# Superfluidity versus Anderson Localization in a Dilute Bose Gas

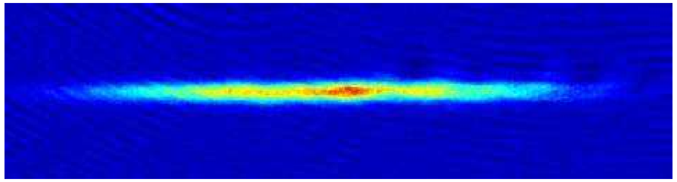
Nicolas Pavloff

Laboratoire de Physique Théorique et Modèles Statistiques
Université Paris-Sud, Orsay

work in collaboration M. Albert, P. Lebœuf, T. Paul and P. Schlagheck:

- Phys. Rev. Lett. **98**, 210602 (2007)
- arXiv : 0803.4116

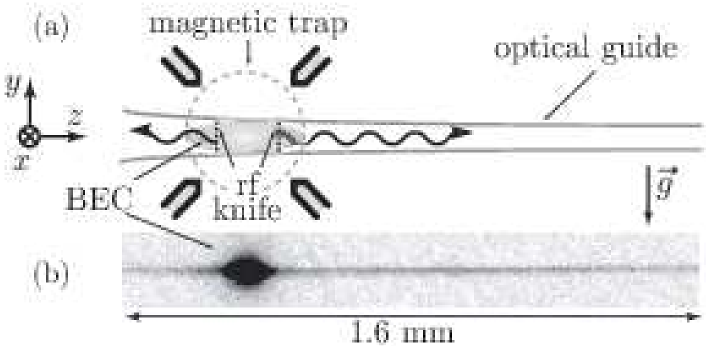
# quasi-1D condensates :

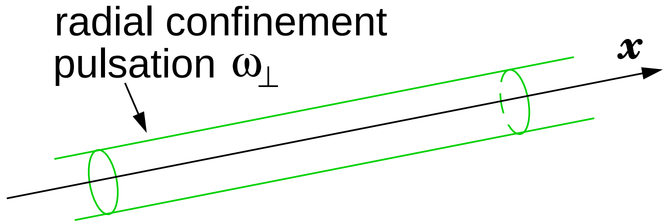


*quasi-1D condensate*

longitudinal size $\sim 10^2 \mu\text{m}$

transverse size $\sim 1\mu\text{m}$




harmonic radial confinement :

$$V_\perp(\vec{r}_\perp) = \frac{1}{2} m \omega_\perp^2 r_\perp^2 \, .$$

W. Guérin *et al.*, Phys. Rev. Lett. **97**, 200402 (2006)

## 1D regime

$a$ : 3D s-wave scattering length ($a > 0$)

$$\frac{a^2 m\,\omega_\perp}{\hbar} \ll n_1 a \ll 1 \,. \tag{1}$$

• The first inequality allows to avoid the Tonks-Girardeau regime and implies that the interaction energy between atoms is weak compared to the kinetic energy. It implies $L_\phi \ggg \xi$ $\quad L_\phi = \xi \exp\left[\pi\sqrt{\frac{\hbar n_1}{2 m a \omega_\perp}}\right]$

• the second inequality allows to avoid the 3D-like transverse Thomas-Fermi regime and implies that the chemical potential $\mu$ (measured relatively to the transverse ground state) is small compared to $\hbar\omega_\perp$.

(1) being fulfilled, one gets into the $1D$ mean field regime where the system is described by $\psi(x,t)$ verifying

$$-\frac{\hbar^2}{2\,m}\partial_x^2\psi + \left(U_{\rm ext}(x) + g\,|\psi|^2\right)\psi = i\hbar\,\partial_t\psi \,, \tag{2}$$

where $|\psi|^2 = n_1(x,t)$ is the longitudinal density of the condensate, and $g = 2\,\hbar\,\omega_\perp\,a = \hbar^2/(m a_1)$, $-a_1$ being the 1D scattering length.

# Experiments on Anderson localization in 1D systems

LINEAR WAVES :

↪ acoustic waves: 1983 — C. H. Hodges & J. Woodhouse, J. Acoust. Soc. Am. **74**, 894 (1983)

↪ $3^{rd}$ sound in $^4$He films: 1988 — D. T. Smith *et al.*, Phys. Rev. Lett. **88**, 1286 (1988)

↪ light: 1994 — see also M. V. Berry & S. Klein, Eur. J. Phys. **18**, 222 (1997)

INTERACTING ELECTRONIC SYSTEMS :

↪ importance of phase coherence: $L \simeq L_{\rm loc} < L_\phi$

↪ First experimental evidence: — Gershenson *et al.*, Phys. Rev. Lett. **79**, 725 (1997)

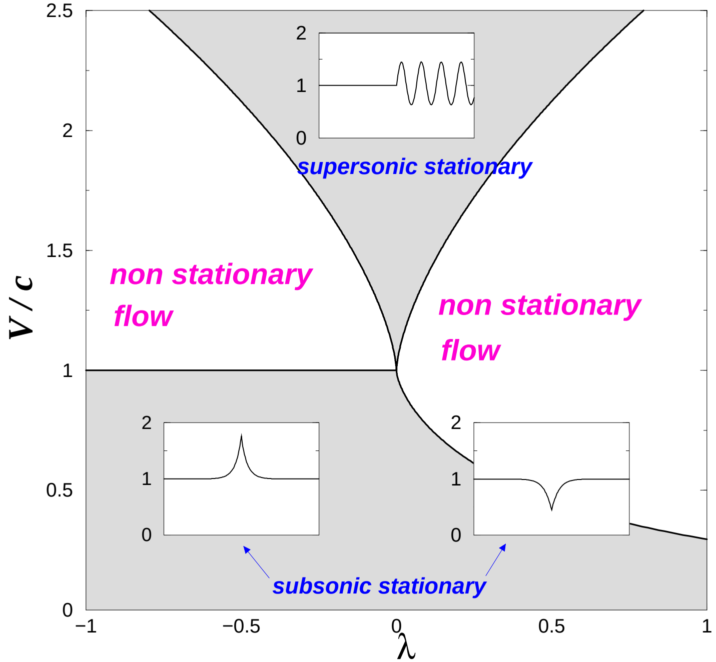
BEC SYSTEMS :

↪ importance of the type of disorder

D. Clément *et al.*, Phys. Rev. Lett. **95**, 170409 (2005)
C. Fort *et al.*, Phys. Rev. Lett. **95**, 170410 (2005)
T. Schulte *et al.*, Phys. Rev. Lett. **95**, 170411 (2005)

# Flow past an impurity

$$U(x) = \lambda\,\mu\,\xi\,\delta(x)\ .$$




P. Leboeuf & N. Pavloff, Phys. Rev. A **64**, 033602 (2001)

Perturbative treatment $(V > c)$ :

