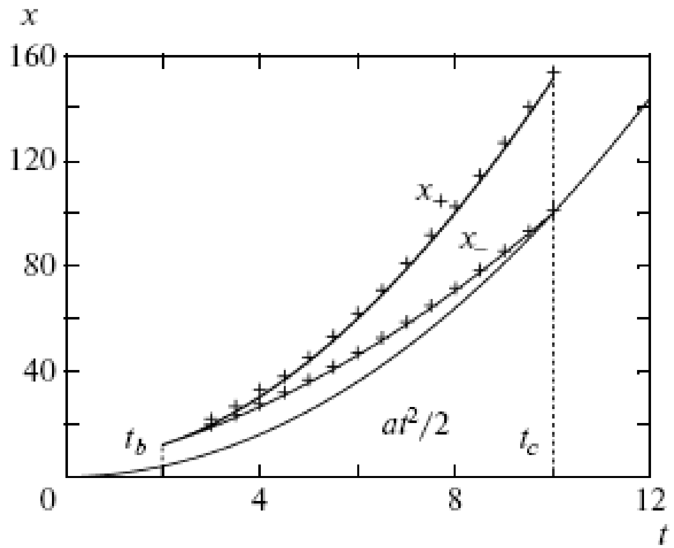
Z. Dutton, M. Budde, C. Slowe, L.V. Hau, Science **293**, 663 (2001) ; M.A. Hoefer *et al.*, PRA **74**, 023623 (2006)

simplest configuration: hard wall moving with constant acceleration

Kamchatnov & Korneev, JETP **110**, 170 (2010)



## Conclusion

This talk focussed not on the specific features of atom laser physics, but rather on some of the physical problems that can be investigated by means of atom lasers.

→ transport in presence of disorder . Effects of interaction lead to qualitatively different phenomena. What is the transmission in the time dependent regime ?

→ Hawking radiation , BECs seem to offer the most promising prospect to observe a fully quantum Hawking radiation.

→ Dispersive shocks , BEC appears to be a versatile tool for studying frictionless nonlinear mechanisms of dissipation.

## BEC in presence of disorder ?

• In the case of strong disorder :

$\hookrightarrow$ phase transition at $T = 0 \rightarrow$ "Bose glass" : non-superfluid.

$\hookrightarrow$ The system can no longer be described by GPE.

• Here we consider only the case of weak disorder.

$\hookrightarrow$ only slightly decreases the condensate and the superfluid fraction

K. Huang & H. F. Meng, Phys. Rev. Lett. **69**, 644 (1992); S. Giorgini, L. Pitaevskii & S. Stringari, Phys. Rev. B **49**, 12938 (1994).

$\hookrightarrow$ more precisely, for $U(x) = \lambda\,\mu\,\xi \sum \delta(x - x_n)$, the depletion of the condensate is proportional to $n_{\rm i}\,\xi\,\lambda^2 \ll 1$ here.

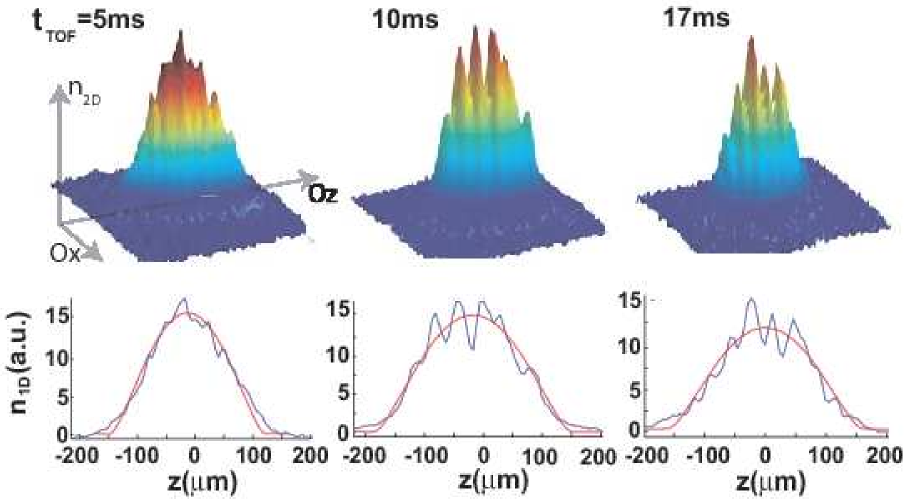
G. E. Astrakharchik & L. P. Pitaevskii, Phys. Rev. A **70**, 013608 (2004)

