Unusual transport phenomena in a guided atom laser

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P. Leboeuf & N. Pavloff, Phys. Rev. A **64**, 033602 (2001)

N. Pavloff, Phys. Rev. A **66**, 013610 (2002)

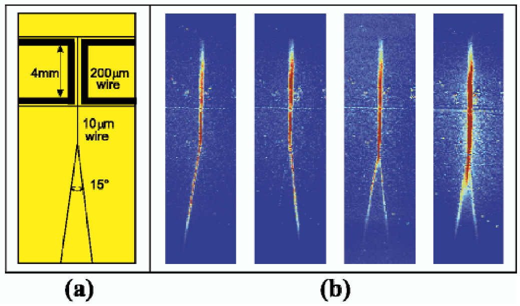
P. Leboeuf, N. Pavloff & S. Sinha, Phys. Rev. A **68**, 063608 (2003)

Cold atoms on a chip :

Prentiss at Harvard, 99

....

recent experiment, Schmiedmayer at Heidelberg :



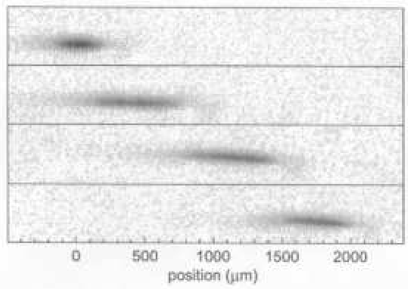
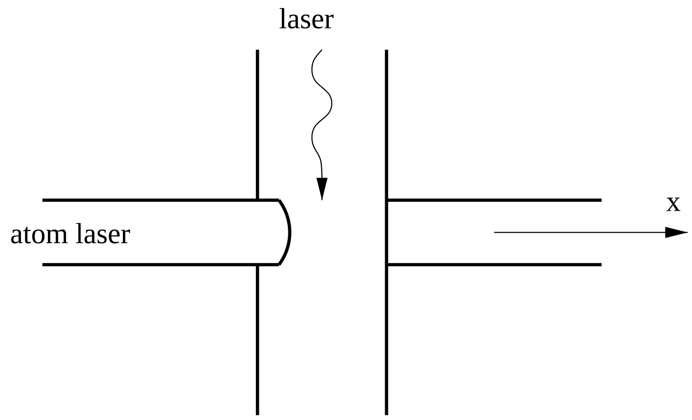


FIG. 3. The time evolution of the Bose-Einstein condensate is shown at $t = 0$ ms, $t = 100$ ms, $t = 200$ ms, and $t = 300$ ms respectively. The trap was turned off 20 ms before the first image of the condensate was obtained and after the potential modification described in the text was complete. The number of atoms in the condensate is estimated to be 1.5×10^5.

mooving BEC on a chip :

Zimmermann at Tübingen →
Hänsch at Munich
Ketterle at M.I.T.

1D geometry :



adiabatic approximation :

$$\boxed{\Psi(\vec{r},t) = \psi(x,t)\ \phi(\vec{r}_\perp\,;n)}$$

where $n(x,t) = \int d^2 r_\perp\ |\Psi|^2 = |\psi(x,t)|^2$ (ϕ being normalized to unity)

hypothesis :

$$\ell_\phi = \begin{bmatrix}\text{phase}\\ \text{coherence}\\ \text{length}\end{bmatrix} \gg \begin{bmatrix}\text{typical size}\\ \text{of the flow}\\ \text{pattern}\end{bmatrix} \quad \text{and} \quad \begin{pmatrix}\text{transverse}\\ \text{scale}\end{pmatrix} \ll \begin{pmatrix}\text{longitudinal}\\ \text{scale}\end{pmatrix}$$

$$\begin{cases} V_\perp(\vec{r}_\perp) : & \text{confining potential} \\ V_\parallel(x,t) : & \text{obstacle} \end{cases}$$

following Jackson *et al.*, Phys. Rev. A **58**, 2417 (1998)

$$-\frac{1}{2}\vec{\nabla}^2_\perp \phi + \left\{ V_\perp(\vec{r}_\perp) + 4\pi a_{sc}\, n\, |\phi|^2 \right\} \phi(\vec{r}_\perp; n) = \epsilon(n)\, \phi(\vec{r}_\perp; n)\ , \qquad (1)$$

