Dipole Oscillations of a Bose-Einstein Condensate in the Presence of Defects and Disorder

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We consider dipole oscillations of a trapped dilute Bose-Einstein condensate in the presence of a scattering potential consisting either in a localized defect or in an extended disordered potential. In both cases the breaking of superfluidity and the damping of the oscillations are shown to be related to the appearance of a nonlinear dissipative flow. At supersonic velocities the flow becomes asymptotically dissipationless.

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One of the most spectacular consequences of phase coherence and interactions in condensed matter is superfluidity, a direct manifestation of which is the capacity of a fluid to move without dissipation. According to the standard Landau criterion, the superfluidity (SF) of a uniform flow of, e.g., liquid ^{4}He, or a Bose-Einstein condensate (BEC) will be broken if an obstacle moves through the fluid with a speed higher than a critical velocity v_c^L. In the case of a BEC v_c^L is the sound velocity. Though this property has been explicitly checked in ^{4}He [1] and in a BEC flow [2] in the presence of small impurities, experiments in superfluid ^{4}He (see, e.g., [3]) and more recently in BEC [4,5] have shown that the critical velocity for breaking SF is generically lower than v_c^L, due to phase slips induced by vortex (or soliton) emission, as originally proposed by Feynman.

Collective oscillations of BEC confined by harmonic traps offer new opportunities to explore the central question of the breaking of SF and of the origin of drag and dissipation in quantum liquids and gases. In a recent series of experiments, damping of the oscillations (such as dipole, quadrupole, or Bloch oscillations) in the presence of a single localized scatterer [6], and disordered [7–9] or quasiperiodic [10] superimposed potentials has been used to investigate different dynamical regimes, including the possibility of a Bose glass, (Anderson) localization or other possible phases. These investigations have clearly shown the experimental relevance of analyzing transport properties of BEC *via* the damping of collective excitations. However the connection of the damping with localization properties still remains to be clarified.

Our purpose here is to provide a global analysis of the damping of dipole oscillations in the presence of a single localized scatterer or a disordered potential. We consider the regime where the experiments have been realized up to now, i.e., a quasi-1D geometry where the chemical potential is larger than the typical amplitude of the perturbing potential. The reason why we treat together the localized defect and the random potential is that, qualitatively, several of the main features of the dynamics are contained in the former case. Its analysis therefore facilitates the comprehension of the latter, and stresses the generic aspects, leading to a unified picture of dissipation (as illustrated in Fig. 1). We find that in both cases there exist SF undamped oscillations at small amplitudes. As the amplitude of oscillation (or the typical size of the perturbation) increases, the system enters a dissipative regime of damped oscillations where solitons and phononlike excitations are emitted (this regime was recently studied experimentally in Ref. [5] for a moving obstacle). In the case of a superimposed disordered potential, this dissipative or resistive phase, where nonlinearities of the system play a crucial role, has no relation with (Anderson) localization.

The system considered is a weakly interacting BEC confined in a cylindrically symmetric 3D harmonic potential $m(\omega_\perp^2 r_\perp^2 + \omega_x^2 x^2)/2$ in presence of an additional potential $U(x)$. In the limit of a highly anisotropic trap, $\omega_\perp \gg \omega_x$, the transverse confinement is such that the quasi-1D regime can be reached. It is important to note that for moderate $U(x)$ (even a disordered one) the phase coherence of the system is preserved as demonstrated in Refs. [8,11]. The system is thus accurately described by a 1D order parameter $\psi(x, t)$, depending on a single spatial

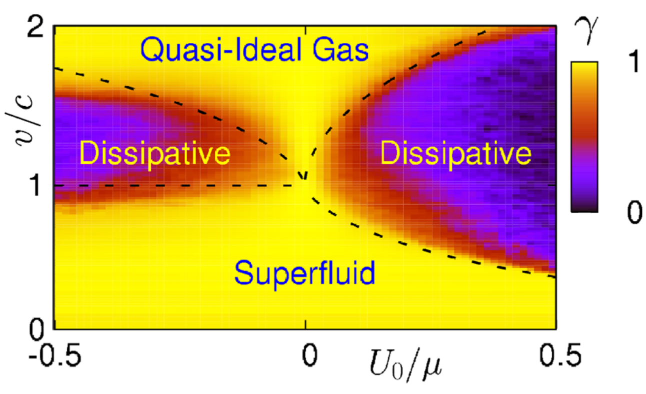

FIG. 1 (color online). Dynamical regimes for dipole oscillations in presence of a localized Gaussian defect. The plot represents the fluidity factor γ (see text) computed after a time $t_f = 25 \times 2\pi/\omega_x$. The yellow (light gray) region correspond to zero damping ($\gamma = 1$). The dashed lines are analytic determinations of the frontiers between the different regimes.