T. Paul, P. Lebœuf, P. Schlagheck & N. Pavloff, Phys. Rev. Lett. **98**, 210602 (2007)

# BEC in presence of disorder ?

experimental evidence of phase coherence in presence of disorder :

Rice and LCFIO




D. Clément, Ph. Bouyer, A. Aspect & L. Sanchez-Palencia, Phys. Rev. A **77**, 033631 (2008)

Yong P. Chen. *et al.*, Phys. Rev. A **77**, 033632 (2008)

# Extreme value statistics

Consider $N$ uncorrelated random variables: $U_1, ...U_N$.
What is the distribution of $U_{\rm m} = \max\{U_1, ..., U_N\}$ ?

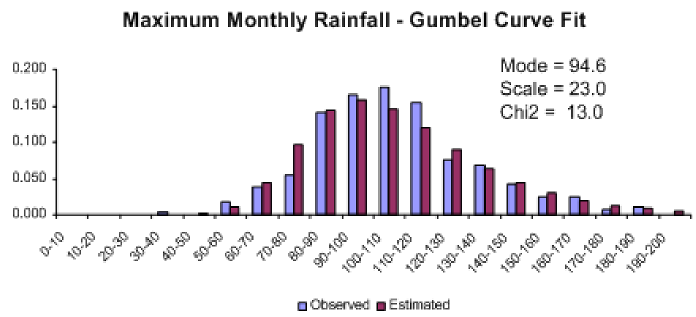
one has: $\text{proba}\,(U_{\rm m} < U) = \text{proba}\,(U_1 < U) \times ... \times \text{proba}\,(U_N < U)$

Hence the cumulative distributions verify $\mathscr{F}(U) = [F(U)]^N$.

If $p(U) = e^{-U} \quad (U \in \mathbb{R}^+)$, then $x = U_{\rm m} - \ln(N)$ is distributed according to the Gumble distribution

$$p(x) = e^{-x} \exp\left\{-e^{-x}\right\} \; .$$

universal provided $p(U)$ decreases at infinity faster than a power.



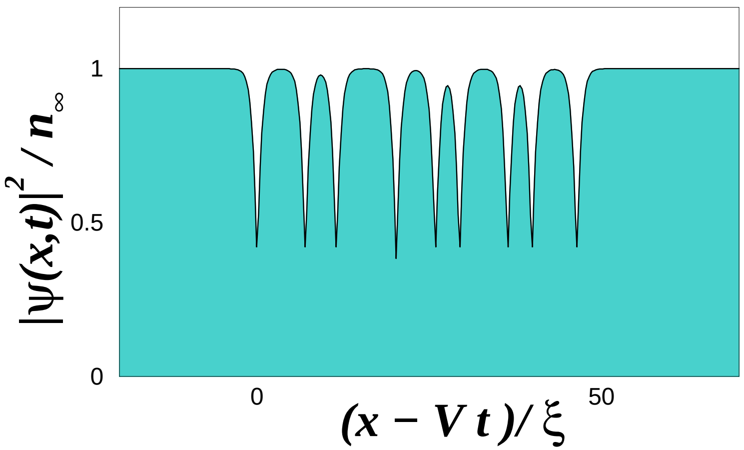

Distribution of the largest monthly rainfall over a period of 291 years at Kew Gardens (London).

## Superfluid (and subsonic) regime

In this regime (stable with respect to time evolution), only local and stationary perturbations around the impurities.

Perfect transmission of the matter wave.

No drag is exerted on the potential, but the flow is associated to a momentum




$$P = \hbar \int_{\mathbb{R}} \mathrm{d}x [n(x) - n_0] \partial_x S \ ,$$

where $S$ is the phase of $\psi$.

This allows to determine the mass of the non superfluid component $M_\mathrm{n} = P/v_\mathrm{beam}$ . Defining $M = m n_0 L$ perturbation theory yields

$$\frac{M_\mathrm{n}}{M} = \frac{m^2}{2\,\hbar^4 \kappa^3 L} \int_{\mathbb{R}^2} \mathrm{d}y_1 \mathrm{d}y_2\, U(y_1) U(y_2) (1 + 2\kappa |y_1 - y_2|) \mathrm{e}^{-2\kappa|y_1 - y_2|} \ .$$

$M_\mathrm{n}/M \ll 1$ when $|\delta n(x)| \ll n_0$. $\kappa = \frac{m}{\hbar} \left| V^2 - c^2 \right|^{1/2}$

