Analogues acoustiques de l'horizon d'un trou noir

vers une mise en evidence du rayonnement de Hawking

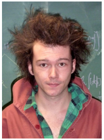
Nicolas Pavloff **LPTMS**, Université Paris-Saclay

"Échos de sciences : à l'écoute des ondes sonores", mars 2018

P. Lebœuf

I. Carusotto

A. Recati



P.-É. Larré

A. Kamchatnov

D. Boiron

S. Fabbri

C. Westbrook

P. Zin

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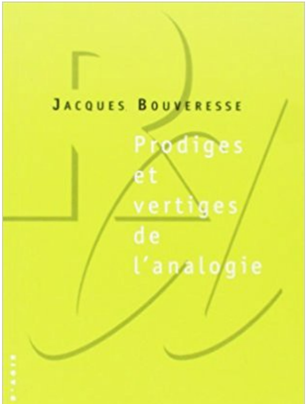
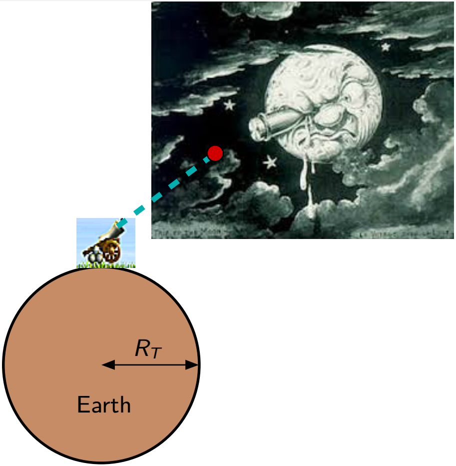


"Chez les déconstructionnistes, on fait de la théorie à peu près comme on fait de la poésie ou de la musique."

18^{th} century : naive Black Hole



1783: John Michell
1796: Laplace

$v_{escape} \propto (M_T/R_T)^{1/2} \propto \rho^{1/2} R_T$

Un astre lumineux, de la même densité que la Terre, et dont le diamètre serait 250 fois plus grand que le Soleil, ne permettrait, en vertu de son attraction, à aucun de ses rayons de parvenir jusqu'à nous. Il est dès lors possible que les plus grands corps lumineux de l'univers puissent, par cette cause, être invisibles

$v_{escape} = \sqrt{2\, g\, R_T} = 11.2$ km/s

where $g = \mathcal{G} M_T R_T^{-2}$

$250 \times 107 \times 11.2 = 299600$ km/s

1915 - Einstein's General relativity

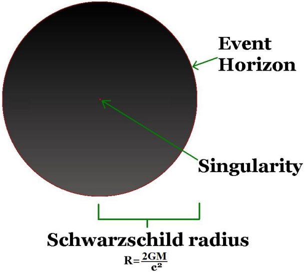
1915 - Schwarzschild solution of Einstein equations. Metric singular at $r = R_s = 2\mathcal{G}M/c^2$ ($R_s(earth) = R_T \times (v_{escape}/c)^2 = 9$ mm)

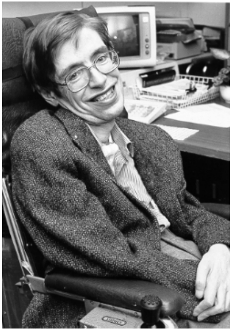
1931 - Chandrasekar: heavy white dwarfs are unstable

1939 - Oppenheimer and Snider: gravitationnal collapse
Oppenheimer and Volkoff : heavy neutron star → black hole

1958 - Finkelstein theorises that R_s is a causality barrier: an event horizon

1974: Hawking radiation





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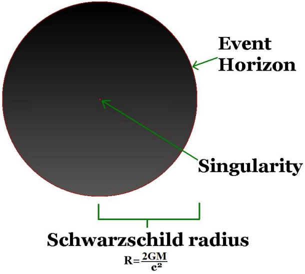

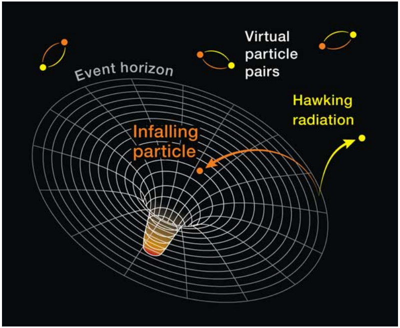

20^{th} century : General Relativity

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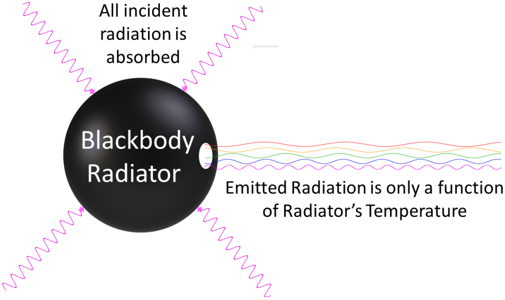

