Quasi-Landau Spectrum of the Chaotic Diamagnetic Hydrogen Atom

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(Received 28 September 1987)

By the employment of "constant-scaled-energy spectroscopy" as a novel spectroscopic technique, the quasi-Landau resonance system of the diamagnetic H atom in even-partity $m=0$ magnetic final states is observed for the first time in its entirety from the regular $1/n$ into the chaotic quasi-Landau regime. It evolves, fully unexpectedly, into a systematically structured hierarchy of generations of resonances, correlated to three physically different types of closed classical orbits.

PACS numbers: 32.80.−t, 05.45.+b, 32.60.+i

The physics of the highly excited diamagnetic hydrogen atom has recently attracted much attention,[1-8] largely because this simple nonseparable quantum system turns classically chaotic as it approaches the ionization limit.[6,9] In this context the quasi-Landau (QL) oscillations and their correlation to classical periodic orbits are of particular interest.[10] Until recently, it was accepted that only one QL resonance type, discovered by Garton and Tomkins,[11] exists. Experiments with the H atom[3-5] and theoretical studies[7,8,12,13] have uncovered further, basically new resonances correlated with three-dimensional orbits. Nevertheless, the central question as to the *entire* set of QL resonances resulting from final states with a given m quantum number and parity evolving from the regular into the chaotic QL regime has remained open.

We have addressed this basic problem and studied the H-atom Balmer spectrum with even-parity $m=0$ magnetic final states as a function of both the excitation energy E and the magnetic field B, employing for the first time "constant-scaled-energy spectroscopy." Different from previous experiments at constant B,[3-5] this technique makes a systematic search for, in principal, *all* possible QL resonances associated with closed classical orbits.[12] In analogy to theoretical work, it is based on the scaling property of the classical Hamiltonian[14]:

$$H(\mathbf{r},\mathbf{p};\gamma)=\gamma^{2/3}\tilde{H}(\tilde{\mathbf{r}},\tilde{\mathbf{p}};\gamma=1),$$

where scaled variables are defined by $\tilde{\mathbf{r}}=\gamma^{2/3}\mathbf{r}$, $\tilde{\mathbf{p}}=\gamma^{-1/3}\mathbf{p}$, and $\gamma=B/(2.35\times10^5\ \mathrm{T})$. The semiclassical Bohr-Sommerfeld quantization condition[15] for the two nonseparable coordinates ρ,z (cylindrical coordinates) is transformed accordingly to scaled form[13]

$$(2\pi)^{-1}\oint_i(\tilde{p}_\rho d\tilde{\rho}+\tilde{p}_z d\tilde{z})=n\gamma^{1/3}=C_i, \tag{1}$$

where i denotes a closed classical orbit. Since the scaled action C depends on the scaled energy $\tilde{E}=E\gamma^{-2/3}$ only, it has a constant value C_i for $\tilde{E}=$const and a given i. In this case, $C_i=n\gamma^{1/3}$ describes a spectrum of *equidistant* lines on a scale $\gamma^{-1/3}$, the Fourier transform of which in the conjugate coordinate, $n\gamma^{1/3}$, consists of *one* resonance for each i, to which is correlated the respective orbit i. By application of these theoretical concepts to experiment, constant-scaled-energy spectra have been taken accordingly to our scanning the field strength linearly with $\gamma^{-1/3}$, simultaneously adjusting E (via the laser wavelength) such that $\tilde{E}=E\gamma^{-2/3}=$const was obeyed. Apart from this novel spectroscopic procedure the experiments have been carried out as previously.[3]

Figure 1(a) shows, as a typical example, a $\gamma^{-1/3}$ spectrum at $\tilde{E}=-0.45$, and Fig. 1(b) the corresponding Fourier-transform $n\gamma^{1/3}$ action spectrum. The orbits shown correlate to the respective resonances, and have been obtained by classical trajectory calculation.[4,13] Such action spectra have been taken (physically with a

