Sonic black holes and Hawking radiation in Bose-Einstein condensates

Nicolas Pavloff

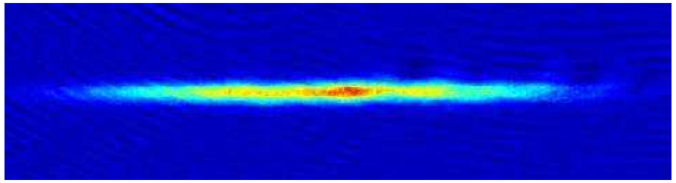
Laboratoire de Physique Théorique et Modèles Statistiques
Université Paris-Sud, CNRS, Orsay

work in collaboration with:



I. Carusotto and A. Recati P.É. Larré

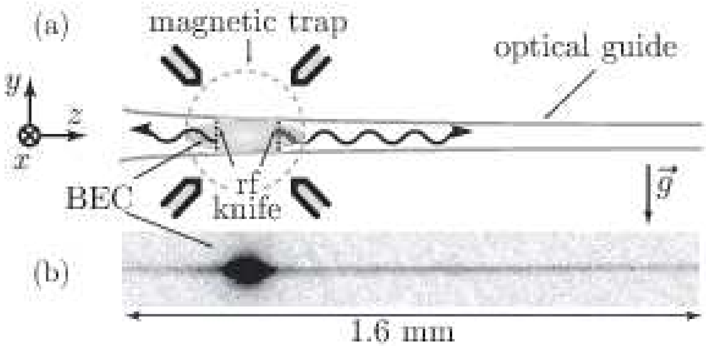
quasi-1D condensates :



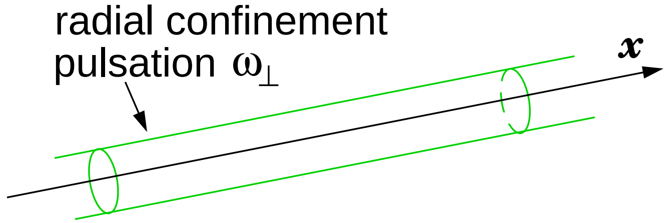
quasi-1D condensate

longitudinal size $\sim 10^2 \mu$m

transverse size $\sim 1 \mu$m



W. Guérin *et al.*, Phys. Rev. Lett. **97**, 200402 (2006)



harmonic radial confinement :

$$V_\perp(\vec{r}_\perp) = \frac{1}{2} m \omega_\perp^2 r_\perp^2 \ .$$

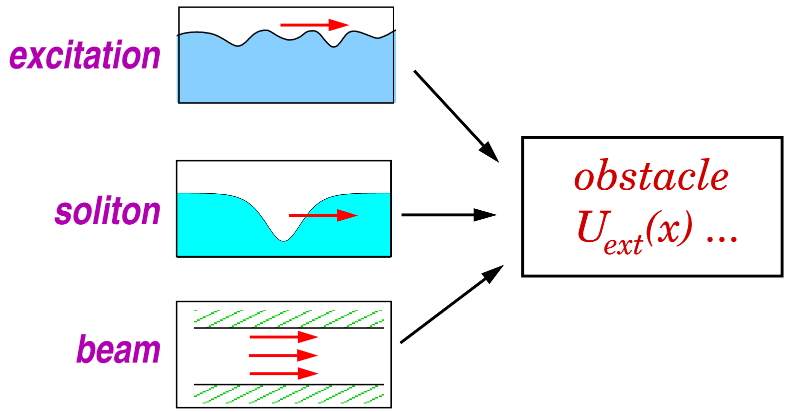
Mesoscopic physics & BECs :

interaction in phase coherent systems,
non-linear transport.

Large range of interaction regimes :
⇝ From "atom lasers" practicaly without interaction → strongly correlated 1D systems
⇝ well defined theoretical framework (Bose-Hubbard/Gross-Pitaevskii)

Situations of 1D transport :

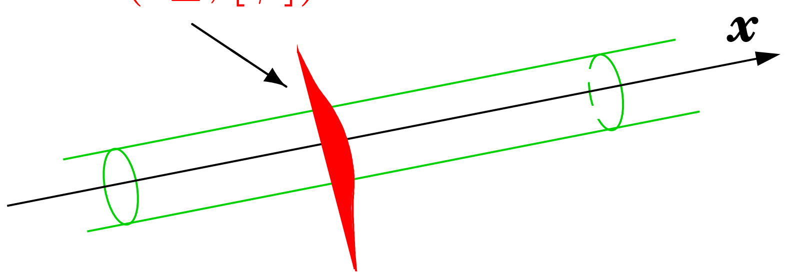
- Propagation of excitations, of (dark) solitons, of a beam ...
- In presence of localized or extended obtacles
- Effects of disorder
- Black-hole configuration
- Dispersive shock waves



1D mean field regime

Born-Oppenheimer :

$\Psi(\vec{r}, t) = \psi(x, t) \times \Phi(\vec{r}_\perp; [\psi])$



$1D$ mean field regime with order parameter $\psi(x, t)$ verifying

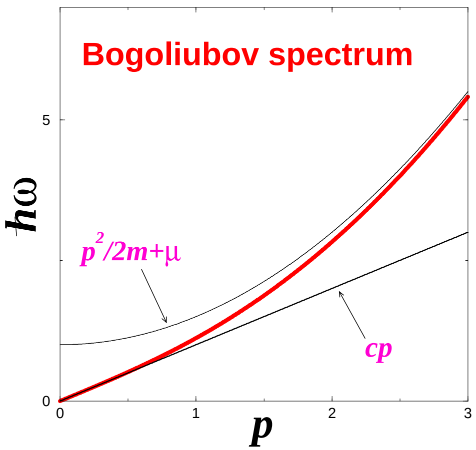
$$-\frac{\hbar^2}{2\,m}\partial_x^2\psi + \left(U_{\text{ext}}(x) + g\,|\psi|^2\right)\psi = i\hbar\,\partial_t\psi \quad \text{or} \quad \mu\,\psi \tag{1}$$