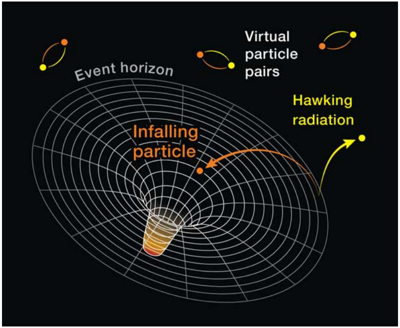

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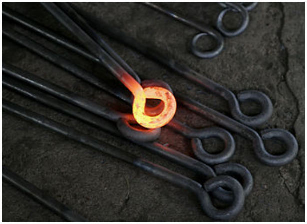
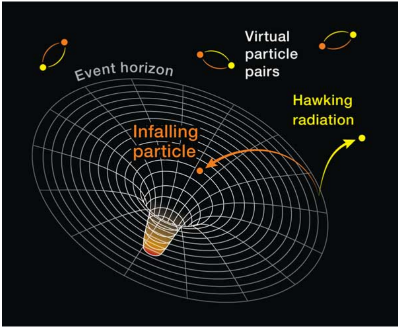
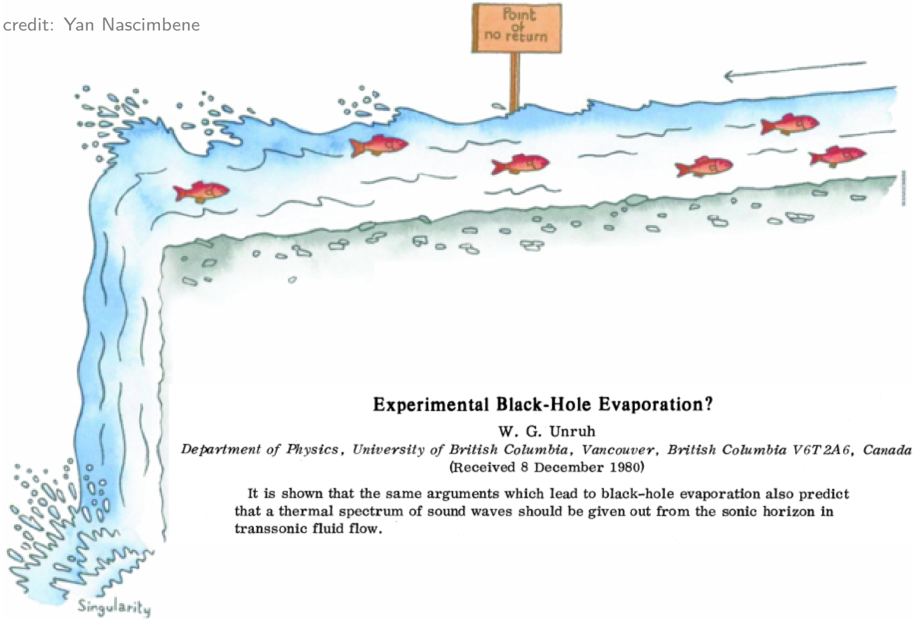
$$T_H = 10^{-7} M_\odot / M \text{ (K)}$$





credit: Yan Nascimbene



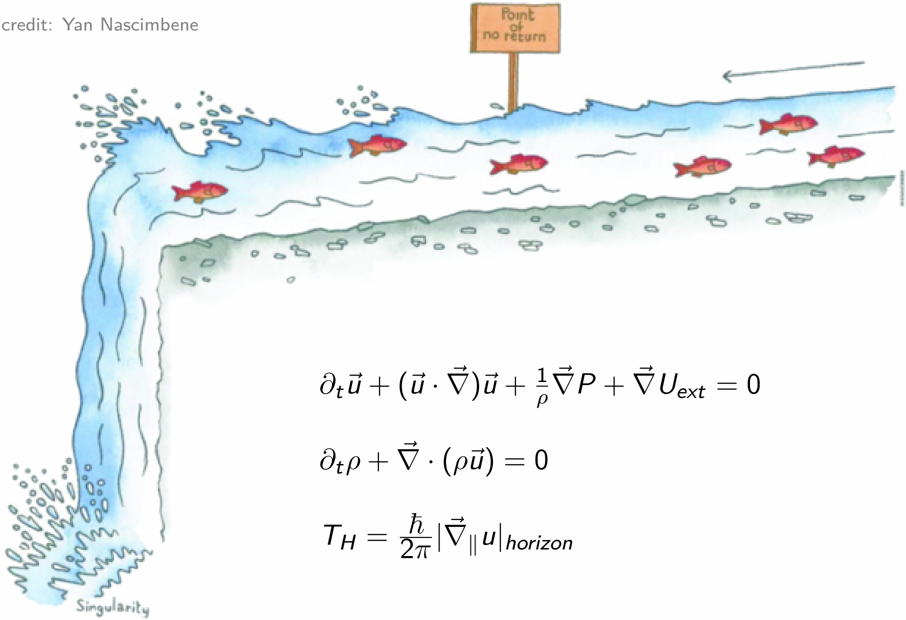
Experimental Black-Hole Evaporation?

W. G. Unruh
Department of Physics, University of British Columbia, Vancouver, British Columbia V6T 2A6, Canada
(Received 8 December 1980)

It is shown that the same arguments which lead to black-hole evaporation also predict that a thermal spectrum of sound waves should be given out from the sonic horizon in transsonic fluid flow.

Unruh, Phys. Rev. Lett. (1981)

credit: Yan Nascimbene

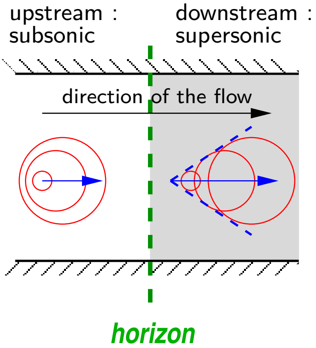


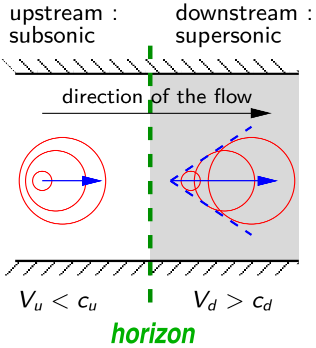
$$\partial_t \vec{u} + (\vec{u}\cdot\vec{\nabla})\vec{u} + \frac{1}{\rho}\vec{\nabla}P + \vec{\nabla}U_{ext} = 0$$

$$\partial_t \rho + \vec{\nabla}\cdot(\rho\vec{u}) = 0$$

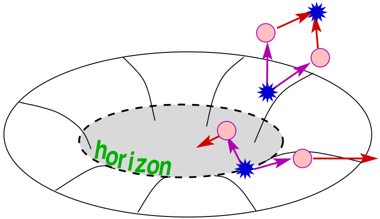
$$T_H = \frac{\hbar}{2\pi}|\vec{\nabla}_{\parallel} u|_{horizon}$$

