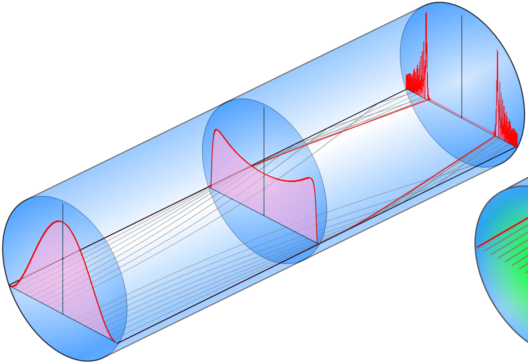
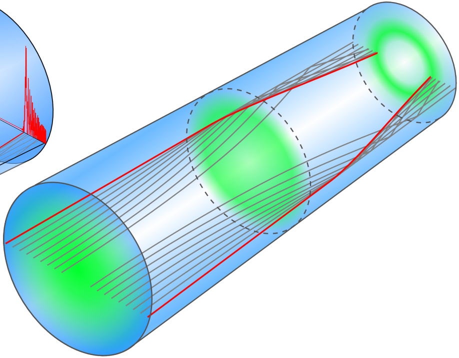
Optical hydrodynamics for nonlinear light propagation

Nicolas Pavloff

LPTMS, Univ. Paris Sud, Université Paris-Saclay, CNRS

Cargèse mai 2018





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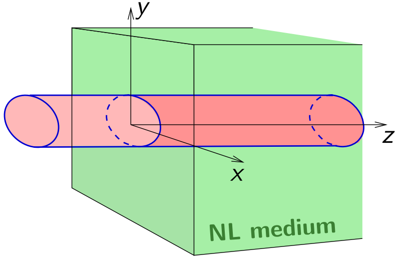
Types of nonlinear transport

- no obstacle
- one obstacle
- many obstacles

$$\vec{\nabla}^2 \vec{E}(\vec{r},t) = \frac{1}{\varepsilon_0 c^2}\partial_t^2 \vec{D}(\vec{r},t)$$



$$\begin{cases} \vec{E}(\vec{r},t) = \hat{x}\left\{\frac{1}{2}A(\vec{r}_\perp,z)\,\mathrm{e}^{\mathrm{i}(\beta_0 z-\omega_0 t)} + \mathrm{c.c.}\right\} \\ \vec{D}(\vec{r},t) = \varepsilon_0\vec{E} + \vec{P}_L(\vec{r},t) + \vec{P}_{NL}(\vec{r},t) \end{cases}$$

$$\begin{cases} \vec{P}_L(\vec{r},t) = \varepsilon_0 \chi^{(1)}_{\omega_0}(\vec{r})\vec{E}(\vec{r},t) \\ \vec{P}_{NL}(\vec{r},t) = \varepsilon_0 \underset{\sim}{\chi}^{(3)} \vdots \vec{E}(\vec{r},t)\vec{E}(\vec{r},t)\vec{E}(\vec{r},t) \\ \qquad = \varepsilon_0 \frac{3}{4}\chi^{(3)}|\vec{E}|^2\vec{E}(\vec{r},t) \end{cases}$$

$$(\chi^{(3)} = \underset{\sim}{\chi}^{(3)}_{xxyy} + \underset{\sim}{\chi}^{(3)}_{xyxy} + \underset{\sim}{\chi}^{(3)}_{xyyx})$$

- linear, homogeneous system: PW with $\beta_0 = \frac{\omega_0}{c}(1+\chi^{(1)}_{\omega_0})^{1/2} = k_0\, n(\omega_0)$

nonlinear, non homogeneous system. paraxial approximation $\partial_z A \ll \beta_0 A$

$$\chi^{(1)}(\vec{r}) = \chi^{(1)}_{\omega_0} + \Delta\chi^{(1)}(\vec{r}_\perp) \qquad \mathrm{i}\partial_z A = -\frac{1}{2\beta_0}\vec{\nabla}^2_\perp A - k_0 \Delta n(\vec{r})A$$

$$\Delta n(\vec{r}) = \Delta n^{(1)}(\vec{r}_\perp) + n_2|A(\vec{r}_\perp,z)|^2 \quad \text{with} \begin{cases} \Delta n^{(1)}(\vec{r}_\perp) = \frac{1}{2}\Delta\chi^{(1)}(\vec{r}_\perp)/n(\omega_0) \\ n_2 = \frac{3}{8}\chi^{(3)}/n(\omega_0) \quad < 0 \quad \text{in the following} \end{cases}$$

I_0 typical light intensity
$Z_{NL} = -1/(n_2 k_0 I_0)$
$\xi_\perp = \sqrt{Z_{NL}/\beta_0}$

$\vec{r}_\perp = \xi_\perp \times \vec{r}_\perp$
$z = Z_{NL} \times z$
$A = \sqrt{I_0} \times A(\vec{r}_\perp, z)$

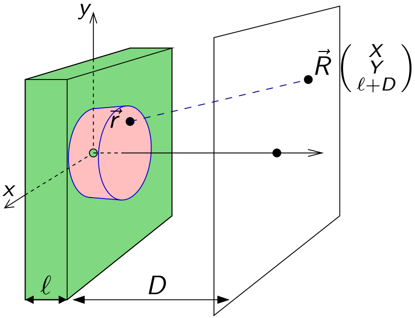
$$\mathrm{i}\partial_z A = -\tfrac{1}{2}\vec{\nabla}_\perp^2 A + |A|^2 A$$

dispersionless hydrodynamics

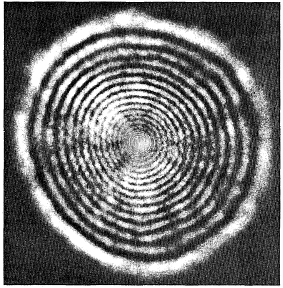
$A(\vec{r}_\perp, z) = \sqrt{\rho}\exp\{\mathrm{i}\, S\}$
$\vec{\nabla}_\perp S = \vec{u}$

$$\begin{cases} \partial_z \rho + \vec{\nabla}_\perp \cdot (\rho \vec{u}) = 0 \\ \partial_z \vec{u} + (\vec{u}\cdot\vec{\nabla}_\perp)\vec{u} + \vec{\nabla}_\perp \rho = 0 \end{cases}$$

for thin NL medium ($z \ll L_\perp$) : $\rho(\vec{r}_\perp, z) \simeq \rho(\vec{r}_\perp, 0)$ and if $\vec{u}$ was initially small, it remains small $\rightsquigarrow$ $S(\vec{r}_\perp, z) = -z \times \rho(\vec{r}_\perp, 0)$



Durbin, Arakelian, Shen (1981)



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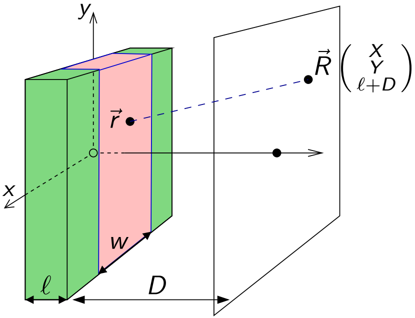


$A(\vec{R}) = \int \mathrm{d}^2 r\, G(\vec{R}, \vec{r}) A(\vec{r})$

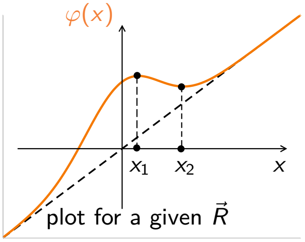
$G(\vec{R}, \vec{r}) \simeq \frac{1}{4\pi D} \exp\{\mathrm{i}k_0 |\vec{R} - \vec{r}|\}$

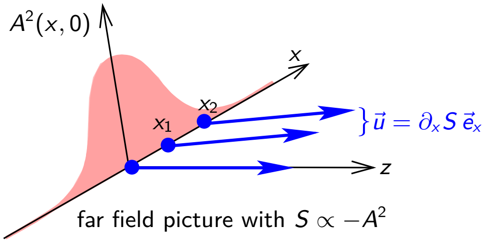
$A(\vec{r}) = A(x, z = 0) \exp\{\mathrm{i}S(x, \ell)\}$

$$A(\vec{R}) = \frac{\mathrm{e}^{\mathrm{i}(k_0 D + \pi/4)}}{\sqrt{8\pi D k_0}} \int \mathrm{d}x\, A(x, 0) \exp\{\mathrm{i}k_0 \frac{(x - X)^2}{2D} + \mathrm{i}S(x, \ell)\}$$

$S(x, \ell) = -\frac{\ell}{Z_{NL}} A^2(x, 0)/I_0$

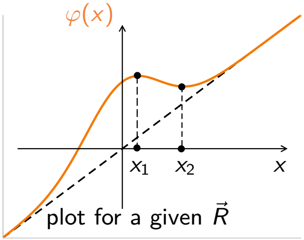
$$A(\vec{R}) \propto \int \mathrm{d}x\, A(x,0)\exp\{-\mathrm{i}\varphi(x)\} \text{ where } \varphi(x) = \frac{k_0 X}{D} x + \frac{\ell}{Z_{NL}} \exp(-2x^2/w^2)$$

