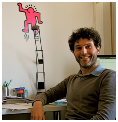
Analogous black-holes in Bose-Einstein condensates

How to create them and how to detect the associated (sonic) Hawking radiation ?

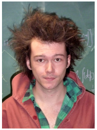
Nicolas Pavloff LPTMS, Université Paris-Saclay

Landau Institute, october 2016

I. Carusotto



A. Recati



P.-É. Larré

A. M. Kamchatnov

D. Boiron

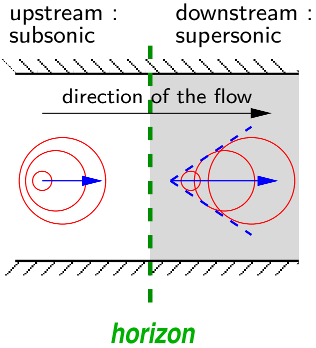
S. Fabbri

C. Westbrook

P. Zin

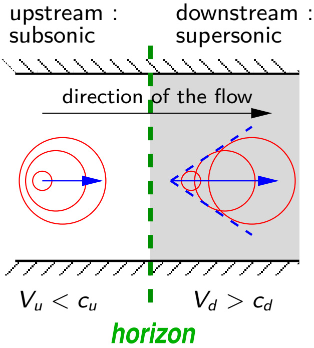
Analogous Hawking radiation

Unruh, Phys. Rev. Lett. (1981)



Analogous Hawking radiation

Unruh, Phys. Rev. Lett. (1981)



upstream : subsonic

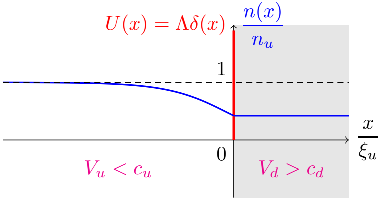
downstream : supersonic

direction of the flow

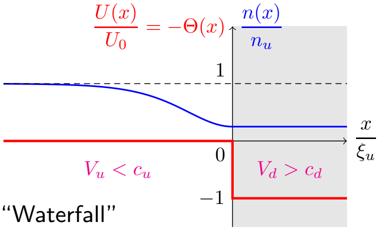
$V_u < c_u$

$V_d > c_d$

horizon



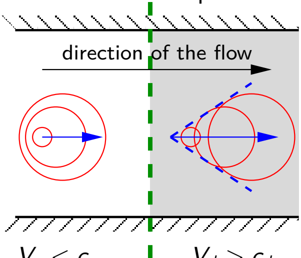
“δ-peak”



“Waterfall”

Analogous Hawking radiation

Unruh, Phys. Rev. Lett. (1981)



upstream :
subsonic

downstream :
supersonic

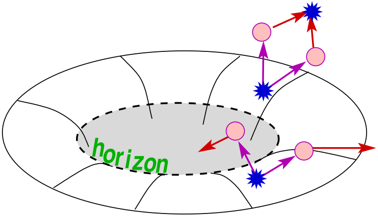
direction of the flow

$V_u < c_u$

$V_d > c_d$

horizon

gravitational black hole

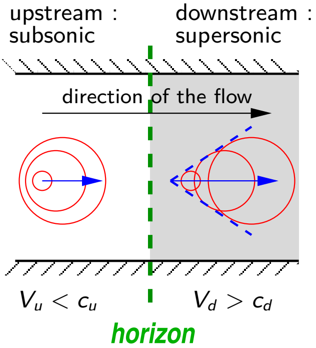


horizon

Hawking radiation 75'

Analogous Hawking radiation

Unruh, Phys. Rev. Lett. (1981)



upstream :
subsonic

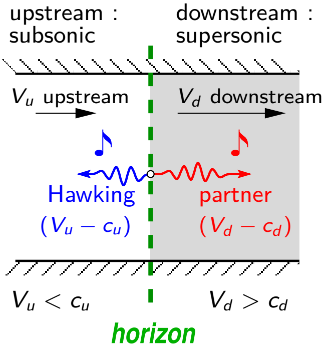
downstream :
supersonic

direction of the flow

$V_u < c_u$

$V_d > c_d$

horizon



upstream :
subsonic

downstream :
supersonic

V_u upstream

V_d downstream

Hawking
$(V_u - c_u)$

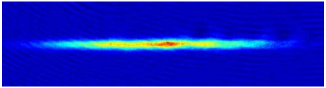
partner
$(V_d - c_d)$

$V_u < c_u$

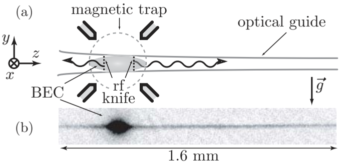
$V_d > c_d$

horizon

quasi-1D Bose-Einstein condensates

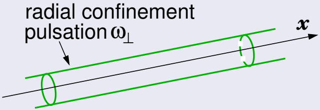


quasi-1D condensate
longitudinal size $\sim 10^2 \mu$m
transverse size $\sim 1\mu$m



Guerin *et al.*, Phys. Rev. Lett. (2006)

tight harmonic radial confinement :



$$V_\perp(\vec{r}_\perp) = \frac{1}{2} m \omega_\perp^2 r_\perp^2 \ .$$

→ **1D model** : $\Psi(x, t)$

1D mean field regime

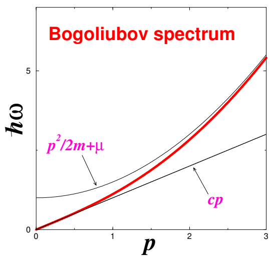
Menotti and Stringari, PRA (2002)

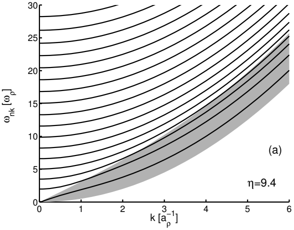
domain of validity

$$\frac{\hbar\,\omega_\perp}{\hbar^2/ma^2} \ll n_1 a \sim \frac{\mu}{\hbar\omega_\perp} \ll 1 \qquad \begin{cases} a &= 3D \text{ scattering length} \\ n_1 &= 1D \text{ linear density} \end{cases}$$

- The first inequality allows to avoid the Tonks-Girardeau regime and implies $E_{\rm int} \ll E_{\rm kin}$. Also $L_\phi \ggg \xi$ $\qquad L_\phi = \xi \exp\left[\pi\sqrt{\frac{\hbar\, n_1}{2m\, a\, \omega_\perp}}\right]$

- the second inequality allows to avoid the 3D-like transverse Thomas-Fermi regime and implies that transverse motion is "frozen" (more precisely: Born-Oppenheimer approximation)





C. Tozzo & F. Dalfovo, PRA (2002)

$\leftarrow \eta = \frac{\mu}{\hbar\omega_\perp} = 9.4$

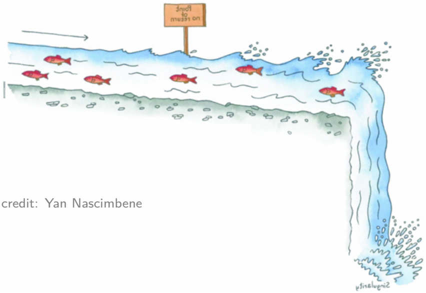
only

axi-symmetric

excitations

included $(m = 0)$

A possible dumb hole configuration



credit: Yan Nascimbene

stationary: $\Psi(x,t) = \psi(x)\exp\{-i\mu t\}$

classical mean field:

$$-\frac{1}{2}\psi_{xx} + \left(U(x) + g|\psi|^2\right)\psi = \mu\,\psi$$

where[a] $g = 2\omega_\perp a$

$n(x) = |\psi|^2$ $\qquad$ $n(x)v(x) = Im(\psi^* \cdot \psi_x)$

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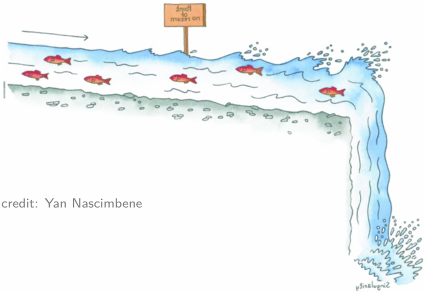
[a] Olshanii, PRL (1998)