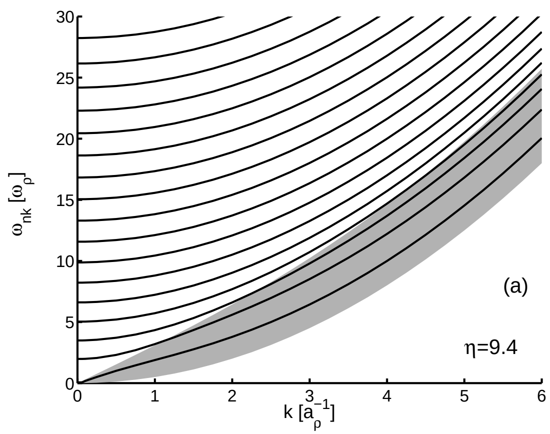
where $|\psi|^2 = n_1(x, t)$ is the longitudinal density of the condensate, and $g = 2\,\hbar\,\omega_\perp\,a$, where a : 3D s-wave scattering length $(a > 0)$

domain of validity : $$\frac{\hbar\,\omega_\perp}{\hbar^2/ma^2} \ll n_1 a \sim \frac{\mu}{\hbar\omega_\perp} \ll 1$$

• The first inequality allows to avoid the Tonks-Girardeau regime and implies $E_{\rm int} \ll E_{\rm kin}$. Also $L_\phi \ggg \xi$ $\quad L_\phi = \xi \exp\left[\pi\sqrt{\frac{\hbar n_1}{2ma\omega_\perp}}\right]$

• the second inequality allows to avoid the 3D-like transverse Thomas-Fermi regime and implies that transverse motion is frozen

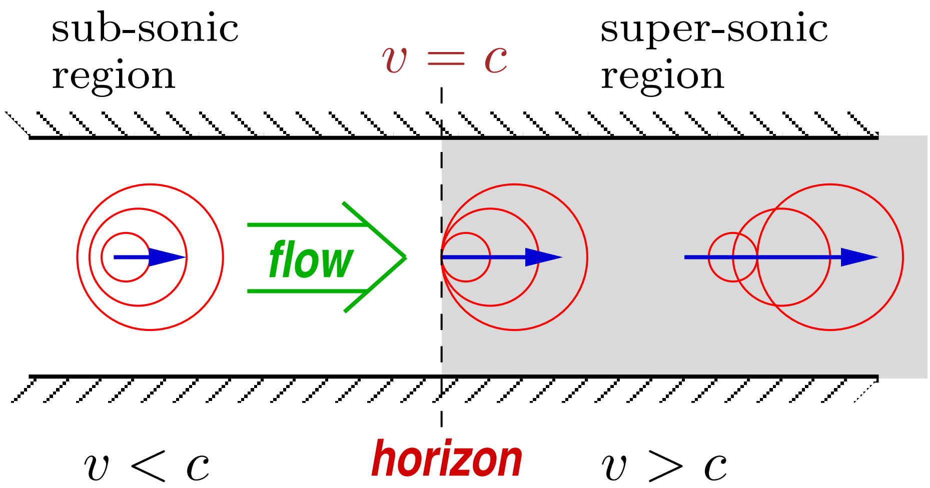




C. Tozzo & F. Dalfovo, Phys. Rev. A (2002)

$\leftarrow \eta = \mu/\hbar\omega_\perp$ only axi-symmetric excitations included $(m = 0)$

Sonic black holes : "dumb holes"



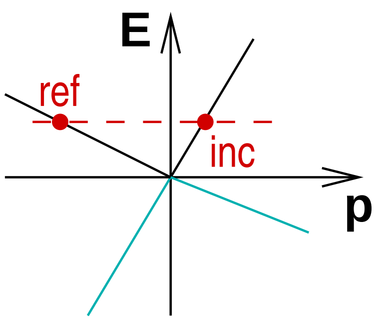
W. G. Unruh, Phys. Rev. Lett. (1981)

even without a source,
vacuum fluctuations
$\rightsquigarrow$ Hawking radiation

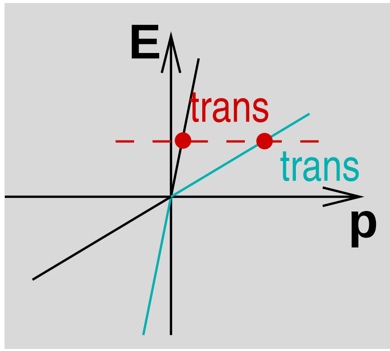
in the laboratory :

$$E(p) = c\,|p| + v\,p$$

$\swarrow$ comoving $\downarrow$ Doppler



subsonic region

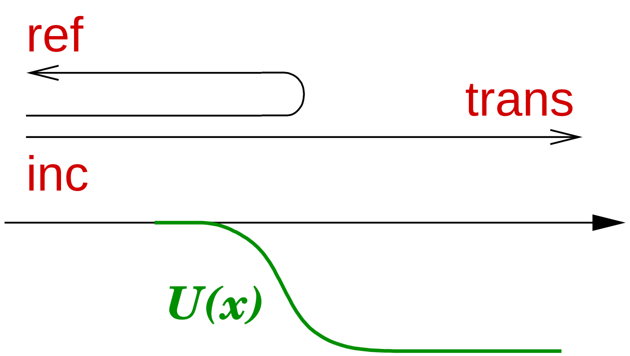


supersonic region

Analogous to tunnel effect : (quantum reflexion)