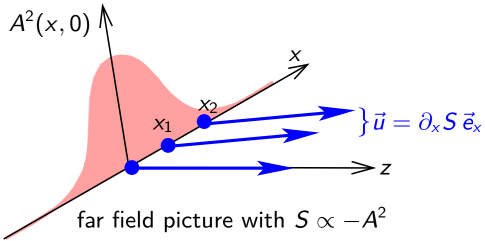




The fellowship of the ring(s): Khokhlov's group 1967

$$A(\vec{R}) \propto \int \mathrm{d}x\, A(x,0)\exp\{-\mathrm{i}\varphi(x)\} \text{ where } \varphi(x) = \frac{k_0 X}{D} x + \frac{\ell}{Z_{NL}} \exp(-2x^2/w^2)$$



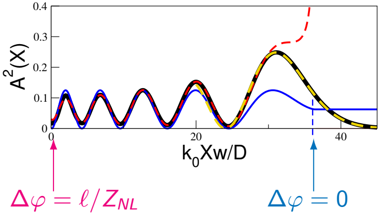


$$A(\vec{R}) = \sum_{\alpha=1,2} \frac{\sqrt{2\pi}A(x_\alpha,0)}{|\varphi''(x_\alpha)|^{1/2}} e^{-\mathrm{i}(\varphi(x_\alpha)+\sigma_\alpha\pi/4)}$$

very rough estimate

$$A^2(\vec{R}) \approx C^{st}\,[1+\sin(\Delta\varphi)]$$

$\Delta\varphi = \varphi(x_1) - \varphi(x_2)$ varies from ℓ/Z_{NL} (for $X=0$) to 0 (when the stationary points merge, at $k_0 X w/D = 2e^{-1/2}\ell/Z_{NL}$).



To count the number of rings, it suffices to count the maxima of $\sin\Delta\varphi$ when $\Delta\varphi$ varies from 0 to ℓ/Z_{NL} ($=30$ on the figure).

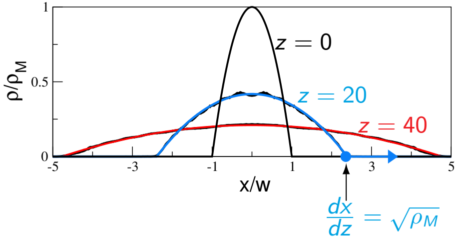
dimensionless units:

$$\rho(\vec{r}_\perp, z=0) = \begin{cases} \rho_M(1-\frac{x^2}{w^2}) \text{ if } |x| < w \\ 0 \text{ otherwise} \end{cases}$$

self-similar profile: Talanov 1965

$$\rho(x,z) = \frac{\rho_M}{f(z)}\left(1 - \frac{x^2}{w^2 \cdot f^2(z)}\right)$$
$$u(x,z) = x \cdot \phi(z)$$

$$\phi = f'/f$$
$$\ln(\sqrt{f}+\sqrt{f-1})+\sqrt{f(f-1)} = 2z\sqrt{\rho_M}/w$$



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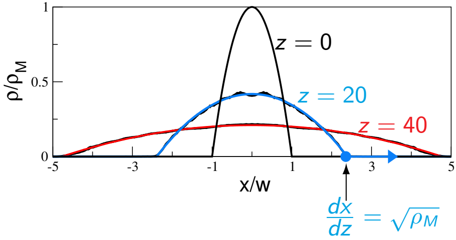
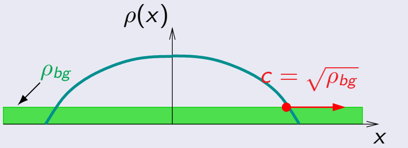
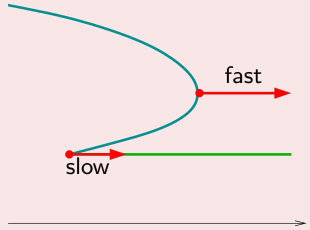


in the presence of ρ_{bg}



dispersive regularization of wave breaking



The fellowship of the ring(s): Fleischer's group 2006

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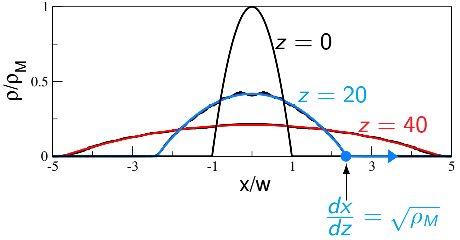

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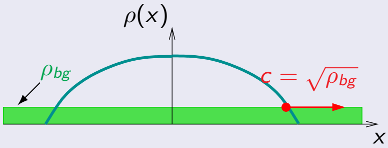

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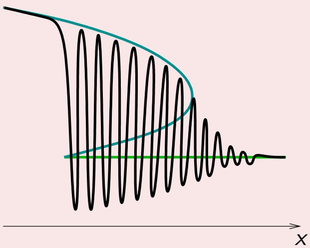
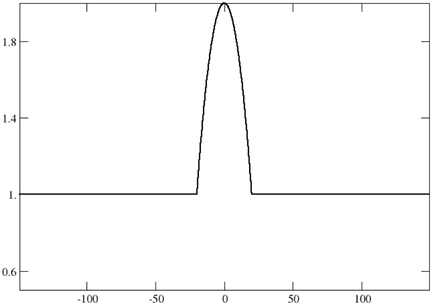
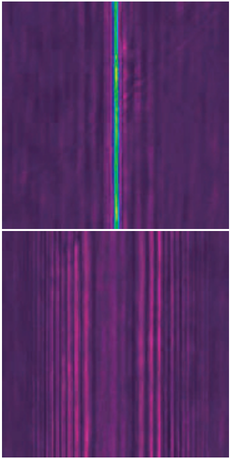




input



output

$0 \leq z \leq 60$

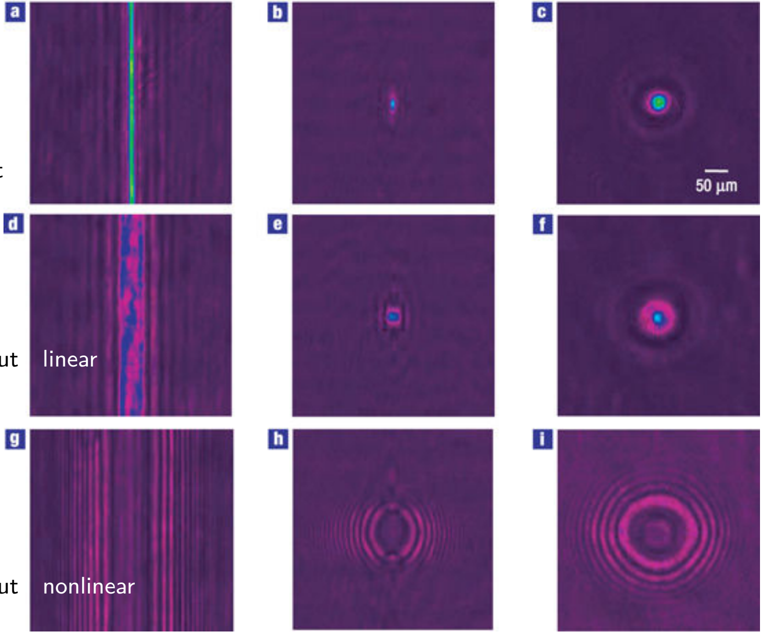
The fellowship of the ring(s): Fleischer's group 2006

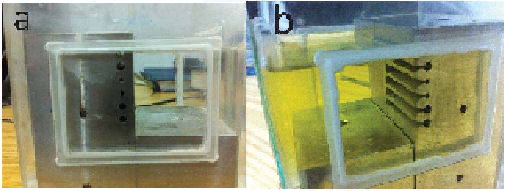
photo-refractive material: NL induced by a voltage bias across the crystal

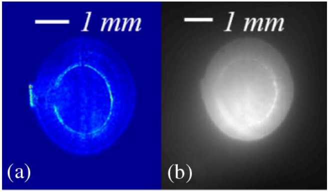
input

output

output





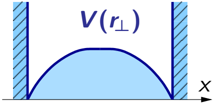


$L_\perp \ll L_{range\ NL} \ll L_z$: highly nonlocal paraxial approximation

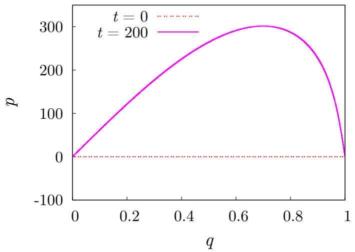
Snyder & Mitchell 1997, Folli & Conti 2012