(1) determines $\epsilon(n)$: effective non-linear potential to be used in the 1D reduction of the Gross-Pitaevskii equation :

$$\boxed{-\tfrac{1}{2}\partial_{xx}\psi + \left\{ V_\parallel(x,t) + \epsilon[n(x,t)] \right\} \psi(x,t) = i\,\partial_t \psi(x,t)\ .} \qquad (2)$$

for parabolic confinement $V_\perp(\vec{r}_\perp) = \frac{1}{2}\,\omega_\perp^2\ r_\perp^2$, and (1) yields

$$\epsilon(n) = \begin{cases} 2\,\omega_\perp\, n\, a_{sc} & \text{when} \quad n\,a_{sc} \ll 1 \quad \text{(dilute regime)} \\[2ex] 2\,\omega_\perp \sqrt{n\, a_{sc}} & \text{when} \quad n\,a_{sc} \gg 1 \quad \text{(Thomas-Fermi regime)} \end{cases}$$

in the following one considers condensed atoms incident on a fixed obstacle

in the stationary regime : $\boxed{\psi(x,t) = \exp\{-i\,\mu\, t\}\, A(x)\, \exp\{i\, S(x)\}}$

the density is $n(x) = A^2(x)$, the velocity $v(x) = dS/dx$

and the flux is a constant : $n(x)v(x) \stackrel{\text{def}}{=} J_\infty$

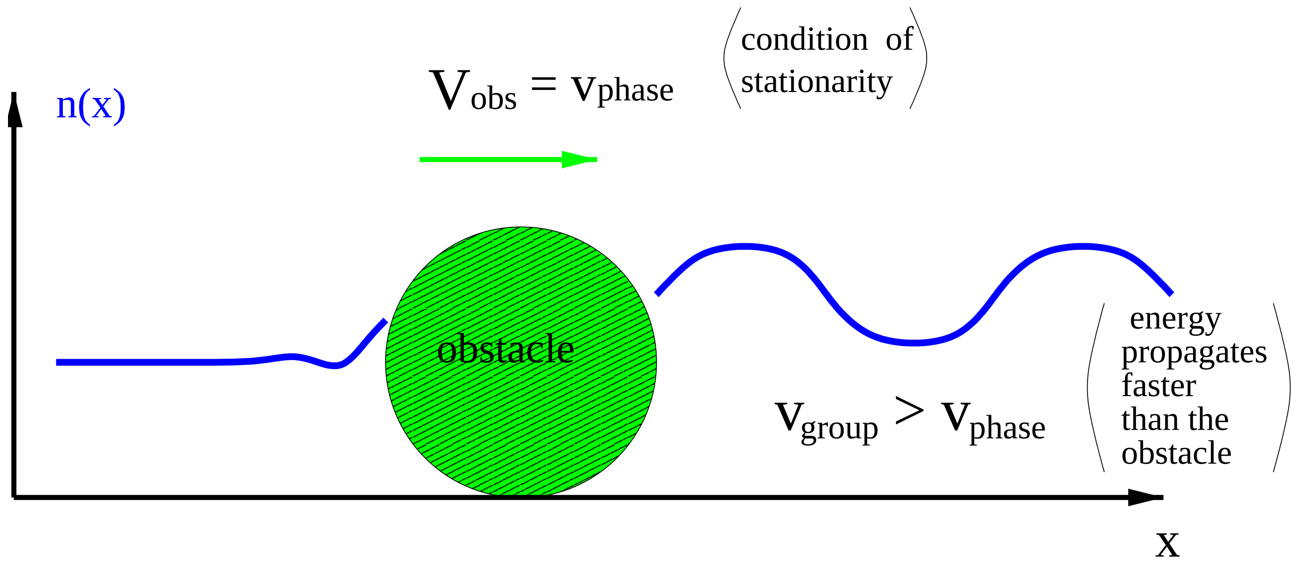
Eq. (2) becomes :

$$-\frac{1}{2}\frac{d^2 A}{dx^2} + \left\{ V_\parallel(x) + \epsilon[n(x)] + \frac{J_\infty^2}{2\, n^2(x)} \right\} A(x) = \mu\, A(x)\,. \tag{3}$$

boundary conditions (stationary regime)

no wake far down-stream because $v_{group}(k) = \frac{d\omega}{dk} > v_{phase}(k) = \frac{\omega(k)}{k}$