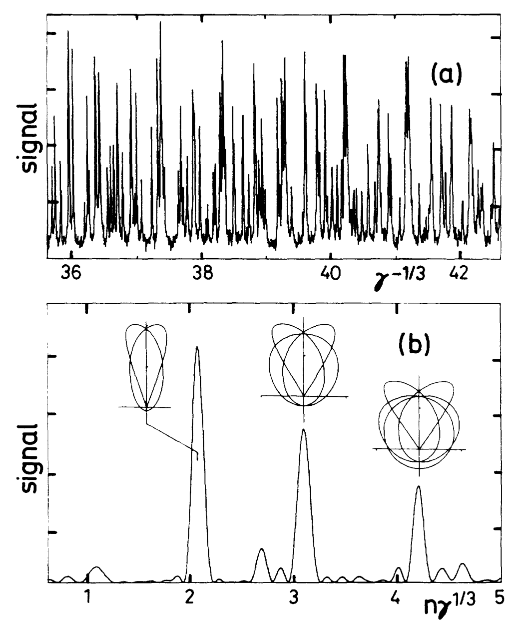

FIG. 1. (a) Scaled-energy spectrum at $\tilde{E}=-0.45$ as a function of $\gamma^{-1/3}$. Range of excitation energy $-77.7\ \mathrm{cm}^{-1}\le E\le -54.3\ \mathrm{cm}^{-1}$ and field strength $5.19\ge B\ge 3.03$ T. (b) Fourier-transformed action spectrum of (a); closed orbits correlated to respective resonances in (ρ,z) projection; z coordinate vertically.

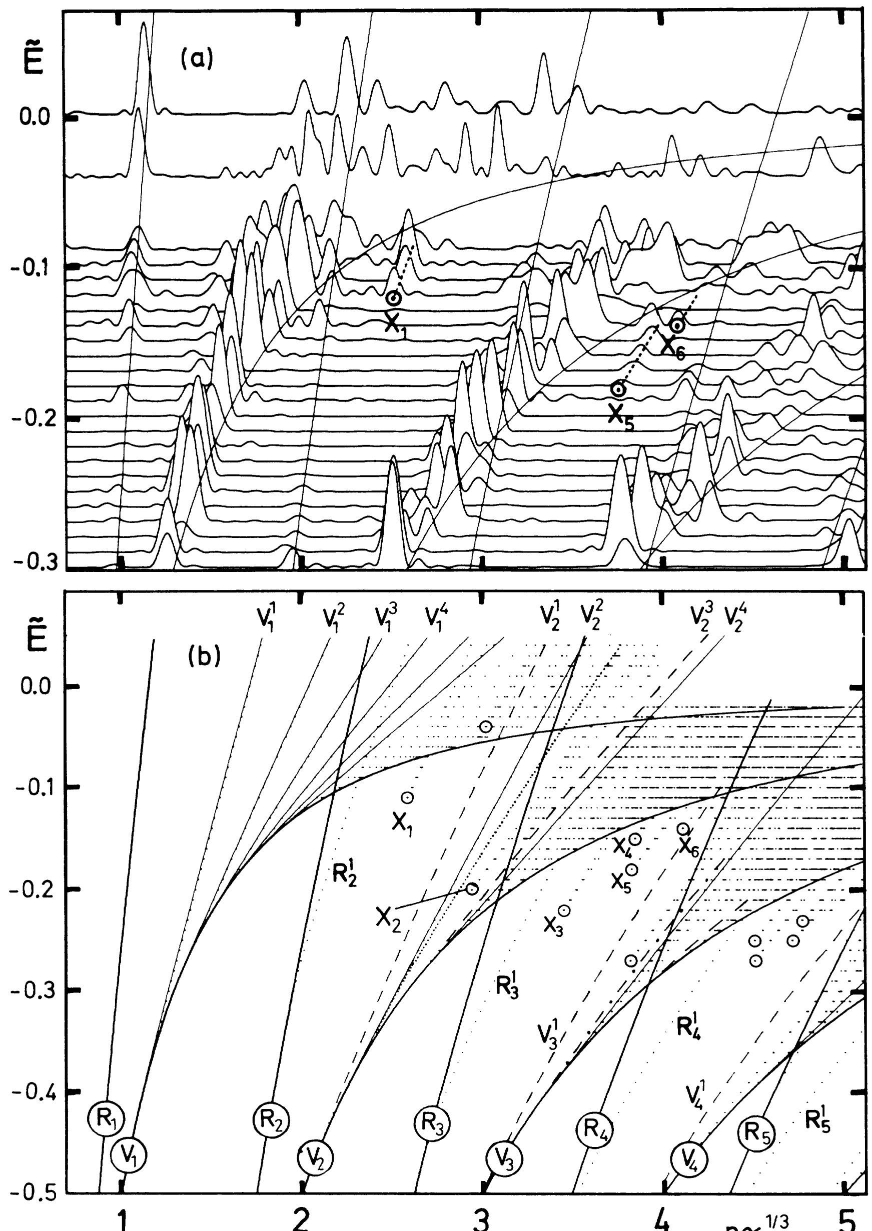

FIG. 2. (a) Experimental quasi-Landau resonance action spectrum as a function of scaled energy $\tilde{E}$ in overlay form. Even-parity, magnetic $m=0$ final state. (b) Semiclassically calculated $(\tilde{E},C)$ spectrum of quasi-Landau resonances correlated to closed classical orbits through origin.