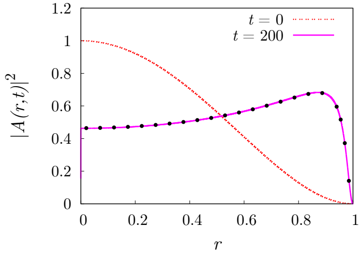
$$\Delta_{NL} n(\vec{r}_\perp, z) = \int \mathrm{d}^2 r'_\perp \chi(\vec{r}'_\perp) A^2(\vec{r}_\perp - \vec{r}'_\perp, z) \simeq \chi(\vec{r}_\perp) \int \mathrm{d}^2 r'_\perp A^2(\vec{r}'_\perp, z)$$
$$= \chi(\vec{r}_\perp) \times C^{st}$$

$$-\mathrm{i}\partial_z A = -\frac{1}{2}\vec{\nabla}_\perp^2 A + V(r_\perp) A$$



lagrangian manifold



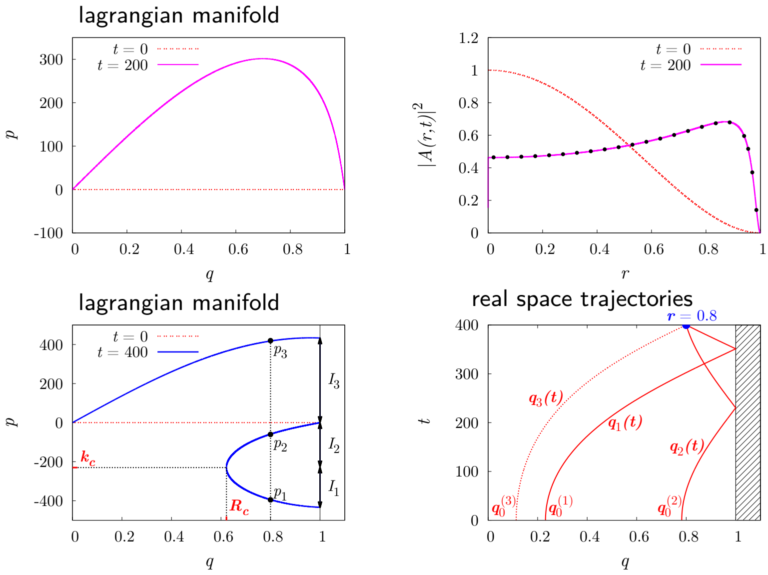


One evolves a swarm of **test points** $(\boldsymbol{r}, \boldsymbol{p})$
in phase space with the Hamilton equations

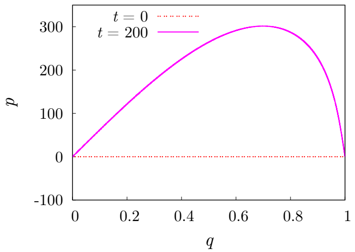
The density conservation eq. gives :

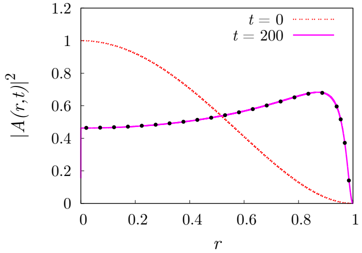
$$\left|A^2\left[\boldsymbol{r}(r_0, t)\right]\right| \mathrm{d}\boldsymbol{r} = \left|A_0^2[\boldsymbol{r}_0]\right| \mathrm{d}\boldsymbol{r}_0$$

$$\Leftrightarrow |A(r,t)| = \left|\frac{\partial \boldsymbol{r}_0}{\partial r}\right|_{\boldsymbol{r}_0(r,t)}^{1/2} |A_0[\boldsymbol{r}_0(r,t)]|$$

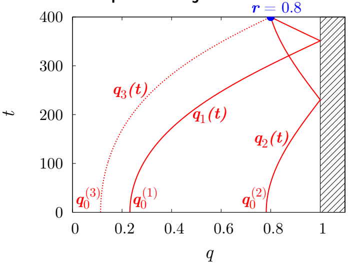

lagrangian manifold
t = 0
t = 200
p
q
|A(r,t)|^2
r
lagrangian manifold
t = 0
t = 400
p_3
p_2
p_1
I_3
I_2
I_1
k_c
R_c
real space trajectories
r = 0.8
q_3(t)
q_1(t)
q_2(t)
q_0^(3)
q_0^(1)
q_0^(2)
t

lagrangian manifold

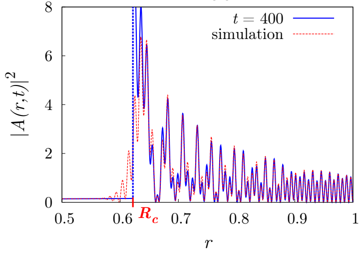




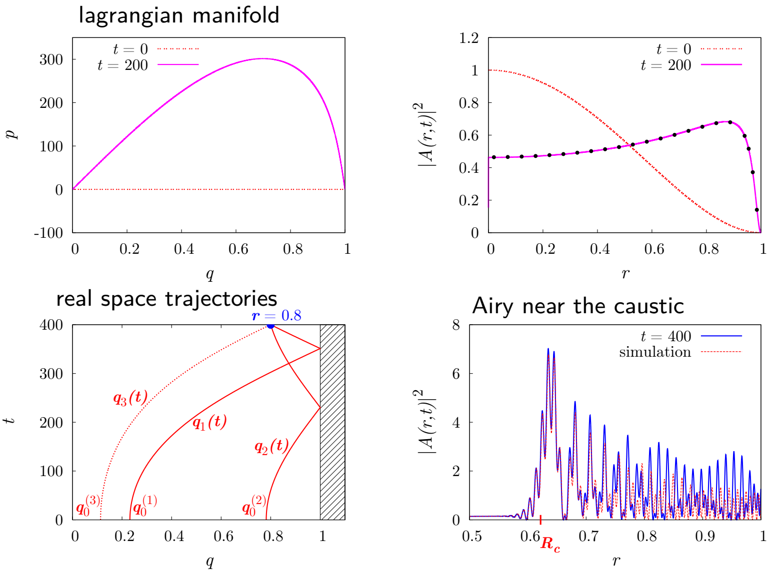
real space trajectories



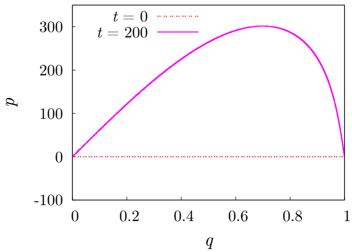
semiclassical approximation

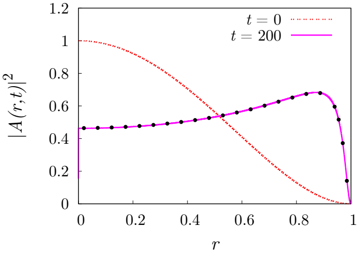


The fellowship of the ring(s): Bar-Ad's group 2015



lagrangian manifold





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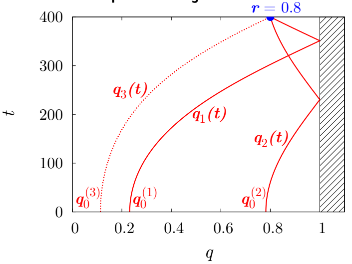
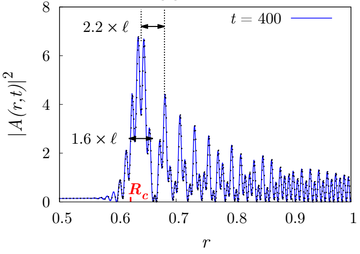
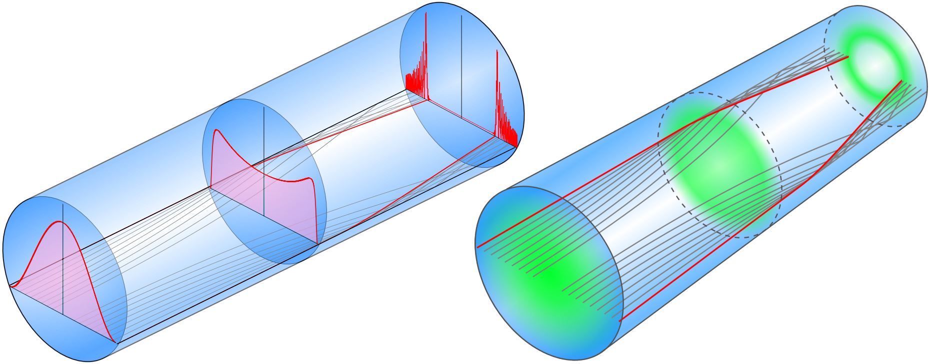
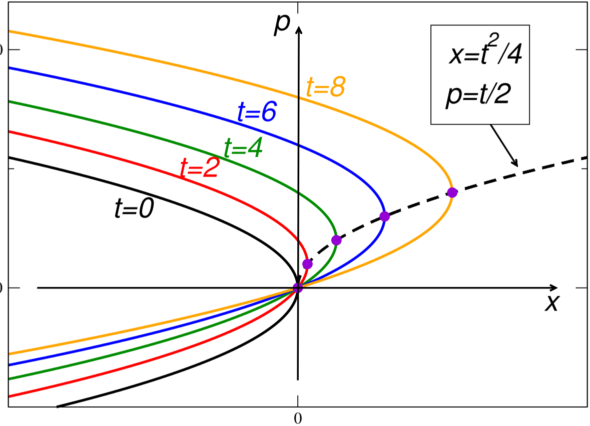


uniform approximation





$$i\partial_t\Phi = -\tfrac{1}{2}\partial_x^2\Phi \quad \rightsquigarrow \quad \Phi(x,t) = Ai\left(x - \frac{t^2}{4}\right)\exp\{i(xt/2 - t^3/6)\}$$