Dumb hole


upstream :
subsonic
downstream :
supersonic
direction of the flow
horizon

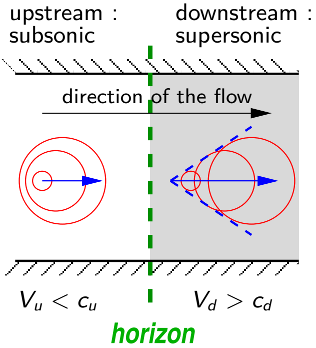


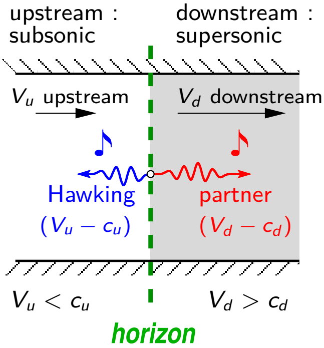
gravitational black hole



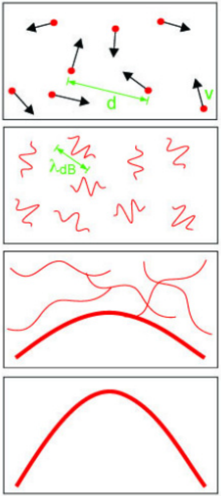
Hawking radiation 74'

Dumb hole


upstream : subsonic
downstream : supersonic
direction of the flow
$V_u < c_u$
$V_d > c_d$
horizon


upstream : subsonic
downstream : supersonic
V_u upstream
V_d downstream
Hawking
$(V_u - c_u)$
partner
$(V_d - c_d)$
$V_u < c_u$
$V_d > c_d$
horizon

Bose-Einstein condensation $1925 \to 1995$

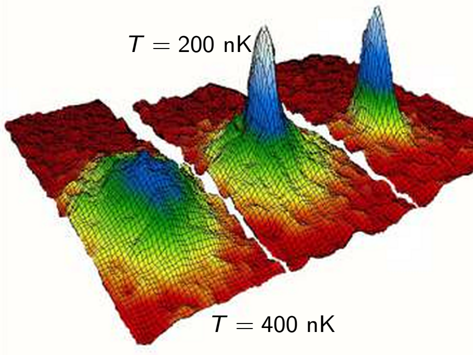


High Temperature T:
thermal velocity v
density d^{-3}
"Billiard balls"

Low Temperature T:
De Broglie wavelength
$\lambda_{dB} = h/mv \propto T^{-1/2}$
"Wave packets"

$T = T_{crit}$:
Bose-Einstein Condensation
$\lambda_{dB} \approx d$
"Matter wave overlap"

T=0:
Pure Bose condensate
"Giant matter wave"



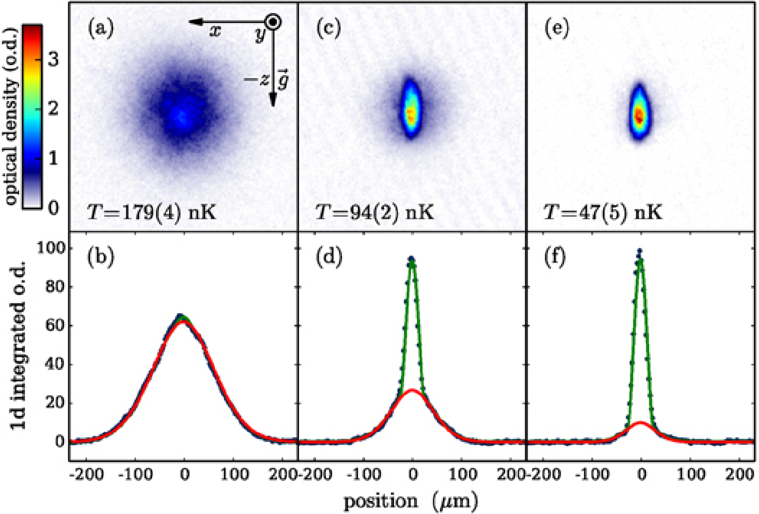
$T = 50$ nK

$T = 200$ nK

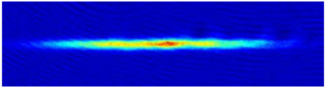
$T = 400$ nK

JILA group, ^{87}Rb, $N \simeq 10^5$ atoms

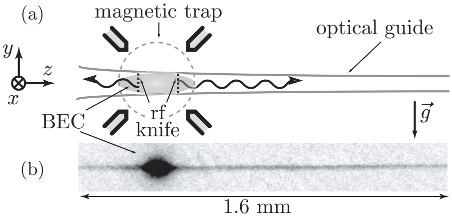
Cornell & Wiemann (JILA), Ketterle (MIT) nobel prize 2001



Y. Tang *et al.*, N. J. Phys. (2015) ^{162}Dy, $N \simeq 10^5$ atoms



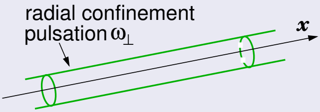
quasi-1D condensate
longitudinal size $\sim 10^2 \mu$m
transverse size $\sim 1\mu$m



Guerin *et al.*, Phys. Rev. Lett. (2006)

low T, quantum, low c, 1D

tight harmonic radial confinement :

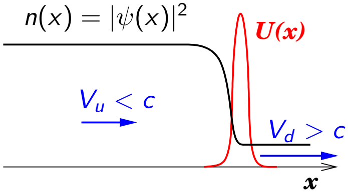


$$V_\perp(\vec{r}_\perp) = \frac{1}{2} m \omega_\perp^2 r_\perp^2 \ .$$

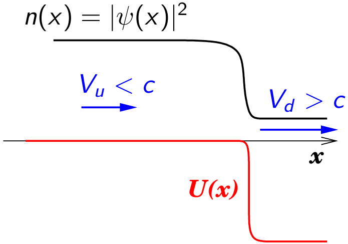
$\rightarrow$ **1D model** : $\Psi(x, t)$

stationnary Gross-Pitaevskii Eq.

$$-\frac{1}{2}\psi_{xx} + \left(U(x) + g|\psi|^2\right)\psi = \mu\,\psi$$



P. Leboeuf and N. Pavloff, Phys. Rev. A (2001)



How to form a sonic horizon ?