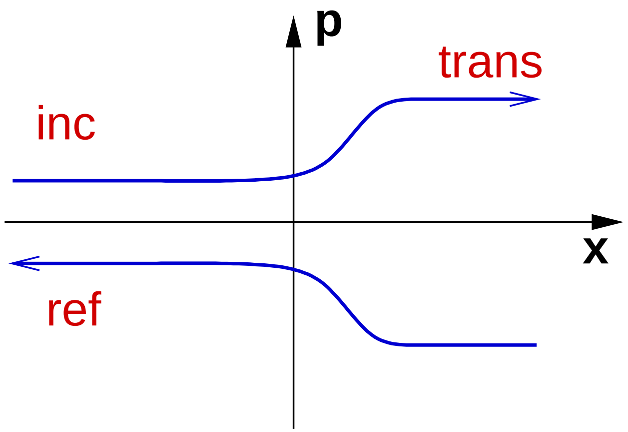
real space

particle incoming from the left with $E > U_{\max}$

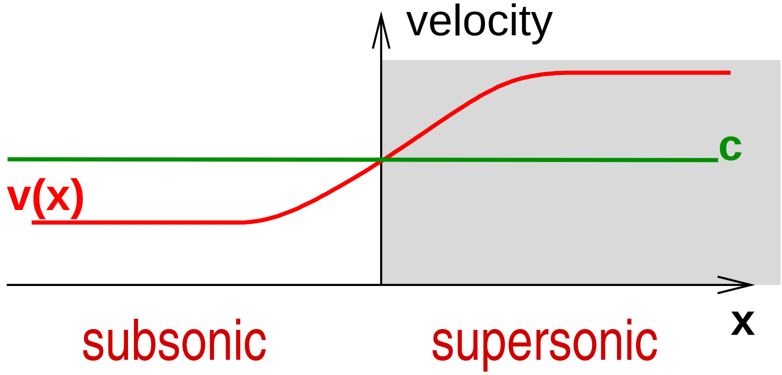


phase space trajectory

$E = p^2/2m + U(x)$



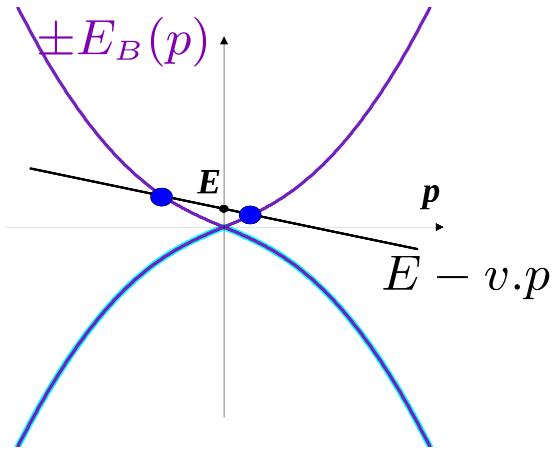
A model configuration :



