Advanced Nonlinear Physics

international Master « Physics of Complex Systems »

Third Lecture

Viscous shocks

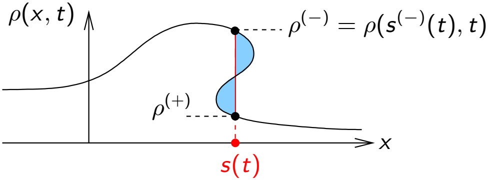
tuesday, september 22$^{\rm nd}$, 2020

website of (the 1st part of) the course:
`http://lptms.u-psud.fr/nicolas_pavloff/enseignement/advanced-nonlinear-physics/`

One still wishes to solve $\rho_t + c(\rho)\rho_x = 0$, i.e., to determine $\rho(x,t)$ for a given $\rho(x,0) \equiv \rho_0(x)$.

We saw last time that, when characteristic cross, wave breaking occurs and a shock is formed, with position $s(t)$ verifying

$$\dot{s} = \frac{Q^{(+)} - Q^{(-)}}{\rho^{(+)} - \rho^{(-)}} \quad (1)$$



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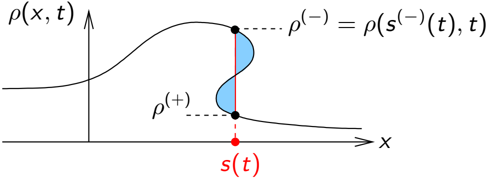
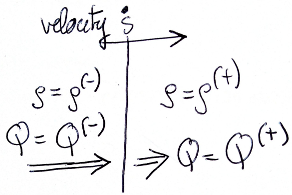
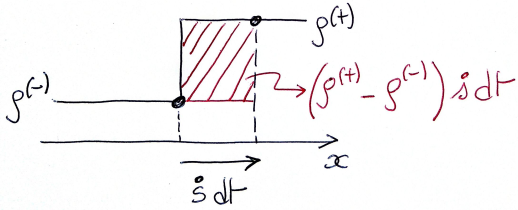
$$\dot{s} = \frac{Q^{(+)} - Q^{(-)}}{\rho^{(+)} - \rho^{(-)}} \quad (1)$$

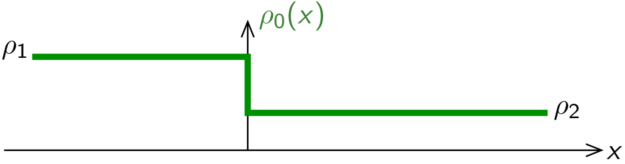


This result is obtained by flux conservation. It can be understood this way by writing $Q^{(+)} = Q^{(-)} + \dot{s} \times (\rho^{(+)} - \rho^{(-)})$





Case where c is an increasing function of ρ. $c_1 = c(\rho_1) > c_2 = c(\rho_2)$.



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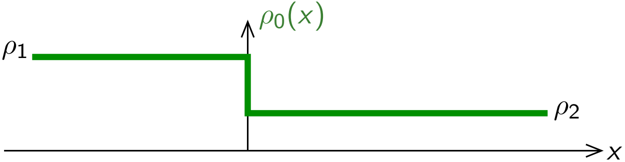

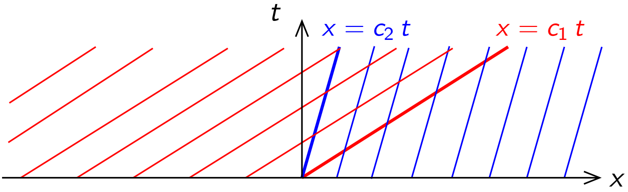

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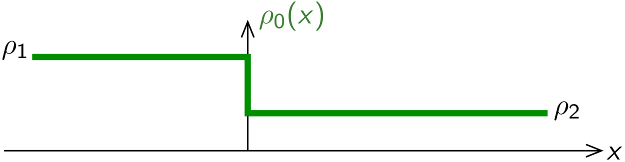

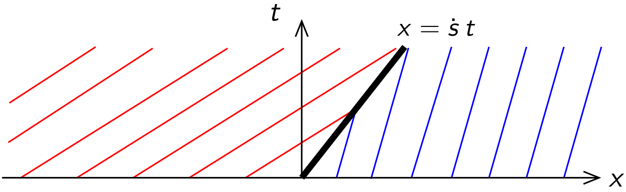

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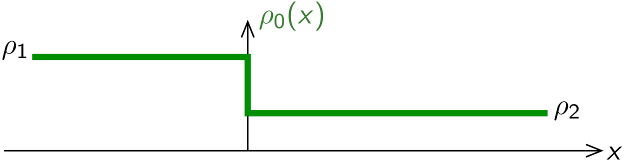