# Damping of dipolar oscillations

M. Albert, T. Paul, N. Pavloff & P. Lebœuf, Phys. Rev. Lett. **100**, 250405 (2008)

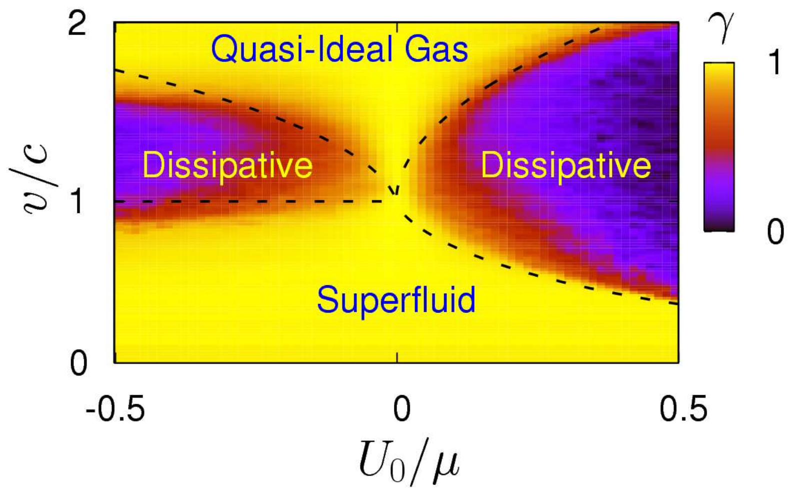
$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \left[\frac{m}{2}\omega_x^2\, x^2 + U(x) + 2\hbar\omega_\perp (an)^\nu\right]\psi\ .$$

• if $U(x) \equiv 0$, center of mass:
$X_t = d_0\ \cos(\omega_x\, t)$

• If $U(x) = U_0\ \exp\{-\frac{x^2}{2\,\sigma^2}\}$,
$X_t \rightsquigarrow d_f\cos(t)$ when $t \to \infty$

• Define $\gamma = d_f / d_0$

$$\begin{cases} \text{no damping:} & \gamma = 1 \\ \text{strong damping:} & \gamma \to 0 \end{cases}$$




$v = d_0\omega_x$

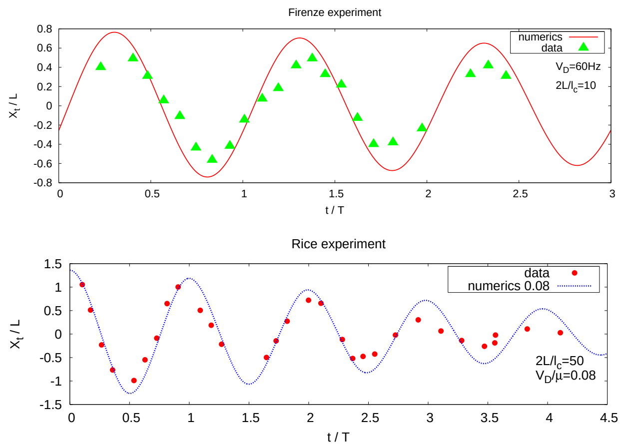
$c$: sound velocity at center of the trap

