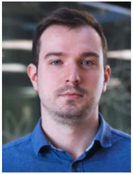
Modulation theory in two-component Bose-Einstein condensates: the ferromagnetic paradigm

T. Congy
Northumbria Univ, Newcastle

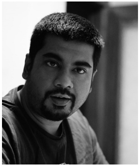


S. Ivanov
ICFO, Barcelona

A. Kamchatnov
ISAN, Troitsk

S. Bresolin
BEC Center, Trento

A. Recati
BEC Center, Trento



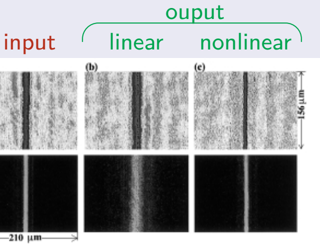
A. Roy
IIT, Mandy

dark-bright solitons in two components systems

Nonlinear optical medium

Segev group, Opt. Lett. 1996

2 carrier waves in a photorefractive crystal



N_1 and N_2 separately conserved

$$i\,\partial_t\psi_1 = -\tfrac{1}{2}\nabla^2\psi_1 + \left(g_{11}|\psi_1|^2 + g_{12}|\psi_2|^2\right)\psi_1$$

$$i\,\partial_t\psi_2 = -\tfrac{1}{2}\nabla^2\psi_2 + \left(g_{12}|\psi_1|^2 + g_{22}|\psi_2|^2\right)\psi_2$$

g_{11}, g_{12} and $g_{22} > 0$

Elongated BECs

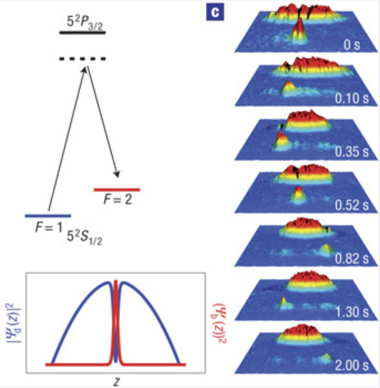
Theory: Busch & Anglin PRL 2001, Manakov regime

Experiment: Cornell group, PRL 2001, immiscible

Sengstock group, Nature Phys. 2008 :

^{87}Rb

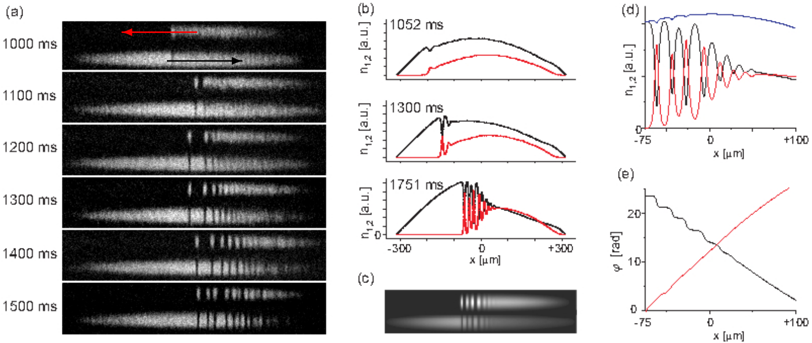
$|F=1, m_F=0\rangle \rightarrow |F=2, m_F=0\rangle$



^{87}Rb $\quad |F, m_F\rangle = |1,1\rangle \equiv |1\rangle$ (70%) and $|2,2\rangle \equiv |2\rangle$ (30%) with $g_{12} < \sqrt{g_{11} g_{22}}$

Dynamical instability if the relative velocity of the two components exceeds a critical value Law et al. PRA 2001

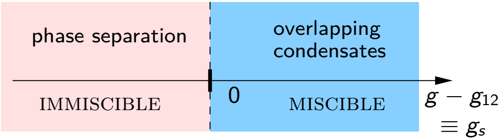
Engels group, PRL 2011 :



for $v \gtrsim v_{crit}$ dark/bright structure with a pedestal
for $v \gg v_{crit}$ dark/dark solitons Engels group PRA 2011