dynamical range of ~ 50) in the $\tilde{E}$ range $-0.50 \leq \tilde{E} \leq +0.20$, at $-0.30 \leq \tilde{E} \leq -0.10$ in steps $\Delta\tilde{E}=0.01$ and at $\tilde{E} < -0.30$ and $\tilde{E} > -0.10$ in steps $\Delta\tilde{E}=0.05$. Figure 2(a) shows the spectra in a concise overlay plot $\tilde{E}$ vs C at $-0.30 \leq \tilde{E} \leq 0.00$. For clarity the original spectra have been adjusted relatively (to within a factor ~ 3) to equal maximum peak height. Although intensities cannot directly be compared, details are lost or obscured, and not all spectra are present in the overlay, Fig. 2(a) clearly exhibits the essential result of this work: the entire QL spectrum as a function of $\tilde{E}$ and C, and its evolution from the regular into the chaotic regime as a remarkably well-structured system of clustered branches of resonances, not previously predicted or anticipated.

To understand the experimental results, we have calculated the complete semiclassical action spectrum, i.e., the *positions* of all resonances in the $(\tilde{E},C)$ plane correlated to closed orbits through the proton by numerical integration of Eq. (1). The result is shown in Fig. 2(b) with $\tilde{E}$ in steps of 0.01. Viewing this figure at glancing angle, one recognizes in regions of low resonance density strings of dots representing the $(\tilde{E},C)$ dependence of in-

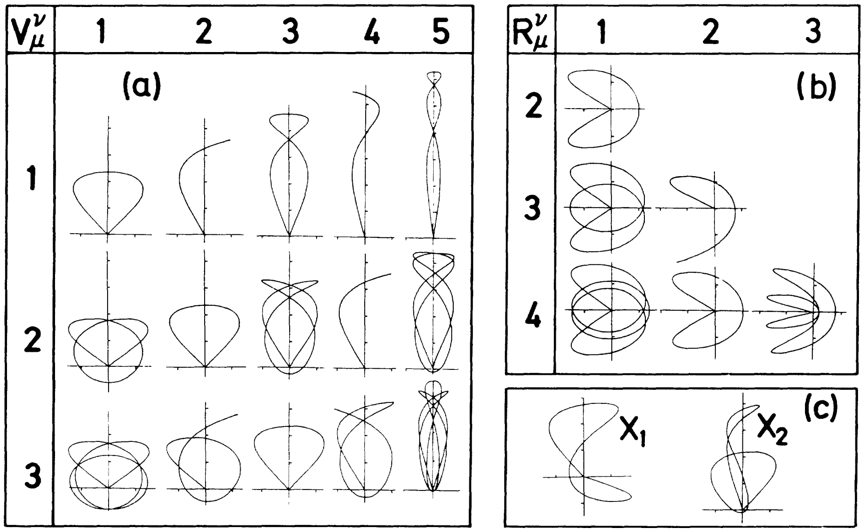

FIG. 3. Closed classical orbits in (ρ,z) projection. (a) Primary vibrators, (b) primary rotators, and (c) "exotics" X_1,X_2 at origin; z coordinate vertically.

dividual resonances. Some examples are indicated by solid or broken lines. Evidently, there exists a striking resemblance of Figs. 2(a) and 2(b) in the overall distribution of resonances in the $(\tilde{E},C)$ plane, in that both spectra evolve fanlike in an obviously systematic fashion of well-distinguished clusters and branches. In fact, the calculations show that the entire manifold of resonances can be traced to roots of three *basic* resonance types with correlated orbits: (a) "vibrators," one-dimensional orbits along the z axis; (b) "rotators," two-dimensional orbits in the $(z=0)$ plane; and (c) "exotics," genuine three-dimensional orbits. From them evolves the whole QL spectrum by sequential bifurcation into a system of complex branches of higher-generation resonances with respective three-dimensional closed orbits.

Rotators.—The *basic rotators*, R_μ $(\mu=1,2,\ldots)$ in Fig. 2(b), form a series of exact harmonics in $n\gamma^{1/3}$, with the fundamental R_1 being the original Garton-Tomkins resonance. Except for R_1, from the harmonics bifurcate directly first-generation resonances, R_μ^ν $(\mu>\nu)$, examples of which $(\nu=1,\ \mu=2,3,4)$ are indicated in Fig. 2(b). These R_μ^ν resonances are correlated to a topologically homologous set of three-dimensional orbits [Fig. 3(b)] characterized by common symmetry in the angle with respect to the $(z=0)$ plane. Only a few rotators (R_1, R_2, and R_3) are observed in this experiment with $m=0$ final states, a fact rationalized below. The strongest one is R_1 observed at $\tilde{E}>-0.02$ generally increasing in strength with $\tilde{E}$. R_2 and R_3 occur only in narrow $\tilde{E}$ regions around $\tilde{E}\sim-0.3$ [Fig. 2(a)] and $\tilde{E}\sim-0.45$ [Fig. 1(b)], respectively.