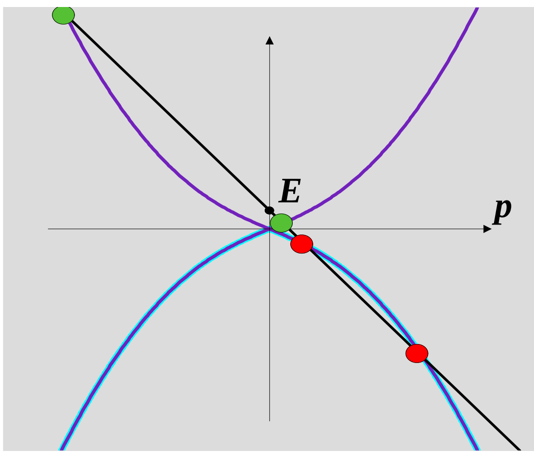
$$E - v(x)p = \pm E_B(p)$$

with

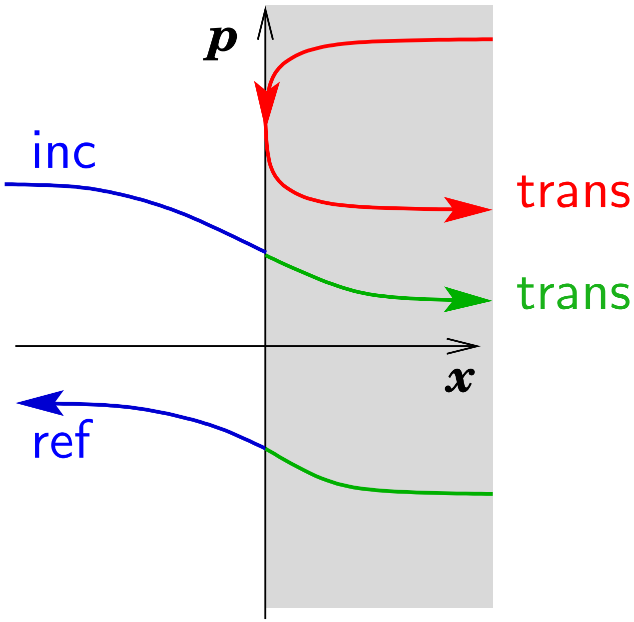
$$E_B(p) = c\,p\,\sqrt{1 + p^2/4}$$



$v < c$



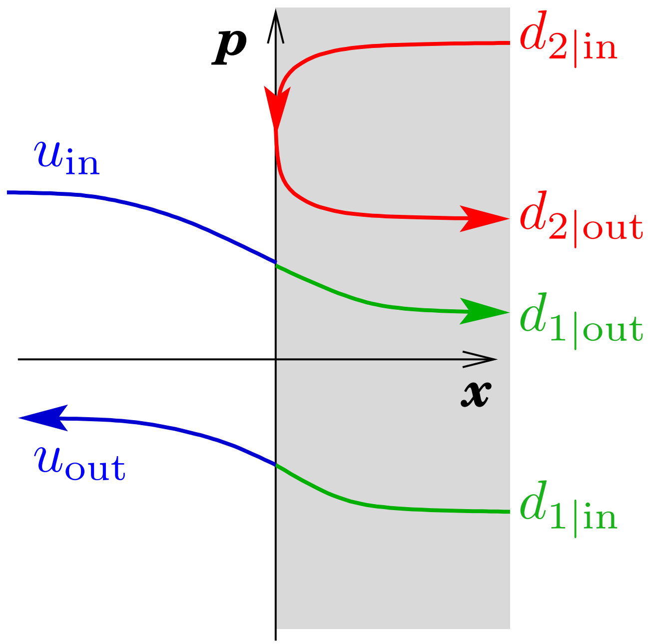
$v > c$

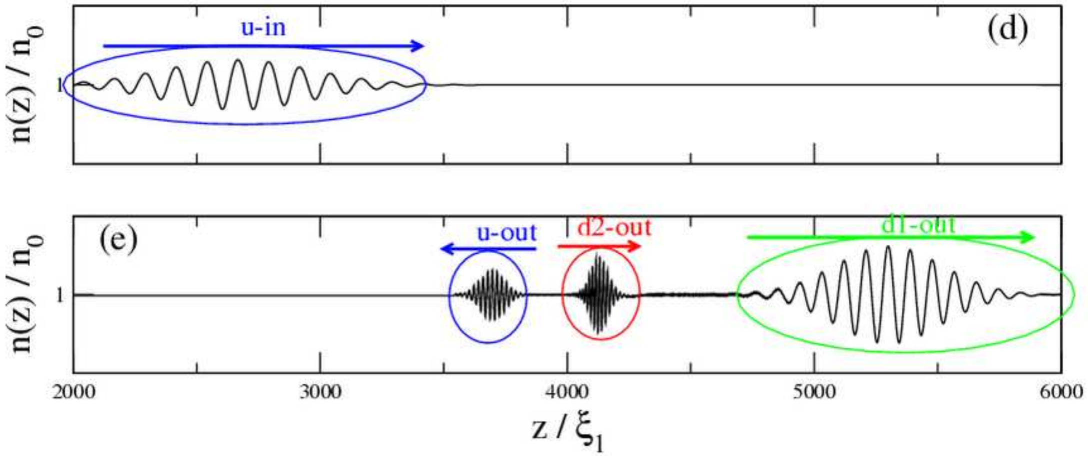


Numerical test, model configuration:

$U(x)$ and $g(x)$ step like with $U(x) + g(x)n_0 = C^{\rm st}$ such that $\psi_0(x) = \sqrt{n_0}\,\exp\{\mathrm{i}k_0\,x\}$, is solution $\forall x$ of

$$-\frac{\hbar^2}{2\,m}\psi_0'' + \left[U(x) + g(x)|\psi_0|^2\right]\psi_0(x) = \mu\,\psi_0(x)\;, \quad C^{\rm st} = \mu - \frac{\hbar^2 k^2}{2\,m}\;.$$



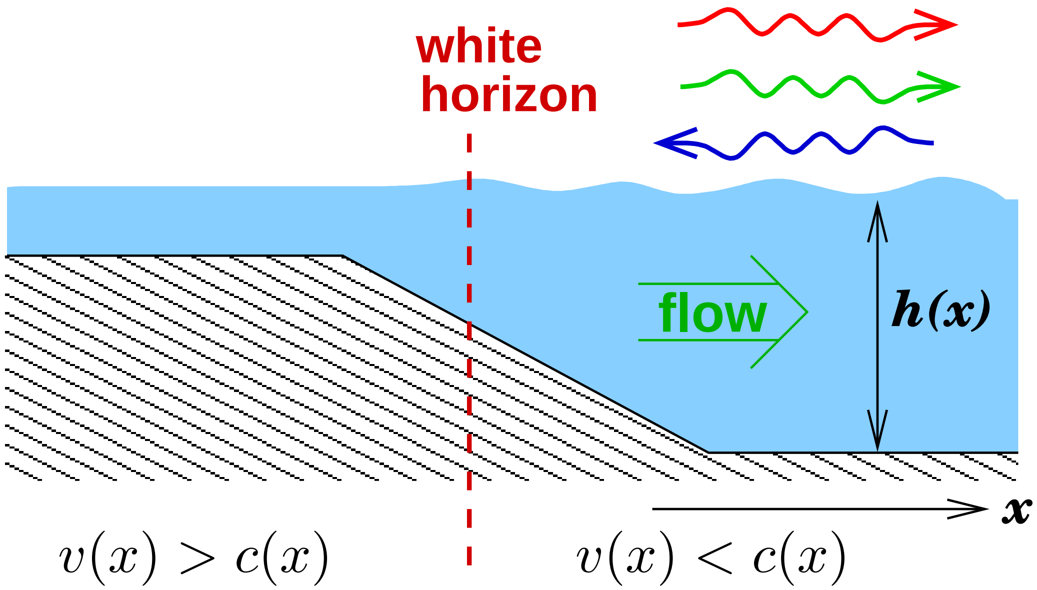


Gravity wave analogues

Schützhold and Unruh, Phys. Rev. D (2002)
Rousseaux *et al.*, New Journal of Physics (2008)
Nardin, Rousseaux, Coullet, Phys. Rev. Lett. (2009)
Weinfurtner *et al.*, Phys. Rev. Lett. (2011)

in a basin of depth h, the dispersion relation of gravity waves is $(\omega - vk)^2 = g\,k\,\tanh(k\,h)$, corresponding to $c = \sqrt{g\,h}$

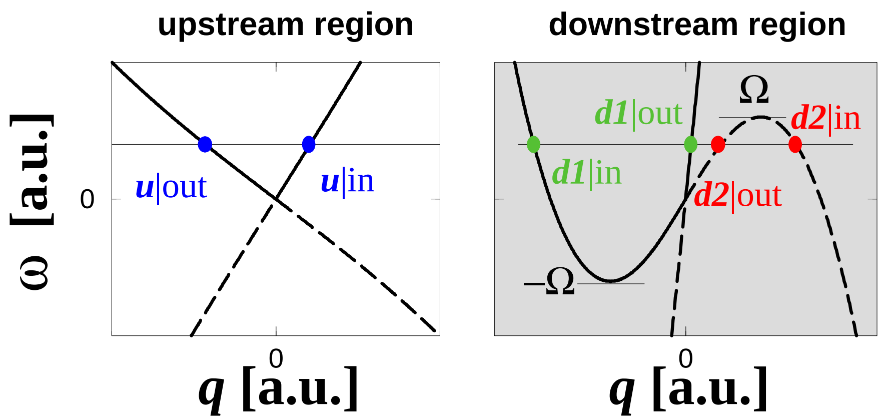
Experimental test of mode conversion :