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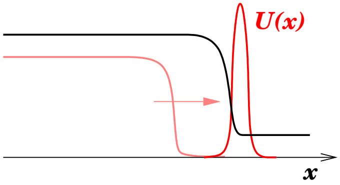

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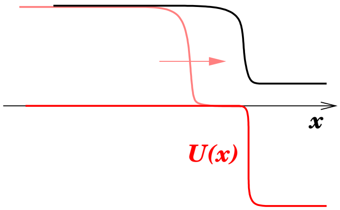

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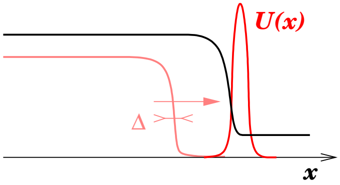
time-dependent Gross-Pitaevskii Eq.

$$-\frac{1}{2}\psi_{xx} + \left(U(x) + g|\psi|^2\right)\psi = i\psi_t$$



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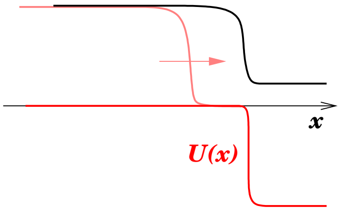

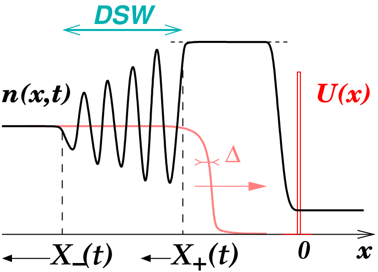

A. Kamchatnov & N. Pavloff, Phys. Rev. A (2012)

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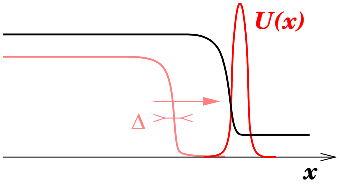
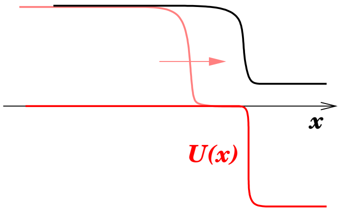
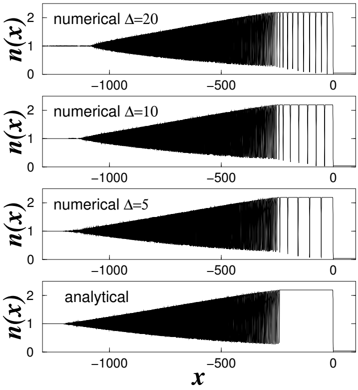
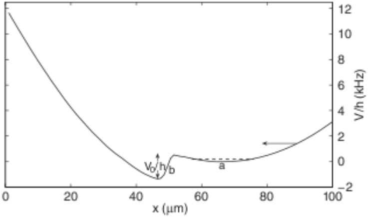


P. Leboeuf and N. Pavloff, Phys. Rev. A (2001)





A. Kamchatnov & N. Pavloff, Phys. Rev. A (2012)



The arrow indicates the direction of the harmonic potential relative to the stationary step like potential ($v \sim 0.3$ mm/s).

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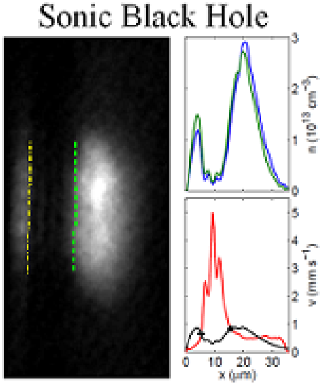
left plot:

$$v(x) = -\frac{1}{n} \int^{x} n_t \, \mathrm{d}x' \; ,$$

$$c(x) = \sqrt{g \, n(x)} \; .$$



green and **blue** curves: density

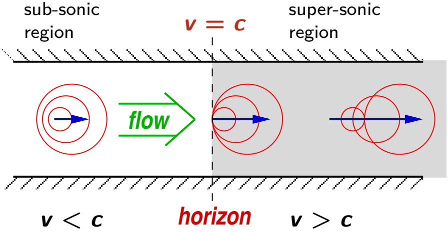
red curve: $-v(x)$

black curve: $c(x)$

velocity

green dashed line: black hole horizon

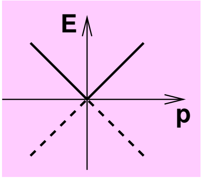
yellow dash-dot: white hole horizon

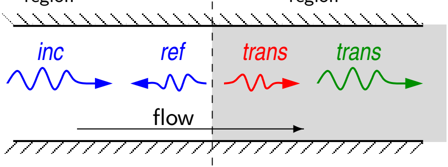


sound waves
in the comoving frame:

$E(p) = c\,|p|$

p: momentum in the
comoving frame

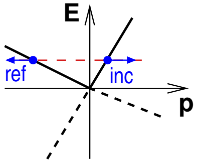




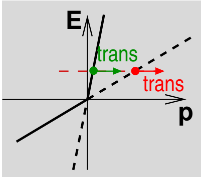
sound waves
in the lab frame:

$$E(p) = c\,|p| + v\,p$$

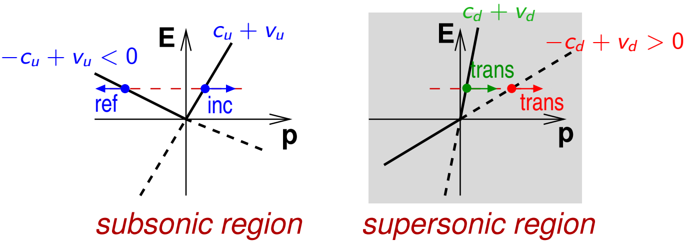
Doppler



subsonic region



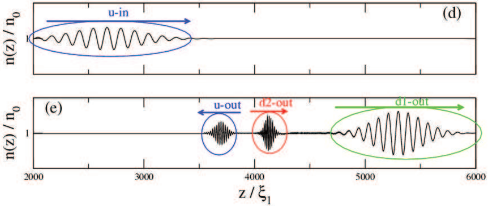
supersonic region



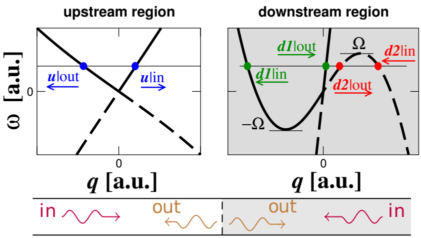
sound waves in the lab frame:

$$E(p) = c\,|p| + v\,p$$

Doppler



Recati, Pavloff, Carusotto, PRA (2009)