Illustration for an obstacle moving at $V_{obs} > 0$ in a beam at rest :



Hence the boundary conditions can only be imposed far down-stream.

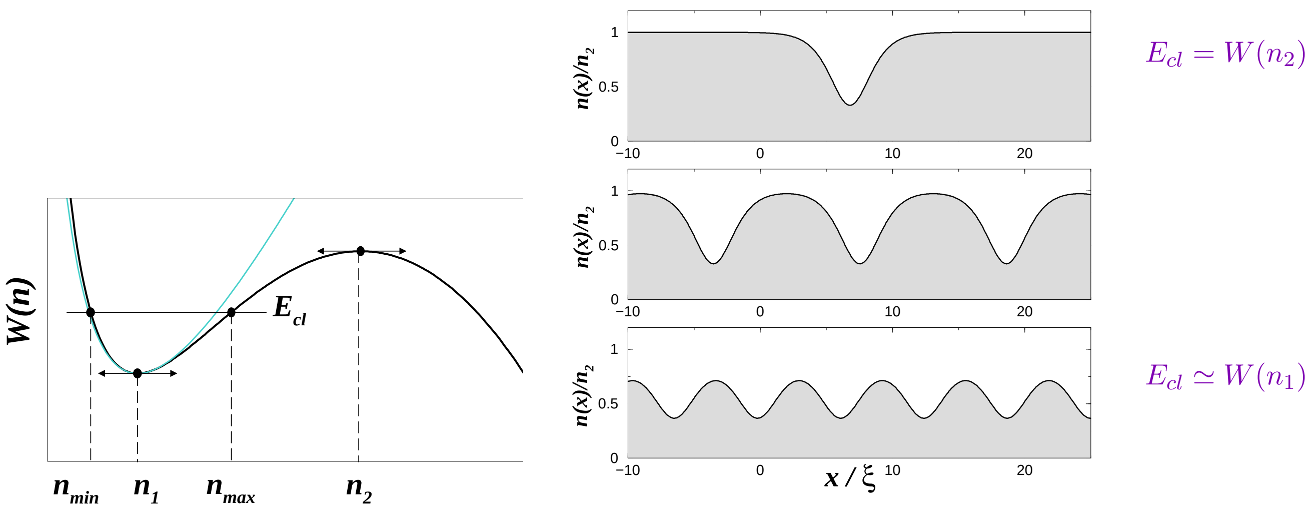
since, because of non-linearity, one cannot disentangle an incident and a reflected part in the perturbed up-stream solution

Is it possible to define a transmission and a reflection coefficient for a non-linear wave ?

In regions where $V_\parallel(x)$ is zero (or constant) (3) admits a first integral

$$\frac{1}{2}A'^2 + W[n(x)] = E_{cl} \quad \text{with} \quad W(n) = -\underbrace{\int_0^n \epsilon(\rho)d\rho}_{\varepsilon(n)} + \mu\, n + \frac{J_\infty^2}{2\, n} \,, \tag{4}$$