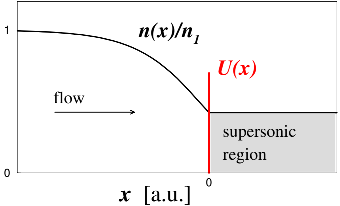
A possible dumb hole configuration

P. Lebœuf & N. Pavloff, Phys. Rev. A (2001)

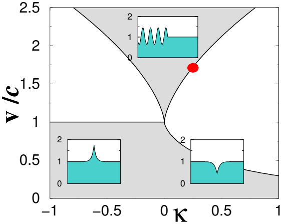


credit: Yan Nascimbene

$U(x) = \kappa\,\delta(x) \qquad (\kappa > 0)$



stationary: $\Psi(x,t) = \psi(x)\exp\{-i\mu t\}$



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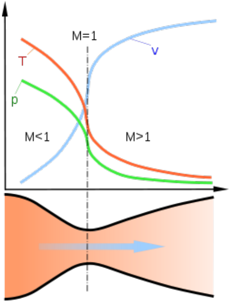
---

[a] Olshanii, PRL (1998)

BEC analogue of a Laval nozzle

A. M. Leszczyszyn *et al.*, Phys. Rev. A (2009)



For a **thick** barrier

$U(x)$ of width $\gg \xi \sim (gn)^{-1/2}$:

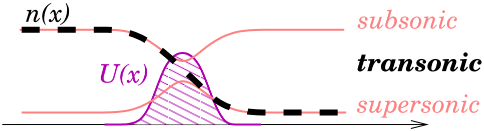
$$\begin{cases} -\frac{(n^{1/2})_{xx}}{2n^{1/2}} + \frac{1}{2} v^2(x) + g\, n(x) + U(x) = C^{st} \,, \\ n(x)v(x) = C^{st} \,. \end{cases}$$

$$\rightsquigarrow \frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}x}\left[v^2 - c^2\right] = \frac{\mathrm{d}U}{\mathrm{d}x} \qquad \text{where } c^2(x) = g\, n(x)$$

$$v(x) \lessgtr c(x) \;\leftrightarrow\; \text{sign}\left(\frac{\mathrm{d}n}{\mathrm{d}x}\right) = \mp\, \text{sign}\left(\frac{\mathrm{d}U}{\mathrm{d}x}\right)$$

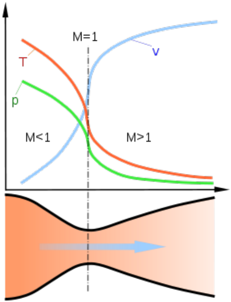
Nozzle of a V2 rocket

$$F = \dot{m}\left(v_{\rm out} - v_{\rm in}\right)$$



BEC analogue of a Laval nozzle

A. M. Leszczyszyn *et al.*, Phys. Rev. A (2009)



Nozzle of a V2 rocket

$$F = \dot{m}\,(v_{\rm out} - v_{\rm in})$$

For a **thick** barrier

$U(x)$ of width $\gg \xi \sim (gn)^{-1/2}$:

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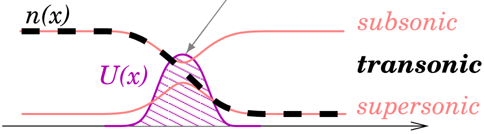
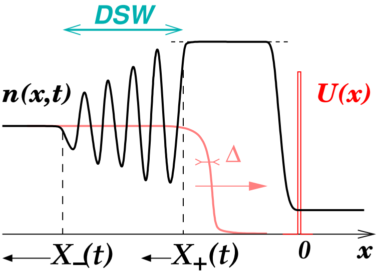
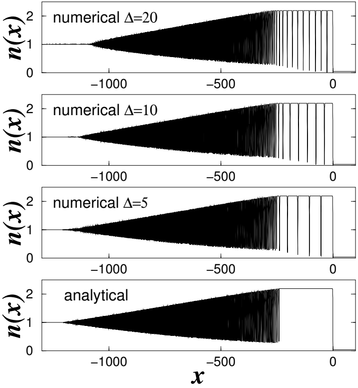
$$\text{transonic :} \left| \begin{array}{l} U_x(x_0) = 0 \\ v(x_0) = c(x_0) \end{array} \right.$$



How to form a sonic horizon ?

A. Kamchatnov & N. Pavloff, Phys. Rev. A (2012)





The profile in the region of the horizon only depends on v_u/c_u, but not of the initial profile. No hair theorem[a] ? not quite, see below.

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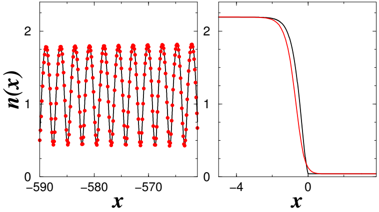
[a] F. Michel, R. Parentani, R. Zegers, Phys. Rev. D (2016)

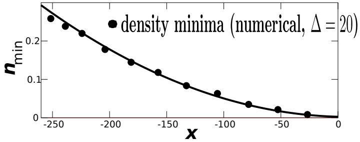
How to form a sonic horizon ?

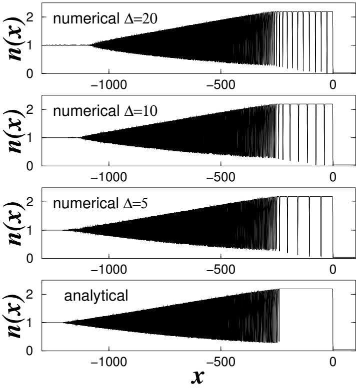
A. Kamchatnov & N. Pavloff, Phys. Rev. A (2012)



red=numerics

black=Whitham+transonic





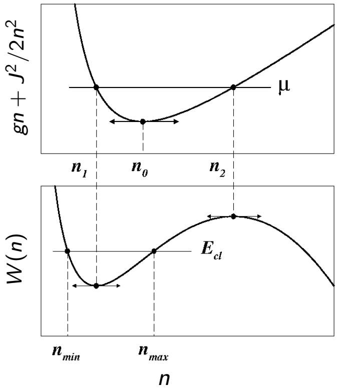
Soliton train in the upstream plateau... experimental problem ?

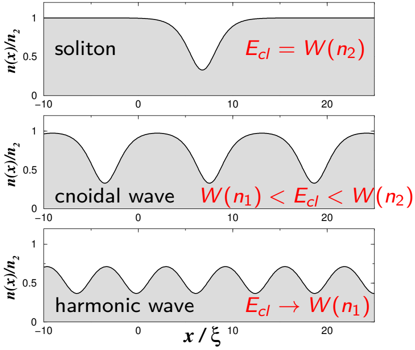
Stationary solutions of the NLS equation

$$-\tfrac{1}{2}A_{xx} + \left[g\,n + \frac{J^2}{2n^2} - \mu\right] A = 0\,, \quad \text{where} \quad J = n(x)v(x) \quad \text{and} \quad A = \sqrt{n}$$

first integral:

$$\tfrac{1}{2}A_x^2 + W(n) = E_{cl}\,, \quad \text{where} \quad W(n) = -\frac{g}{2}n^2 + \mu n + \frac{J^2}{2n}\,.$$



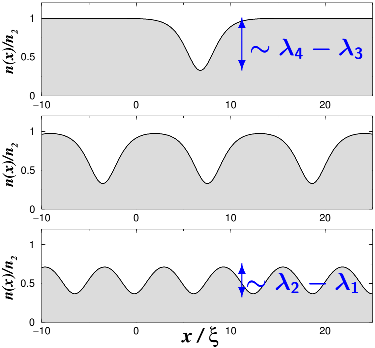


Stationary solutions of the NLS equation

$$n(x,t) = \tfrac{1}{4}(\lambda_4 - \lambda_3 - \lambda_2 + \lambda_1)^2 + (\lambda_4 - \lambda_3)(\lambda_2 - \lambda_1)\mathrm{sn}^2\left(k\left(x - Vt\right), m\right) \ ,$$

$$k = \sqrt{(\lambda_4 - \lambda_2)(\lambda_3 - \lambda_1)}, \quad V = \tfrac{1}{4}(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4), \quad m = m\{\lambda_i\} \in [0,1],$$