One-body Hawking signal at equilibrium

linear relation connecting the operators of the out-going modes $\hat{b}_{u,d1,d2}$ to the in-going $\hat{a}_{u,d1,d2}$ ones

$$\begin{pmatrix} \hat{b}_u(\omega) \\ \hat{b}_{d_1}(\omega) \\ \hat{b}^\dagger_{d_2}(\omega) \end{pmatrix} = \mathbf{S}(\omega) \begin{pmatrix} \hat{a}_u(\omega) \\ \hat{a}_{d_1}(\omega) \\ \hat{a}^\dagger_{d_2}(\omega) \end{pmatrix}$$



$$\begin{aligned} I_u^{\text{out}} = \langle \hat{b}_u^\dagger \hat{b}_u \rangle &= |\mathbf{S}_{uu}|^2 \langle \hat{a}_u^\dagger \hat{a}_u \rangle + |\mathbf{S}_{ud_1}|^2 \langle \hat{a}_{d_1}^\dagger \hat{a}_{d_1} \rangle + |\mathbf{S}_{ud_2}|^2 \langle \hat{a}_{d_2} \hat{a}_{d_2}^\dagger \rangle \\ &= |\mathbf{S}_{uu}|^2 I_u^{\text{in}} + |\mathbf{S}_{ud_1}|^2 I_{d_1}^{\text{in}} + |\mathbf{S}_{ud_2}|^2 (I_{d_2}^{\text{in}} + 1) \end{aligned}$$

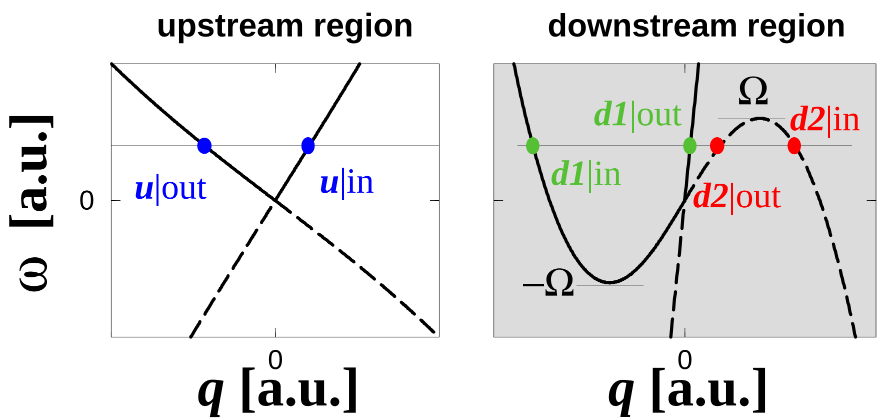
at $T = 0$: $I_u^{\text{out}}(\omega) = |\mathbf{S}_{ud_2}|^2$ needs $\begin{cases} u \rightleftharpoons d_2 \text{ mode conversion} \\ d_2\text{-ingoing mode !} \end{cases}$

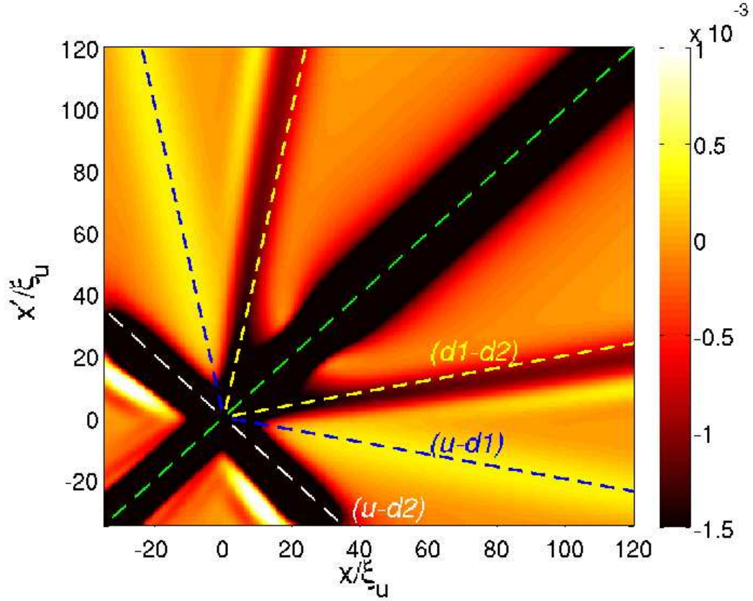
New theoretical and experimental interest

study of density correlation on each side of the horizon

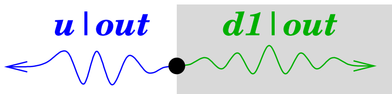
$$G^{(2)}(x,x') = \frac{\langle : n(x) n(x') : \rangle}{\langle n(x') \rangle \langle n(x) \rangle} - 1$$

Balbinot, Carusotto, Fabbri, Fagnocchi, Recati
Phys. Rev. A (2008) & New J. Phys. (2008)





example :