coordinate x along the axial direction of the trap. $\psi(x,t)$ obeys the nonlinear Schrödinger equation [12,13]

$$i\hbar\frac{\partial\psi}{\partial t}=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\frac{m}{2}\omega_x^2x^2+U(x)+2\hbar\omega_\perp(an)^\nu\right]\psi. \tag{1}$$

Here, $n(x,t)\equiv|\psi(x,t)|^2$ is the condensate density per unit of longitudinal length and $a>0$ is the 3D s-wave scattering length. In the low-density regime (LDR, $an\ll 1$) the density profile in the transverse direction is Gaussian shaped and $\nu=1$, whereas $\nu=1/2$ in the opposite high density regime (HDR, $an\gg 1$) where the Thomas-Fermi approximation holds for the transverse degree of freedom. Equation (1) does not account for transverse excitations which may be relevant in the HDR. We checked that it nonetheless gives an excellent account of the experimental result on dipole oscillations of the Florence and Rice groups [7,8] performed in the HDR.

After preparing the condensate in the ground state of the trap [with density $n_0(x)$, chemical potential μ and, at the center of the cloud, speed of sound $c=(\nu\mu/m)^{1/2}$], dipole oscillations are excited by a sudden displacement d_0 of the harmonic potential. For $U(x)\equiv 0$ the center of mass oscillates freely with frequency ω_x, and acquires a velocity $v=\omega_x d_0$ when passing through the origin. The time evolution of the density reads $n(x,t)=n_0(x-X_t)$ where $X_t=d_0\cos(\omega_x t)$ is the position of the center of mass. For a finite $U(x)$, which is turned on simultaneously with the sudden displacement of the trap, $X_t=\frac{1}{N}\int_{\mathbb{R}}xn(x,t)dx$ is computed numerically up to a time t_f chosen such that $X_{t>t_f}$ assumes an oscillatory pattern of roughly constant amplitude which we denote d_f. In order to measure the damping of the dipole oscillations we define a fluidity factor $\gamma=d_f/d_0$ ($\gamma=1$ in the absence of damping and $\gamma\to 0$ for strong damping).

Localized defect.—We start by considering a Gaussian-shaped defect $U(x)=U_0\exp(-x^2/2\sigma^2)$. The fluidity factor γ is plotted in Fig. 1 as a function of the normalized defect strength U_0/μ and velocity $v/c=\omega_x d_0/c$. The numerical calculations were performed for a BEC in the LDR with chemical potential $\mu=40\hbar\omega_x$ selected to have a Thomas-Fermi like density-profile along the axial direction. In this case $n_0(x)=\mathcal{N}_\nu\Theta(L-|x|)[L^2-x^2]^{1/\nu}$ where the factor $\mathcal{N}_\nu$ normalizes the density to the number of atoms N, $\Theta(x)$ is the Heavyside step function and $L=\sqrt{2\mu/m\omega_x^2}$ is half the longitudinal size of the condensate. The parameters are $N=1.5\times 10^4$ ^{87}Rb atoms, $\omega_x=2\pi\times 9\ \mathrm{s}^{-1}=\omega_\perp/10$ and $\sigma=\xi=0.28\ \mu$m, where $\xi=\hbar/\sqrt{2m\mu}$ is the healing length at the center of the condensate. The qualitative structure of Fig. 1 is generic and does not depend on the specific values of σ and μ, and is also observed in the HDR ($\nu=1/2$).