- in $1D$, $F \propto |\hat{U}(\kappa)|^2$

where $\kappa = \frac{m}{\hbar}|V^2 - c^2|^{1/2}$

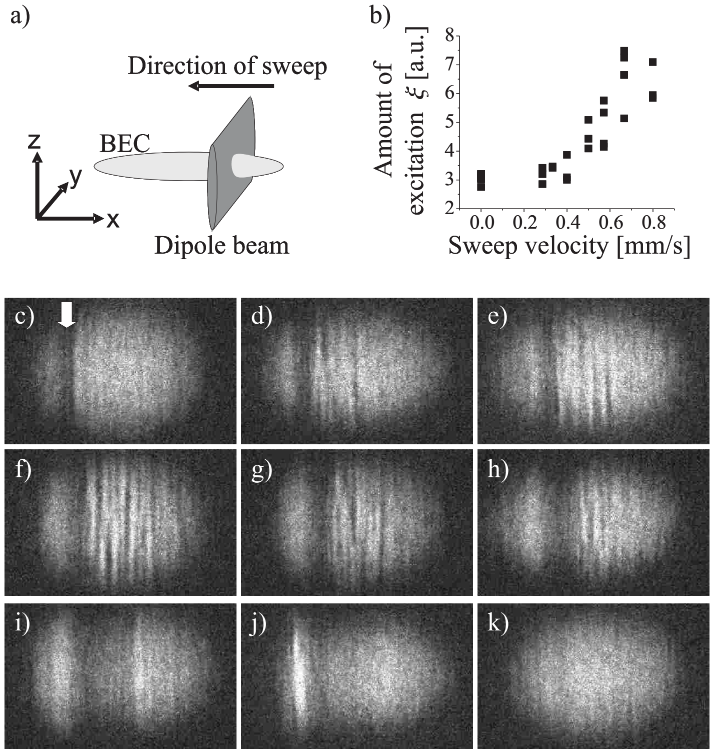
- For a $\delta$ impurity :

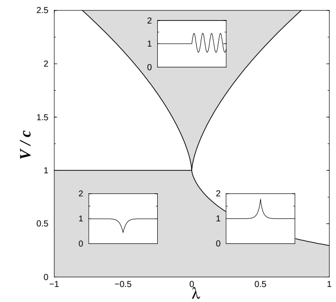
$$\begin{cases} F \propto C^{\rm st} & 1D \\ F \propto (V^2 - c^2)/V & 2D \\ F \propto V^2\,(1 - c^2/V^2)^2 & 3D \end{cases}$$

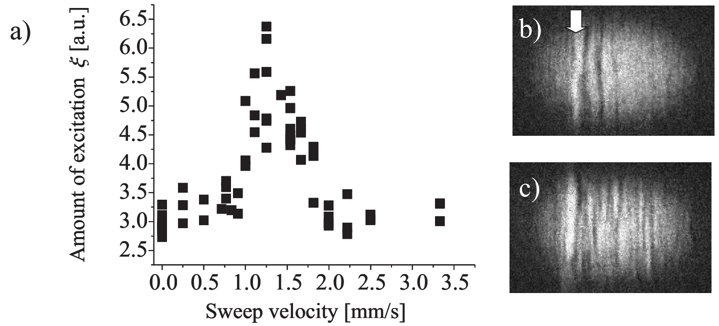
N. Pavloff, Phys. Rev. A **66**, 013610 (2002)

G. E. Astrakharchik & L. P. Pitaevskii,
Phys. Rev. A **70**, 013608 (2004)

# Recent experimental study




Repulsive potential

$U_{\max}/\mu \simeq 0.24$, $c = 2.1$ mm/s

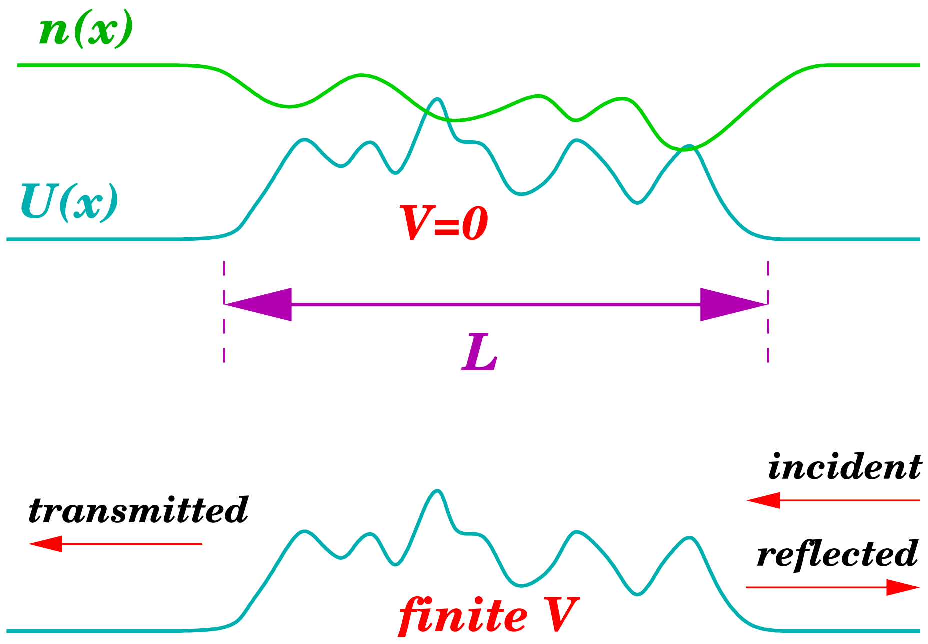
$V$ =0.4 – 0.8, 1, 1.3, 2, 3.3 mm/s

Attractive potential $V$ =1.25 mm/s, $c = 2.1$ mm/s

$|U_{\min}|/\mu \sim 0.17, 0.32$

P. Engels & C. Atherton, Phys. Rev. Lett. **99**, 160405 (2007)

## A (nonlinear) beam incident on a disordered region of size $L$




What are the density profile, the transmission coefficient and the drag exerted on the obstacle when the velocity $V$ of the beam with respect to the obstacle is finite ?

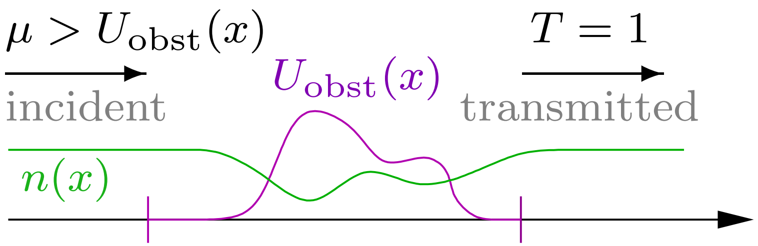
How do these properties scale with $L$?