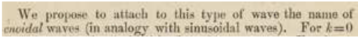
$$v(x,t) = V - \frac{C\{\lambda_i\}}{n(x,t)}.$$



soliton limit: $\lambda_2 \to \lambda_3$

cnoidal wave: $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$

KdV 1895



We propose to attach to this type of wave the name of *cnoidal* waves (in analogy with sinusoidal waves). For $k=0$

sinusoidal limit: $\lambda_2 \to \lambda_1$

Whitham modulation equations

Forest & Lee (1986), Pavlov (1987)

slow modulations $\lambda_i \to \lambda_i(x,t)$ with

$$\frac{\partial \lambda_i}{\partial t} + \mathscr{V}_i(\{\lambda_j\})\,\frac{\partial \lambda_i}{\partial x} = 0$$

$$\mathscr{V}_1(\{\lambda_j\}) = \tfrac{1}{2}\sum_{i=1}^{4}\lambda_i - \frac{(\lambda_4-\lambda_1)(\lambda_3-\lambda_1)K(m)}{(\lambda_4-\lambda_1)K(m)-(\lambda_3-\lambda_1)E(m)} \qquad m = \frac{(\lambda_2-\lambda_1)(\lambda_4-\lambda_3)}{(\lambda_4-\lambda_2)(\lambda_3-\lambda_1)}$$

Gurevich-Pitaevskii problem

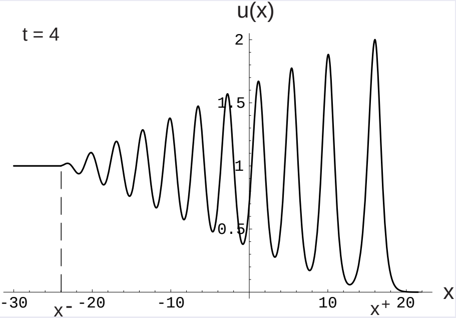
Gurevich & Pitaevskii (1973)

simple case: decay of a initial discontinuity → dispersive shock wave

no characteristic length : self-similar solution depending on $\zeta = x/t$ and matching to the right and left boundaries with a non dispersive flow.

$$u_t + uu_x + u_{xxx} = 0$$

$$u(x,t=0) = \begin{cases} 1 \text{ if } x < 0\ , \\ 0 \text{ if } x > 0\ . \end{cases}$$



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Forest & Lee (1986), Pavlov (1987)

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Gurevich-Pitaevskii problem

Gurevich & Pitaevskii (1973)

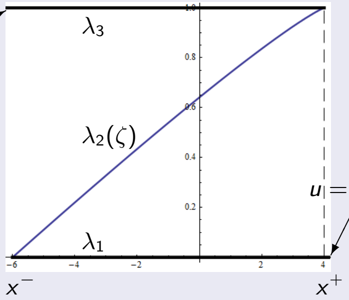
simple case: decay of a initial discontinuity → dispersive shock wave

no characteristic length : self-similar solution depending on $\zeta = x/t$ and matching to the right and left boundaries with a non dispersive flow.

$$(\mathscr{V}_i - \zeta)\,\frac{\mathrm{d}\lambda_i}{\mathrm{d}\zeta} = 0$$

$$u(x,t=0) = \begin{cases} 1 \text{ if } x < 0\,, \\ 0 \text{ if } x > 0\,. \end{cases}$$

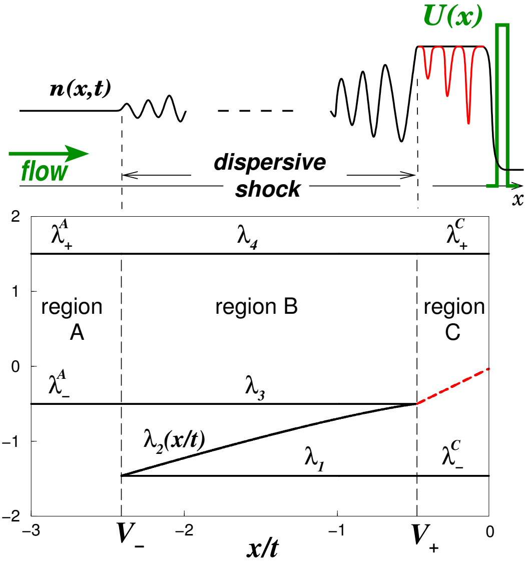
$u = \lambda_3 = 1$



$u = \lambda_1 = 0$

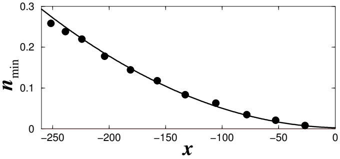
Dispersive shock + soliton train

A. Kamchatnov & N. Pavloff, Phys. Rev. A (2012)



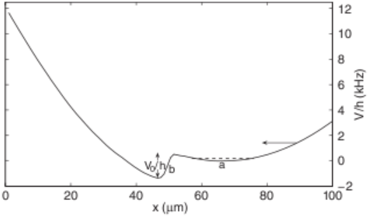
soliton train:

- ★ λ_1, λ_4: fixed
  $\lambda_2(\frac{x}{t}) = \lambda_3(\frac{x}{t})$
- ★ $\frac{x}{t} = \frac{1}{2}\sum_{i=1}^{4} \lambda_i$
  yields → $\lambda_{2,3}\left(\frac{x}{t}\right)$
- ★ $n_{\min} = f(\lambda_i\,'\mathrm{s})$
  $= f(\frac{x}{t})$



Technion experiment

Lahav *et al.* PRL (2010)



The arrow indicates the direction of the harmonic potential relative to the stationary step like potential ($v \sim 0.3$ mm/s).

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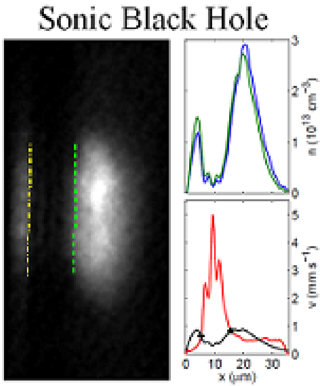
left plot:

$$v(x) = -\frac{1}{n}\int^x n_t \, \mathrm{d}x' \ ,$$

$$c(x) = \sqrt{g\, n(x)} \ .$$



green and **blue** curves: density

red curve: $-v(x)$

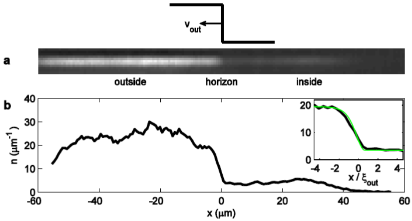
black curve: $c(x)$

velocity

green dashed line: black hole horizon

yellow dash-dot: white hole horizon

Steinhauer, Nature Physics 2016 :

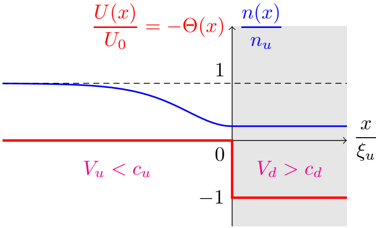


profile near the horizon $\simeq$ waterfall

$n_u/n_d = 5.55$ 5.55 $\quad c_u/c_d = 2.4$ 2.36

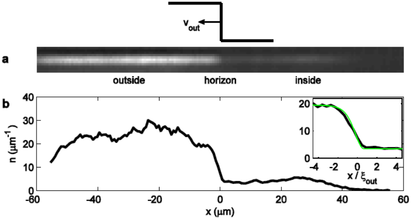
$V_u/c_u = 0.375$ 0.4245 $\quad V_d/c_d = 3.25$ 5.55

theoretical “waterfall”



$$\frac{V_d}{V_u} = \frac{n_u}{n_d} = \left(\frac{c_u}{V_u}\right)^2 = \frac{V_d}{c_d} = \left(\frac{c_u}{c_d}\right)^2$$

Steinhauer, Nature Physics 2016 :

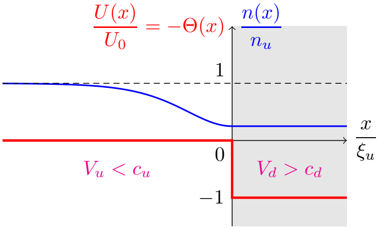


profile near the horizon $\simeq$ waterfall

$n_u/n_d = 5.55$ 5.55 $c_u/c_d = 2.4$ 2.36

$V_u/c_u = 0.375$ 0.4245 $V_d/c_d = 3.25$ 5.55

theoretical “waterfall”

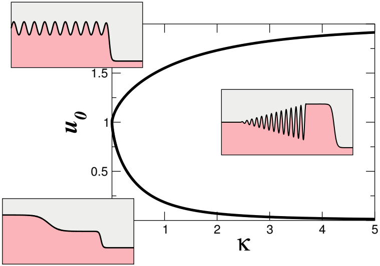


$$\frac{V_d}{V_u} = \frac{n_u}{n_d} = \left(\frac{c_u}{V_u}\right)^2 = \frac{V_d}{c_d} = \left(\frac{c_u}{c_d}\right)^2$$

Beware of fluctuations !