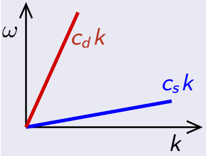
mean field miscibility criterion: $g_{12} < \sqrt{g_{11} g_{22}}$

consider the (realistic) case $g_{11} = g_{22} \equiv g$. Miscibility $\iff$ $g_{12} < g$



^{23}Na, $\vert 1, \pm 1\rangle$		$\frac{g_s}{g} \simeq 7$ %
^{87}Rb, $\vert 1, 1\rangle$	$\vert 2, 2\rangle$	$\frac{g_s}{g} \simeq 1$ %
^{87}Rb, $\vert 1, \pm 1\rangle$		$\frac{g_s}{g} \simeq -0.9$ %

U(1)×U(1) symmetry: two Goldstone modes $\quad |g_s| \ll g$



$$\begin{cases} m c_d^2 = \frac{n_0}{2}(g + g_{12}) \simeq g n_0 \\ m c_s^2 = \frac{n_0}{2}(g - g_{12}) = \frac{n_0}{2} g_s \leftarrow \end{cases}$$

length scale
$\xi_s = \hbar \left(2 m n_0 |g_s|\right)^{-1/2}$

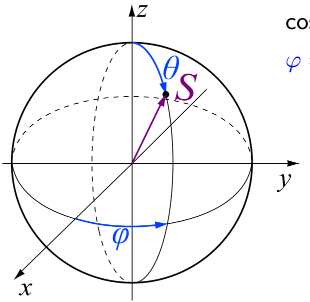
time scale
$\tau_s = \hbar / (n_0 |g_s|)$

Son & Stephanov 2002, Qu, Pitaevskii & Stringari 2016:
the degrees of freedom associated to **total and relative density fluctuations decouple**, even at the nonlinear level.

Mapping to Landau-Lifshitz equation

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \qquad i\,\partial_t \Psi = \left(-\frac{\nabla^2}{2} + \frac{\Omega_R}{2}\,\sigma_x + \frac{\delta}{2}\,\sigma_z \right) \Psi + \begin{pmatrix} g\,|\psi_1|^2 & g_{12}\psi_2^*\psi_1 \\ g_{12}\psi_1^*\psi_2 & g\,|\psi_2|^2 \end{pmatrix} \Psi$$

Relative density fluctuations $\iff$ spin degree of freedom



Bloch sphere

$$\cos\theta = (|\psi_1|^2 - |\psi_2|^2)/n_0$$

$$\varphi = \arg\psi_1 - \arg\psi_2$$

$$\boldsymbol{S} = \left| \begin{array}{l} \cos\varphi\,\sin\theta \\ \sin\varphi\,\sin\theta \\ \cos\theta \end{array} \right.$$

(dissipationless) Landau-Lifshitz eq.

$$\partial_t \boldsymbol{S} = \boldsymbol{S} \times (\boldsymbol{H}_{eff} + \boldsymbol{H}_{ext})$$

$$\boldsymbol{H}_{eff} = \nabla^2 \boldsymbol{S} - \epsilon\, S_z \boldsymbol{e}_z$$

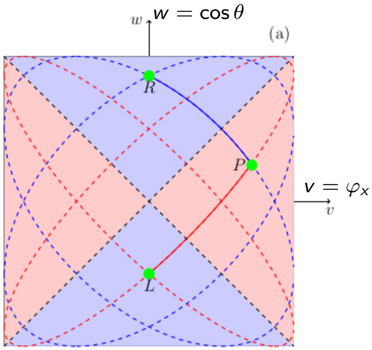
$$\boldsymbol{H}_{ext} = \omega_D \boldsymbol{e}_z + \omega_R \boldsymbol{e}_x$$

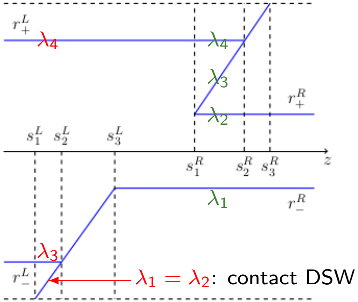
$$\omega_D = \delta/(|g_s|n_0) \quad \omega_R = \Omega/(g_s|n_0|)$$