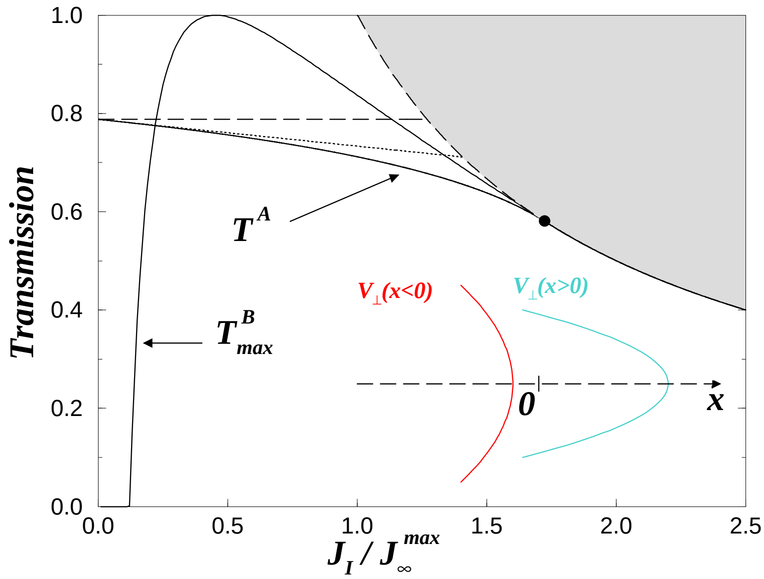
where E_{cl} is an integration constant.



incident current : J_I transmitted current : $J_T = T J_I = J_\infty$

$E_{cl} = W(n_1) + 2\frac{v_1^2 - c_1^2}{v_1} J_I(1-T)$ up-stream : $V_\parallel(x \to +\infty) \to V_0$ and Eq. (4) still holds (E_{cl} takes a different value E_{cl}^0 and μ is repaced by $\mu - V_0$)

Since $n(x \to +\infty)$ should be constant, it is of "type n_2" or of "type n_1"

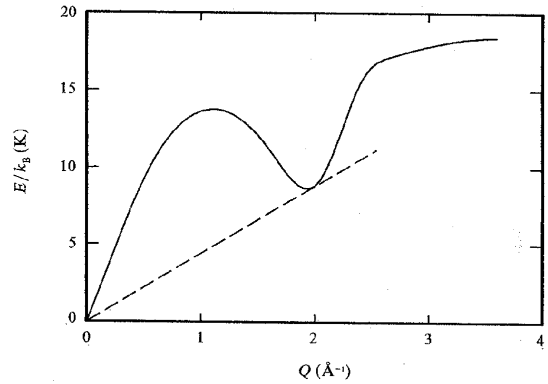


Transmission
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
J_I / J_∞^{max}
T^A
T^B_{max}
$V_\perp(x<0)$
$V_\perp(x>0)$
0
x

Landau criterion

in ^{4}He

$$v_{Landau} = \min \frac{\varepsilon(k)}{k} \simeq 60 \text{ m/s}$$

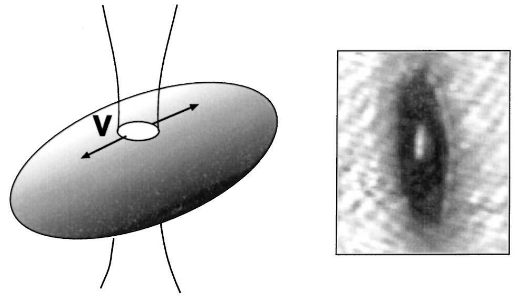


due to vortex formation,
in most experiments :
$1 \text{ mm/s} \lesssim v_{crit,exp} \lesssim 5 \text{ m/s}$

in BEC

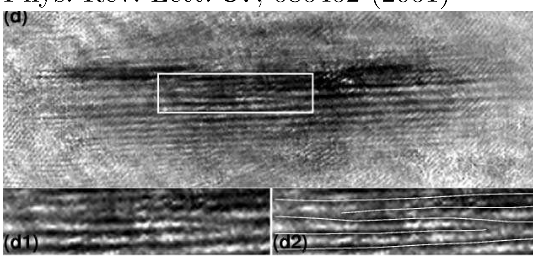
M.I.T. experiment
$\omega^2(k) = k^2\,(c^2 + k^2/4)$
$\rightarrow v_{Landau} = c = 6.2 \text{ mm/s}$
$v_{crit,exp} = 1.6 \text{ mm/s}$

Phys. Rev. Lett. **83**, 2502 (1999)



Evidence of
vortex formation :

Phys. Rev. Lett. **87**, 080402 (2001)