In the frame where the beam is at rest :

$$-\frac{\hbar^2}{2\,m}\partial_x^2\psi + \left[U(x - V\,t) + g\,|\psi|^2\right]\psi = i\hbar\,\partial_t\psi\;,$$

# Two contrasting phenomena

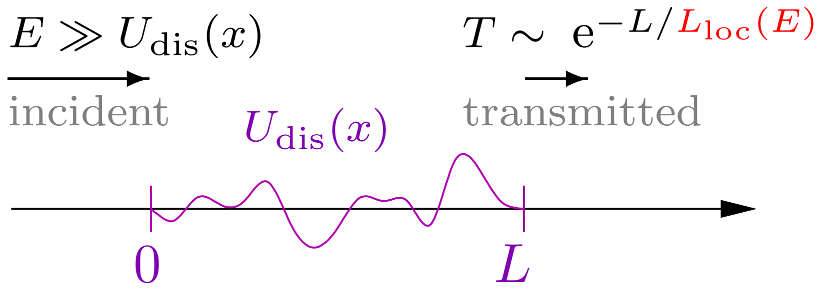
## Superfluidity




Perfect transmission

No drag, no dissipation

## Anderson localization




Large $L$ : no transmission

interaction $\Longleftrightarrow$ disorder

# Model disorder

The disordered potential reads U. Gavish & Y. Castin, Phys. Rev. Lett. **95**, 020401 (2005)

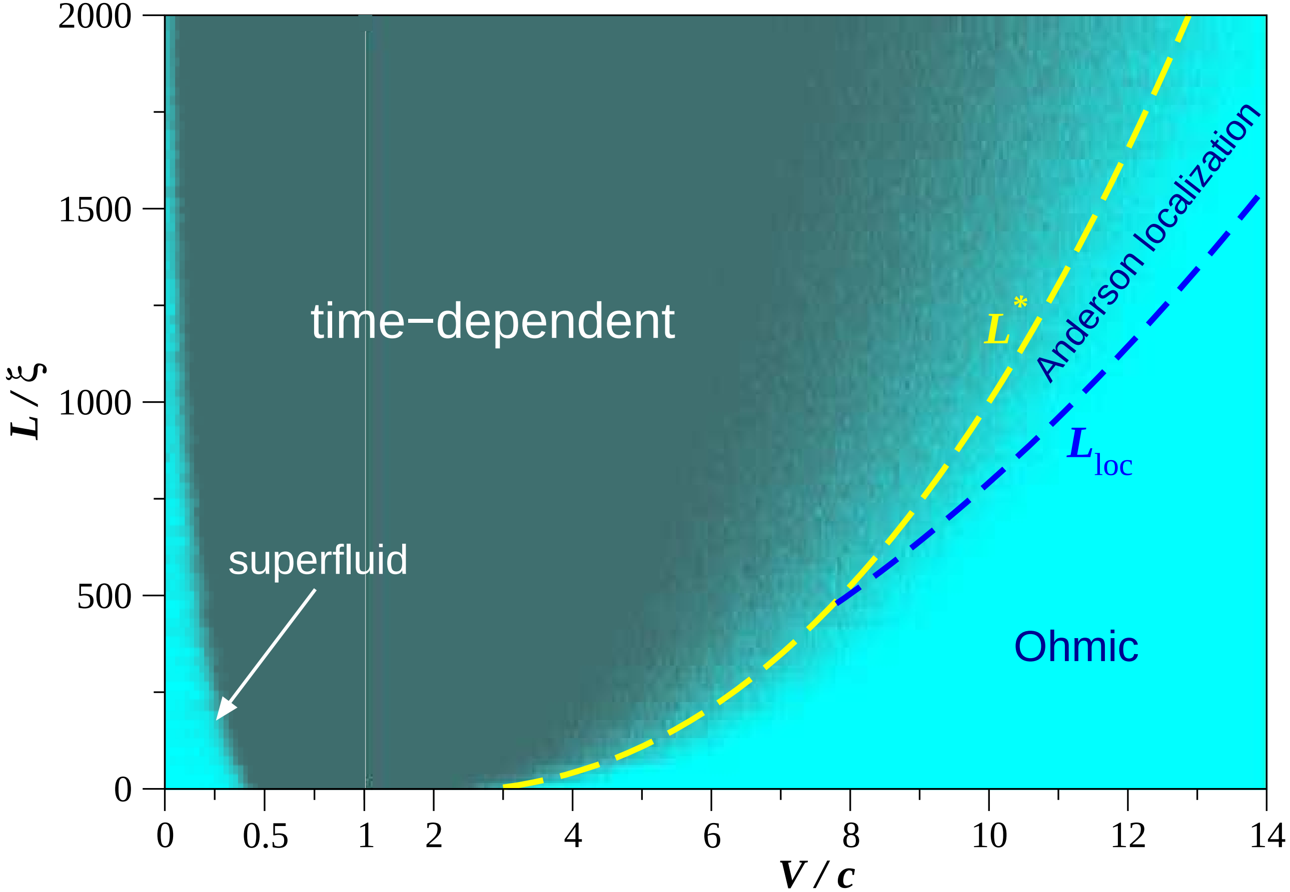
$$U(x) = \lambda\,\mu\,\xi \sum_n \delta(x - x_n)\ , \tag{3}$$

with $x_n$'s: uncorrelated random position of the impurities
$0 = x_1 \leq x_2 \leq x_3 \ldots$, with mean density $n_{\mathrm{i}}$

One has $\langle U(x) \rangle = \lambda\,\mu\,(n_{\mathrm{i}}\,\xi)$ and
$\langle U(x) U(x') \rangle - \langle U(x) \rangle \langle U(x') \rangle = \left(\frac{\hbar^2}{m}\right)^2 \sigma\,\delta(x - x')\ ,$

with $\sigma = n_{\mathrm{i}}\,\lambda^2/\xi^2$. $[\sigma] =$ length$^{-3}$.

## Global Picture : conflict between superfluidity and localization



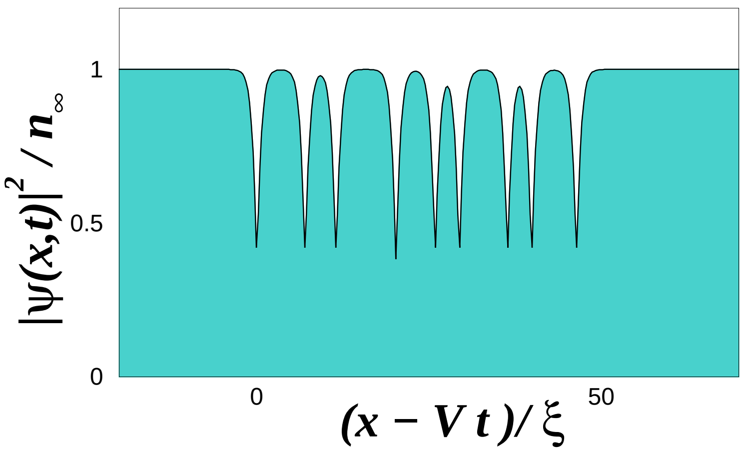

disorder of type (3) with $\lambda = 0.5$ and $n_{\rm i}\xi = 0.5$ $\quad(\mu \gg \langle U \rangle)$.

## Superfluid (and subsonic) regime

In this regime (stable with respect to time evolution), only local and stationary perturbations around the impurities.

Perfect transmission of the matter wave.

No drag is exerted on the potential, but the flow is associated to a momentum




$$P = \hbar \int_{\mathbb{R}} \mathrm{d}x [n(x) - n_0] \partial_x S \ ,$$

where $S$ is the phase of $\psi$.