In the deep subsonic limit $v/c\ll 1$, the Gaussian scatterer induces no observable damping of the dipole oscillations. Numerical results show that the oscillating condensate is only locally perturbed in the vicinity of the defect: a dip or a peak appear in the condensate density for $U_0>0$ and $U_0<0$, respectively [see Fig. 2(a)]. These are characteristic features of a superfluid flow, with no energy dissipation, nor drag exerted [14], and with no damping of the oscillations (perfect transmission through the scatterer potential). In this regime, a perturbative treatment of Eq. (1) and a local density approximation yield a condensate density of the form

$$n(x,t)=n_0(x-X_t)[1+\delta n(x,t)], \tag{2}$$

with

$$\delta n(x,t)=-\frac{2m}{\hbar^2\kappa}\int_{-\infty}^{+\infty}dy e^{-\kappa|x-y|}U(y), \tag{3}$$

where $\kappa=\frac{2m}{\hbar}|c_0^2-\dot{X}_t^2|^{1/2}$, c_0 being the unperturbed local sound velocity: $mc_0^2=2\nu\hbar\omega_\perp[an_0(x-X_t)]^\nu$. The accuracy of this approximation is shown in Fig. 2(a); it is well justified if $mU_0\sigma/(\hbar^2\kappa)\ll 1$ when $\kappa\sigma\ll 1$, or $mU_0/(\hbar\kappa)^2\ll 1$ when $\kappa\sigma\gg 1$ [13], and if $\sigma\ll L$.

For weak defect potentials, $|U_0|\ll\mu$, and if the TF size L is large compared to the dipole oscillation amplitude, the center of mass position X_t can be computed analytically. To lowest order, the solution of the small-amplitude linearization yields $X_t=d_0\cos[(\omega_x+\delta\omega)t]$ where the defect-induced frequency shift reads

$$\delta\omega=\frac{-1}{2m\omega_x}\int_{-\infty}^{+\infty}dx\frac{dn_0(x)}{dx}\frac{dU(x)}{dx}. \tag{4}$$

This gives $\delta\omega=-(4-3\nu)\frac{3U_0\sigma}{8\mu^{3/2}}\omega_x^2\sqrt{\pi m}$ for a Gaussian defect, in excellent agreement with our numerical results. Hence the analytical evaluations of the density profile (2)

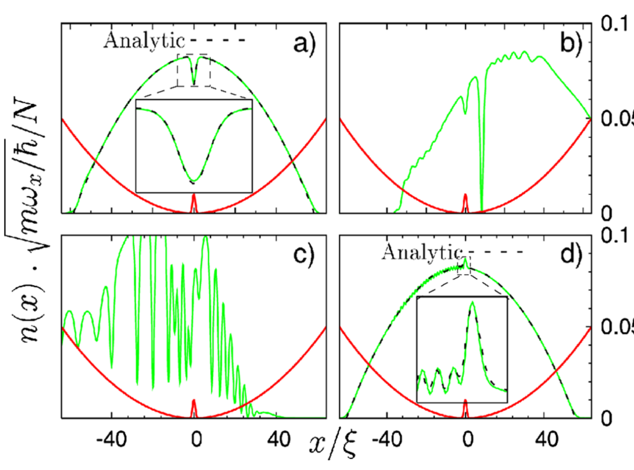

FIG. 2 (color online). Density profile after a time $t=3/4\times 2\pi/\omega_x$ for $U_0/\mu=0.24$ at different initial velocities: (a) $v/c=0.1$, (b) $v/c=0.67$, (c) $v/c=1.2$, (d) $v/c=2.5$. The confining potential is represented as a full (red) curve. Insets blow up the density around the defect. (Other parameters as in Fig. 1) and of the center of mass motion confirm the superfluid behavior of the oscillations in the deep subsonic regime.

The situation changes as the velocity increases at fixed U_0/μ or as U_0/μ is increased at constant velocity. In the former case, at some critical velocity $v_c \leq v_c^L = c$, that depends on the strength of the defect potential, the system looses SF, damping is observed and the fluidity factor diminishes. Using a local Landau criterion [15] we identify the border between the SF and this "dissipative region" as the locus of points where the maximum local condensate velocity $v(x, t)$ equals the local speed of sound $c(x, t) = (2\nu\hbar\omega_\perp/m)^{1/2}[an(x, t)]^{\nu/2}$. The former can be computed from mass conservation and Eq. (2). For an impurity localized at $x = 0$ this yields

$$\frac{v_c}{c} = [1 + \delta n_c]^{1/2+1/\nu} \quad \text{if} \quad U_0 > 0, \qquad \frac{v_c}{c} = 1 \quad \text{if} \quad U_0 < 0, \tag{5}$$

where δn_c is the factor $\delta n(x, t)$ [Eq. (3)] evaluated when $x = X_t = 0$. The lower dashed lines in Fig. 1 show that these estimates coincide well with the numerical findings.