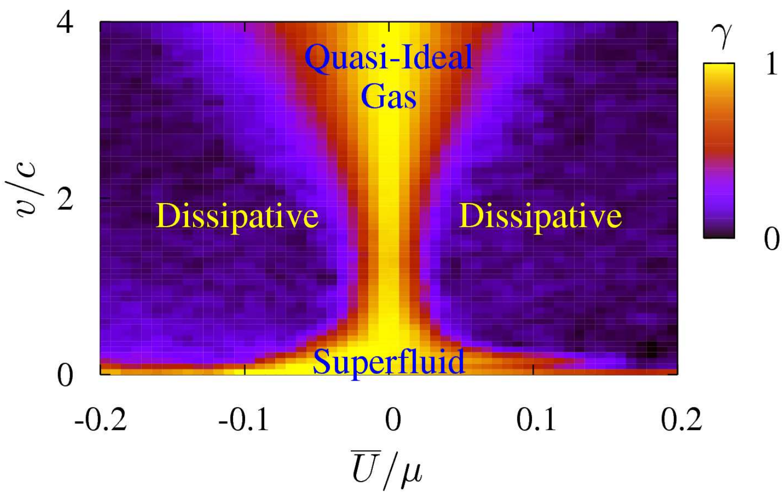
$\mu$: chemical potential

# In presence of disorder :

J. E. Lye *et al.*, Phys. Rev. Lett. **95**, 070401 (2005)

Y. P. Chen *et al.*, Phys. Rev. A **77**, 033632 (2008)




Rice : $d_0 = 700\,\mu\text{m}$,

$L_x = 1000\ \mu\text{m}$

$v/c = 2.8$ and $\bar{U}/\mu = 0.04$

whereas the damping threshold is at

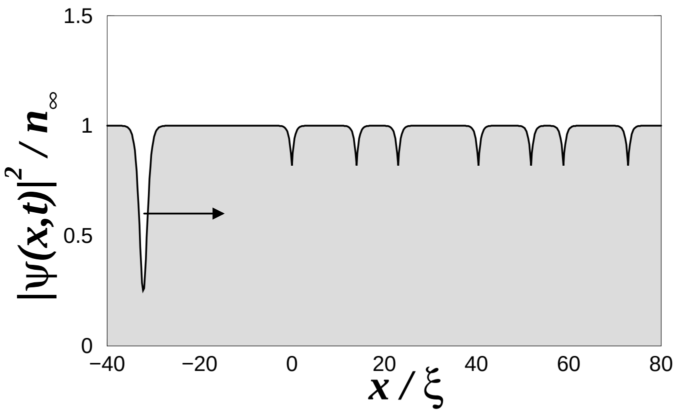
$\bar{U}/\mu = 0.008$

**Possible effects of localization... but subtle**

# Scattering of a dark soliton

One considers a dark soliton incident on a disordered region

N. Bilas & N. Pavloff, Phys. Rev. Lett. **95**, 130403 (2005)




The disordered potential reads[a] :

$$U(x) = \lambda \, \mu \, \xi \sum_n \delta(x - x_n) \; , \qquad (4)$$

with $x_n$'s: uncorrelated random position of the impurities with mean density $n_{\rm i}$ $0 = x_1 \leq x_2 \leq x_3 \ldots$

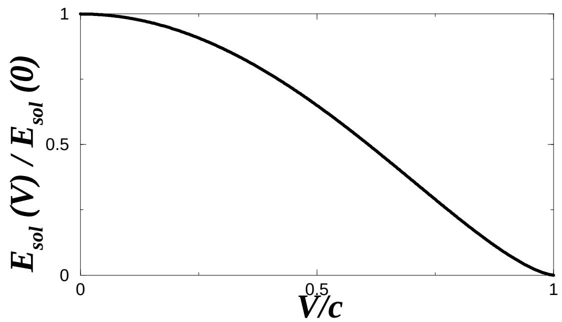
[a] cf. Y. S. Kivshar, S. A. Gredeskul, A. Sánchez & L. Vázquez, Phys. Rev. Lett. **64**, 1693 (1990)

One has $\langle U(x) U(x') \rangle - \langle U(x) \rangle \langle U(x') \rangle = \left( \frac{\hbar^2}{m} \right)^2 \sigma \, \delta(x - x') \; ,$

with $\sigma = n_{\rm i} \, \lambda^2 / \xi^2$.

A dark soliton with velocity $V$ has an energy $E_{\rm sol}$

$$E_{\rm sol} = \frac{4}{3}\,\mu\left(\frac{a_1}{\xi}\right)\left(1-\frac{V^2}{c^2}\right)^{3/2} .$$




In the limit $\lambda \ll 1$ [a] and $V^2 \gg \lambda\, c^2$ [b] a soliton scattering on **a single impurity** radiates an energy $E^{+}_{\rm rad} + E^{-}_{\rm rad}$ with

$$E^{\pm}_{\rm rad} = \mu\,\lambda^2\, F^{\pm}(V/c) ,$$

where for $v = V/c \in [0,1]$

$$F^{\pm}(v) = \frac{\pi}{16\,v^6}\int_0^{+\infty} dy\, \frac{y^4\left(-v \pm \sqrt{1+y^2/4}\right)}{\sinh^2\left[\frac{\pi\, y\sqrt{1+y^2/4}}{2v\sqrt{1-v^2}}\right]} .$$

N. Bilas & N. Pavloff, Phys. Rev. A **72**, 033618 (2005)

[a] This ensures that the impurity only weakly perturbs the constant density profile.

