

Advanced Nonlinear Physics  
international Master « Physics of Complex Systems »

Third Lecture

**Viscous shocks**

tuesday, september 22<sup>nd</sup>, 2020

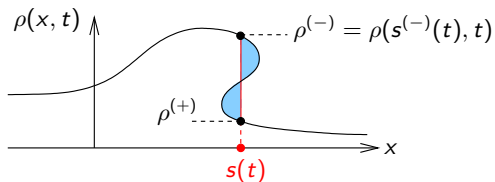
website of (the 1st part of) the course:

[http://lptms.u-psud.fr/nicolas\\_pavloff/enseignement/  
advanced-nonlinear-physics/](http://lptms.u-psud.fr/nicolas_pavloff/enseignement/advanced-nonlinear-physics/)

One still wishes to solve  $\rho_t + c(\rho)\rho_x = 0$ , i.e., to determine  $\rho(x, t)$  for a given  $\rho(x, 0) \equiv \rho_0(x)$ .

We saw last time that, when characteristic cross, wave breaking occurs and a shock is formed, with position  $s(t)$  verifying

$$\dot{s} = \frac{Q^{(+)} - Q^{(-)}}{\rho^{(+)} - \rho^{(-)}} \quad (1)$$



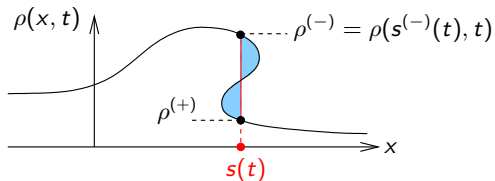
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# Shock velocity

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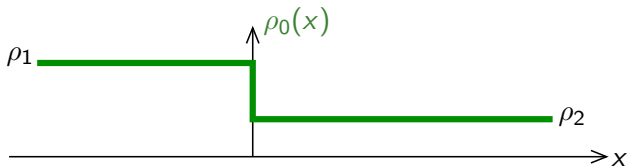
This result is obtained by flux conservation. It can be understood this way by writing  $Q^{(+)} = Q^{(-)} + \dot{s} \times (\rho^{(+)} - \rho^{(-)})$

Handwritten diagram illustrating flux conservation across a shock. The vertical axis is labeled "velocity  $\dot{s}$ ". The horizontal axis represents the shock position. On the left side, the shock position is  $s = s^{(-)}$  and the flux is  $Q = Q^{(-)}$ . On the right side, the shock position is  $s = s^{(+)}$  and the flux is  $Q = Q^{(+)}$ . Arrows indicate the direction of flow and the shock velocity.

Handwritten diagram illustrating flux conservation across a shock. The vertical axis is labeled  $\rho^{(+)}$  and  $\rho^{(-)}$ . The horizontal axis is labeled  $x$ . A control volume is shown with a width of  $\dot{s} dt$  and a height of  $\rho^{(+)} - \rho^{(-)}$ . The flux difference is labeled  $(\rho^{(+)} - \rho^{(-)}) \dot{s} dt$ .

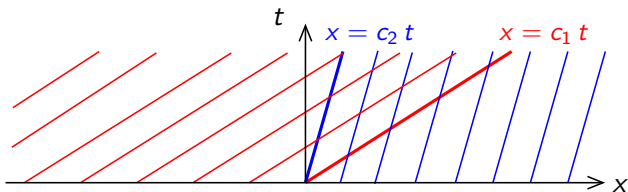
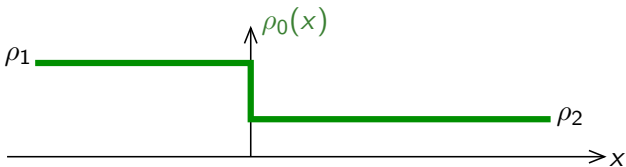
## the Riemann problem

Case where  $c$  is an increasing function of  $\rho$ .  $c_1 = c(\rho_1) > c_2 = c(\rho_2)$ .