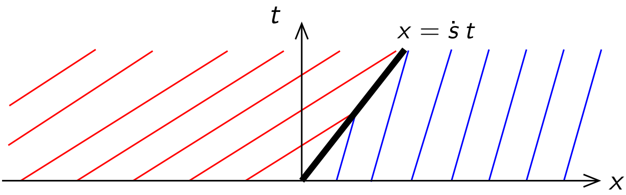

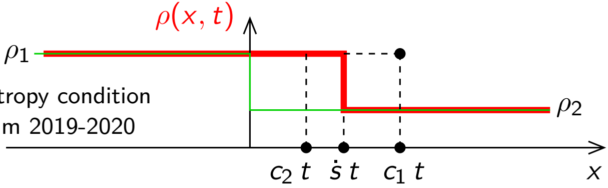

Lax entropy condition
see exam 2019-2020

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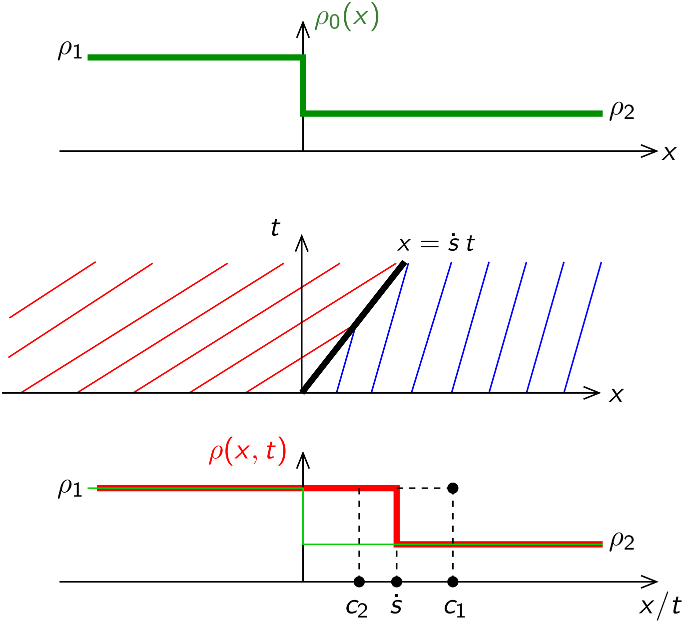
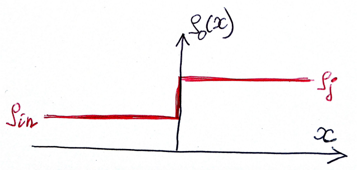
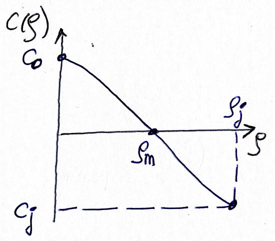


Assuming that there is a constant incoming flow of vehicles, density ρ_{in}, flux $Q_{in} = Q(\rho_{in})$, with $\rho_{in} < \rho_m$.





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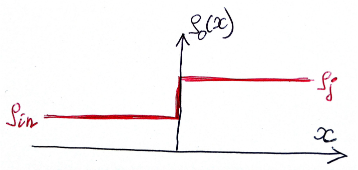
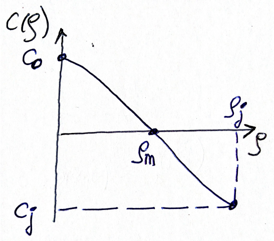
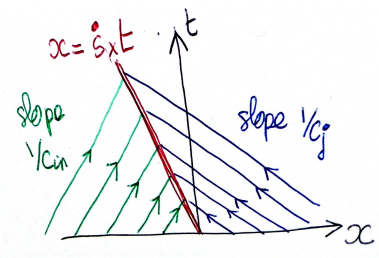


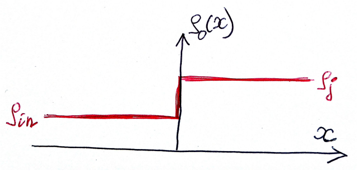


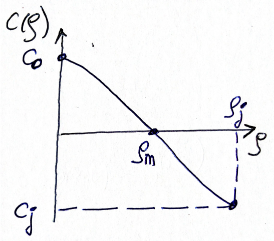
$\dot{s} = -Q_{in}/(\rho_j - \rho_{in}) = cst$ [remember $Q(\rho_j) = 0$]



Describe the motion of the end of a traffic jam

Assuming that there is a constant incoming flow of vehicles, density ρ_{in}, flux $Q_{in} = Q(\rho_{in})$, with $\rho_{in} < \rho_m$.





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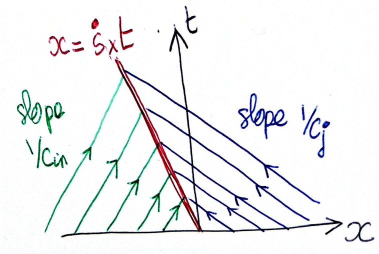

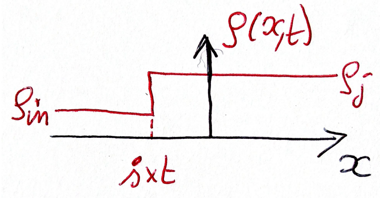

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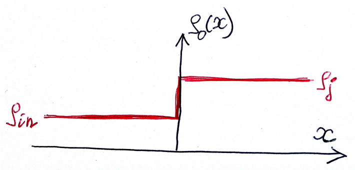

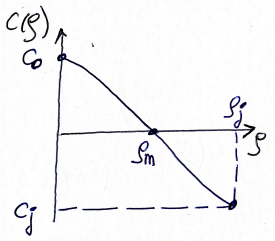

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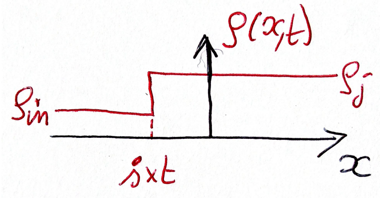
during dt, $Q_{in}dt$ cars arrive and pile up with an increase of density $\rho_j - \rho_{in}$:

$$\begin{cases} Q_{in}dt = (\rho_j - \rho_{in})|dx| \\ \text{with } |dx| = -\dot{s}\, dt \end{cases}$$



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