- initial swarm of particles $x_0 = -p_0^2$
- free propagation

$$\begin{cases} x(t) = x_0 + p_0 t \\ p(t) = p_0 \end{cases}$$

$$x(t) = -\left(p(t) - \frac{t}{2}\right)^2 + \frac{t^2}{4}$$

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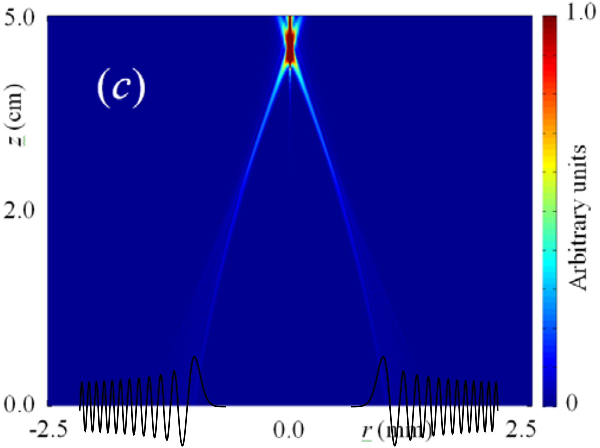

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simple and cheap alternative to a spatial light modulator

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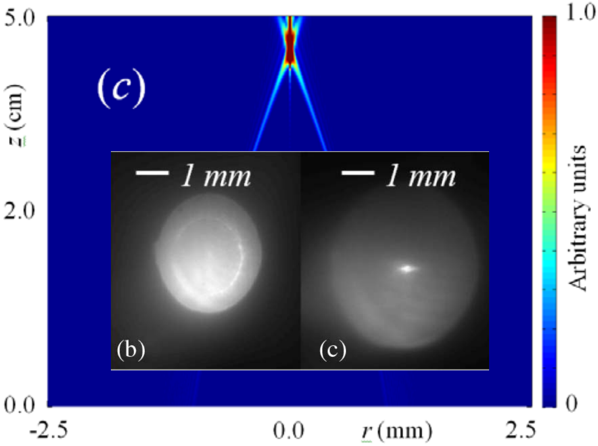

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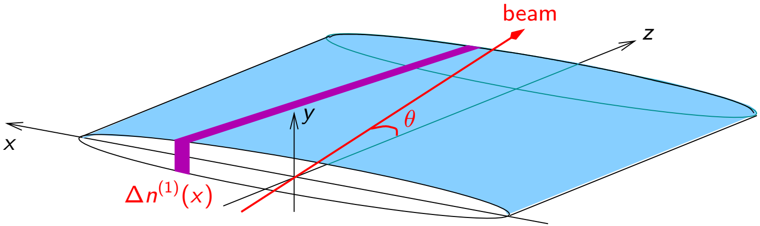


NLS in the presence of an obstacle

$$i\partial_z A = -\tfrac{1}{2}\partial_{xx}A + (U_{ext}(x) + |A|^2)A$$

model potential:
$U_{ext}(x) = \lambda\,\delta(x)$

"stationary" solutions $A(x,z) = e^{i\mu z}a(x)e^{iS(x)}$

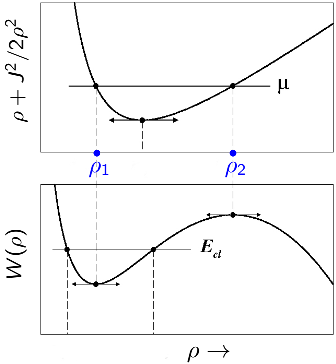
$\rho(x) = a^2(x)$ and $v(x) = \dfrac{dS}{dx}$ current conservation $\rightsquigarrow$ $\rho(x)v(x) = C^{st} \equiv J$

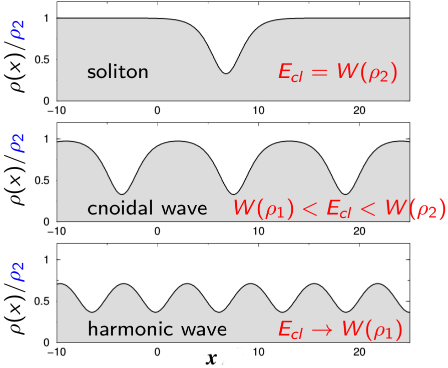
Stationary solutions of the NLS equation in the absence of $U(x)$

$$-\frac{1}{2}a_{xx} + \left[\rho + \frac{J^2}{2\rho} - \mu\right] a = 0\,, \quad \text{where} \quad J = \rho(x)v(x) \quad \text{and} \quad a(x) = \sqrt{\rho}$$

first integral:

$$\frac{1}{2}a_x^2 + W(\rho) = E_{cl}\,, \quad \text{where} \quad W(\rho) = -\frac{\rho^2}{2} + \mu\rho + \frac{J^2}{2\rho}\,.$$



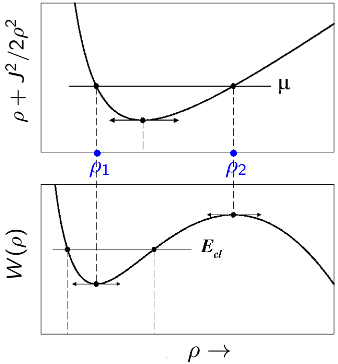


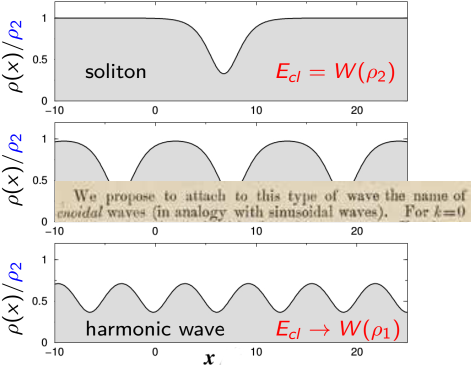
Stationary solutions of the NLS equation in the absence of $U(x)$

$$-\tfrac{1}{2}a_{xx} + \left[\rho + \frac{J^2}{2\rho} - \mu\right] a = 0 \,, \quad \text{where} \quad J = \rho(x)v(x) \quad \text{and} \quad a(x) = \sqrt{\rho}$$

first integral:

$$\tfrac{1}{2}a_x^2 + W(\rho) = E_{cl} \,, \quad \text{where} \quad W(\rho) = -\frac{\rho^2}{2} + \mu\rho + \frac{J^2}{2\rho} \,.$$

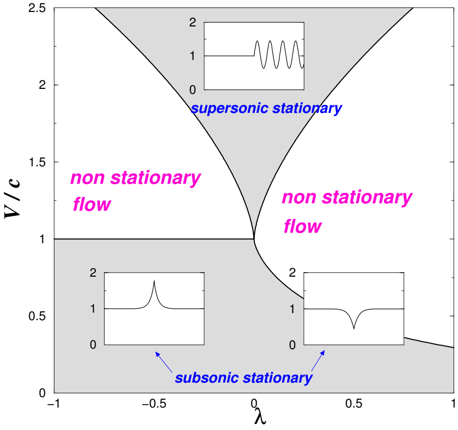




model 1D case : Flow past a point-like impurity

$$-\frac{1}{2}\partial_{xx}A+(U_{\rm ext}(x)+|A|^2)A = {\rm i}\,\partial_t A\ ,$$

$$U_{\rm ext}(x) = \lambda\,\delta(x)\ .$$



$$F = \int_{\mathbb{R}} dx\, \rho(x)\, \frac{dU_{ext}}{dx}$$

Perturbative treatment

$v > c = \sqrt{\rho(-\infty)}$

- in $1D$, $F \propto |\langle -\kappa | U_{\rm ext} | \kappa\rangle|^2$

where $\kappa = |v^2 - c^2|^{1/2}$

• For a δ impurity :

$$\begin{cases} F \propto C^{\rm st} & 1D \\ F \propto (v^2 - c^2)/v & 2D \\ F \propto v^2\,(1 - c^2/v^2)^2 & 3D \end{cases}$$

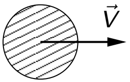
Hakim, PRE (1997)

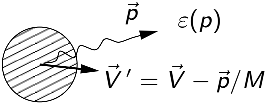
Lebœuf & Pavloff, PRA (2001)

Pavloff, PRA (2002)

Astrakharchik & Pitaevskii, PRA (2004)

Landau criterion (1941)



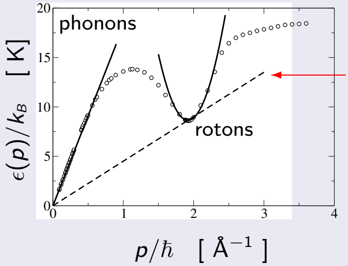


- Energy and momentum conservation: $\frac{M}{2} V^2 = \frac{M}{2}\left(\vec{V} - \frac{\vec{p}}{M}\right)^2 + \varepsilon(p)$.