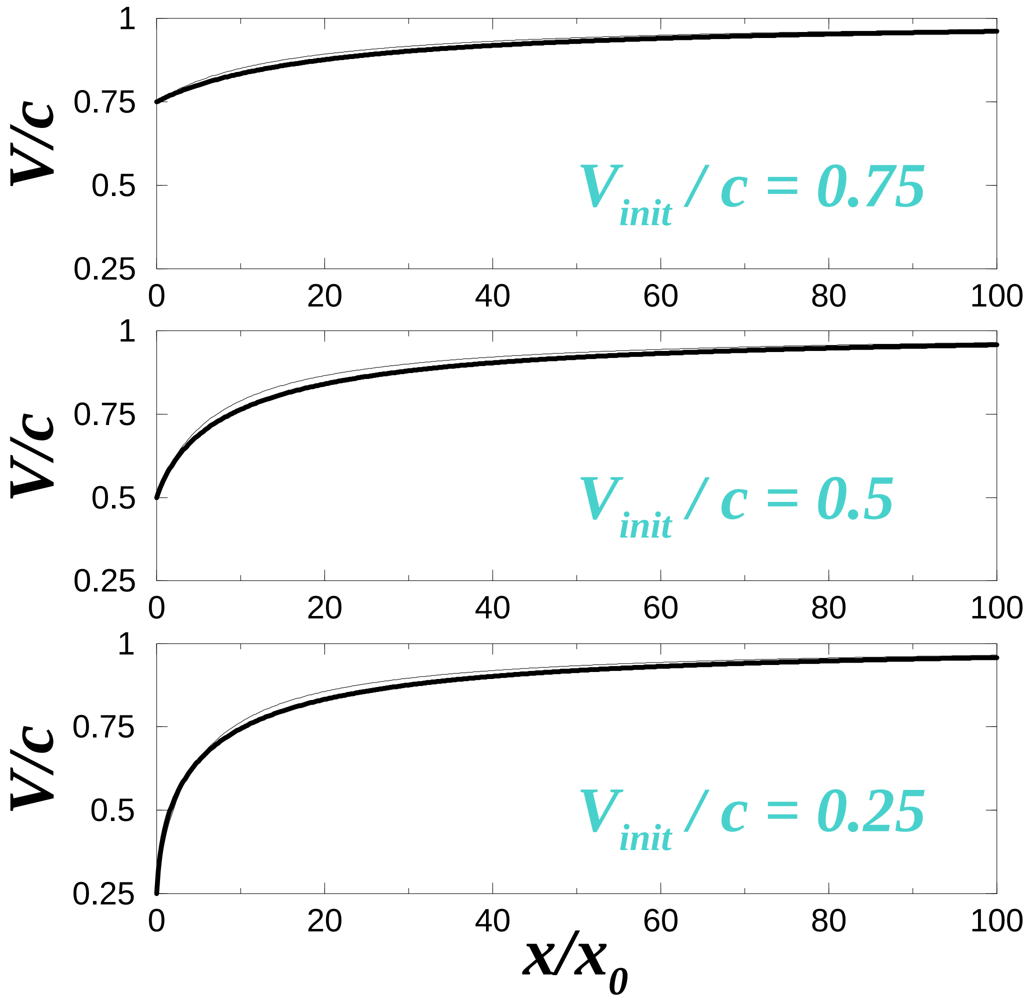
[b] This ensures that the scattering process can be treated perturbatively.

In the limit $\xi \ll \frac{1}{n_{\rm i}}$. the scattering of the soliton by the impurities can be treated as a sequence of independent events. This leads to

$$\frac{dV}{dx} = \frac{c}{4\,x_0}\,\frac{F^+(V/c) + F^-(V/c)}{\frac{V}{c}\sqrt{1-(V/c)^2}} \quad \text{with} \quad x_0 = \frac{a_1}{\sigma\,\xi^3}$$

If $v = V/c \to 1$ one has $F^+(v) + F^-(v) = \frac{4}{15}\left(1 - v^2\right)^{5/2}$.
This yields :

$$V(x) = c\sqrt{1 - \frac{1 - V_{\rm init}^2/c^2}{1 + (1 - V_{\rm init}^2/c^2)\,\frac{2\,x}{15\,x_0}}}\;.$$




In these plots

$$x_0 = \frac{a_1}{\sigma\,\xi^3}\ .$$

For $x \gg x_0$ one has

$$V(x) \simeq c\left(1 - \frac{15\,x_0}{4\,x}\right),$$

independent of $V_{\rm init}$.