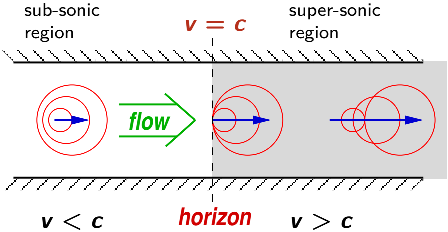
cf. case of δ-peak configuration:

$U(x) = \kappa\,\delta(x)$



Sonic black holes : "dumb holes"

Unruh (1981)



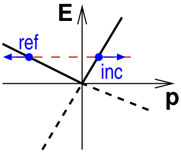
stimulated Hawking radiation:

in the laboratory :

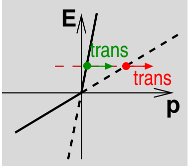
$$E(p) = c\,|p| + v\,p$$

$\swarrow$ $\downarrow$

comoving Doppler



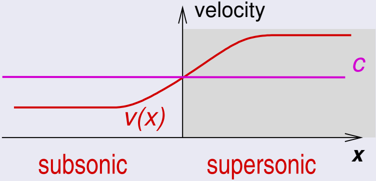
subsonic region

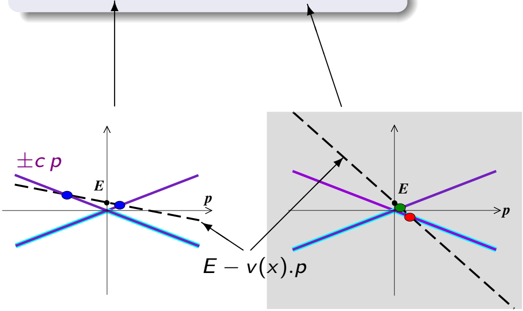


supersonic region

the position of the horizon is energy-dependent

model configuration :





$v(x) < c$

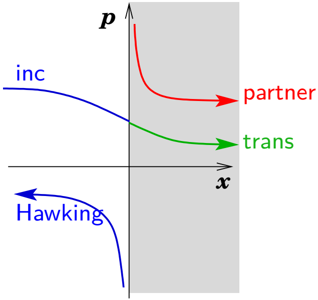
region $x < 0$

$v(x) > c$

region $x > 0$

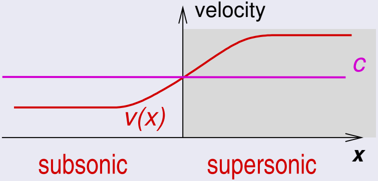
$$E - v(x).p = \pm\, c\, p$$

phase space :



the position of the horizon is energy-dependent

model configuration :

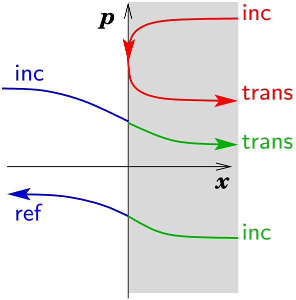


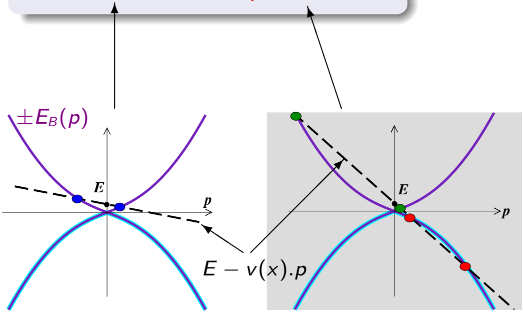
$$E - v(x).p = \pm E_B(p)$$

with

$$E_B(p) = c\,p\,\sqrt{1+\xi^2 p^2/4}$$

phase space :





$v(x) < c$

region $x < 0$

$v(x) > c$

region $x > 0$

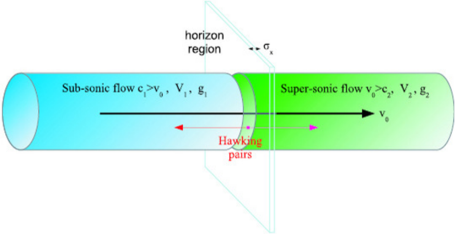
Numerical test, model configuration:

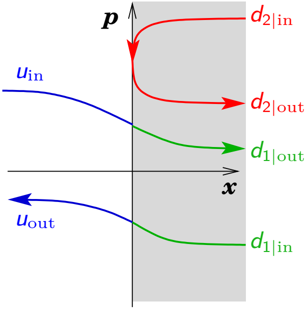
Recati, Pavloff & Carusotto, Phys. Rev. A (2009)

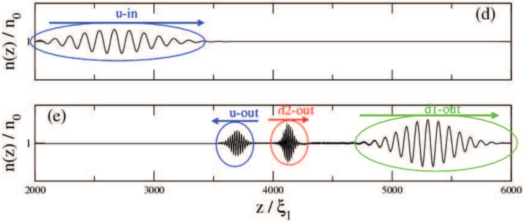
$U(x)$ and $g(x)$ **step like** with
$U(x) + g(x) n_0 = C^{\rm st}$ such that
$\psi_0(x) = \sqrt{n_0}\, \exp\{i k_0\, x\}$, verifies $\forall x$

$$-\tfrac{1}{2}\psi_0'' + \left[U(x) + g(x)|\psi_0|^2\right] \psi_0 = \mu\, \psi_0 \ ,$$

$$C^{\rm st} = \mu - \frac{k_0^2}{2}\ .$$







Partial conclusion :

Up to now, only classical description (= wave mechanics).

Realistic “dumb hole” configuration: i.e., formation of a sonic event horizon

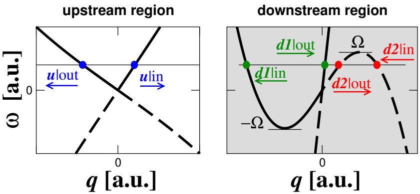
“δ-peak “: dynamical process well described by Whitham approach

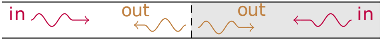
“waterfall” configuration: co-validated by experiment

“stimulated Hawking radiation”: quantum reflection + mode conversion

One-body Hawking signal at equilibrium

$$\omega - Vq = \pm E_B(q)$$





linear relation connecting the out-going modes to the in-going ones

$$\begin{pmatrix} u|\mathrm{out} \\ d_1|\mathrm{out} \\ (d_2|\mathrm{out})^\dagger \end{pmatrix} = \mathbf{S}(\omega) \begin{pmatrix} u|\mathrm{in} \\ d_1|\mathrm{in} \\ (d_2|\mathrm{in})^\dagger \end{pmatrix}$$

$$\langle u^\dagger_{\mathrm{out}}\, u_{\mathrm{out}} \rangle = |\mathbf{S}_{uu}|^2 \langle u^\dagger_{\mathrm{in}} u_{\mathrm{in}} \rangle + |\mathbf{S}_{ud_1}|^2 \langle d^\dagger_{1\mathrm{in}}\, d_{1\mathrm{in}} \rangle + |\mathbf{S}_{ud_2}|^2 \langle d_{2\mathrm{in}}\, d^\dagger_{2\mathrm{in}} \rangle$$

at $T = 0$: $\langle u^\dagger_{\mathrm{out}}\, u_{\mathrm{out}} \rangle = |\mathbf{S}_{ud_2}(\omega)|^2$ needs $\begin{cases} u \rightleftharpoons d_2 \text{ mode conversion} \\ d_2\text{-ingoing mode} \end{cases}$