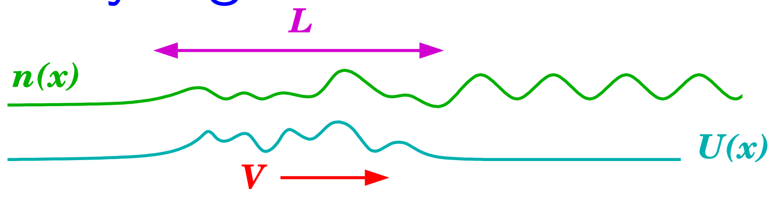
This allows to determine the mass of the non superfluid component $M_\mathrm{n} = P/v_\mathrm{beam}$ . Defining $M = m n_0 L$ perturbation theory yields

$$\frac{M_\mathrm{n}}{M} = \frac{m^2}{2\,\hbar^4 \kappa^3 L} \int_{\mathbb{R}^2} \mathrm{d}y_1 \mathrm{d}y_2 U(y_1) U(y_2) (1 + 2\kappa |y_1 - y_2|) \mathrm{e}^{-2\kappa|y_1 - y_2|} \ .$$

$M_\mathrm{n}/M \ll 1$ when $|\delta n(x)| \ll n_0$. $\kappa = \frac{m}{\hbar} \left| V^2 - c^2 \right|^{1/2}$

# Supersonic stationary regime

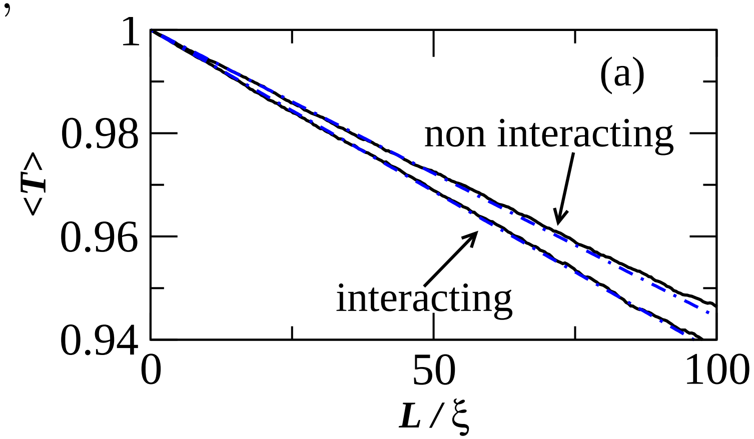
## Ohmic ($\equiv$ perturbative) region




$$\delta n(X) \simeq \frac{2\, m\, n_0}{\hbar^2\, \kappa} \int_{-\infty}^{X} \mathrm{d}y\, U(y)\, \sin[2\kappa(X - y)]\ ,$$

where $X = x - Vt$. This yields $\langle T \rangle \simeq 1 - L/L_{\rm loc}$ where

$$L_{\rm loc}(\kappa) = \frac{\kappa^2}{\sigma}\ . \qquad (4)$$

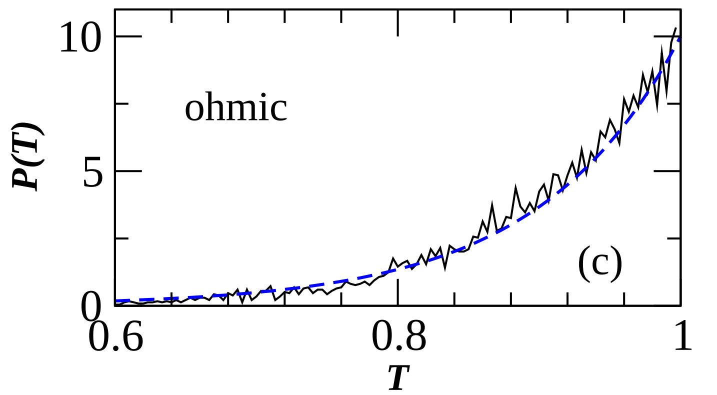



and $\kappa = \dfrac{m}{\hbar} \left| V^2 - c^2 \right|^{1/2}$ . (5)

probability distribution of $T$ :

$$P(T) = \frac{L_{\rm loc}}{L} \exp\left\{ -(1 - T) \frac{L_{\rm loc}}{L} \right\}\ .$$

*bottom plot* : $L/L_{\rm loc} = 0.1 \longrightarrow$



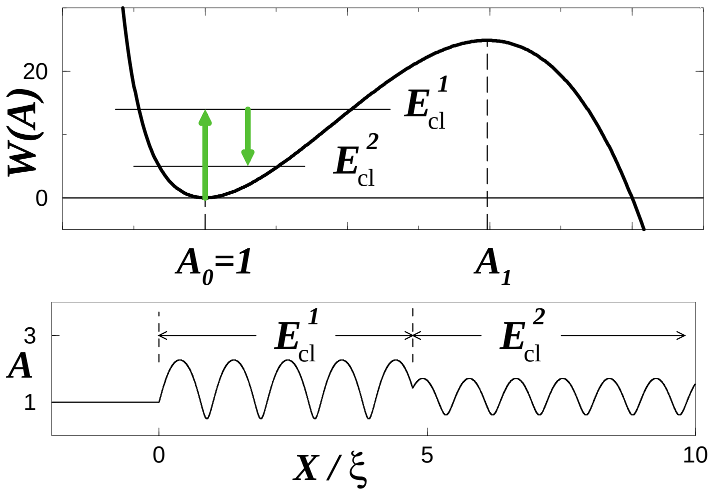
# Diffusion equation for the transmission

First integral in regions where $U(x) \equiv 0$ (between $x_n$ and $x_{n+1}$ say)

$$\frac{\xi^2}{2}\left(\frac{\mathrm{d}A}{\mathrm{d}X}\right)^2 + W[A(X)] = E_{\rm cl}^n \; ,$$

where $A = |\psi|/\sqrt{n_0}$, $E_{\rm cl}^n$ is a constant and $W(A) = \frac{1}{2}(A^2-1)(1+v^2-A^2-v^2/A^2)$. From the final $E_{\rm cl}^{N_{\rm i}}$ one computes the transmission[a]

$$T = \frac{1}{1+(2\kappa^2\,\xi^2)^{-1}E_{\rm cl}^{N_{\rm i}}} \; .$$




*Upper panel:* $W(A)$ *(drawn for* $v = V/c = 4$*).* $A_0(=1)$ *and* $A_1$ *are the zeros of* $\mathrm{d}W/\mathrm{d}A$*. The fictitious particle is initially at rest with* $E_{\rm cl}^0 = 0$*. The value of* $E_{\rm cl}$ *changes at each impurity. The lower panel displays the corresponding oscillations of* $A(X)$*, with two impurities (vertical dashed lines) at* $x_1 = 0$ *and* $x_2 = 4.7\,\xi$*.*

[a] P. Lebœuf, N. Pavloff & S. Sinha, Phys. Rev. A **68**, 063608 (2003)