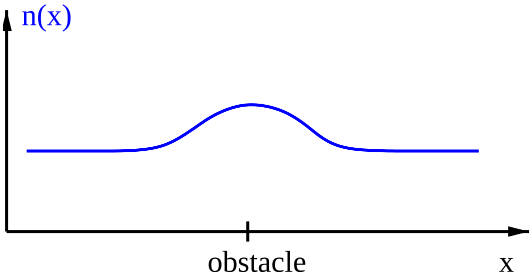


Non-linear effects alter the perturbative Landau criterion

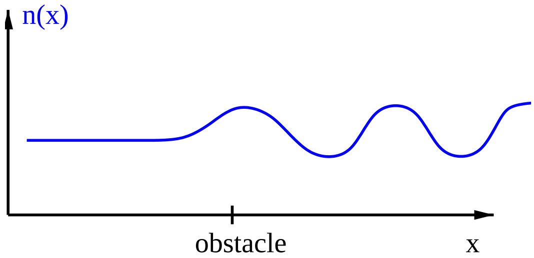
Perturbative treatment :

$n(x) = n_\infty + \delta n(x)$. equivalent to Landau criterion.
One always finds a stationary solution :

$v_\infty < c_\infty$ then $F_d = 0$
the flow is superfluid
typically :



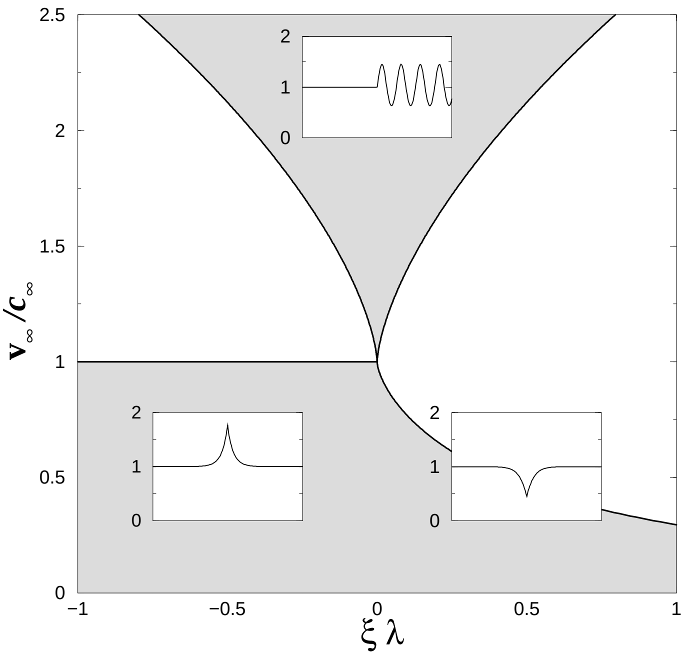
$v_\infty > c_\infty$ then $F_d = -2\, n_\infty |\hat{V}_\parallel(\kappa)|^2$
the flow is dissipative
typically :



This behavior is generic : low velocity $\longrightarrow$ superfluidity and high velocity $\longrightarrow$ dissipation.

BUT ! the perturbative approach fails when $v_\infty \simeq c_\infty$. In this case, the flow is non-stationary and besides nonlinearities play an important role.

As an illustration, consider the simple case $V_\parallel(x) = \lambda\, \delta(x)$. The different types of stationary flow are :



The shaded zone of the plane (v_∞, λ) is the domain of existence of stationary solutions occurring for a potential $V_\parallel(x) = \lambda\, \delta(x)$. The axis are labelled in dimensionless units. The insets represent density profiles $n(x)/n_\infty$ typical for the different flows (the condensed beam is incident from the right). Each inset is located at values of v_∞ and λ typical for the flow it displays. The left (right) lower one is a superfluid flow across an attractive (repulsive) potential. The upper one is a dissipative flow.