Spontaneous Hawking radiation

Spectrum of Hawking radiation

$$\mathrm{i}\,\partial_t\hat{\Psi} = -\tfrac{1}{2}\partial_x^2\hat{\Psi} + \left[U(x) + g\,\hat{\Psi}^\dagger\hat{\Psi}\right]\hat{\Psi}$$

Energy current associated to emission of elementary excitations

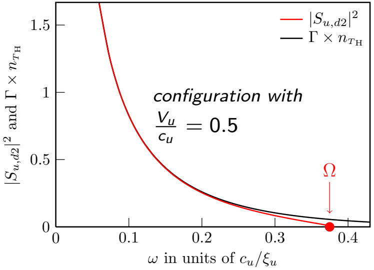
$$\hat{\Pi} = -\tfrac{1}{2}\partial_t\hat{\Psi}^\dagger\partial_x\hat{\Psi} + \mathrm{h.c.}$$

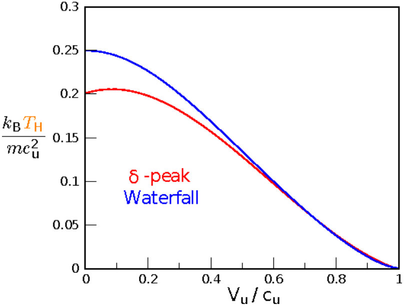
at $T = 0$, $\langle\hat{\Pi}\rangle = -\int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\,\hbar\omega\,|S_{u,d2}|^2$

Hawking radiation in the $u|\mathrm{out}$ channel. Equivalent to a black body radiation of temperature $T_H \sim 10\%\ \mu$

$$|S_{u,d2}|^2 \overset{\omega\to 0}{\simeq} \frac{\Gamma}{\exp\{\hbar\omega/k_B T_H\} - 1}$$

$$\to \frac{k_B T_H}{g n_u} \sim 0.1 \to T_H \sim 5 - 10\ \mathrm{nK}$$

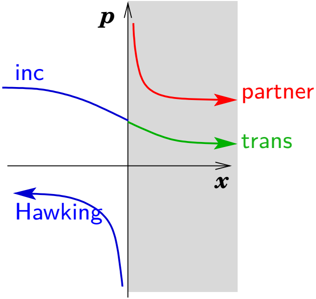




Hand waving mathematics

long wave length limit : $E - v(x) \cdot p = \pm c(x) \cdot p \quad \rightarrow \quad p = \frac{E}{v(x) \pm c(x)}$

phase space :



Tunnel probability

$P \propto e^{-2S/\hbar}$ where $S = \left| Im \int p(x)\, dx \right|$

near the horizon

$$\frac{E}{v(x)-c(x)} \simeq \frac{E}{(x \pm i\epsilon)\, \frac{d}{dx}(v-c)\big|_0} \rightarrow \pm i\delta(x) \frac{E}{\frac{d}{dx}(v-c)\big|_0}$$

$$S \simeq \frac{\pi\, E}{\frac{d}{dx}(v-c)\big|_0}$$

Hawking temperature

$P \propto e^{-E/(k_B T_H)}$ with $k_B T_H = \frac{\hbar}{2\pi} \left| \frac{d}{dx}(v-c) \right|_0$

Example of the waterfall configuration

linear relation connecting the out-going modes to the in-going ones

$$\begin{pmatrix} u|\mathrm{out} \\ d_1|\mathrm{out} \\ (d_2|\mathrm{out})^\dagger \end{pmatrix} = \mathbf{S}(\omega) \begin{pmatrix} u|\mathrm{in} \\ d_1|\mathrm{in} \\ (d_2|\mathrm{in})^\dagger \end{pmatrix}$$

at $T = 0$ outgoing energy current

$$\langle \hat{\Pi} \rangle = - \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\, \hbar\omega\, |S_{u,d2}|^2$$

$$S_{u,d2} = \frac{f_{u,d2}}{\sqrt{\omega/gn_u}} + h_{u,d2}\sqrt{\omega/gn_u} + \mathcal{O}(\omega^{3/2})$$

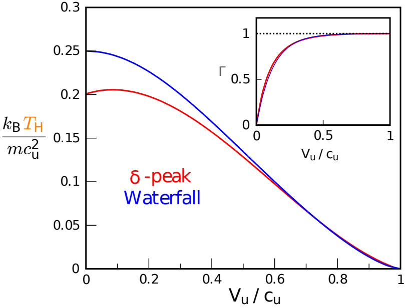
$$|S_{u,d2}|^2 = \Gamma(\omega) n_{therm}(\omega)$$

$$= (\Gamma + \mathcal{O}(\omega^2))(\frac{k_B T_H}{\omega} + \frac{1}{2} + \mathcal{O}(\omega))$$

in the waterfall configuration

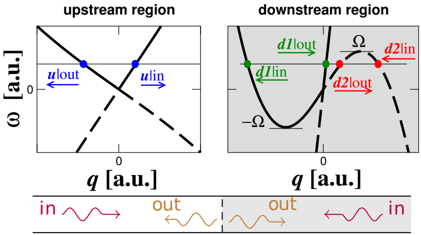
$$\frac{k_B T_H}{gn_u} = \frac{1}{2} \frac{(1 - M_u^4)^{3/2}}{(2 + M_u^2)(1 + 2M_u^2)}$$

where $M_u = V_u/c_u$



Density correlations

Balbinot, Carusotto, Fabbri, Fagnocchi, Recati, Phys. Rev. A & New J. Phys. (2008)

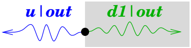


New theoretical and experimental interest:

study of density correlation on each side of the horizon

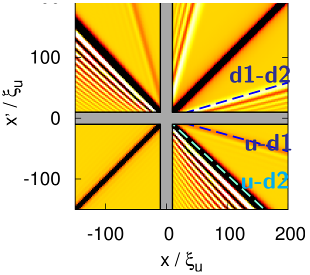
$$g^{(2)}(x,x') = \frac{\langle : n(x) n(x') : \rangle}{\langle n(x') \rangle \langle n(x) \rangle} - 1$$

★ example of induced correlation:



$x = (v_d + c_d)t$ correlates with
$x' = (v_u - c_u)t$

★ affects the density correlation pattern

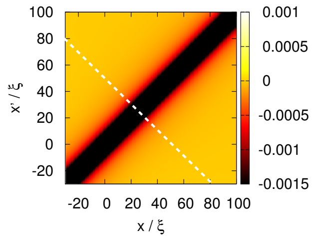


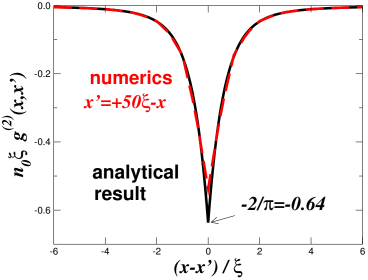
Larré *et al.*, Phys. Rev. A (2012)

Uniform 1D Condensate (no black hole)

$$g^{(2)}(x,x') = \frac{1}{n_0\,\xi}\,F\left(\frac{|x-x'|}{\xi}\right) \qquad \text{where} \qquad \begin{cases} F(X) = -\dfrac{1}{\pi\,X}\displaystyle\int_0^\infty \mathrm{d}t\,\frac{\sin(2tX)}{(1+t^2)^{3/2}} \\ F(0) = -2/\pi\;. \end{cases}$$

$$\left(\mu = g\,n_0 = \frac{1}{\xi^2}\right)$$



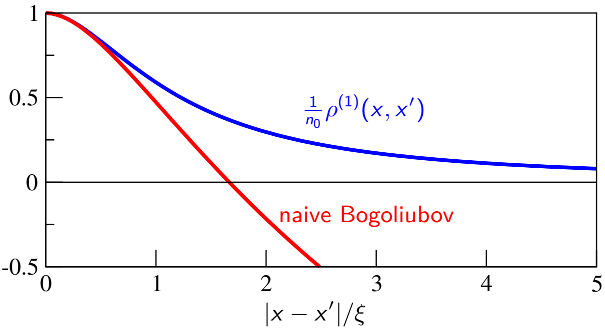


Uniform 1D Condensate

Popov result ($T = 0$)

$$\frac{1}{n_0}\rho^{(1)}(x,x') = exp\left\{-\frac{F\left(|x-x'|/\xi\right)}{2\pi n_0 \xi}\right\} \qquad \text{where}$$