## Anderson localization

$L > L_{\rm loc}$ : non perturbative. The diffusion equation for $T$ yields (for $L \gg L_{\rm loc}$)

$$\langle \ln T \rangle = -L/L_{\rm loc}(\kappa)\ ,$$

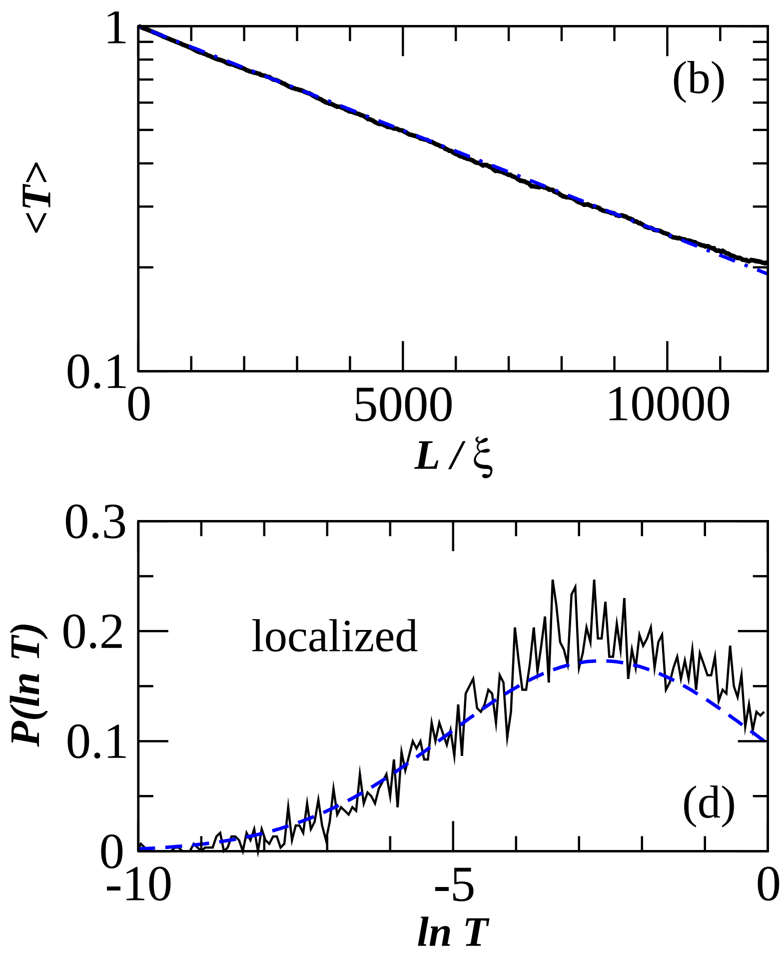
where $L_{\rm loc}(\kappa)$ is given by Eqs. (4,5).

The probability distribution reads

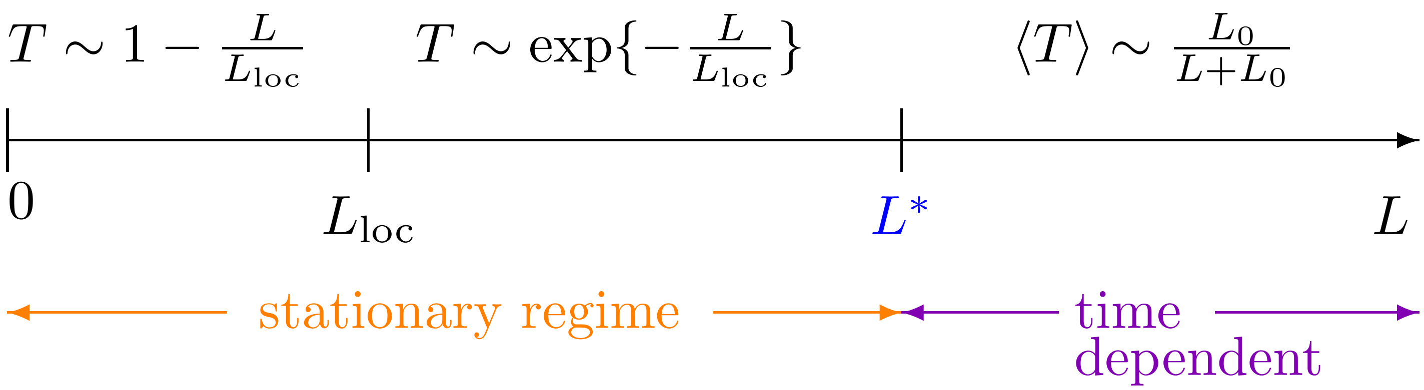
$$P(\ln T) = \sqrt{\frac{L_{\rm loc}}{4\pi L}}\ {\rm e}^{-\frac{L_{\rm loc}}{4L}\left(\frac{L}{L_{\rm loc}}+\ln T\right)^2}\ .$$

*figure drawn for* $V/c = 30 \longrightarrow$

*bottom plot :* $L/L_{\rm loc} = 2.4$



## Picture in the supersonic regime :




- $L_{\rm loc}$ has the same expression as for non-interacting particles with

$$\frac{m\,V}{\hbar} = k \to \kappa = \frac{m}{\hbar}\sqrt{V^2 - c^2} = \sqrt{k^2 - \frac{1}{\xi^2}}\ .$$

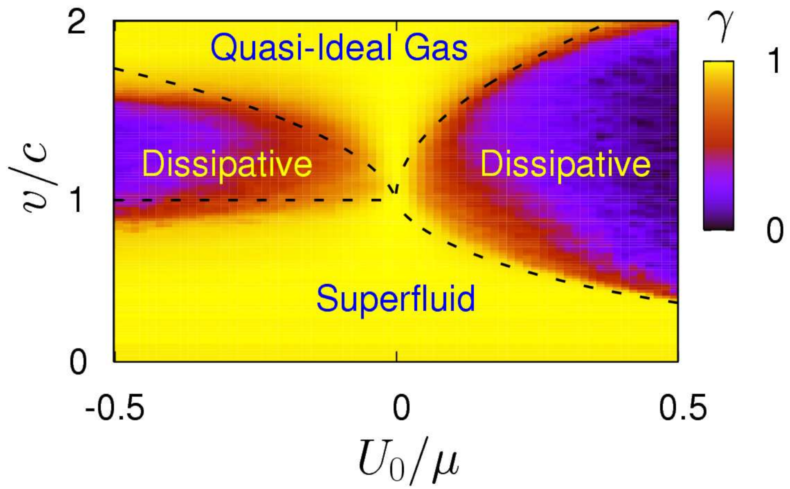
- $$L^* = \frac{1}{2} L_{\rm loc}(\kappa)\ \ln\left(\frac{V^2}{8\,c^2}\right)\ .$$

# Damping of dipolar oscillations

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \left[\frac{m}{2}\omega_x^2\,x^2 + U(x) + 2\hbar\omega_\perp(an)^\nu\right]\psi\ .$$

- if $U(x) \equiv 0$, center of mass: $X_t = d_0\,\cos(\omega_x\,t)$
- If $U(x) = U_0\,\exp\{-\frac{x^2}{2\,\sigma^2}\}$, $X_t \rightsquigarrow d_f\cos(t)$ when $t \to \infty$
- Define $\gamma = d_f/d_0$

$$\begin{cases} \text{no damping:} & \gamma = 1 \\ \text{strong damping:} & \gamma \to 0 \end{cases}$$




$v = d_0\omega_x$

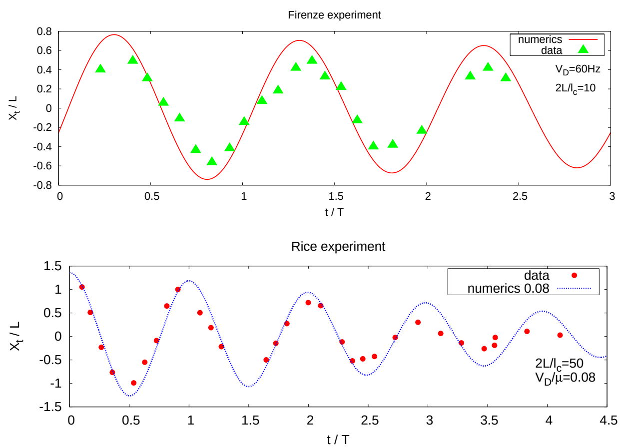
$c$: sound velocity at center of the trap