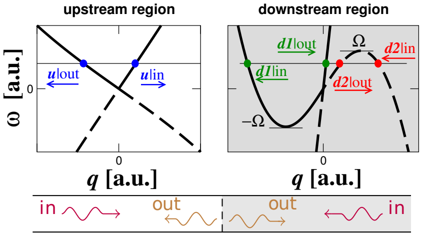
Hawking temperature

$$T_H \simeq 10\ nK \ll T_{exp} \quad !$$

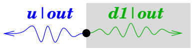
New theoretical and experimental interest:

study of density correlation on each side of the horizon

$$g^{(2)}(x, x') = \langle n(x) n(x') \rangle - \langle n(x') \rangle \langle n(x) \rangle$$



★ example of induced correlation:



$x = (v_d + c_d)t$ correlates with

$x' = (v_u - c_u)t$

★ affects the density correlation pattern

Hawking temperature

$$T_H \simeq 10\ nK \ll T_{exp} \ \ !$$

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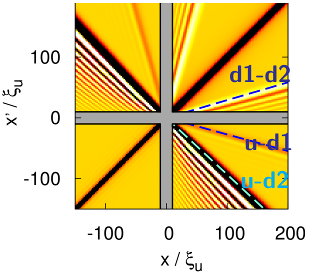
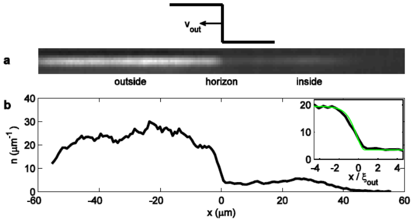
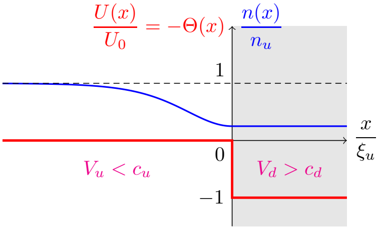


Larré, Recati, Carusotto, Pavloff, Phys. Rev. A (2012)

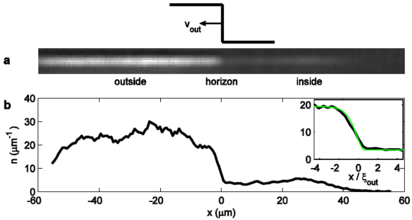


profile near the horizon $\simeq$ waterfall
$n_u/n_d = 5.55$ 5.55 $c_u/c_d = 2.4$ 2.36
$V_u/c_u = 0.375$ 0.4245 $V_d/c_d = 3.25$ 5.55

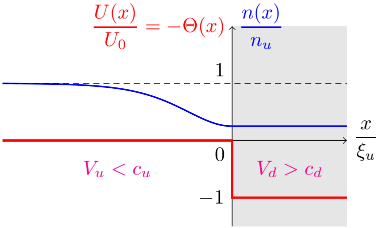
theoretical "waterfall"



$$\frac{V_d}{V_u} = \frac{n_u}{n_d} = \left(\frac{c_u}{V_u}\right)^2 = \frac{V_d}{c_d} = \left(\frac{c_u}{c_d}\right)^2$$



theoretical "waterfall"



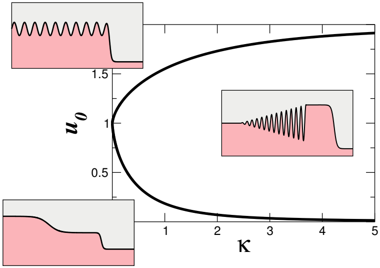
profile near the horizon ≃ waterfall
$n_u/n_d = 5.55$ 5.55 $c_u/c_d = 2.4$ 2.36
$V_u/c_u = 0.375$ 0.4245 $V_d/c_d = 3.25$ 5.55

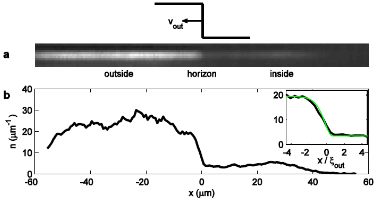
$$\frac{V_d}{V_u} = \frac{n_u}{n_d} = \left(\frac{c_u}{V_u}\right)^2 = \frac{V_d}{c_d} = \left(\frac{c_u}{c_d}\right)^2$$

Beware of fluctuations !

cf. case of δ-peak configuration:

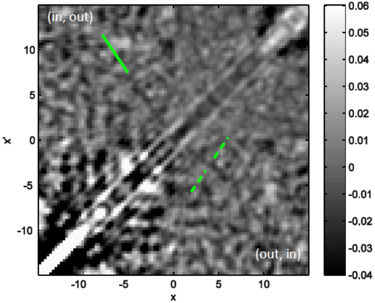
$U(x) = \kappa\,\delta(x)$

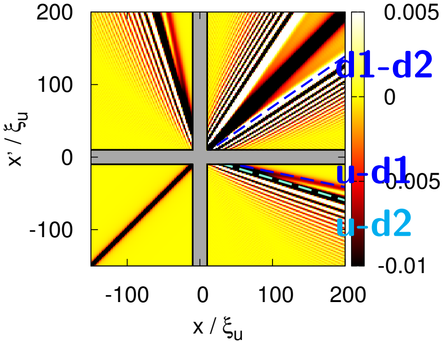




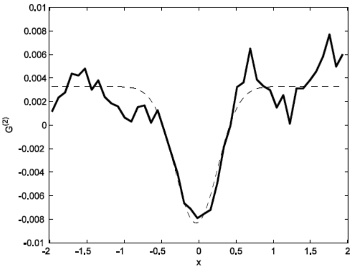
density profile near the horizon $\simeq$
waterfall $n_u/n_d = 5.55$ 5.55
$c_u/c_d = 2.4$ 2.36
$V_u/c_u = 0.375$ 0.4245 $V_d/c_d = 3.25$ 5.55

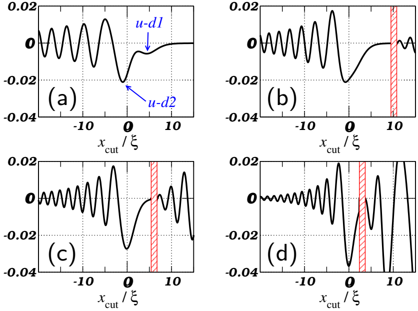
$T_H = 1.0$ nK | $T_H/(gn_u) = 0.36$?
$T_H/(gn_u)|_{theo} \lesssim 0.25$





correlation profile

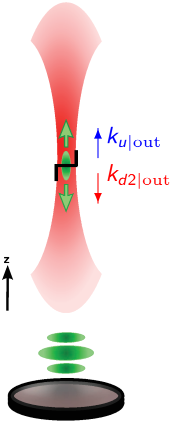


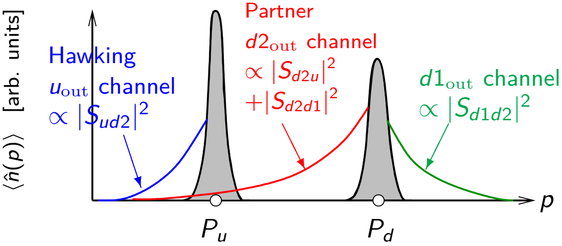


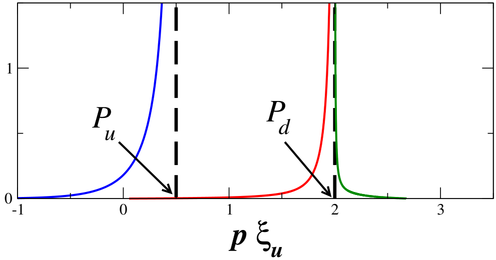
correlations $g_2(x, x')$ along a cut $x + x' = C^{\rm st}$. For all the plots: the abscissa is the coordinate $x_{\rm cut}$ along the cut in unit of $\xi = \sqrt{\xi_u \xi_d}$ and the ordinate is $\sqrt{n_u n_d \xi_u \xi_d}\, g_2(x, x')$. Left plot: experimental results of Steinhauer. Right plots: theoretical results in the waterfall configuration with $V_u/c_u = 0.4245$ along different cuts. Figs. (a), (b), (c) and (d): cases when the cut $x + x' = C^{\rm st}$ intercepts the $u - d2$ correlation lines at $x/\xi_u = 100$, 50, 30 and 15. The (red) shaded zone is the forbidden zone around $x' = 0$.

$T = 0$, adiabatic opening of the trap

Boiron *et al.* PRL (2015)



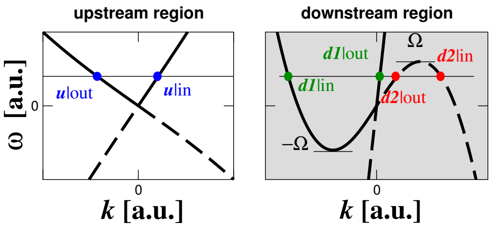




p, q : absolute momenta in units of ξ_u^{-1}

right plot: $g_2(p,q) \rightarrow$

where $g_2(p,q) = \frac{\langle : \hat{n}(p)\hat{n}(q) : \rangle}{\langle \hat{n}(p)\rangle\langle \hat{n}(q)\rangle}$

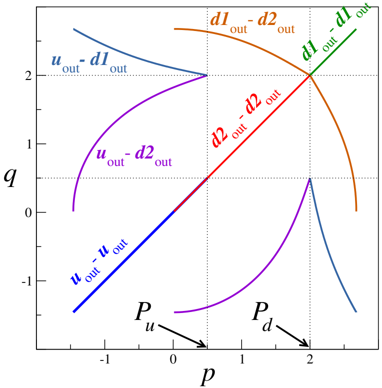


k : momentum relative to the condensate

$p = k + P_{(u/d)}$ where $P_{(u/d)} = mV_{(u/d)}$

$T = 0$ adiabatic opening

Boiron *et al.* PRL (2015)

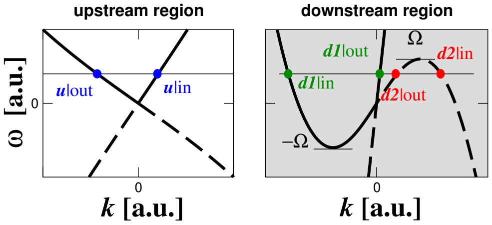


without horizon: $g_2 \equiv 1$

p, q : absolute momenta in units of ξ_u^{-1}

right plot: $g_2(p,q) \rightarrow$

where $g_2(p,q) = \dfrac{\langle : \hat{n}(p)\hat{n}(q) : \rangle}{\langle \hat{n}(p)\rangle\langle \hat{n}(q)\rangle}$