for $M \gg m$ this reads $\varepsilon(p) = \vec{V}.\vec{p}$, hence $V \cos\theta = \varepsilon(p)/p$

emission of excitations possible only if $$V > v_{\mathrm{L}} = \min\left[\frac{\varepsilon(p)}{p}\right]$$

- Excitation spectrum **of superfluid ^{4}He**

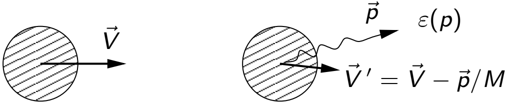


slope : $v_{\mathrm{L}} = 59$ m/s

due to vortex formation, in most experiments :

$1\ \mathrm{mm/s} \lesssim v_{\mathrm{crit,exp}} \lesssim 5\ \mathrm{m/s}$

Landau criterion (1941)



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emission of excitations possible only if $\boxed{V > v_{\mathrm{L}} = \min\left[\frac{\varepsilon(p)}{p}\right]}$

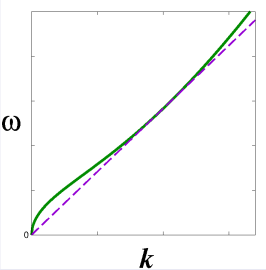
onset of Wave resistance

gravity-capillary waves at the surface of water:

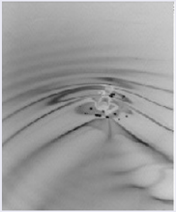
$\omega^2 = k(g + \frac{\sigma}{\rho}k^2)$

$v_{\mathrm{L}} = \left(\frac{4\,g\,\sigma}{\rho}\right)^{1/4} = 23$ cm/s

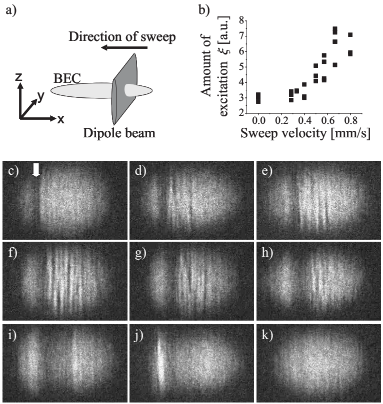
Kelvin (1871)

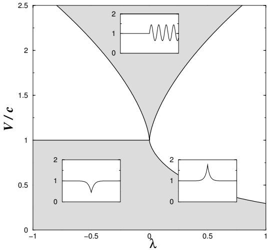


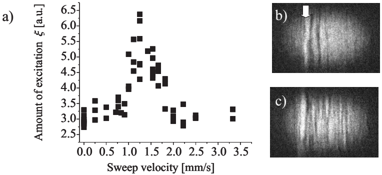
Burghelea and Steinberg, PRL 2001



$V = 25.33$ cm/s







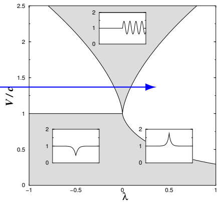
Repulsive potential

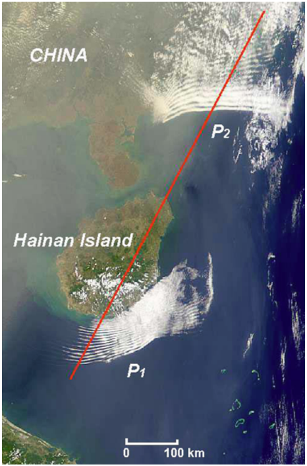
$U_{\max}/\mu \simeq 0.24$, $c = 2.1$ mm/s
$v = 0.4 - 0.8$, 1, 1.3, 2, 3.3 mm/s

Attractive potential

$v = 1.25$ mm/s, $c = 2.1$ mm/s
$|U_{\min}|/\mu \sim 0.17, 0.32$

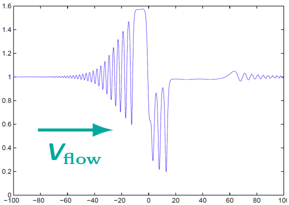
Non-stationary regime



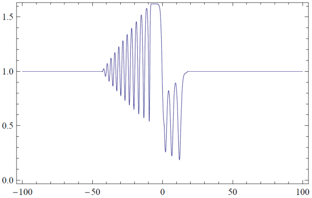


Waves generated by wind South China sea

numerical solution



analytic solution



The obstacle (located at $x \simeq 0$) typically emits 2 **dispersive shock waves**

Flow around an impenetrable cylinder (no damping, no polarization)

0.37 — 1.0 — 1.44 → V_{flow}/c

vortices appear

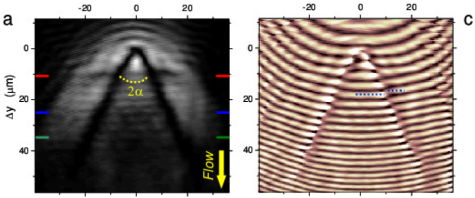
Frisch, Pomeau, Rica PRL (1992)
Huepe, Brachet, CRAS (1997)
Stießberger, Zwerger, PRA (2000)
Rica, Physica D (2001)
Berloff, Roberts, J Phys A (2001)

Cerenkov radiation sets in

oblique solitons

Kamchatnov,Pitaevskii PRL (2008)

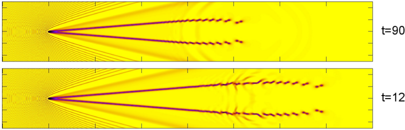
supersonic flow



LKB group, Science (2011)

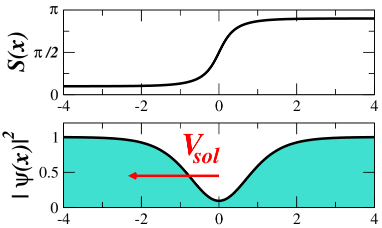
convective instability
of oblique dark solitons

El,Gammal,Kamchatnov PRL (2006)



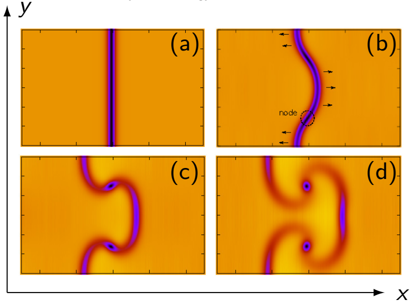
Dark solitons :

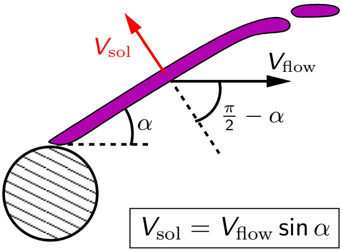
1D objects: $V_{sol} < c$



in 2D, snake instability:

courtesy of T. Congy

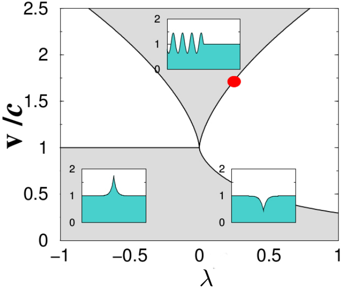


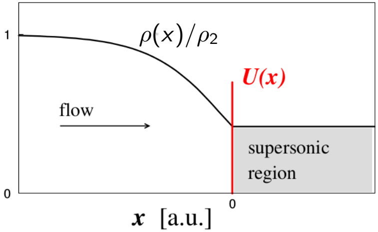


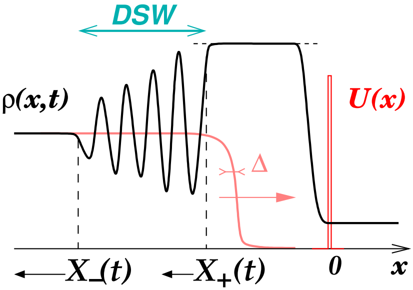
How to form a sonic horizon ?