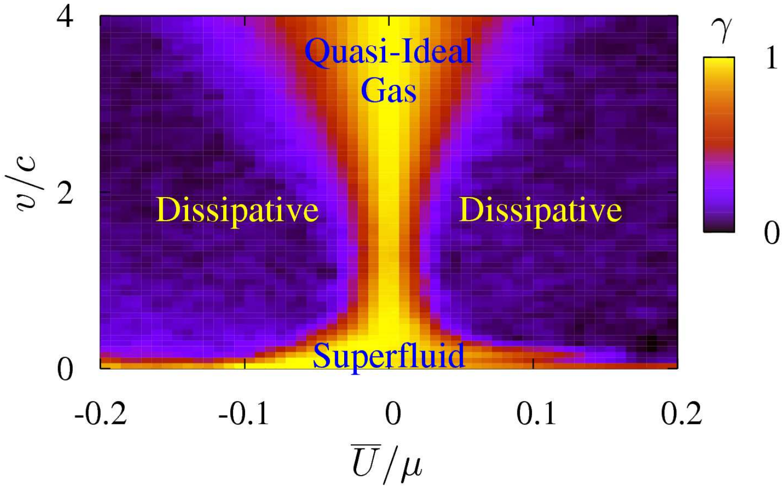
$\mu$: chemical potential

# In presence of disorder :

J. E. Lye *et al.*, Phys. Rev. Lett. **9**5, 070401 (2005)

Y. P. Chen *et al.*, arXiv:0710.5187




Rice : $d_0 = 700\,\mu$m,
$L_x = 1000\ \mu$m
$v/c = 2.8$ and $\bar{U}/\mu = 0.04$

whereas the damping threshold is at $\bar{U}/\mu = 0.008$

Possible effects of localization... but subtle

# Conclusion

Different types of set-ups lead to a large variety of phenomena :

→ Algebraic decay of a dark soliton.

→ Anderson localization : non-interacting elementary excitations or supersonic beams in presence of interaction.

→ For a beam :

- $L_{\text{loc}}$ is renormalized in presence of interaction
- time dependent regime (for $L \geq L^*$ )
- different regimes ⇒ different heating rates

→ Dipolar oscillations : possible indirect evidences...

# BEC in presence of disorder ?

• In the case of strong disorder :

$\hookrightarrow$ phase transition at $T = 0 \rightarrow$ "Bose glass" : non-superfluid.

$\hookrightarrow$ The system can no longer be described by GPE.

• Here we consider only the case of weak disorder.

$\hookrightarrow$ only slightly decreases the condensate and the superfluid fraction

K. Huang & H. F. Meng, Phys. Rev. Lett. **69**, 644 (1992); S. Giorgini, L. Pitaevskii & S. Stringari, Phys. Rev. B **49**, 12938 (1994).

$\hookrightarrow$ more precisely, for $U(x) = \lambda \, \mu \, \xi \sum \delta(x - x_n)$, the depletion of the condensate is proportional to $n_{\rm i} \, \xi \, \lambda^2 \ll 1$ here.

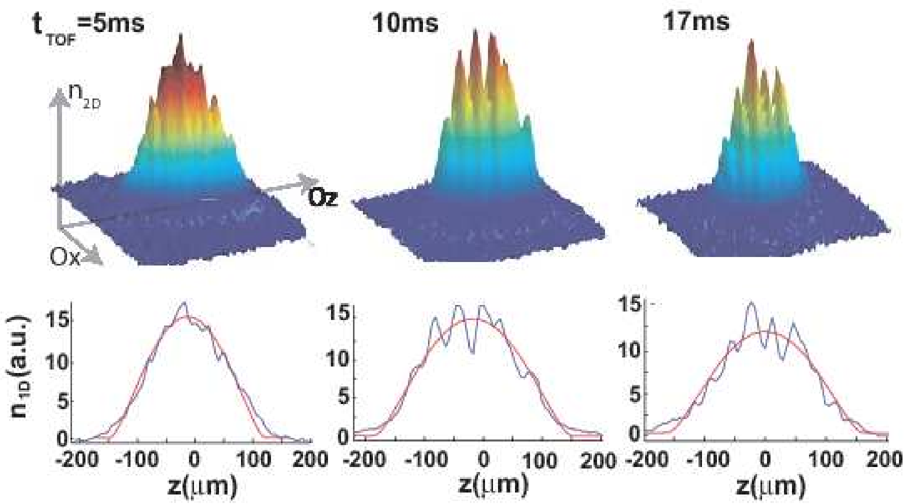
G. E. Astrakharchik & L. P. Pitaevskii, Phys. Rev. A **70**, 013608 (2004)

T. Paul, P. Lebœuf, P. Schlagheck & N. Pavloff, Phys. Rev. Lett. **98**, 210602 (2007)

# BEC in presence of disorder ?

experimental evidence of phase coherence in presence of disorder :

Rice and LCFIO

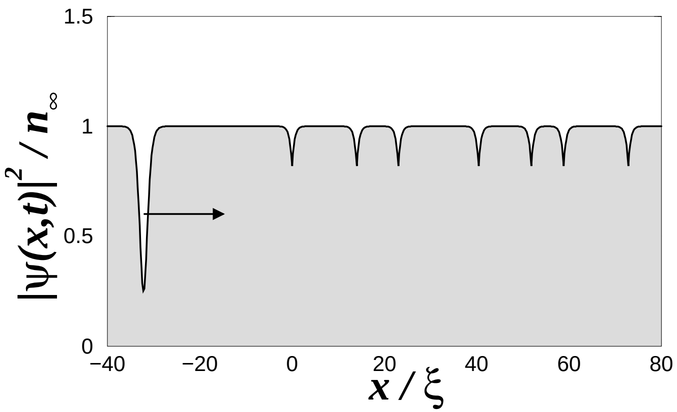



D. Clément, Ph. Bouyer, A. Aspect, L. Sanchez-Palencia, arXiv:0710.1984

# Scattering of a dark soliton

One considers a dark soliton incident on a disordered region

N. Bilas & N. Pavloff, Phys. Rev. Lett. **95**, 130403 (2005)




The disordered potential reads[a] :

$$U(x) = \lambda\, \mu\, \xi \sum_n \delta(x - x_n) \; , \qquad (6)$$

with $x_n$'s: uncorrelated random position of the impurities with mean density $n_{\mathrm{i}}$ $0 = x_1 \le x_2 \le x_3 \ldots$

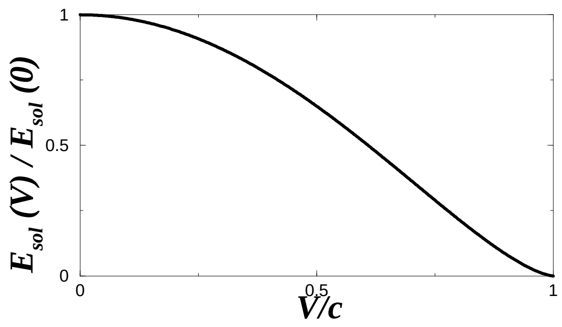
[a] cf. Y. S. Kivshar, S. A. Gredeskul, A. Sánchez & L. Vázquez, Phys. Rev. Lett. **64**, 1693 (1990)

One has $\langle U(x) U(x') \rangle - \langle U(x) \rangle \langle U(x') \rangle = \left( \frac{\hbar^2}{m} \right)^2 \sigma\, \delta(x - x') \; ,$

with $\sigma = n_{\mathrm{i}}\, \lambda^2 / \xi^2$.