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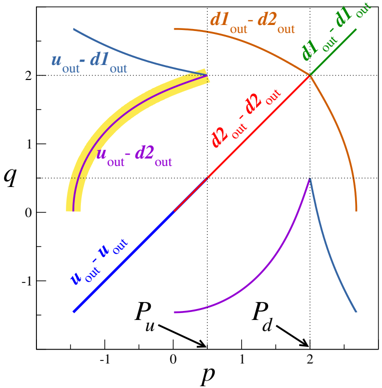
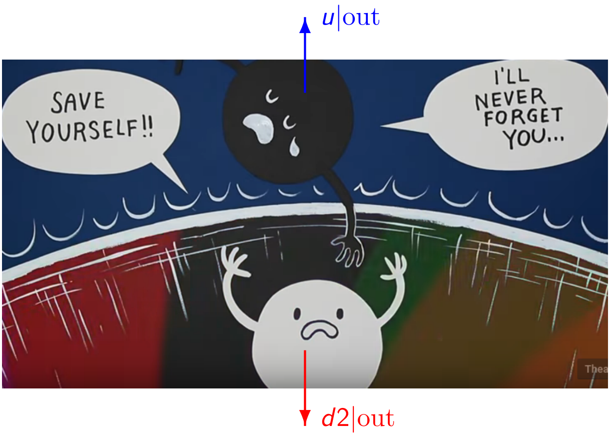


without horizon: $g_2 \equiv 1$

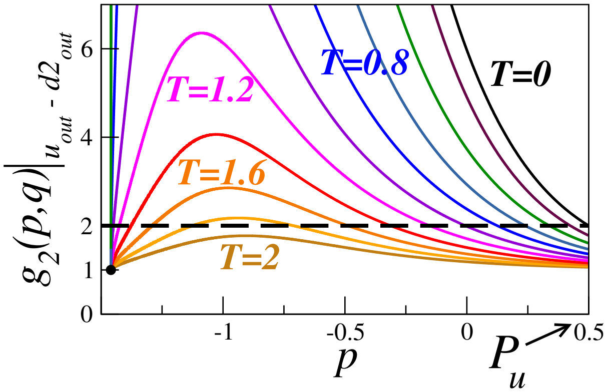
Violation of Cauchy-Schwarz inequality ($T \neq 0$)

$$\text{C.-S. violation : } g_2(p,q)\Big|_{u_{\text{out}}-d2_{\text{out}}} > \sqrt{g_2(p,p)\Big|_{u_{\text{out}}} \times g_2(q,q)\Big|_{d2_{\text{out}}}} \equiv 2$$



C.-S. violation : $g_2(p,q)\Big|_{u_{\rm out} - d2_{\rm out}} > \sqrt{g_2(p,p)\Big|_{u_{\rm out}} \times g_2(q,q)\Big|_{d2_{\rm out}}} \equiv 2$

Boiron *et al.* PRL (2015)



T in units of μ

$T_H = 0.13$

$V_u/c_u = 0.5$

$V_d/c_d = 4$

$V_d/V_u = 4$

$n_u/n_d = 4$

Schützhold and Unruh, Phys. Rev. D (2002)

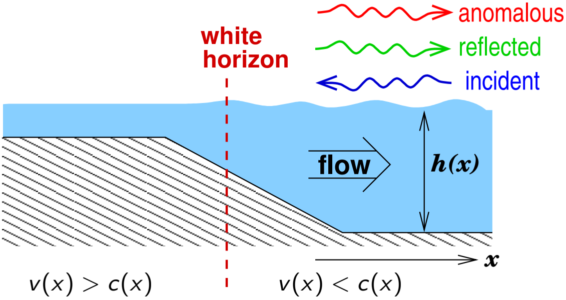
Weinfurtner *et al.*, Phys. Rev. Lett. (2011)

Rousseaux *et al.*, New Journal of Physics (2008)

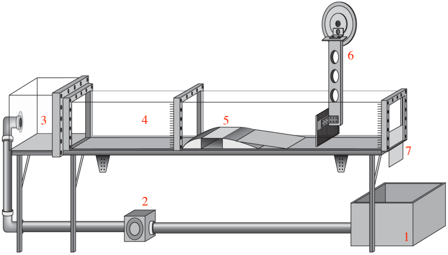
Euvé *et al.*, Phys. Rev. D (2016), Phys. Rev. Lett. (2016)

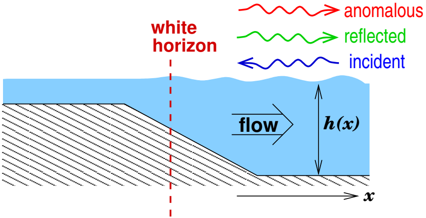
in a basin of depth h, the dispersion relation of gravity waves is
$(\omega - Vk)^2 = g\,k\,\tanh(k\,h)$, corresponding to $c = \sqrt{g\,h}$

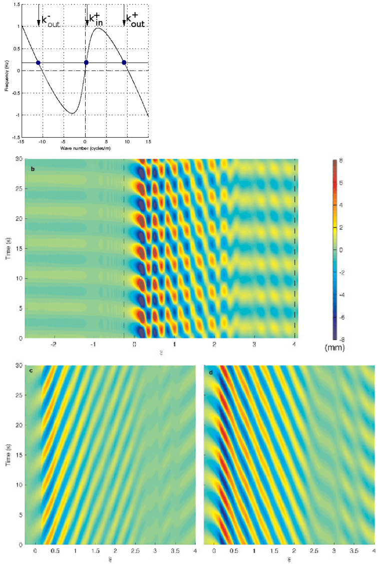
Experimental test of mode conversion :



Weinfurtner *et al.*, Phys. Rev. Lett. (2011)