The soliton has disappeared when $\Delta N \sim 1$. This happens for a critical velocity $V_{\rm cr} = c[1-(\xi/2a_1)^2]^{1/2}$. Hence the distance covered by the soliton in the disordered region before decaying is

$$L = 30\, a_1 \left(\frac{a_1}{\xi}\right)^2 \times \frac{1}{\sigma \xi^3}.$$

## Partial Conclusion

(1) The soliton is accelerated until it reaches the speed of sound and disappears.

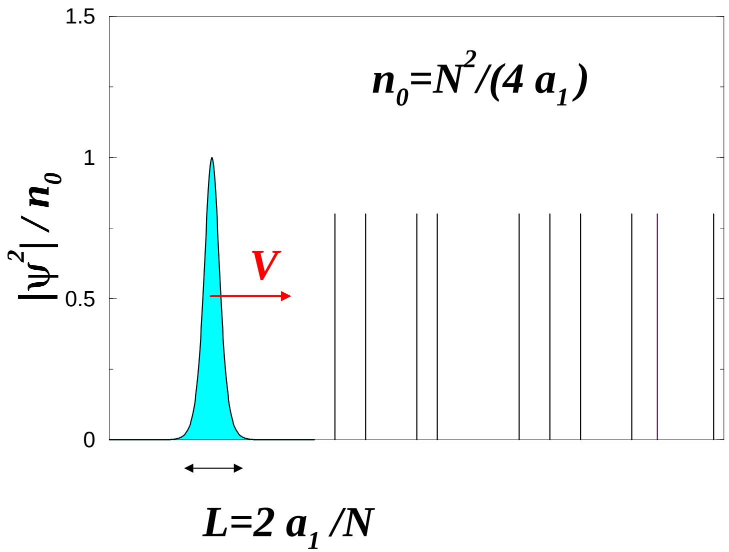
(2) Its decay is algebraic and not exponential.

(3) The length covered in the disordered region before decaying is independent of the initial velocity of the soliton (as is the corresponding travelling time).

# Bright soliton incident on a disordered potential

attractive effective interaction ($a_1 \to -a_1$). A bright soliton is characterized by 2 parameters : $N$ and $V$. It has an energy $E_{\rm sol}$ with

$$\frac{E_{\rm sol}}{N} = \frac{1}{2} m V^2 - \frac{1}{3} \frac{\hbar^2}{m a_1^2} N^2 \; .$$

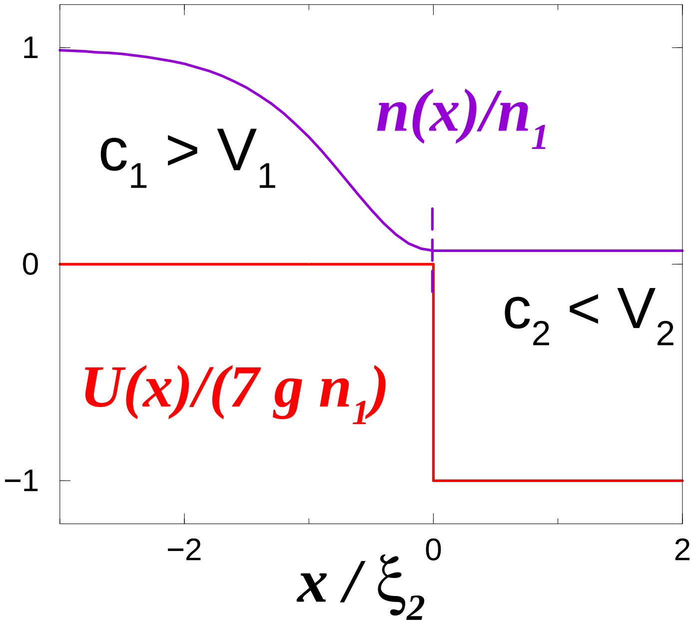



if $mV^2 \gg \hbar^2 N^2/(m a_1^2)$ : $V \sim C^{\rm st}$ and $N$ decreases exponentially.

if $mV^2 \ll \hbar^2 N^2/(m a_1^2)$ : $V$ and $N$ tend to a $C^{\rm st}$.