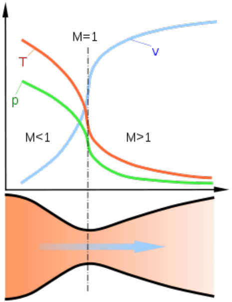
A. Kamchatnov & N. Pavloff, Phys. Rev. A (2012)

$U(x) = \lambda\,\delta(x) \qquad (\lambda > 0)$









Nozzle of a V2 rocket

$F = \dot{m}\,(v_{\rm out} - v_{\rm in})$

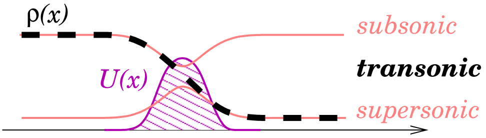
For a **thick** barrier

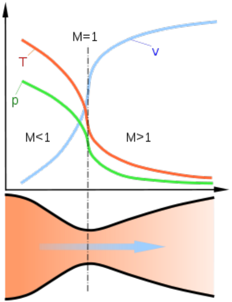
$U(x)$ of width $\gg \xi \sim \rho^{-1/2}$:

$$\begin{cases} -\frac{(\rho^{1/2})_{xx}}{2\rho^{1/2}} + \frac{1}{2}v^2(x) + \rho(x) + U(x) = C^{st} \,, \\ \rho(x)v(x) = C^{st} \,. \end{cases}$$

$$\rightsquigarrow \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}x}\left[v^2 - c^2\right] = \frac{\mathrm{d}U}{\mathrm{d}x} \qquad \text{where } c^2(x) = \rho(x)$$

$$v(x) \lesseqgtr c(x) \;\leftrightarrow\; \mathrm{sign}\left(\frac{\mathrm{d}\rho}{\mathrm{d}x}\right) = \mp\,\mathrm{sign}\left(\frac{\mathrm{d}U}{\mathrm{d}x}\right)$$





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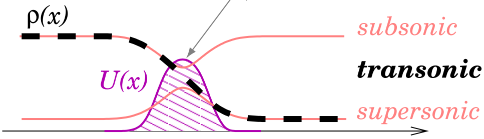
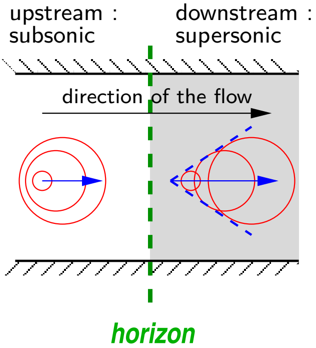
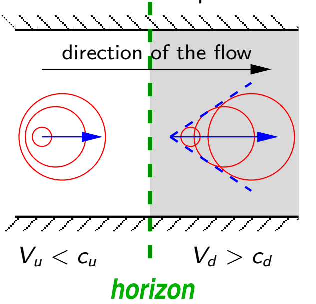
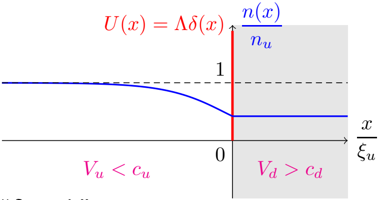
transonic : $\frac{dU}{dx}(x_0) = 0$, $v(x_0) = c(x_0)$

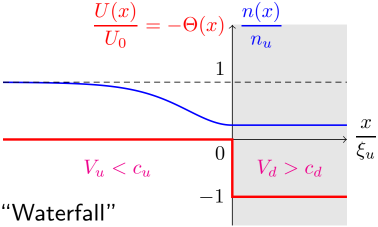



upstream :
subsonic
downstream :
supersonic
direction of the flow
horizon

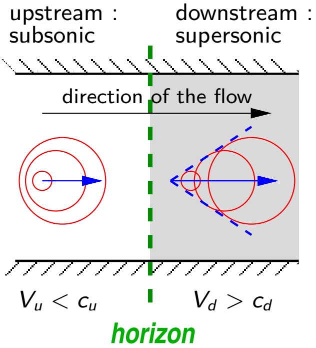




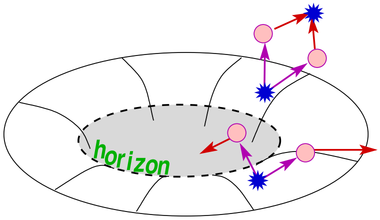
"δ-peak"



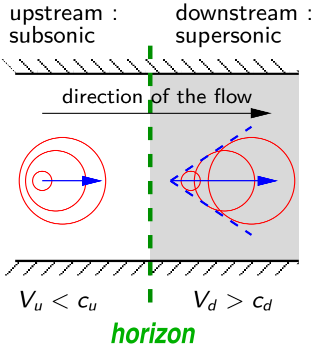
"Waterfall"

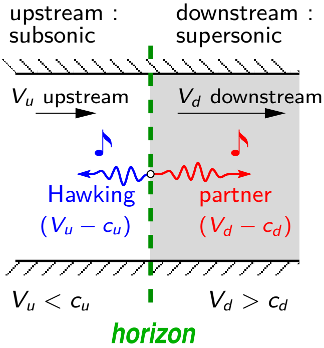


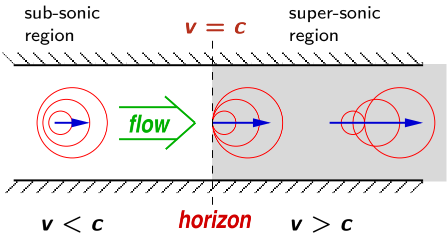
gravitational black hole



Hawking radiation 74'


upstream : subsonic
downstream : supersonic
direction of the flow
$V_u < c_u$
$V_d > c_d$
horizon


upstream : subsonic
downstream : supersonic
V_u upstream
V_d downstream
Hawking
$(V_u - c_u)$
partner
$(V_d - c_d)$
$V_u < c_u$
$V_d > c_d$
horizon

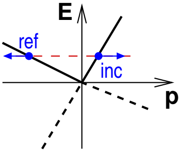


stimulated Hawking radiation

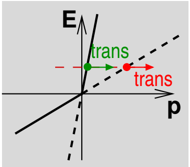
in the laboratory :

$$E(p) = c\,|p| + v\,p$$

$c\,|p|$ ↙ comoving; $v\,p$ ↓ Doppler



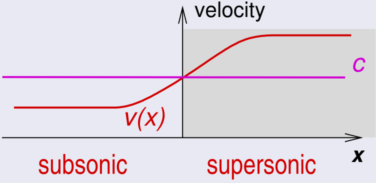
subsonic region



supersonic region

the position of the horizon is energy-dependent

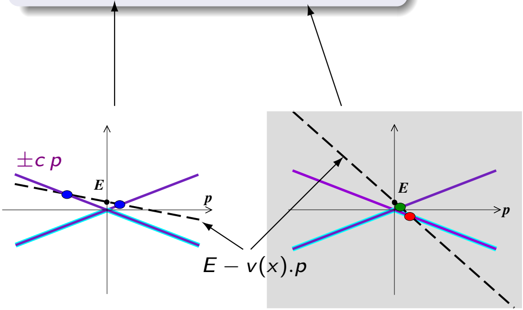
model configuration :



$$E - v(x).p = \pm\, E_s(p)$$

with

$$E_s(p) = c\, p$$



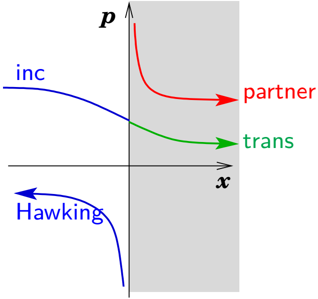
$v(x) < c$

region $x < 0$

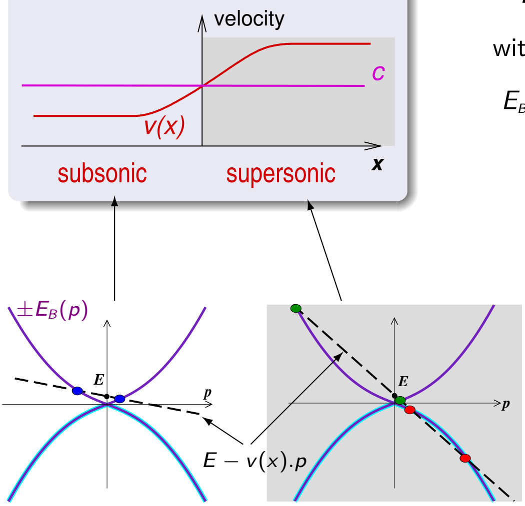
$v(x) > c$

region $x > 0$

phase space :



model configuration :



$v(x) < c$ — region $x < 0$

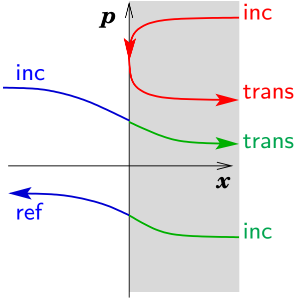
$v(x) > c$ — region $x > 0$

$$E - v(x).p = \pm E_B(p)$$

with

$$E_B(p) = c\, p \sqrt{1 + \xi^2 p^2/4}$$

phase space :

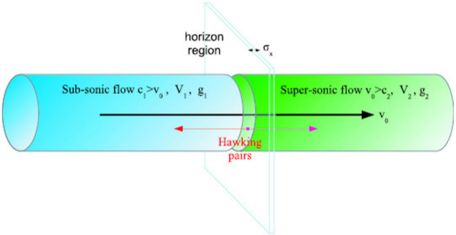


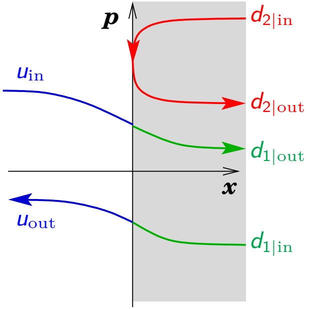
Numerical test, model configuration:

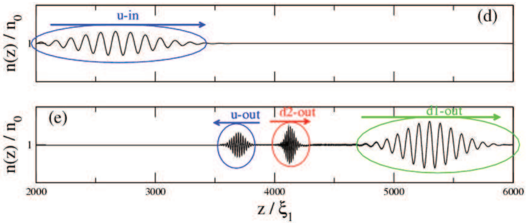
$U(x)$ and $g(x)$ **step like** with
$U(x) + g(x)n_0 = C^{\text{st}}$ such that
$\psi_0(x) = \sqrt{n_0}\,\exp\{ik_0\,x\}$, verifies $\forall x$

$$-\frac{1}{2}\psi_0'' + \left[U(x) + g(x)|\psi_0|^2\right]\psi_0 = \mu\,\psi_0\ ,$$

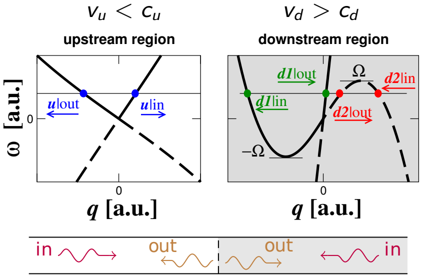
$$C^{\text{st}} = \mu - \frac{k_0^2}{2}\ .$$



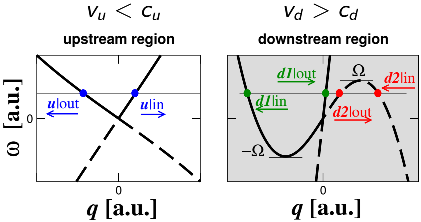


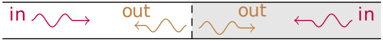


$$\omega = vk \pm \omega_B(k)$$

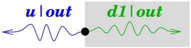


$$\omega = vk \pm \omega_B(k)$$



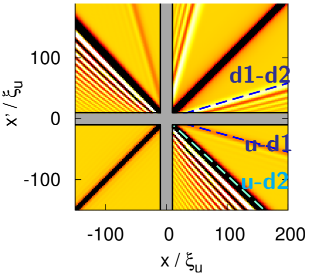


★ example of induced correlation:



$x = (v_d + c_d)t$ correlates with $x' = (v_u - c_u)t$

★ affects the density correlation pattern

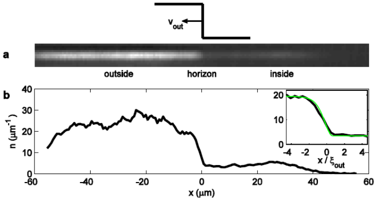


Larré *et al.*, Phys. Rev. A (2012)

New theoretical and experimental interest:

study of density correlation on each side of the horizon

$$g^{(2)}(x,x') = \frac{\langle : n(x) n(x') : \rangle}{\langle n(x') \rangle \langle n(x) \rangle} - 1$$



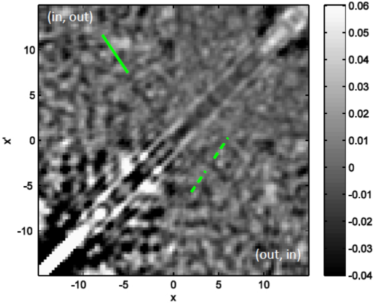
density profile near the horizon $\simeq$ waterfall $n_u/n_d = 5.55$ 5.55

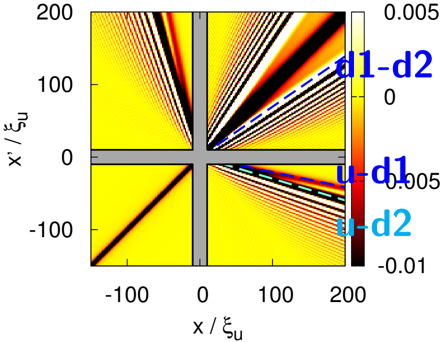
$c_u/c_d = 2.4$ 2.36

$V_u/c_u = 0.375$ 0.4245 $V_d/c_d = 3.25$ 5.55

$T_H = 1.0$ nK | $T_H/(gn_u) = 0.36$?

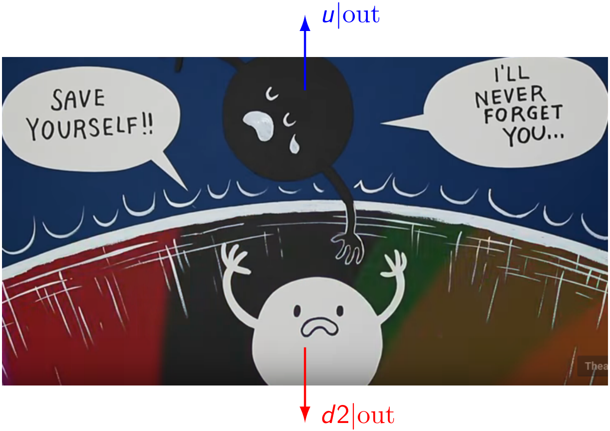
$T_H/(gn_u)|_{theo} \leq 0.25$





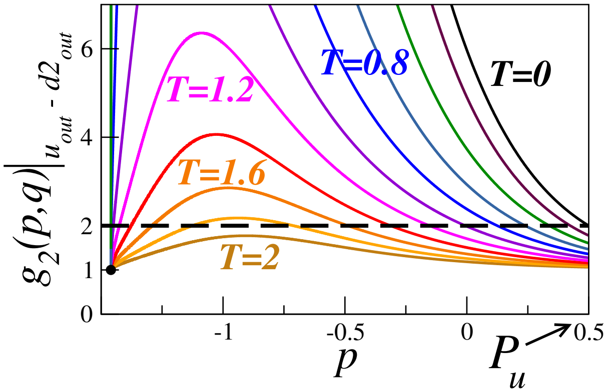
Violation of Cauchy-Schwarz inequality ($T \neq 0$)

C.-S. violation : $g_2(p,q)\Big|_{u_{\rm out} - d2_{\rm out}} > \sqrt{g_2(p,p)\Big|_{u_{\rm out}} \times g_2(q,q)\Big|_{d2_{\rm out}}} \equiv 2$



$$\text{C.-S. violation}: \; g_2(p,q)\Big|_{u_{\rm out}-d2_{\rm out}} > \sqrt{g_2(p,p)\Big|_{u_{\rm out}} \times g_2(q,q)\Big|_{d2_{\rm out}}} \equiv 2$$

Boiron *et al.* PRL (2015)



T in units of μ

$T_H = 0.13$

$V_u/c_u = 0.5$

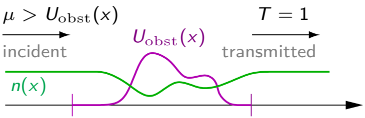
$V_d/c_d = 4$

$V_d/V_u = 4$

$n_u/n_d = 4$

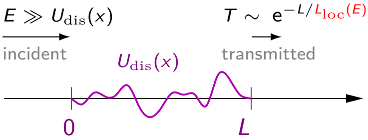
Two contrasting phenomena

Superfluidity



Perfect transmission
No drag, no dissipation

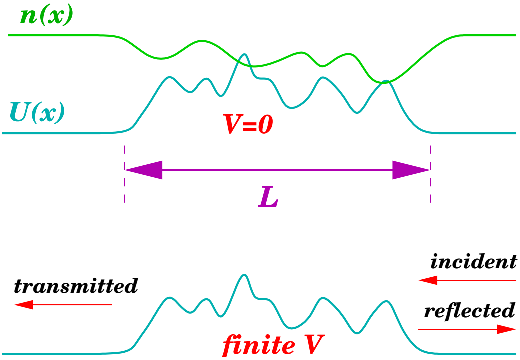
Anderson localization



Large L : no transmission

interaction $\Longleftrightarrow$ disorder

A (nonlinear) beam incident on a disordered region of size L



What are the density profile, the transmission coefficient and the drag exerted on the obstacle when the velocity V of the beam with respect to the obstacle is finite ?

How do these properties scale with L?

In the frame where the beam is at rest :

$$-\frac{\hbar^2}{2\,m}\partial_x^2\psi + \left[U(x - V\,t) + g\,|\psi|^2\right]\psi = i\hbar\,\partial_t\psi \;,$$

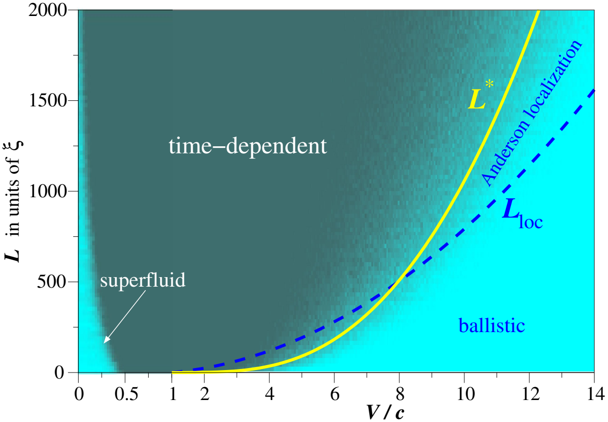
model disordered potential

$$U(x) = \lambda\,\mu\,\xi \sum_n \delta(x - x_n)\ ,$$

x_n's: uncorrelated random position of the impurities $0 = x_1 \leq x_2 \leq x_3 \ldots,$ with mean density $n_{\rm i}$

One has $\langle U(x)\rangle = \lambda\,\mu\,(n_{\rm i}\,\xi)$ and
$\langle U(x)U(x')\rangle - \langle U\rangle^2 = \left(\frac{\hbar^2}{m}\right)^2 \sigma\,\delta(x-x')$
with $\sigma = n_{\rm i}\,\lambda^2/\xi^2$. $[\sigma] =$ length^{-3}.