Vibrators.—The *basic vibrators*, V_μ $(\mu=1,2,\ldots)$ in Fig. 2(b), form also a harmonic series with V_1 the fundamental. Correlated to the V_μ's are the one-dimensional orbits along the z axis.[4,13] From them bifurcate directly first-generation vibrators, indicated by V_μ^ν $(\nu=1,2,\ldots)$ in Fig. 2(b). The first series, V_1^ν, is just the one previously discovered[4,13] and called there "regular" (or "ν type"). From V_2 bifurcates V_2^ν, a subseries $(\nu=2,4,\ldots)$ of which is the exact first harmonic of V_1^ν; from V_3 bifurcates V_3^ν, a subseries $(\nu=3,6,\ldots)$ of which is the exact second harmonic of V_1^ν; and so on. The systematics of the whole first-generation vibrators, V_μ^ν, is strikingly mirrored by the geometrical nature of the correlated classical closed orbits: They form a topologically homologous matrix set of three-dimensional orbits [Fig. 3(a)] with characteristic common symmetry in the absolute azimuthal angle with respect to the z axis.

As to the further evolution of vibrators, the calculations show that sequential bifurcation into higher generations increases rapidly in the higher clusters. However, any systematics gets quickly lost with increasing resonance density. Also, the correlated closed orbits do not show more topological systematics.

Identification of *individual* experimental resonances with calculated ones is, in general, not possible except for a few isolated features in regions of sparse density. What can be said, however, is that the gross overall fanlike structure within the well-separated clusters along the left-hand side of the respective basic V_μ lines is, on the whole, determined by three-dimensional vibrators. Furthermore, the fact that well-developed structures occur even in regions with high resonance density [Fig. 2(b)] shows that only relatively few vibrators are actually excited. Concerning specifically the first cluster (along the left of V_1), the observed resonances can be vibrators of the V_1^ν series since only the V_1^ν's do not bifurcate further. This is true at least in the region $C\lesssim3$ and except for a few intersparsed rotators and exotics (see below).

Exotics.—Surprisingly, some resonances are observed in regions where, according to the calculations, neither rotators nor vibrators exist that originate from the basic ones, directly or by sequential bifurcation. Examples of such "exotic" resonances are indicated in Fig. 2(a) (cross, circles, and dotted lines). Consistent with the experimental observation, the calculations also reveal the existence of exotics. Figure 2(b) (circles and dotted lines) shows examples in sparse-density regions definitely free of ordinary rotators and vibrators. According to the calculations, "exotics" have the following characteristics: They appear suddenly at singular $(\tilde{E},C)$ points with evidently random distribution in the $(\tilde{E},C)$ plane. The correlated closed orbits bifurcate right at the point of origin and generally have no characteristic symmetry. For illustration, orbits correlated to X_1 and X_2 at origin are shown in Fig. 3(c).

Concerning the intensity of QL resonances, this is generally determined by the angular dependence of the excited final wave function and the stability of the correlated orbits.[5,16] According to previous work,[6] the transition from regularity to irregularity occurs roughly at $-0.5 \lesssim \tilde{E} \lesssim -0.14$. In the present case, where the final-state orbital $|d;m=0\rangle$ is mostly oriented in the z direction, one expects preferential excitation of resonances correlated to orbits generally centered along the z direction, that is, orbits originating from basic vibrators, V_μ. Consistent with this model, in the observed spectrum vibrators are, on the whole, more strongly excited than rotators. Excitation of final states with orbital wave functions oriented preferentially in the $z=0$ plane (e.g., $|m|=2$) will thus be expected to result in distributions shifted more towards the rotators.

In summary, employing "constant-scaled-energy spectroscopy" as a novel technique, we have investigated the magnetized H-atom QL spectrum for the first time systematically as a function of both excitation energy and magnetic field strength from the regular into the completely chaotic regime. Furthermore, systematic semiclassical calculations have resulted in the position spectrum of *all* QL resonances in this regime. The main result is the discovery of an overall structure of the whole QL spectrum as a quasiperiodically ordered system of clusters and branches of resonances. They evolve from three physically distinct *basic* resonance types with correlated closed classical orbits: one-dimensional "vibrators," two-dimensional "rotators," and genuinely three-dimensional "exotic" orbits.

We thank the Deutsche Forschungsgemeinschaft for supporting this project.

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