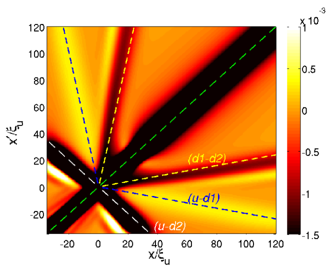
$$F(X) = \int_0^\infty \mathrm{d}t \left(\frac{t^2+2}{t\sqrt{t^2+4}} - 1\right)\Big(1-\cos(X\,t)\Big)$$

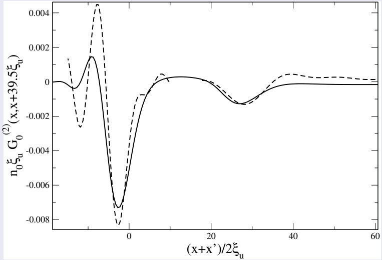


Two-body Hawking signal

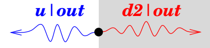
Recati, N. Pavloff & I. Carusotto, Phys. Rev. A (2009)

Comparison of numerical and analytic results (model configuration)





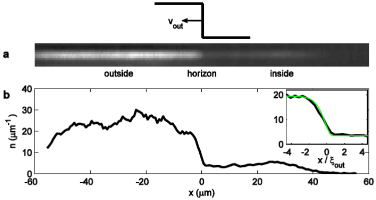
main correlation signal :



$x = V_{\mathrm{d2|out}}\ t$ correlates with $x' = V_{\mathrm{u|out}}\ t$

where $V_{\mathrm{d2|out}} = V_d - c_d$ and $V_{\mathrm{u|out}} = V_u - c_u$

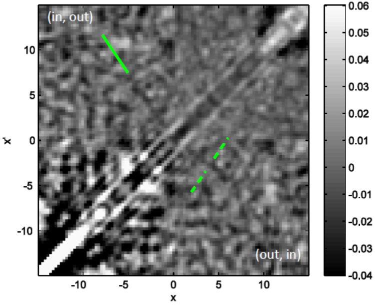
Steinhauer, Nature Physics 2016 :

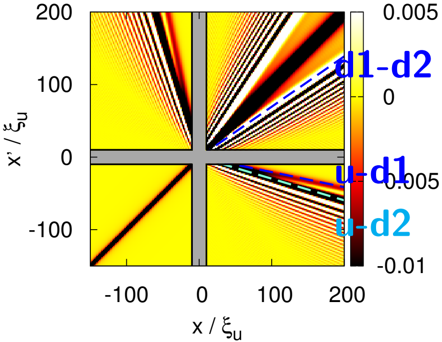


density profile near the horizon $\simeq$
waterfall $n_u/n_d = 5.55$ 5.55
$c_u/c_d = 2.4$ 2.36
$V_u/c_u = 0.375$ 0.4245 $V_d/c_d = 3.25$ 5.55

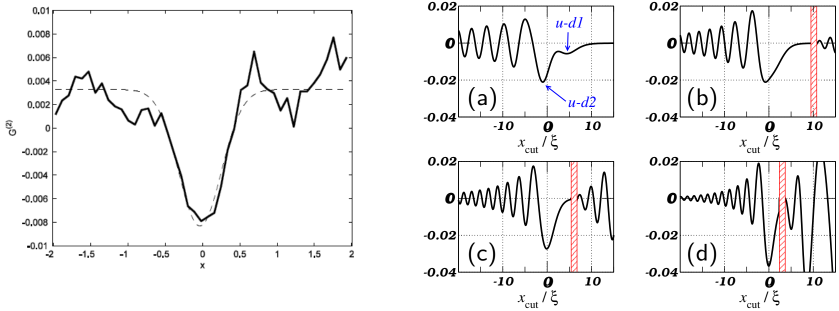
$T_H = 1.0$ nK

$T_H/(gn_u) = 0.36$?
$T_H/(gn_u)|_{theo} \leq 0.25$





correlation profile

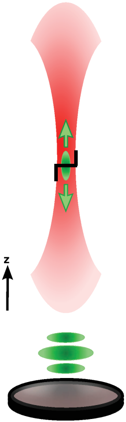


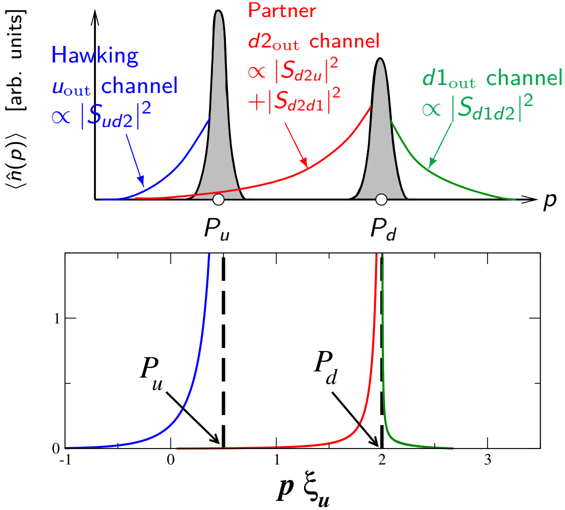
correlations $g_2(x, x')$ along a cut $x + x' = C^{\rm st}$. For all the plots: the abscissa is the coordinate $x_{\rm cut}$ along the cut in unit of $\xi = \sqrt{\xi_u \xi_d}$ and the ordinate is $\sqrt{n_u n_d \xi_u \xi_d}\, g_2(x, x')$. Left plot: experimental results of Steinhauer. Right plots: theoretical results in the waterfall configuration with $V_u/c_u = 0.4245$ along different cuts. Figs. (a), (b), (c) and (d): cases when the cut $x + x' = C^{\rm st}$ intercepts the $u - d2$ correlation lines at $x/\xi_u = 100$, 50, 30 and 15. The (red) shaded zone is the forbidden zone around $x' = 0$.

One body momentum distribution in the presence of a horizon

$T = 0$, adiabatic opening of the trap

Boiron *et al.* PRL (2015)



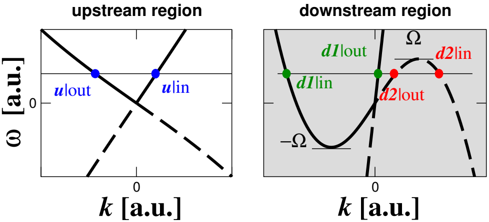


Two body momentum distribution in the presence of a horizon

p, q : absolute momenta in units of ξ_u^{-1}

right plot: $g_2(p,q) \rightarrow$

where $g_2(p,q) = \dfrac{\langle :\hat{n}(p)\hat{n}(q): \rangle}{\langle \hat{n}(p) \rangle \langle \hat{n}(q) \rangle}$

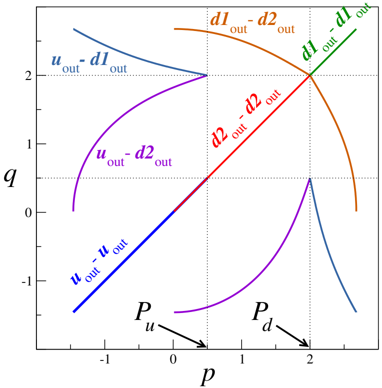


k : momentum relative to the condensate

$p = k + P_{(u/d)}$ where $P_{(u/d)} = mV_{(u/d)}$

$T = 0$ adiabatic opening

Boiron *et al.* PRL (2015)