Determination of the drag

the drag exerted by the atom laser on the obstacle is :

$$F_d(t) = \int_{-\infty}^{+\infty} dx \; n(x,t) \, \frac{dV_\parallel(x)}{dx} \, . \tag{5}$$

$F_d(t)$ can also be computed via the stress tensor $T(x,t)$:

$T(x,t) = -\,\mathrm{Im}\,(\psi^* \partial_t \psi) + \frac{1}{2}|\partial_x \psi|^2 - \varepsilon[n(x,t)] - V_\parallel(x)\, n(x,t)$
where $\quad \varepsilon(n) = \int_0^n \epsilon(\rho)\, d\rho$,

the total impulsion of the beam is $P(t) = \displaystyle\int_{-\infty}^{+\infty} dx \; \mathrm{Im}(\psi^* \partial_x \psi)$. It is related to $T(x,t)$ and $F_d(t)$ through

$$\frac{d\,P}{dt} = T(-\infty,t) - T(+\infty,t) - F_d(t) \underset{\text{in the stationary regime}}{\longrightarrow} F_d = T(-\infty) - T(+\infty) \tag{6}$$

Eq. (6) allows for an analytical determination of the drag in simple stationary cases and is a test of the numerical accuracy of the numerical procedure.

Finally, note that for stationary flows, the stress tensor reads

$$T(x) = \frac{1}{2}\left(\frac{dA}{dx}\right)^2 + W[n(x)] - V_\parallel(x)\, n(x) \quad \text{where} \quad W(n) = -\varepsilon(n) + \mu\, n + \frac{J_\infty^2}{2n} \tag{7}$$

In regions where the spatial variations of $V_\parallel(x)$ are negligeable, $T(x)$ is a constant (as easily seen from (3)).

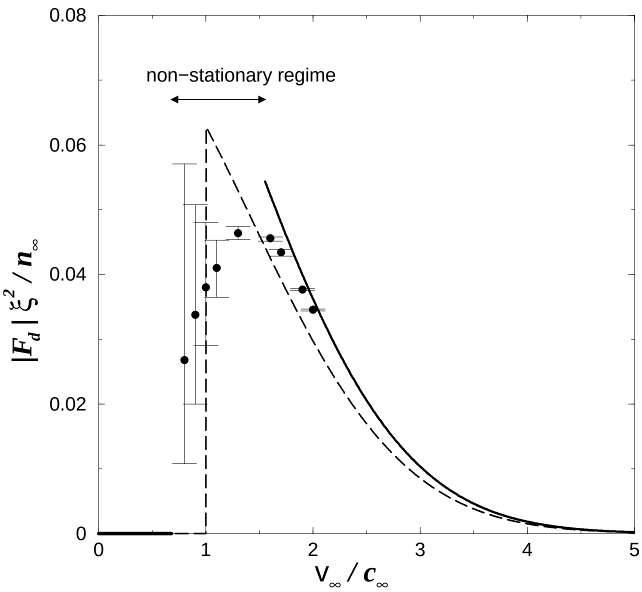
As an illustration, (6) and (7) yield for stationary dissipative flow over a delta peak potential : $F_d = -2\, n_\infty\, \lambda^2$ for $V_\parallel(x) = \lambda\, \delta(x)$.
In this way, analytical expressions (computed via a quadrature) can also be obtained for the drag exerted by a square well potential (attractive or repulsive).

Gaussian potential : $V_{\parallel}(x) = V_0 \exp\left\{-x^2/\sigma^2\right\}$

Same behavior as the delta peak and square well potentials :

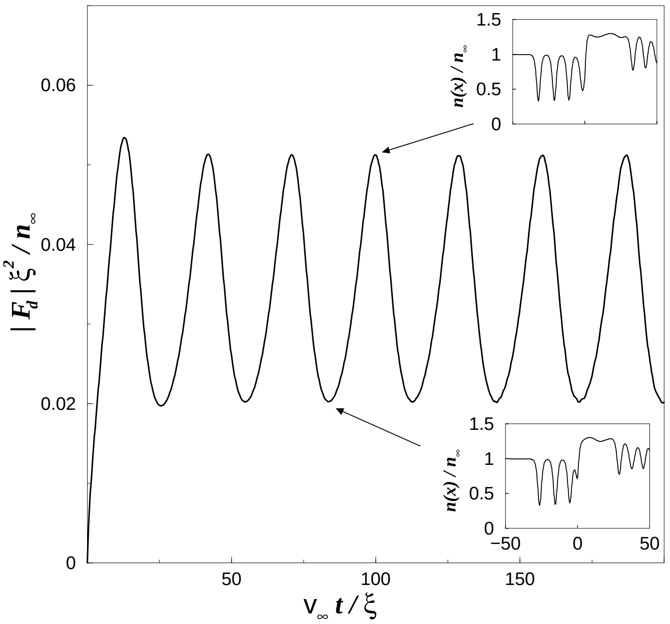
Two critical velocities : $v_{crit,1}$ $(\leqslant c_\infty)$ for the onset of dissipation
$v_{crit,2}$ $(> c_\infty)$ for a new stationary regime