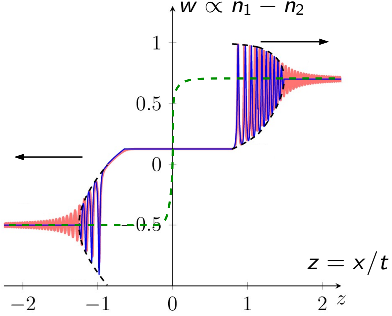
decoupled degrees of freedom

$$g_s \equiv g - g_{12}\,, \quad |g_s| \ll g$$

$$\epsilon = \frac{g_s}{|g_s|} = \begin{cases} +1 & \text{miscible, easy plane} \\ -1 & \text{immiscible, easy axis} \end{cases}$$

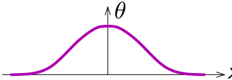


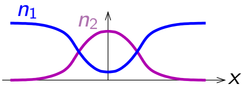


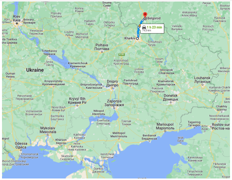


- finite region of hyperbolicity
- RI depend non-monotonously on physical variables
- the hyperbolocity region is divided in monotonicity sectors where the system is genuinely nonlinear
- Riemann problem →

Kosevich, Gann, Zhukov, Voronov (1998)



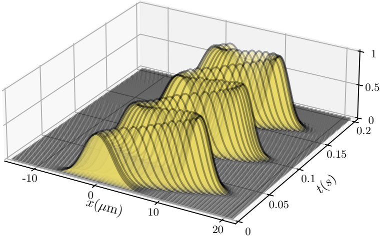




Kharkov – Belgorod collaboration

constant force:

$$\boldsymbol{H}_{ext} = \eta\, x\, \boldsymbol{e}_z$$

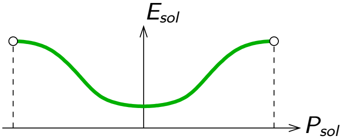


$$E = \int dx \left[|\partial_x \boldsymbol{S}|^2 - \boldsymbol{S} \cdot (\boldsymbol{H}_{eff} + \boldsymbol{H}_{ext}) \right]$$
$$= E_{sol} - \eta N_2 X_{sol}$$

$$P_{sol} = \int dx (\cos\theta - 1)\varphi_x$$
$$E_{sol} = A_{N_2} + B_{N_2} \times \sin^2(P_{sol}/4)$$

$$\frac{d}{dt} P_{sol} = -\partial E / \partial X_{sol} = \eta N_2$$

$$\frac{d}{dt} X_{sol} = \partial E / \partial P_{sol} = \partial E_{sol} / \partial P_{sol}$$



The minority component acts as a barrier for the majority component

$$\psi_1(x,t) = \sqrt{n_1}\,\exp(i\phi_1)$$

$$P_{sol} = \int dx\,(1-\cos\theta)\varphi_x = \phi_1\Big|_{-\infty}^{\infty} \equiv \Delta\phi_1$$

$$N_1^{right} = \int_{X_{sol}}^{\infty} n_1(x,t)dx$$

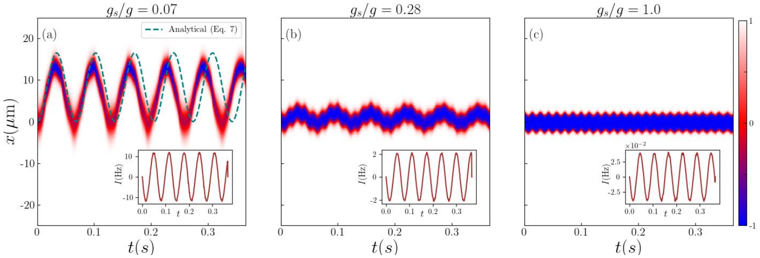
$$I_1(t) \equiv \tfrac{d}{dt}N_1^{right} = -n_0\,\tfrac{d}{dt}X_{sol}$$

where $n_0 = n_1 + n_2 = constant$

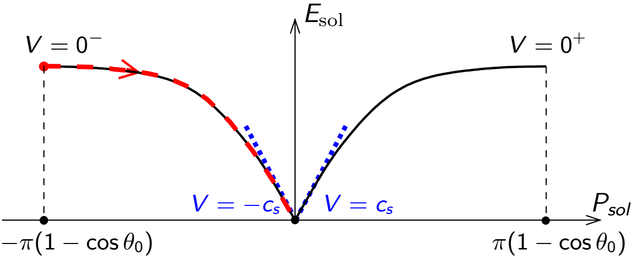
Josephson equations !