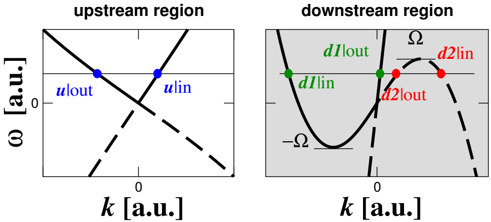
without horizon: $g_2 \equiv 1$

Two body momentum distribution in the presence of a horizon

p, q : absolute momenta in units of ξ_u^{-1}

right plot: $g_2(p,q) \rightarrow$

where $g_2(p,q) = \dfrac{\langle : \hat{n}(p)\hat{n}(q) : \rangle}{\langle \hat{n}(p)\rangle\langle \hat{n}(q)\rangle}$



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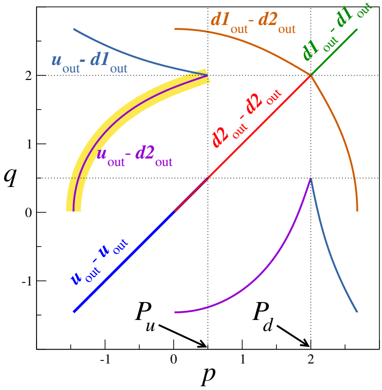
Boiron *et al.* PRL (2015)

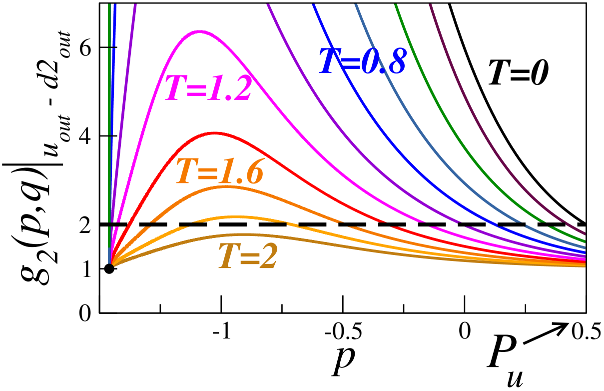


without horizon: $g_2 \equiv 1$

Violation of Cauchy-Schwarz inequality ($T \neq 0$)

$$\text{C.-S. violation}: \ g_2(p,q)\Big|_{u_{\rm out}-d2_{\rm out}} > \sqrt{g_2(p,p)\Big|_{u_{\rm out}} \times g_2(q,q)\Big|_{d2_{\rm out}}} \equiv 2$$

Boiron *et al.* PRL (2015)



T in units of μ

$T_H = 0.13$

$V_u/c_u = 0.5$

$V_d/c_d = 4$

$V_d/V_u = 4$

$n_u/n_d = 4$

Quantum effects within 1D mean field ?

Popov, Teor. Mat. Fiz. (1971)

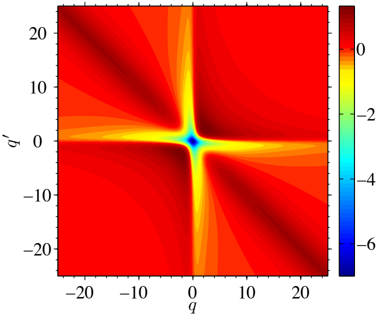
The NLS ↔ Gross-Pitaevskii eq. is a nonlinear **quantum field** eq. :

$$-\tfrac{1}{2}\partial_x^2\hat{\psi} + g\,\hat{\psi}^\dagger\hat{\psi}\,\hat{\psi} = i\,\partial_t\hat{\psi}\ , \quad \text{with} \quad \left[\hat{\psi}(x,t),\hat{\psi}^\dagger(y,t)\right] = \delta(x-y)\ .$$

BEC : macroscopic occupation of the lowest quantum state: $\hat{\psi}(x,t) = \underline{\psi_{(0)}(x,t)} + \underline{\hat{\phi}(x,t)}$ (Bogoliubov 1947)

$\psi_{(0)}$	: solution of the (classical) NLS
$\hat{\phi}$	: solution of a linearized (quantum) eq.

makes it possible to consider vacuum fluctuations. In particular : **Hawking radiation** in a stationary, non uniform setting.



Mathey, Vishwanath, Altman, PRA (2009)

Bouchoule, Arzamasovs, Kheruntsyan, Gangardt, PRA (2012)

Phonon evaporation and adiabatic trap opening

$$\begin{aligned}\hat{\phi}(x) &= e^{\mathrm{i}P_{(u/d)}x}\int_0^\infty \frac{\mathrm{d}\omega}{\sqrt{2\pi}}\sum_{L\in\{U,D1\}}\left[u_L(x,\omega)\hat{a}_L(\omega)+v_L^*(x,\omega)\hat{a}_L^\dagger(\omega)\right]\\ &+ e^{\mathrm{i}P_{(u/d)}x}\int_0^\Omega \frac{\mathrm{d}\omega}{\sqrt{2\pi}}\left[u_{D2}(x,\omega)\hat{a}_{D2}^\dagger(\omega)+v_{D2}^*(x,\omega)\hat{a}_{D2}(\omega)\right] .\end{aligned}$$

- If $x \ll -\xi_u$: $u_U(x) = \mathcal{U}_{u|\mathrm{in}}e^{\mathrm{i}q_{u|\mathrm{in}}x} + S_{uu}\,\mathcal{U}_{u|\mathrm{out}}e^{\mathrm{i}q_{u|\mathrm{out}}x}$,
- If $x \gg \quad \xi_d$: $u_U(x) = S_{d1,u}\,\mathcal{U}_{d1|\mathrm{out}}e^{\mathrm{i}q_{d1|\mathrm{out}}x} + S_{d2,u}\,\mathcal{U}_{d2|\mathrm{out}}e^{\mathrm{i}q_{d2|\mathrm{out}}x}$.

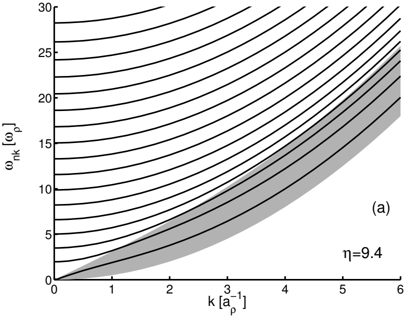
adiabatic opening of the trap: $\begin{pmatrix}\mathcal{U}(\omega)\\ \mathcal{V}(\omega)\end{pmatrix}_{\mathrm{mode}} \to \begin{pmatrix}1\\0\end{pmatrix}$ or $\begin{pmatrix}0\\1\end{pmatrix}$ for $d2|out$

see also: de Nova, Sols & Zapata PRA (2014)

- adiabaticity **always violated** for long wave-lengths, when $\omega \times t_{\mathrm{char}} \ll 1$

truly 1D ? No : transverse excitations

when $\hbar\boldsymbol{\omega}_\perp \leq \boldsymbol{\mu}$:



Zaremba, PRA (1998)
Stringari, PRA (1998)
Fedichev & Shlyapnikov, PRA (2001)
Tozzo & Dalfovo, PRA (2002)

modified dispersion relation :

$$\omega_0^2(q) = c_{1d}^2\, q^2 \left(1 - \tfrac{1}{48}(qR_\perp)^2 + \ldots\right)$$

new channels :

$$\omega_{n\geq 1}^2(q) = 2n(n+1)\,\omega_\perp^2 + \tfrac{1}{4}(qR_\perp\omega_\perp)^2 + \ldots$$

these new channels will be populated

at $T = 0$

mass term ≠ Klein-Gordon

→ new "in" modes

Conclusion

BECs offer interesting prospects to observe analogous Hawking radiation

[Steinhauer, Nature Physics]

general perspective : **quantum effects** with nonlinear **matter** waves

One- and two-body momentum distributions accessible by present day experimental techniques provide clear direct evidences

⇨ of the occurrence of a sonic horizon.

➥ of the associated acoustic Hawking radiation.

☞ of the quantum nature of the Hawking process.

☺ The signature of the quantum behavior persists even at temperatures larger than the chemical potential.