the flow is time dependent in the region $v_{crit,1} < v_\infty < v_{crit,2}$
The drag is :



Drag exerted by a low density beam on a Gaussian potential ($V_0\,\xi^2 = 0.2$, $\sigma = 0.5\,\xi$), as a function of the beam velocity. The dashed line is the perturbative result. The solid line is the exact drag evaluated in the stationary regime. The circles correspond to the drag evaluated in the time-dependent regime. The error bars correspond to the extremal values of the time dependent $F_d(t)$.

The "error bars" correspond to the extremal values of $F_d(t)$:



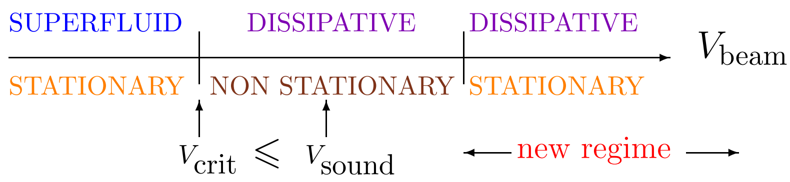
Time evolution of the drag exerted by a low density beam (of velocity $v_\infty = 0.9\,c_\infty$) on a Gaussian potential ($V_0\,\xi^2 = 0.2$, $\sigma = 0.5\,\xi$). F_d and t are expressed in dimensionless units. The upper (lower) inset represents a density profile observed when the drag is maximum (minimum).

CONCLUSION

I Narrowing guide :

For μ fixed, there always exists a special value of J_I such that $T = 1$.
Besides the corresponding flow is dynamically stable.

II Obstacle :



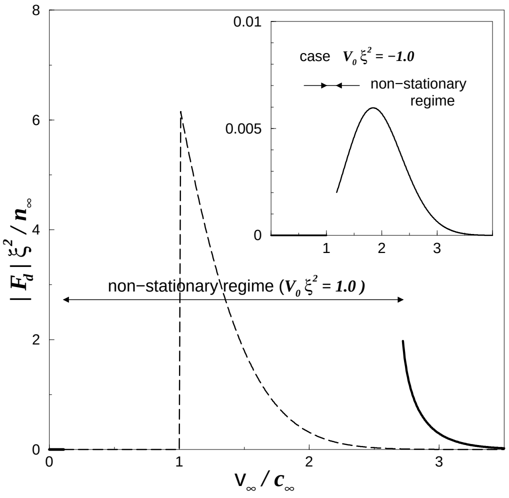
The perturbative regime is valid far from the velocity of sound . Besides, when $V_{\text{beam}} \gg V_{\text{sound}}$ the flow is quasi-superfluid (dissipation goes to zero).

- non-linear effects :

(1) $V_{\text{crit}} < V_{\text{Landau}}$ for *repulsive* potentials

BUT Landau critical velocity is always reached for *attractive* potentials

(2) enormous difference in drag between attractive and repulsive potentials :



Drag exerted by a low density beam on a Gaussian potential ($V_0\,\xi^2 = \pm 1.0$, $\sigma/\xi = 1.0$), as a function of the beam velocity. The main figure displays the drag for a repulsive potential (solid line) together with the perturbative result (dashed line) which is not affected by the sign of the potential. The inset displays an enlargement of the main figure allowing to see the (very small) drag for an attractive potential.