When the system enters the dissipative regime, one or a few gray solitons detach from the defect during the first oscillations, as well as some phononlike excitations [see Fig. 2(b)]. As time goes on, the interactions of the solitons among them, with the defect and with the phononlike excitations produce time-dependent fluctuations of the shape. During this process the condensate does not loose phase coherence, but the center of mass motion looses collectivity, part of the kinetic collective energy being transformed into density fluctuations. The damping process continues until the center of mass velocity becomes comparable with v_c. Thereupon, though presenting local density fluctuations, the amplitude of the oscillations remain constant in time. Deeper in the dissipative regime, an increased emission of gray solitons and phononlike excitations is observed, leading to a massive distortion of the initial condensate profile [see Fig. 2(c)]. The BEC enters a strongly irregular time-dependent regime, the collectivity of the dipole motion is totally lost, and the damping increases drastically.

Finally, at sufficiently high supersonic velocities, a different phase is reached where the damping tends again to zero ($\gamma \to 1$). In this regime, that we denote as "quasi-ideal", the kinetic energy of the condensate is large compared to the strength of the external potential, and interactions tend to be negligible. We find, in agreement with previous theoretical studies [13,14], a strong suppression of dissipation as the velocity increases. The condensate density is again only locally distorted in the vicinity of the defect (cf. Figure 2(d)]. This local distortion is very well described by applying the combination of perturbative and local density approach already used in the SF regime. The density is of the same form as in Eq. (2) with here [13]

$$\delta n(x, t) = \frac{4m}{\hbar^2\kappa}\left[\frac{U(x)}{\kappa} + \Theta(-x\dot{X}_t)\mathrm{Im}\{e^{i\kappa x}\hat{U}(\kappa)\}\right], \tag{6}$$

where $\hat{U}$ is the Fourier transform of U. This form indeed corresponds to almost perfect transmission, with small reflexion on the defect (see the inset of Fig. 2(d)], the amount of which decreases at large velocity ($\kappa \to \infty$).

The frontier between the dissipative and the quasi-ideal region can be estimated by studying the related problem of a homogeneous condensate flowing through a barrier potential [16]. In this simplified configuration it is possible to determine analytically the velocity at which the system undergoes a transition from a local perturbation to an irregular fluctuating density profile [13]. We find that this estimate fits very well the numerically determined supersonic frontier (see Fig. 1). This stresses the qualitative similarities between Fig. 1 and the phase diagram obtained for a defect moving through a homogeneous fluid [13,14]. Interestingly, the existence of the three regions (SF, nonlinear dissipative, and quasi-ideal weakly-damped) has been recently observed experimentally for a localized defect in Ref. [5].

Disordered potential.—Keeping the same parameters as in Fig. 1, we now replace the single localized impurity by a disordered potential and compute, as before, the fluidity factor as a function of velocity and of the intensity of the (now random) potential. Figure 3 corresponds to the case where $U(x)$ is an optical speckle potential of mean value $\bar{U}$ with a correlation length l_c such that $L/l_c = 30$, typical of experimental configurations [17]. The picture is generic and does not depend on the details of the disorder. The main result that emerges from the comparison of Figs. 1 and 3 is that, in the weak-disorder limit ($\bar{U}/\mu \ll 1$), the global properties of the damping phase diagram are qualitatively similar in both cases. The same three phases observed for the localized defect are again present. However, their relative importance is quite different. One observes a considerable shrinking of the SF and quasi-ideal weakly damped regions, compared to the nonlinear dissipative one. The symmetry of γ with respect to the sign of $\bar{U}$

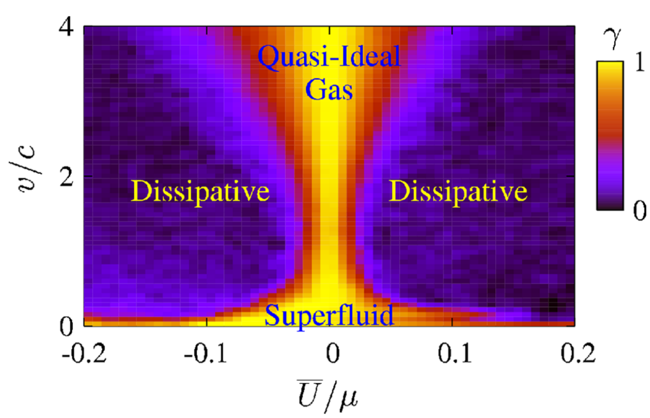

FIG. 3 (color online). Fluidity factor γ of dipole oscillations in presence of a speckle potential.