- $\frac{d}{dt}\Delta\phi_1 = \eta N_2 \equiv \Delta\mu$
- $I_1 = \mathcal{I}_0 \sin(\Delta\phi_1)$

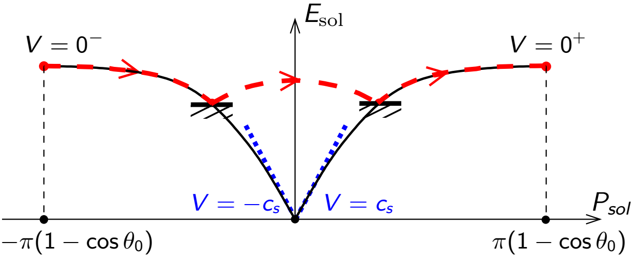
where $\mathcal{I}_0 = 2n_0/\sinh(N_2/2)$

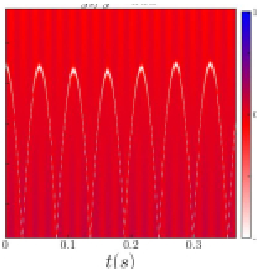


Periodic dispersion relation (50%-50% mixture $E_{sol} = |2\sin(P_{sol}/2)|$)



Periodic dispersion relation $\left(\text{50\%-50\% mixture } E_{sol} = |2\sin(P_{sol}/2)| \right)$

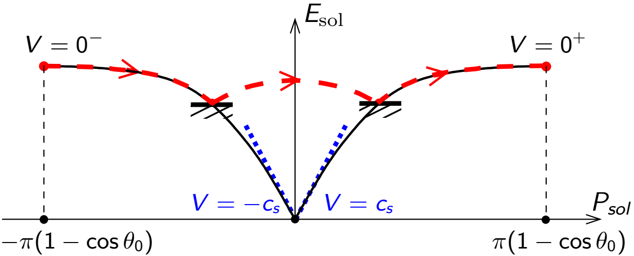


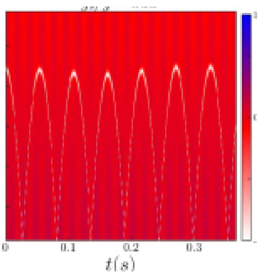


$\leftarrow \eta = 10^{-3}$

What about the easy plane soliton ?

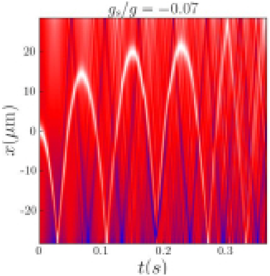
Periodic dispersion relation $\left(\text{50\%-50\% mixture } E_{sol} = |2\sin(P_{sol}/2)| \right)$





$\leftarrow \eta = 10^{-3}$

$\eta = 3 \times 10^{-2} \rightarrow$



CONCLUSION

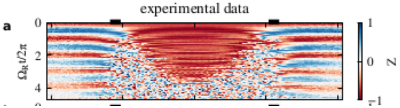
J. C. Maxwell (1870)

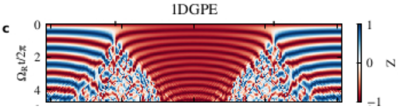
The recognition of the formal analogy between two systems leads to a knowledge of both, more profound than could be obtained by studying each system separately.

BEC as a platform to study ferromagnetic systems ☺

(almost) **perfect coherence** and **no dissipation** + tunable parameters

➪ Possible development: coherently coupled BEC $\Longleftrightarrow$ $\boldsymbol{H}_{ext} \parallel \boldsymbol{e}_x$





Farolfi et al. (2021)   Kamchatnov & Shaykin (2021)