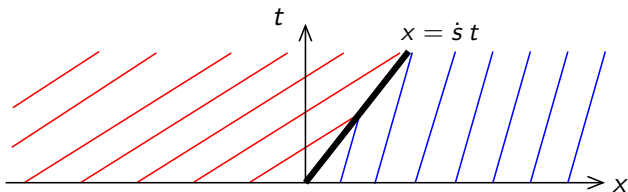
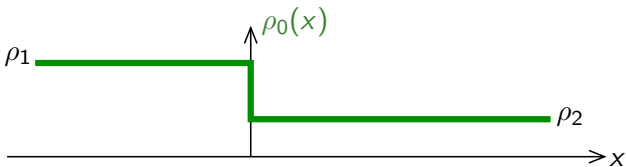
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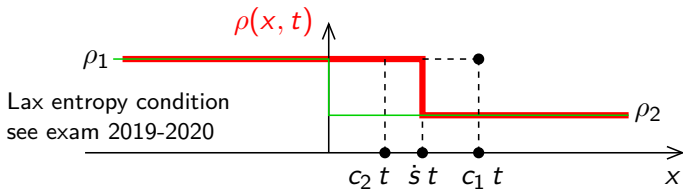
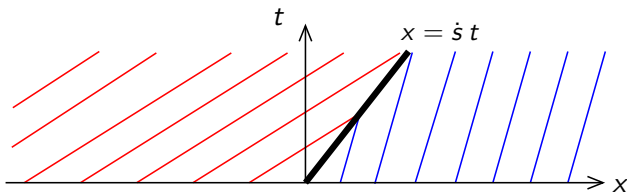
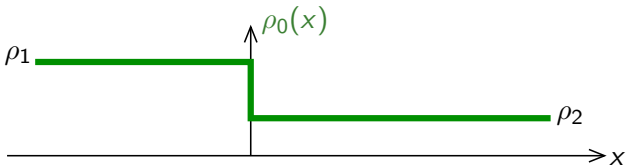
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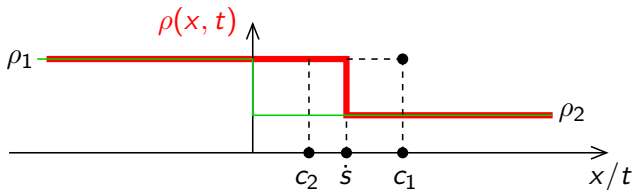
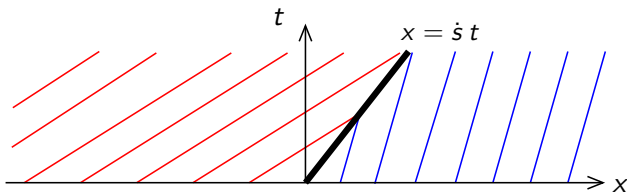
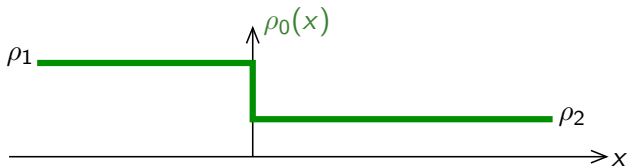
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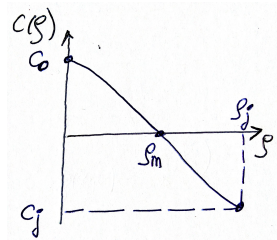
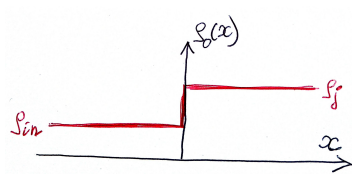
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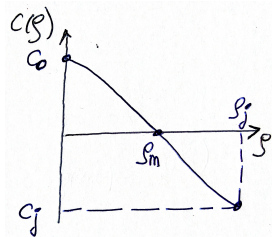
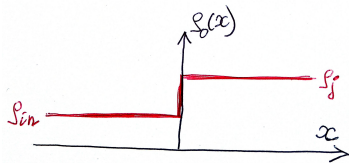
# Describe the motion of the end of a traffic jam

Assuming that there is a constant incoming flow of vehicles, density  $\rho_{in}$ , flux  $Q_{in} = Q(\rho_{in})$ , with  $\rho_{in} < \rho_m$ .

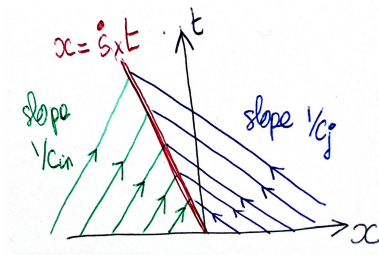


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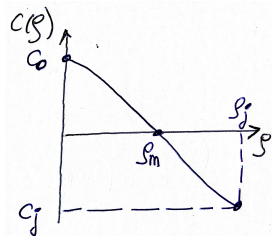
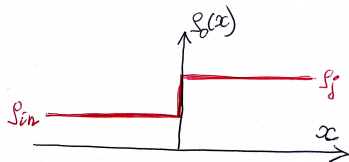


$$\dot{s} = -Q_{in}/(\rho_j - \rho_{in}) = cst \quad [\text{remember } Q(\rho_j) = 0]$$

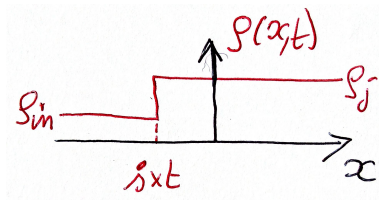
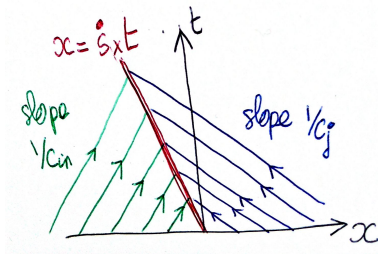


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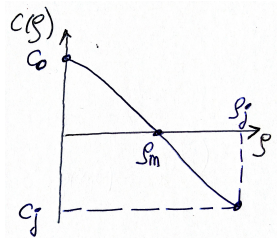
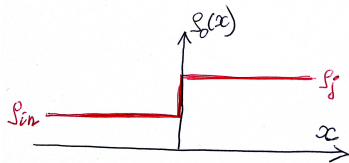


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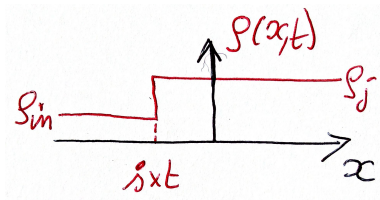
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during  $dt$ ,  $Q_{in}dt$  cars arrive and pile up with an increase of density  $\rho_j - \rho_{in}$  :

$$\begin{cases} Q_{in}dt = (\rho_j - \rho_{in})|dx| \\ \text{with } |dx| = -\dot{s} dt \end{cases}$$



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