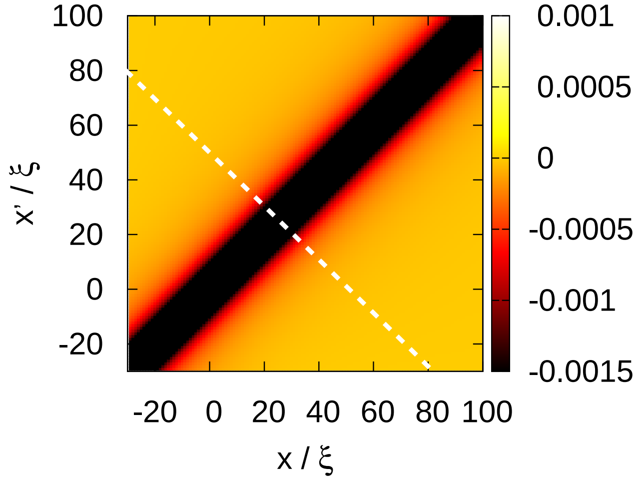


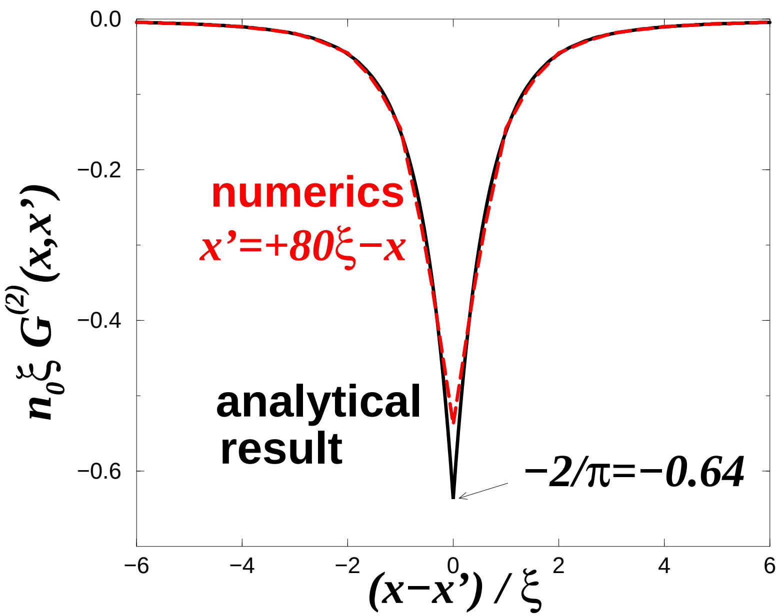
$x = (v_d + c_d)t$ correlates with $x' = (v_u - c_u)t$

Uniform 1D Condensate (no black hole)

$$G^{(2)}(x,x') = \frac{-2}{\pi\, n_0\, \xi}\, F\left(\frac{|x-x'|}{\xi}\right) \quad \text{where} \quad \begin{cases} F(X) = \dfrac{1}{2\,X}\displaystyle\int_0^\infty \mathrm{d}t\, \frac{\sin(2tX)}{(1+t^2)^{3/2}} \\ F(0) = 1\ . \end{cases}$$

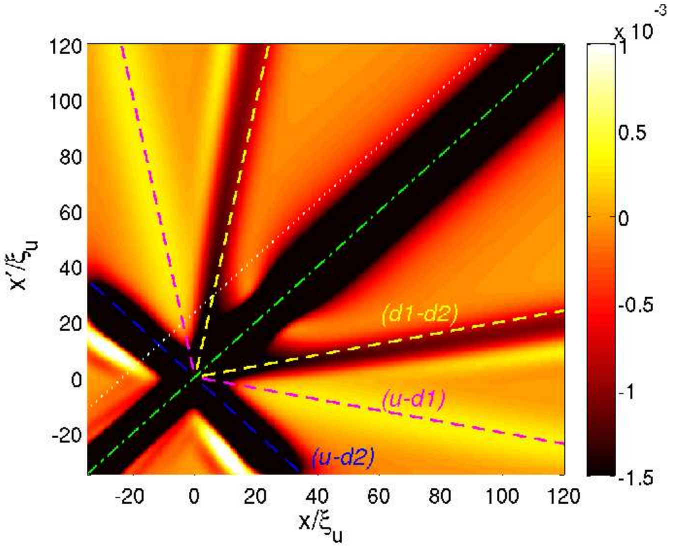
$$\left(\mu = g\, n_0 = \frac{\hbar^2}{m\,\xi^2}\right)$$

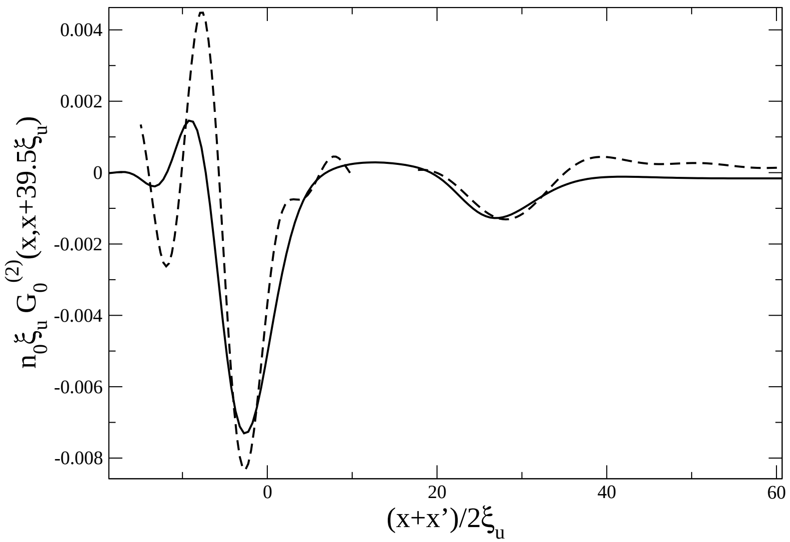




Two-body Hawking signal

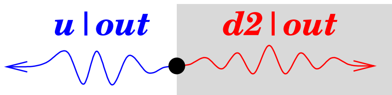
Comparison of numerical and analytic results (stationary phase neglecting interferences between the correlation signals) :





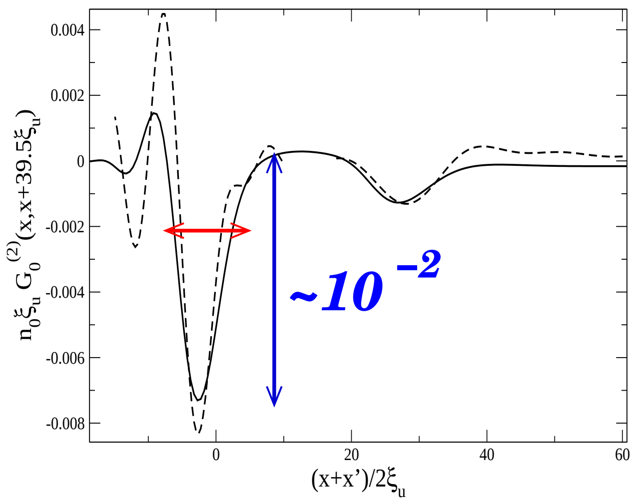
A. Recati, N. Pavloff & I. Carusotto, Phys. Rev. A **80**, 043603 (2009)

main correlation signal :