A dark soliton with velocity $V$ has an energy $E_{\rm sol}$

$$E_{\rm sol} = \frac{4}{3}\,\mu\left(\frac{a_1}{\xi}\right)\left(1-\frac{V^2}{c^2}\right)^{3/2}.$$




In the limit $\lambda \ll 1$ [a] and $V^2 \gg \lambda\, c^2$ [b] a soliton scattering on **a single impurity** radiates an energy $E^+_{\rm rad} + E^-_{\rm rad}$ with

$$E^{\pm}_{\rm rad} = \mu\,\lambda^2\, F^{\pm}(V/c)\ ,$$

where for $v = V/c \in [0,1]$

$$F^{\pm}(v) = \frac{\pi}{16\,v^6}\int_0^{+\infty} dy\ \frac{y^4\left(-v \pm \sqrt{1+y^2/4}\right)}{\sinh^2\left[\frac{\pi\,y\sqrt{1+y^2/4}}{2v\sqrt{1-v^2}}\right]}\ .$$

N. Bilas & N. Pavloff, Phys. Rev. A **72**, 033618 (2005)

[a] This ensures that the impurity only weakly perturbs the constant density profile.

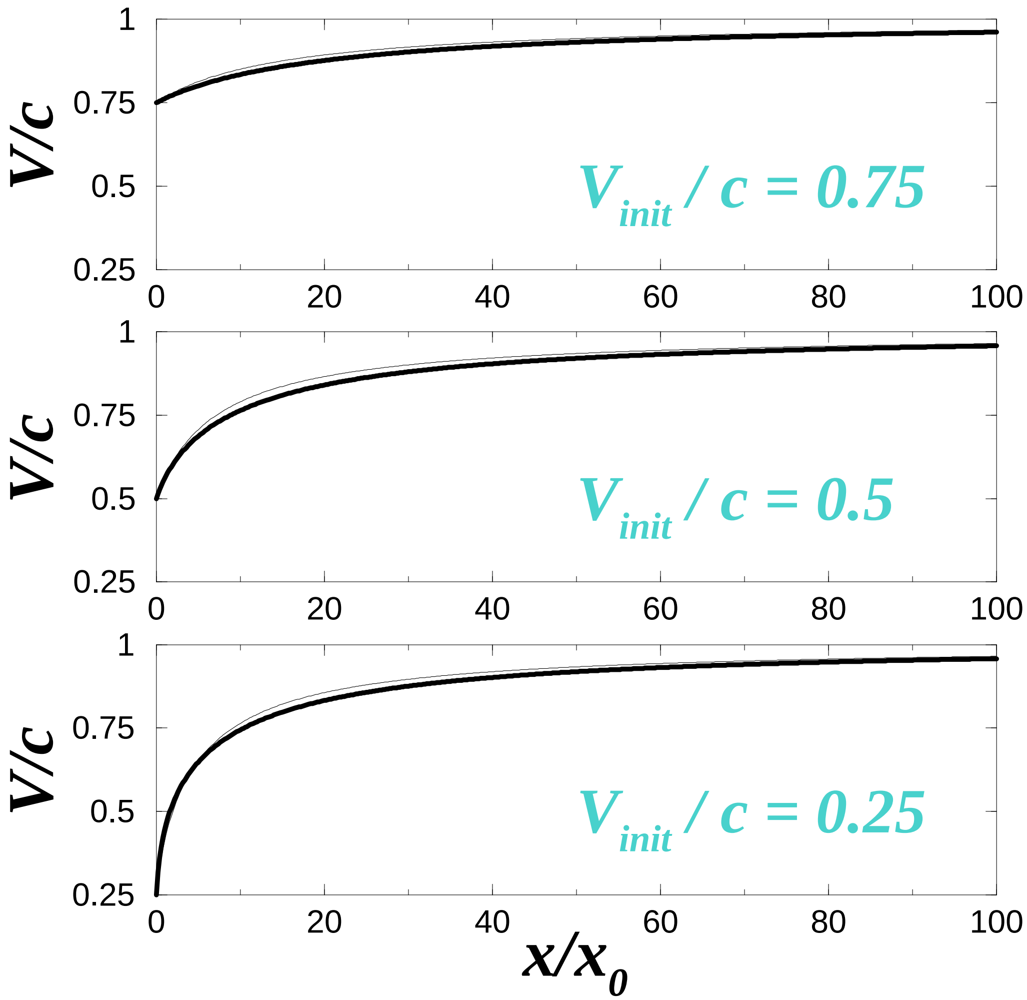
[b] This ensures that the scattering process can be treated perturbatively.

In the limit $\xi \ll \frac{1}{n_{\rm i}}$. the scattering of the soliton by the impurities can be treated as a sequence of independent events. This leads to

$$\frac{dV}{dx} = \frac{c}{4\,x_0} \frac{F^+(V/c) + F^-(V/c)}{\frac{V}{c}\sqrt{1-(V/c)^2}} \quad \text{with} \quad x_0 = \frac{a_1}{\sigma\,\xi^3}$$

If $v = V/c \to 1$ one has $F^+(v) + F^-(v) = \frac{4}{15}\left(1 - v^2\right)^{5/2}$.
This yields :

$$V(x) = c\sqrt{1 - \frac{1 - V_{\rm init}^2/c^2}{1 + (1 - V_{\rm init}^2/c^2)\frac{2\,x}{15\,x_0}}}\ .$$




In these plots

$$x_0 = \frac{a_1}{\sigma\,\xi^3} .$$

For $x \gg x_0$ one has

$$V(x) \simeq c\left(1 - \frac{15\,x_0}{4\,x}\right),$$

independent of $V_{\text{init}}$.

The soliton has disappeared when $\Delta N \sim 1$. This happens for a critical velocity $V_{\rm cr} = c[1 - (\xi/2a_1)^2]^{1/2}$. Hence the distance covered by the soliton in the disordered region before decaying is

$$L = 30\, a_1 \left(\frac{a_1}{\xi}\right)^2 \times \frac{1}{\sigma \xi^3}.$$

## Partial Conclusion

(1) The soliton is accelerated until it reaches the speed of sound and disappears.