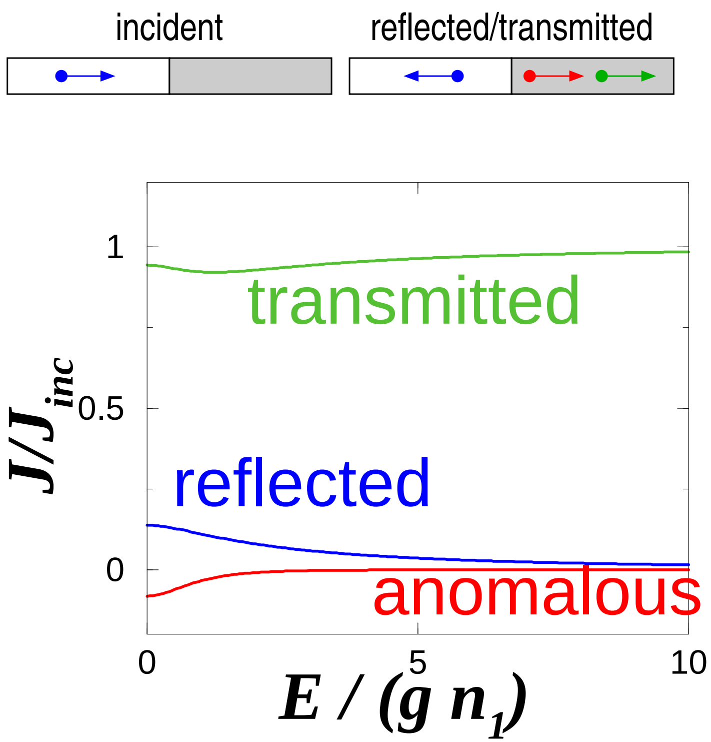
Y. S. Kivshar, S. A. Gredeskul, A. Sánchez & L. Vázquez, Phys. Rev. Lett. **64**, 1693 (1990).

# Waterfall configuration

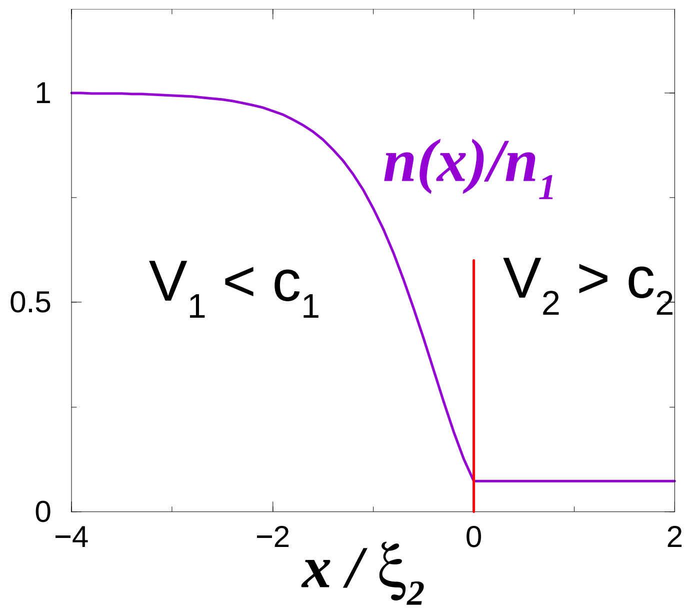



$V_1/c_1 = 0.25 \quad V_2/c_2 = 16$

for instance :