$x = V_{\mathrm{d2|out}}\, t$ correlates with $x' = V_{\mathrm{u|out}}\, t$

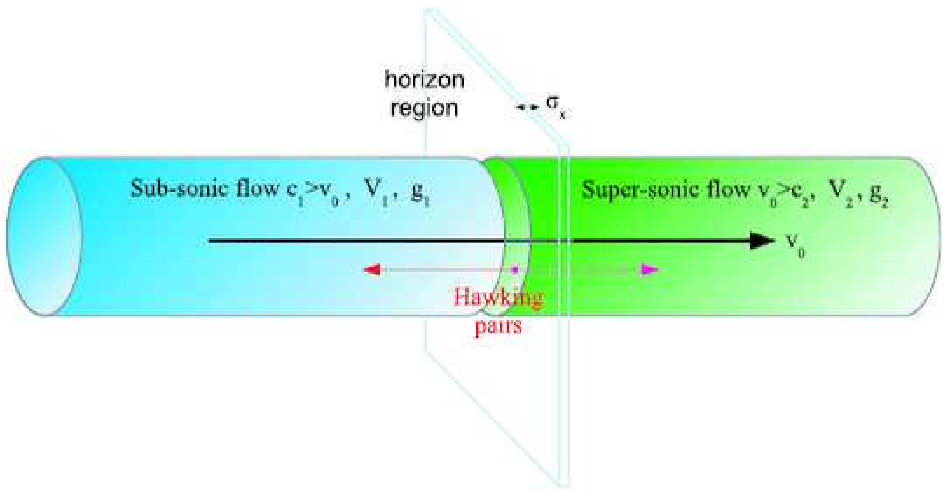
orders of magnitude :



^{87}Rb

	laser	cigar
	$n_0 \sim 50\ \mu\mathrm{m}^{-1}$	$100\ \mu\mathrm{m}^{-1}$
	$c \sim 1\ \mathrm{mm/s}$	$3\ \mathrm{mm/s}$
	$\xi \sim 1\ \mu\mathrm{m}$	$0.1\ \mu\mathrm{m}$

$$\left|G^{(2)}\right|_{\max} \sim 2 \times 10^{-4} \leftrightarrow 10^{-3}$$



need for new dumb-hole configurations !

Conclusion

BECs offer interesting prospects to observe a fully quantum Hawking radiation. Density correlations appear as promising tools for identifying Hawking radiation ...with some unessential limitations.

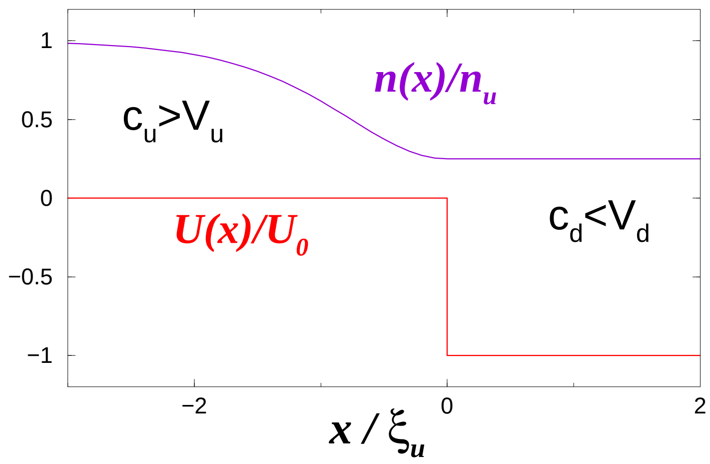
→ Clear signal , well understood. One knows where to look, and at which quantity.

→ Poorly affected by noise and finite T .

→ Drawbacks :
- weak signal intensity.
- awkward configuration.
- what about transverse degrees of freedom ?

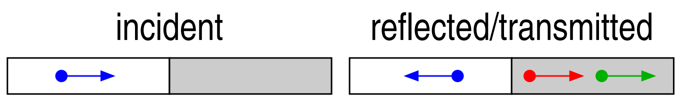
→ What comes next ?
- more realistic dumb hole configurations,
- white hole stability ...

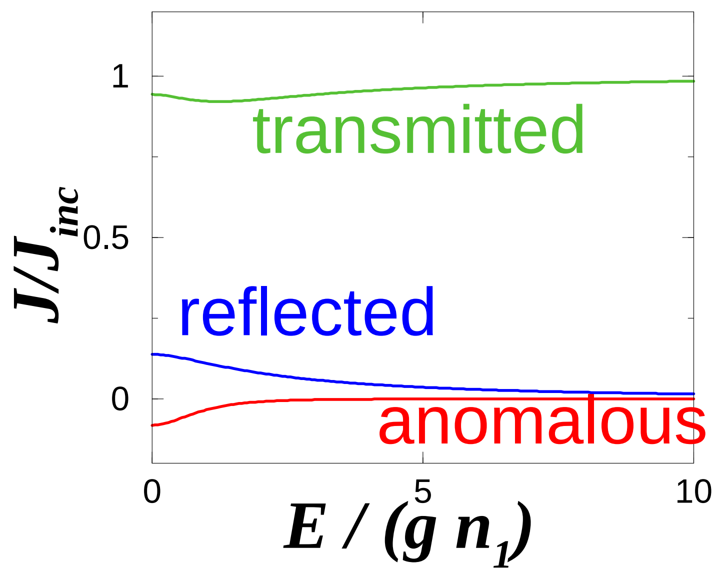
Waterfall configuration



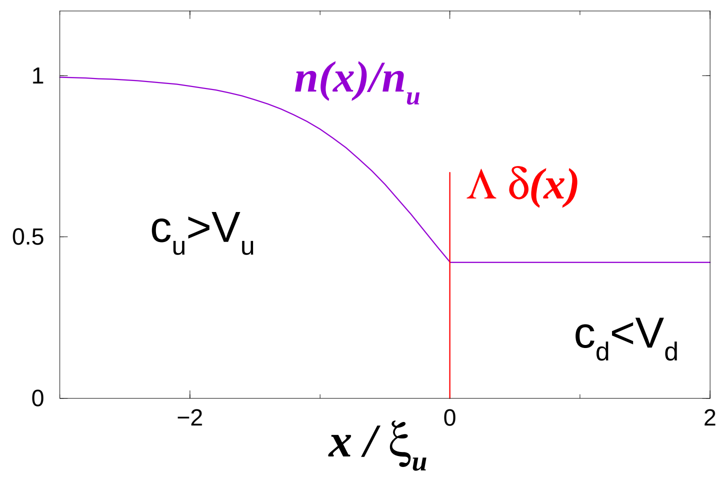
$V_d/c_d = 0.25 \quad V_d/c_d = 16$

for instance :