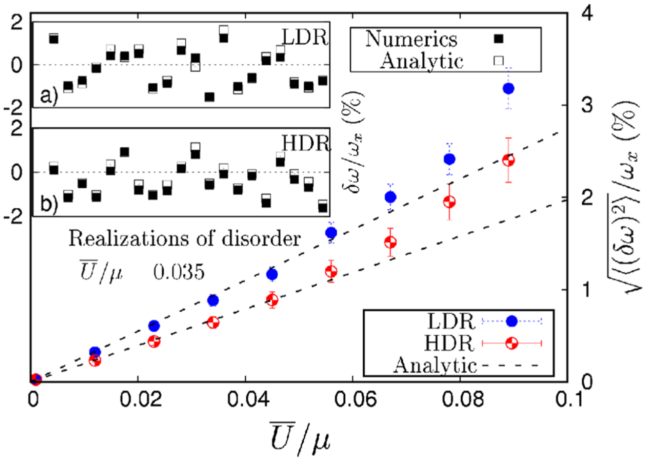

FIG. 4 (color online). Relative standard deviation $\omega_x^{-1}\langle\delta\omega^2\rangle^{1/2}$ as function of average speckle intensity. The analytical results correspond to Eq. (7). The insets compare numerical and analytical frequency shifts for different realization of disorder for $\bar{U}/\mu = 0.035$ in the HDR and LDR.

is greatly enhanced in the disordered case (Fig. 3). We interpret this as an effect of particle number conservation [18].

In the presence of disorder, important experimental efforts have been undertaken for understanding the possible connection of damping of dipole oscillations with Anderson localization. Our analysis shows that damping occurs in the dissipative phase. However, the dissipative mechanism observed in this phase is—as in the case of a localized defect—connected to the loss of collectivity due to the emission of solitons and linear excitations, and not to a localization phenomenon. From what is known in the case of a homogeneous flow, genuine Anderson localization might only occur in the deep supersonic regime $v/c \gg 1$ [19]. Without going into a detailed analysis of the experimental results [7,8], let us mention that the displacement of the harmonic potential (around 700 μm) used at Rice University gives $v/c \approx 2.8$, while the lowest speckle height considered is $\bar{U}/\mu \approx 0.04$, which locates the system in the dissipative phase (under the experimental conditions, we find that at $v/c \approx 2.8$ the dissipative phase border is at $\bar{U}/\mu \approx 0.008$). Similarly, from the data published in Ref. [7] our analysis shows that the experiments at Florence were also performed in the supersonic dissipative region. These simple remarks explain the experimentally observed damping, and locate the experimental configurations in a regime where the effects of Anderson localization are at best indirect.

In the subsonic SF regime it is possible to compute the frequency shift due to the disordered potential similarly as in the one-peak case. The approach is found to be equivalent to the sum rule approach developed in [7]. One can also derive a simple relation for the variance of the frequency shift:

$$\langle(\delta\omega)^2\rangle = \iint_{\mathbb{R}^2} \frac{dxdx'}{4m^2\omega_x^2} \frac{d^2 n_0(x)}{dx^2} \frac{d^2 n_0(x')}{dx'^2} \langle U(x)U(x')\rangle, \tag{7}$$

where $\langle\ldots\rangle$ denotes ensemble average. For small $\bar{U}/\mu$ Eq. (7) is found to be in very good agreement with numerical integration of Eq. (1), as seen in Fig. 4.

To conclude, we have presented a comprehensive picture of the damping properties of dipole oscillations of BEC in the presence of a scattering potential. Strong analogies are stressed between different types of potentials. Three different phases are shown to exist: superfluid ($v/c < 1$), nonlinear dissipative ($v/c \sim 1$) and quasi-ideal ($v/c > 1$). The mechanism that breaks SF and leads to damped oscillations is shown to correspond to a generic onset of dissipation which, in the presence of disorder, is unrelated to localization properties. As the strength of the disorder potential increases the nonlinear dissipative phase occupies most of the phase diagram. Our findings allow us to give a simple interpretation of experimental results.

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