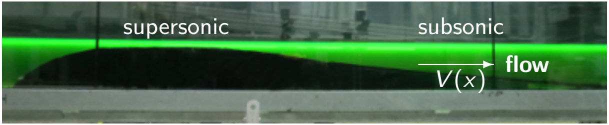




Gravity waves in a flume

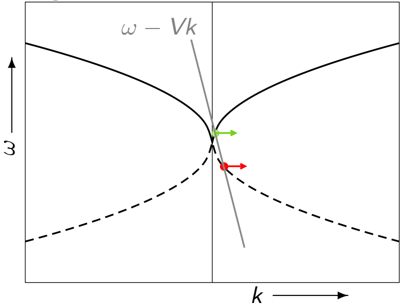
Euvé *et al.*, Phys. Rev. D (2016)

$$\omega - Vk = \pm\sqrt{gk\tanh(hk)}$$

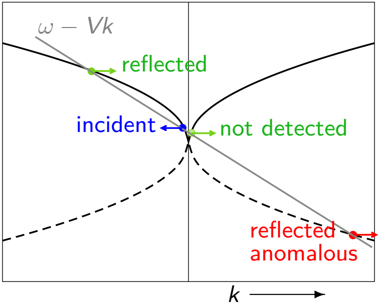


Poitiers experiment

supersonic : $V > c$



subsonic : $V < c$



Gravity waves in a flume

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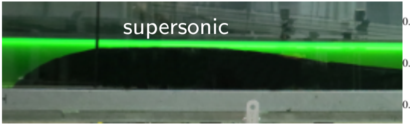
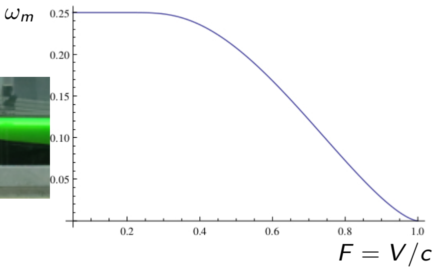


Poitiers experiment



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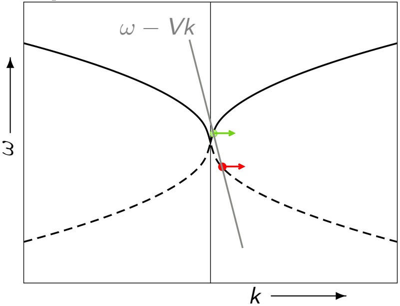

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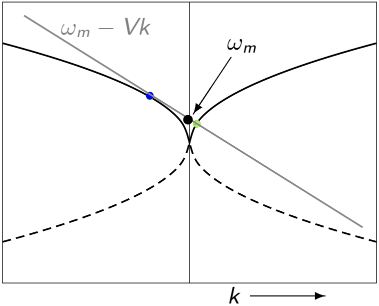


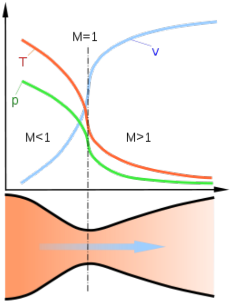
BECs offer interesting prospects to observe analogous Hawking radiation

[Steinhauer, Nature Physics]

general perspective : **quantum effects** with nonlinear **matter** waves

One- and two-body momentum distributions accessible by present day experimental techniques provide clear direct evidences

- of the occurrence of a sonic horizon.
- of the associated acoustic Hawking radiation.
- of the quantum nature of the Hawking process.
  - ☺ The signature of the quantum behavior persists even at temperatures larger than the chemical potential.



Nozzle of a V2 rocket

$F = \dot{m}\,(v_{\mathrm{out}} - v_{\mathrm{in}})$

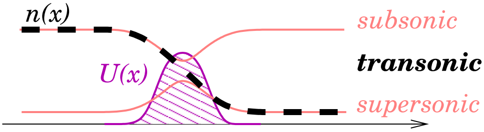
For a **thick** barrier

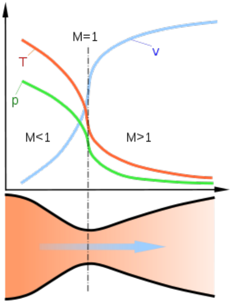
$U(x)$ of width $\gg \xi \sim (gn)^{-1/2}$:

$$\begin{cases} -\frac{(n^{1/2})_{xx}}{2n^{1/2}} + \frac{1}{2}v^2(x) + g\,n(x) + U(x) = C^{st}\,, \\ n(x)v(x) = C^{st}\,. \end{cases}$$

$$\rightsquigarrow \frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}x}\left[v^2 - c^2\right] = \frac{\mathrm{d}U}{\mathrm{d}x} \qquad \text{where } c^2(x) = g\,n(x)$$

$$v(x) \lessgtr c(x) \;\leftrightarrow\; \mathrm{sign}\left(\frac{\mathrm{d}n}{\mathrm{d}x}\right) = \mp\,\mathrm{sign}\left(\frac{\mathrm{d}U}{\mathrm{d}x}\right)$$





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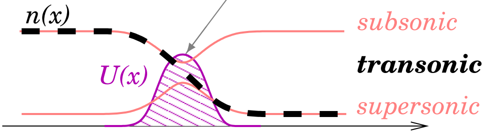
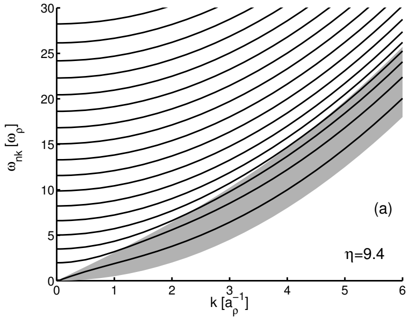
transonic : $U_x(x_0) = 0$, $v(x_0) = c(x_0)$



when $\hbar\omega_\perp \leq \mu$:



Zaremba, PRA (1998)
Stringari, PRA (1998)
Fedichev & Shlyapnikov, PRA (2001)
Tozzo & Dalfovo, PRA (2002)

modified dispersion relation :

$$\omega_0^2(q) = c_{1d}^2\, q^2 \left(1 - \tfrac{1}{48}(qR_\perp)^2 + \ldots\right)$$

new channels :

$$\omega_{n\geq 1}^2(q) = 2n(n+1)\,\omega_\perp^2 + \tfrac{1}{4}(qR_\perp\omega_\perp)^2 + \ldots$$

these new channels will be populated at $T = 0$

mass term $\neq$ Klein-Gordon

$\rightarrow$ new “in” modes