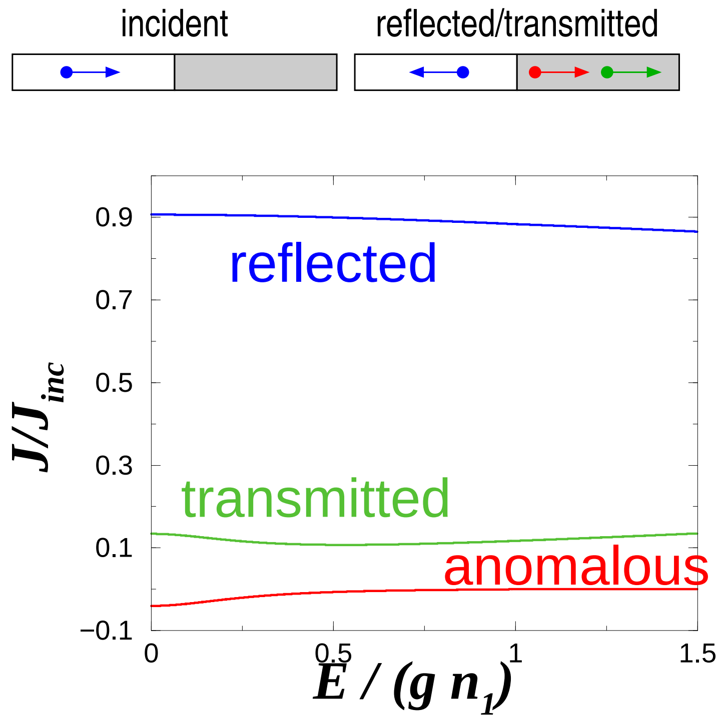




Localized obstacle



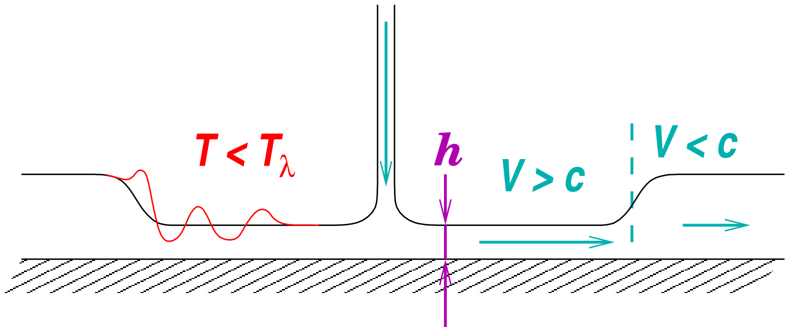
for instance :



$V_u/c_u = 0.1 \qquad V_d/c_d = 5.0$

At longer term ...

The hydraulic jump is a stable white hole (Volovik JETP 2005)



$$c = \sqrt{gh}$$

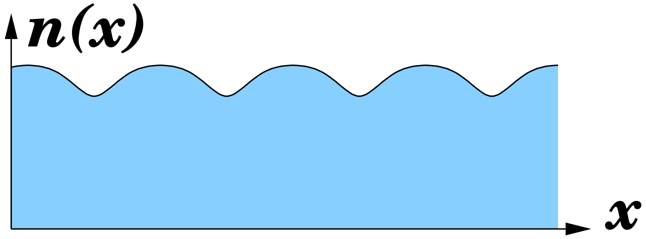
$$(\omega - \vec{k}.\vec{v})^2 = c^2\, k^2 \left[1 + h^2 k^2 \left(-\frac{1}{3} + \frac{\sigma}{\rho g h^2}\right) + ...\right]$$

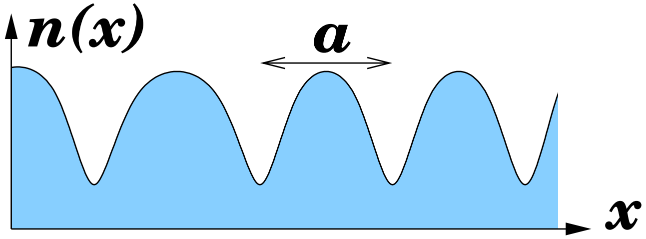
appearance of oscillations in the superfluid phase ?

cf, Pitaevskii striped phase ?

1D Super-solid

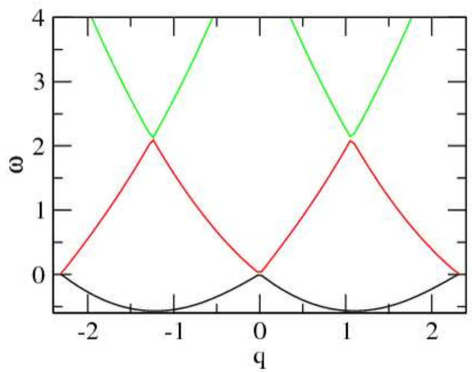
L. Pitaevskii (JETP 84): above the Landau critical velocity, a super-sonic superfluid forms a "striped phase"

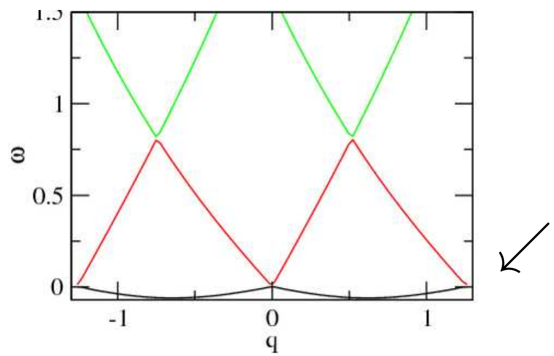




Question: is this "supersolid" phase superfluid ?

One has to study the excitation spectrum :





$$\omega \simeq -\sqrt{\frac{k}{m}} \left|\sin\left(\frac{qa}{2}\right)\right|$$