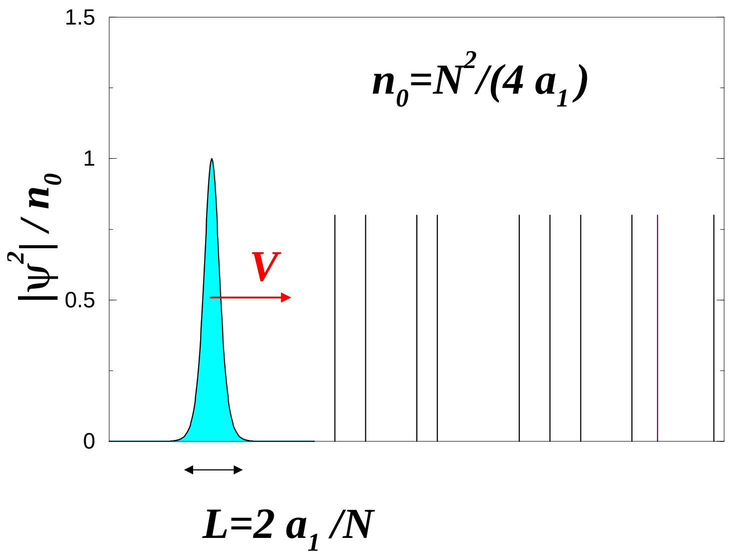
(2) Its decay is algebraic and not exponential.

(3) The length covered in the disordered region is independent of the initial velocity of the soliton (as is the traveling time).

## Bright soliton incident on a disordered potential

attractive effective interaction ($a_1 \to -a_1$). A bright soliton is characterized by 2 parameters : $N$ and $V$. It has an energy $E_{\rm sol}$ with

$$\frac{E_{\rm sol}}{N} = \frac{1}{2}\, m\, V^2 - \frac{1}{3}\, \frac{\hbar^2}{m a_1^2}\, N^2 \, .$$




if $\quad mV^2 \gg \hbar^2 N^2/(m a_1^2)$ : $\;V \sim C^{\rm st}$ and $N$ decreases exponentially.

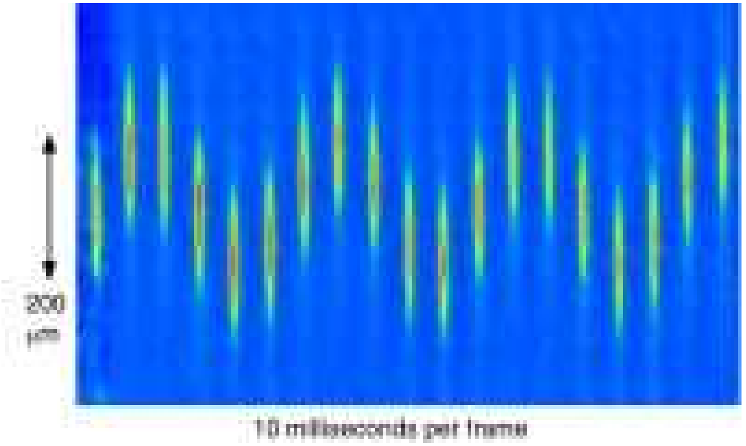
if $\quad mV^2 \ll \hbar^2 N^2/(m a_1^2)$ : $\;V$ and $N$ tend to a $C^{\rm st}$.

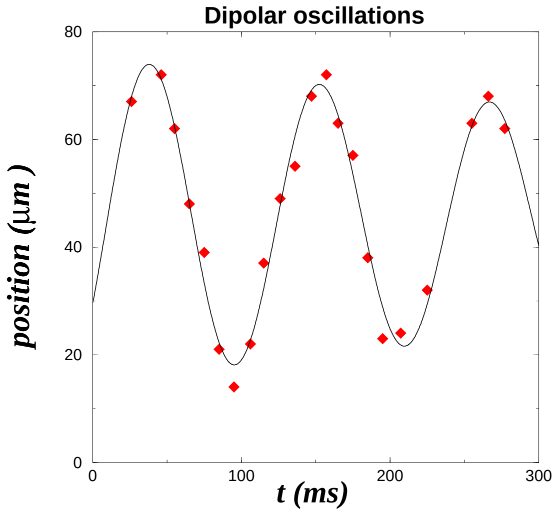
Y. S. Kivshar, S. A. Gredeskul, A. Sánchez & L. Vázquez, Phys. Rev. Lett. **64**, 1693 (1990).

# Experimental results

## LENS - University of Firenze

• Study of discrete collective modes (dipolar and quadrupolar) in the transverse Thomas-Fermi regime.




J. E. Lye *et al.*, Phys. Rev. Lett. **95**, 070401 (2005).

$\hookrightarrow$ dipolar excitation ($\omega = \omega_{\rm long} = 2\pi \times 8.74$ Hz, the longitudinal trapping frequency) one observes a damping over a typical length $L_{\rm loc}^{\rm exp} \simeq 1$ mm (for $\langle U \rangle / \mu = 0.06$). $L_{\rm loc}^{\rm exp} \gg L_{\rm long}$ ($\simeq 0.1$ mm).

$\hookrightarrow$ In this regime the localization length reads :

$$L_{\rm loc} = \frac{\xi^2}{2\, r_c} \left(\frac{\mu}{\langle U\rangle}\right)^2 \left(\frac{\mu}{\hbar\omega}\right)^2 \left(1 - \frac{2\,\langle U\rangle}{\mu}\right)^3 , \tag{7}$$

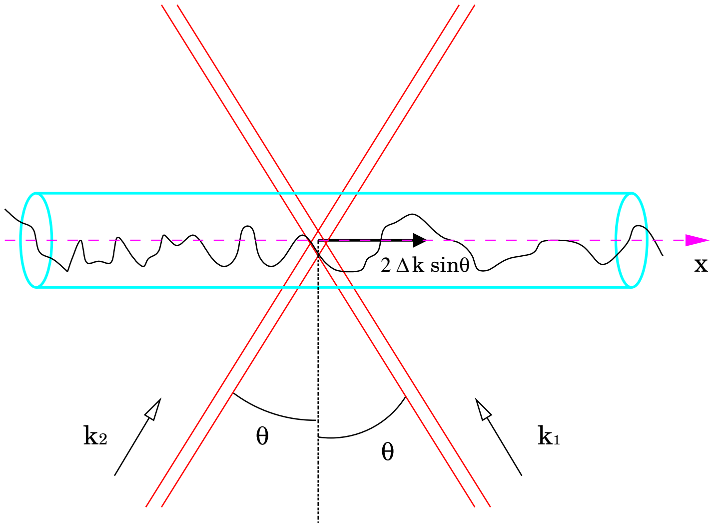
$r_c$ being the correlation length of $U(x)$, defined as
$\int_{\mathbb{R}} {\rm d}x \langle U_1(x) U_1(0)\rangle = r_c \langle U\rangle^2$ where $U_1(x) = U(x) - \langle U\rangle$.

For $\omega = \omega_{\rm long}$, (7) leads to $L_{\rm loc}^{\rm theo} \simeq 7$ mm !!

## IOTA - Orsay-Palaiseau

• quasi-1D BEC, in the transverse Thomas-Fermi regime, with a length $L_{\rm long} = 300\,\mu$m

$\langle U \rangle / \mu = 0.2$, $r_c = 5.2$ $\mu$m and $\xi = 0.16$ $\mu$m.




• If $\omega = \omega_{\rm long} = 2\pi \times 6.7$ Hz (dipole), one gets $L_{\rm loc} = 6$ mm !

• But if $\omega = 8 \times \omega_{\rm long}$, then $L_{\rm loc} \sim 275$ $\mu$m $< L_{\rm long}$.