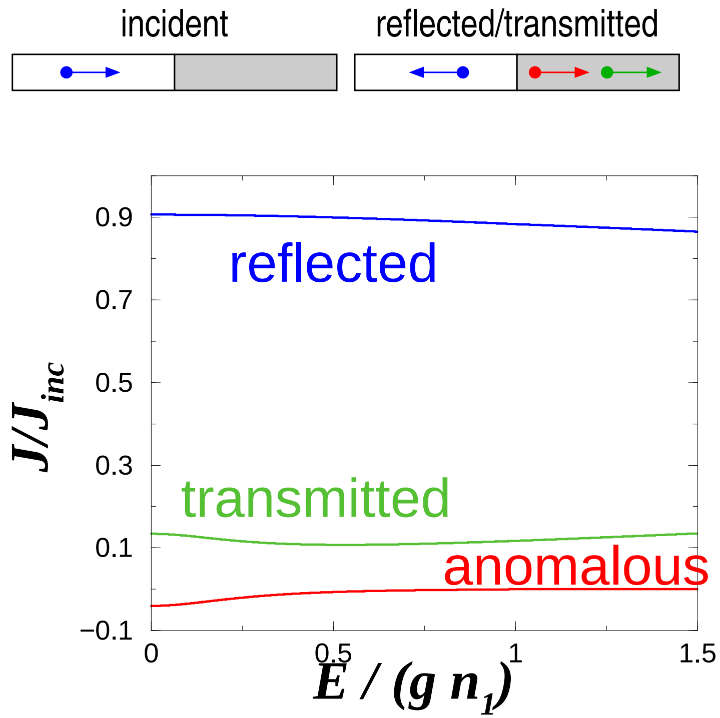


# Localized obstacle



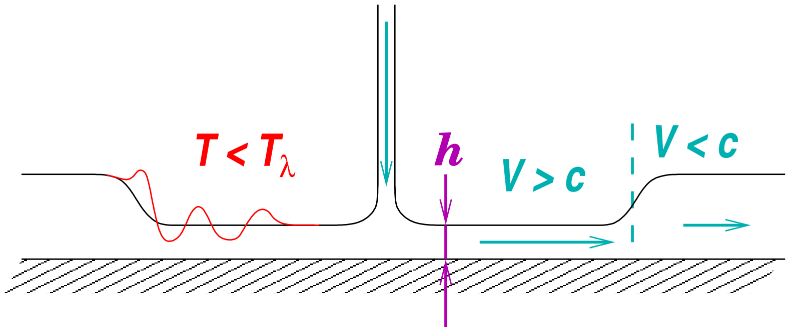

$V_1/c_1 = 0.1 \qquad V_2/c_2 = 5.0$

for instance :



# At longer term ...

The hydraulic jump is a stable white hole (Volovik JETP 2005)



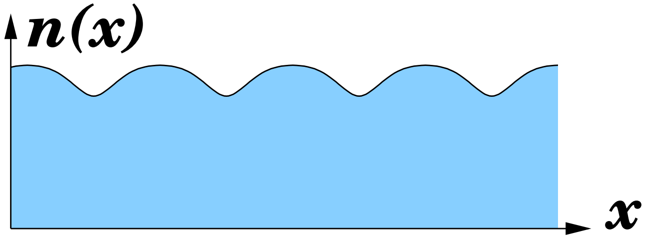

$$c = \sqrt{gh}$$

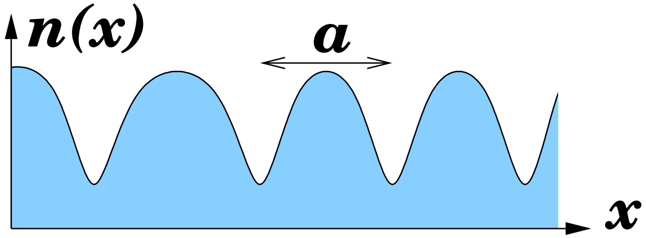
$$(\omega - \vec{k}.\vec{v})^2 = c^2\, k^2 \left[1 + h^2 k^2 \left(-\frac{1}{3} + \frac{\sigma}{\rho g h^2}\right) + ...\right]$$

appearance of oscillations in the superfluid phase ?
cf, Pitaevskii striped phase ?

# 1D Super-solid

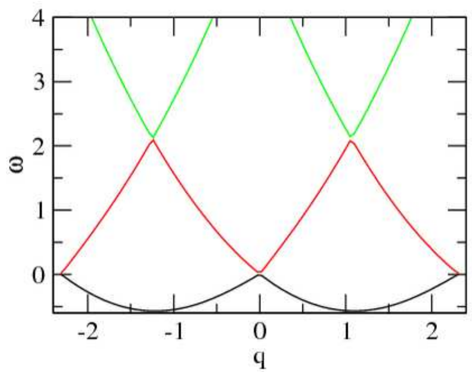
L. Pitaevskii (JETP 84): above the Landau critical velocity, a super-sonic superfluid forms a “striped phase”

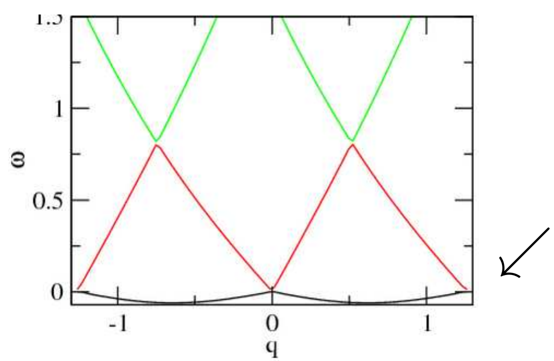



Question: is this “supersolid” phase superfluid ?

One has to study the excitation spectrum :




$$\omega \simeq -\sqrt{\frac{k}{m}}\left|\sin\left(\frac{qa}{2}\right)\right|$$