• Other disordered potentials: Gaussian (white or correlated) noise, Speckle potential.

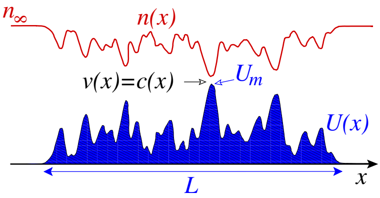


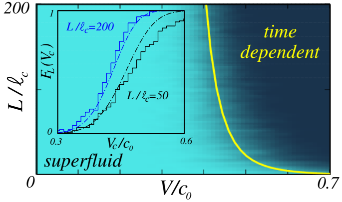
disordered delta peaks with $\lambda = 0.5$ and $n_\mathrm{i}\xi = 0.5$ $\quad (\mu \gg \langle U \rangle)$.

model disordered potential : $U(x) = \lambda\,\mu\,\xi \sum_n \delta(x - x_n)$,
x_n's: uncorrelated random position of the impurities, with mean density n_i.
One has $\langle U(x) \rangle = \lambda\,\mu\,(n_\mathrm{i}\,\xi)$

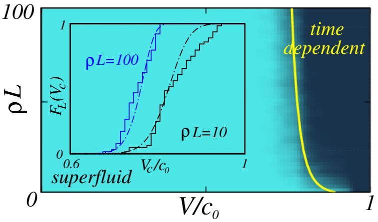
- Similar to the non-disordered case.

  V. Hakim, Phys. Rev. E **55**, 2835 (1997)
- Linked to statistics of extremes of the random potential.
- One obtains analytical results in two limiting cases:





Smooth disorder $L_{\text{typ}} \gg \xi$



random delta peaks

$F_L(V_{crit})$: cumulative probability distribution of V_{crit}

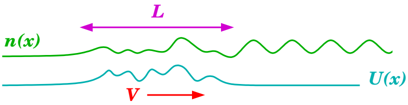
Ballistic ($\equiv$ perturbative) region

$$\delta n(\zeta) \simeq \frac{2mn_0}{\hbar^2\kappa}\int_{-\infty}^{\zeta} \mathrm{d}y\, U(y)\, \sin[2\kappa(\zeta - y)]$$

where $\zeta = x - Vt$. This yields $\langle T\rangle \simeq 1 - L/L_{\rm loc}$ where

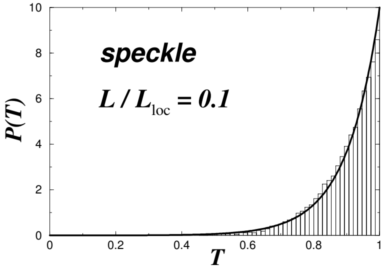
$$L_{\rm loc}(\kappa) = \frac{\kappa^2}{\sigma}\ . \qquad (1)$$

and $$\kappa = \frac{m}{\hbar}\left|V^2 - c^2\right|^{1/2}\ . \qquad (2)$$



probability distribution of T :

$$P(T) = \frac{L_{\rm loc}}{L}\exp\left\{-(1-T)\frac{L_{\rm loc}}{L}\right\}\ .$$



$L > L_{\rm loc}$: non perturbative

$P(\lambda, t)$ ($\lambda = T^{-1} - 1,\ t = L/L_{\rm loc}$) is solution of the **DMPK** (Fokker-Planck) equation:

$$\partial_t P = \partial_\lambda [\lambda(\lambda+1)\partial_\lambda P]$$

- This implies that

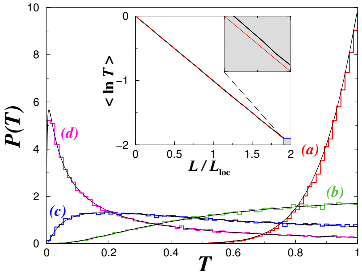
  $$\langle \ln T \rangle = -L/L_{\rm loc}(\kappa) ,$$

  where $L_{\rm loc}(\kappa)$ is given by Eqs. (1,2).

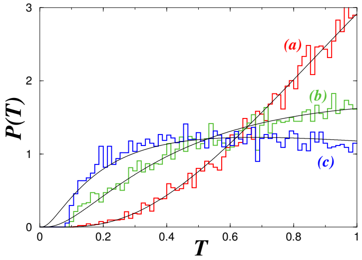
- and that the asymptotic probability distribution is log-normal

$$P(\ln T, t = \frac{L}{L_{\rm loc}}) = \frac{1}{\sqrt{4\pi t}}\, e^{-\frac{1}{4t}(t + \ln T)^2}$$

δ-peaks $t = 0.1$, 0.5, 1 and 2



speckle, $t = 0.31$, 0.52, 0.68



First integral in regions where $U(x) \equiv 0$

(between x_n and x_{n+1} say)

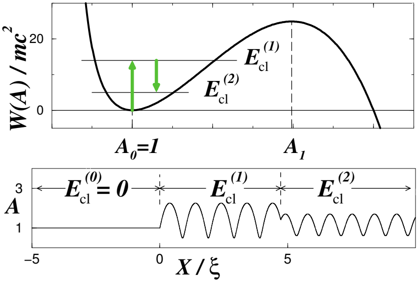
$$\frac{\xi^2}{2}\left(\frac{\mathrm{d}A}{\mathrm{d}X}\right)^2 + W[A(X)] = E_{\rm cl}^{(n)} \,,$$

where $A = |\psi|/\sqrt{n_0}$, $E_{\rm cl}^{(n)}$ is a constant and
$W(A) = \frac{1}{2}(A^2-1)(1+v^2-A^2-v^2/A^2)$.

From the final $E_{\rm cl}^{(N_{\rm i})}$ one computes the transmission

$$T = \frac{1}{1+(2\kappa^2\,\xi^2)^{-1}E_{\rm cl}^{(N_{\rm i})}} \,.$$

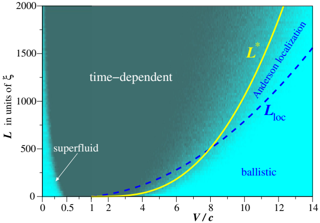
previous slide : $\lambda^{(n)} = \frac{m}{2\hbar^2\kappa^2}\,E_{\rm cl}^{(n)}$



Upper panel: $W(A)$ (drawn for $v = V/c = 4$). $A_0 (= 1)$ and A_1 are the zeros of $\mathrm{d}W/\mathrm{d}A$. The fictitious particle is initially at rest with $E_{\rm cl}^{(0)} = 0$. The value of $E_{\rm cl}$ changes at each impurity. The lower panel displays the corresponding oscillations of $A(X)$, with two impurities (vertical dashed lines) at $x_1 = 0$ and $x_2 = 4.7\,\xi$.

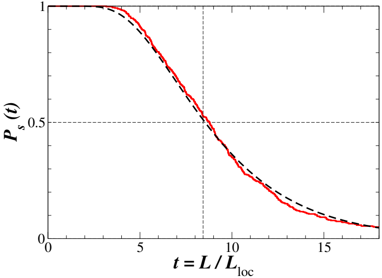
Upper threshold

for the supersonic stationary regime



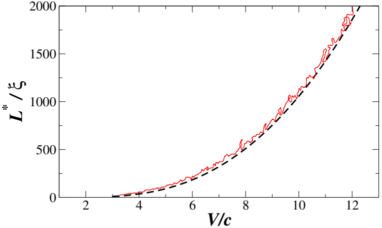
One solves the DMPK equation with the boundary condition that there exists a $\lambda_{\max}$ [corresponding to $E_{\rm cl}^{max} = W(A_1)$] at which $P(\lambda_{\max}, t) = 0$: i.e., this upper boundary is a "sink".

fixed V ($V/c = 450$)



Fraction of stationary solutions

phase diagram



rich analogy between nonlinear optics and hydrodynamics

- modulational instability
- observation of dispersive shock waves
- analogy with superfluid motion
- in the presence of disorder : competition between SF and AL
- possible formation of “sonic” horizon

Conclusion

rich analogy between nonlinear optics and hydrodynamics

- modulational instability
- observation of dispersive shock waves
- analogy with superfluid motion
- in the presence of disorder : competition between SF and AL
- possible formation of "sonic" horizon

1D integrable turbulence in a nonlinear fiber (focusing NLS)

- Whitham theory helpful in the initial stage of development of integrable turbulence (stationary PDF of Riemann invariants) Randoux, Gustave, Suret, El, PRL 2017
- also helpful at much later stage: soliton gas

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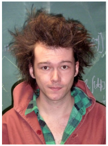

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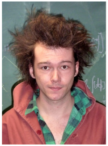
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Thank you for your attention