Cauchy-Schwarz : a wrong theorem ?... cf. sub-Poissonian fluctuations

$\langle ... \rangle \overset{def}{=} \mathrm{Tr}(\rho ...)$

$$\begin{aligned} \langle \hat{n}^2 \rangle &= \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle \\ &= \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle + \langle \hat{a}^\dagger 1\, \hat{a} \rangle \\ &= \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle + \langle \hat{n} \rangle \end{aligned}$$

$$\begin{aligned} 0 \leq \langle \delta n^2 \rangle &\overset{def}{=} \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 \\ &= \underbrace{\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle - \langle \hat{n} \rangle^2}_{sign?} + \langle \hat{n} \rangle \end{aligned}$$

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Cauchy-Schwarz: $|\langle \hat{\boldsymbol{A}} \rangle|^2 \leq \langle \hat{\boldsymbol{A}}^\dagger \hat{\boldsymbol{A}} \rangle$

Hence	$|\langle \hat{a}\hat{a} \rangle|^2 \leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle$
But	$|\langle \hat{a}^\dagger \hat{a} \rangle|^2 \not\leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle$

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stupid theoretical example

average over a number state: $\rho \equiv |n\rangle\langle n|$

$$\begin{aligned} \langle \hat{a}\hat{a} \rangle^2 &= 0 \\ \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle &= n(n-1) \\ \langle \hat{a}^\dagger \hat{a} \rangle^2 &= n^2 \not\leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle \end{aligned}$$

a number state is clearly sub-Poissonian !

Cauchy-Schwarz : a wrong theorem ?... cf. sub-Poissonian fluctuations

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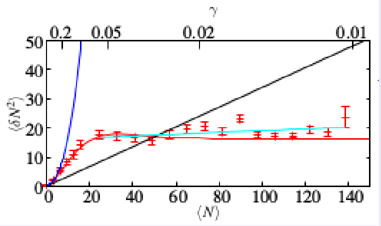
experimental results



— Poissonian limit : $\langle \delta N^2 \rangle = 0.34 \langle N \rangle$

— Ideal Bose gas

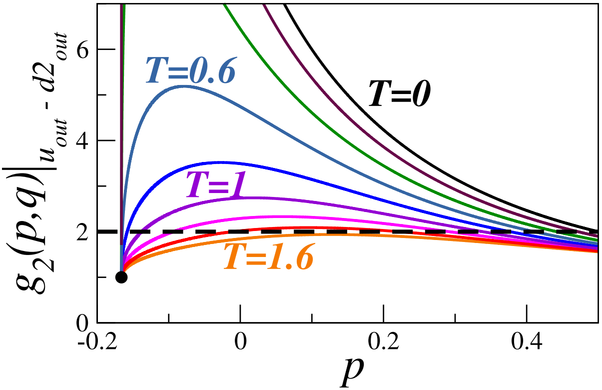
— Yang-Yang

— Quasi-cond.

Jacqmin *et al.*, *PRL (2011)*

Violation of Cauchy-Schwarz inequality (δ peak $T \neq 0$)

C.-S. violation : $g_2(p,q)\Big|_{u_{\rm out}-d2_{\rm out}} > \sqrt{g_2(p,p)\Big|_{u_{\rm out}} \times \, g_2(q,q)\Big|_{d2_{\rm out}}} \equiv 2$



T in units of μ

$T_H = 0.12$

$V_u/c_u = 0.5$

$V_d/c_d = 1.83$

$V_d/V_u = 2.34$

$n_u/n_d = 2.37$

Gravity wave analogues

Schützhold and Unruh, Phys. Rev. D (2002)

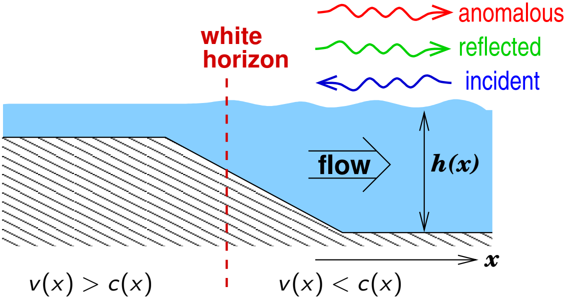
Rousseaux *et al.*, New Journal of Physics (2008)

Weinfurtner *et al.*, Phys. Rev. Lett. (2011)

Euvé *et al.*, Phys. Rev. D (2016)

in a basin of depth h, the dispersion relation of gravity waves is $(\omega - Vk)^2 = g\,k\,\tanh(k\,h)$, corresponding to $c = \sqrt{g\,h}$

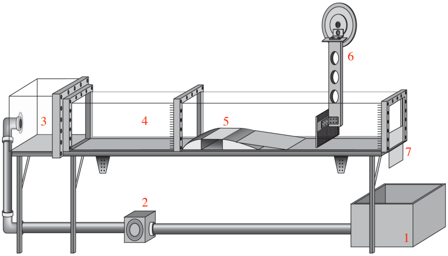
Experimental test of mode conversion :

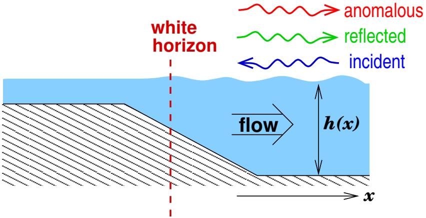


$v(x) > c(x)$ $v(x) < c(x)$

Vancouver experiment

Weinfurtner *et al.*, Phys. Rev. Lett. (2011)





white horizon

anomalous

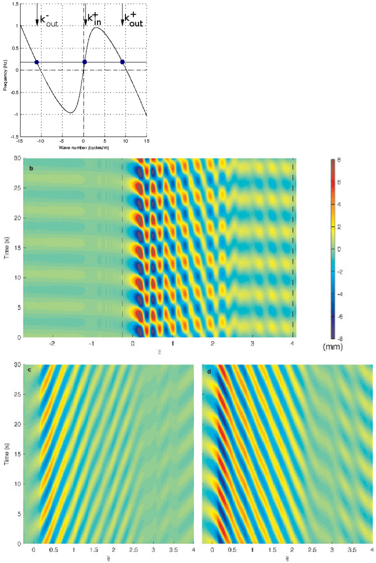
reflected

incident

flow

$h(x)$

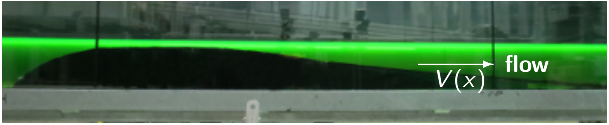
x



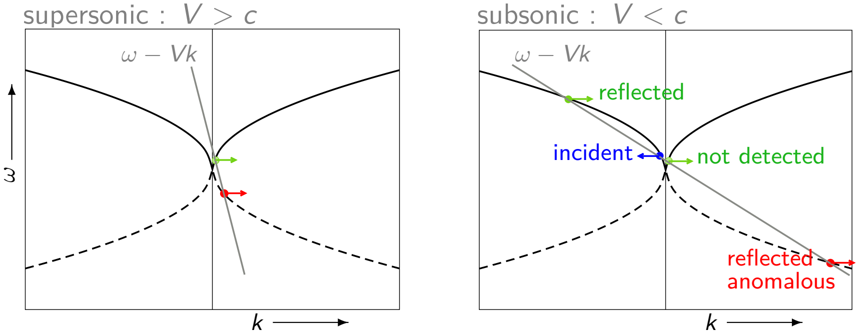
Gravity waves in a flume

Euvé *et al.*, Phys. Rev. D (2016)

$$\omega - Vk = \pm\sqrt{gk\tanh(hk)}$$



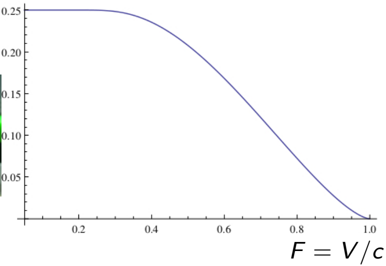
Poitiers experiment



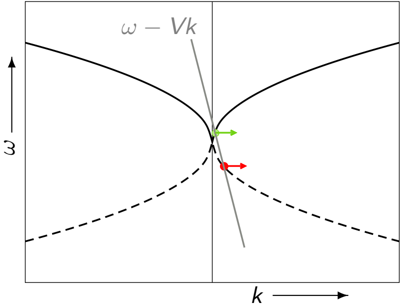
Gravity waves in a flume

$$\omega - Vk = \pm\sqrt{gk\tanh(hk)}$$

Poitiers experiment



supersonic : $V > c$



subsonic : $V < c$