Letter

Backreaction equations for 1 + 1 dimensional BEC sonic black holes

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As in the gravitational context, one of the most challenging open questions in analogue black holes formed in Bose-Einstein condensates concerns the backreaction of Hawking-like radiation on the condensate and its subsequent evolution. In this work we derive the basic equations describing this backreaction within the density-phase formalism, which avoids infrared divergences and is particularly well suited to one-dimensional configurations.

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In 1974 Hawking [1,2] showed that, taking into account quantum mechanics, black holes (BHs) are not at all "black" as believed but emit thermal radiation and hence "evaporate." The evolution of a BH driven by the emission of quantum fields is commonly referred to as the "backreaction."

In the absence of a complete and self consistent quantum gravity theory, this backreaction is described within the framework of quantum field theory in curved spacetime by the "semiclassical" Einstein equations [3]

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G \langle T_{\mu\nu}(\hat{\phi}, g_{\mu\nu}) \rangle, \tag{1}$$

where on the left-hand side (lhs) we have the Einstein tensor while the right-hand side (rhs) is the expectation values in the Unruh state [4] of the stress energy tensor operator for the quantum fields $\hat{\phi}$ (matter and gauge fields) evaluated on the spacetime metric $g_{\mu\nu}$ which is treated as a classical (not quantum) field. These equations, together with the field equation $F(\hat{\phi}, g_{\nu\nu}) = 0$ for the quantum fields $\hat{\phi}$, have to be solved self consistently for the metric $g_{\mu\nu}$ which would then describe the spacetime of the evaporating BH.

One expects this scheme to give a truthful description of the process valid on scales much bigger than the Planck scale ($\sim 10^{-33}$ cm) where quantum effects associated to the gravitational field can no longer be neglected. Unfortunately, even in the simplest case of spherical symmetry one does not know $\langle T_{\mu\nu}(\hat{\phi}, g_{\mu\nu}) \rangle$ for an arbitrary spherically symmetric BH metric. Some insight into the backreaction has come by considering a perturbative approach (*à la* Hartree-Fock) to Eq. (1). One considers first the classical Einstein vacuum equations $R_{\mu\nu} = 0$ (i.e., Eq. (1) with no quantum source on the rhs) leading to a stationary BH solution $g^0_{\mu\nu}$. In spherical symmetry $g^0_{\mu\nu}$ would be the metric of a Schwarzschild BH. Then one sets $g_{\mu\nu} = g^0_{\mu\nu} + \delta g_{\mu\nu}$, where $\delta g_{\mu\nu}$ represents the (supposed) small semiclassical correction to the background $g^0_{\mu\nu}$, and rewrite the backreaction equations (1) as

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G \langle T_{\mu\nu}(\hat{\phi}, g^0_{\mu\nu}) \rangle, \tag{2}$$

where the lhs has to be linearized in $\delta g_{\mu\nu}$ and on the rhs we have the expectation value of the energy-momentum tensor evaluated on the background $g^0_{\mu\nu}$ using for $\hat{\phi}$ the quantum field equation $F(\hat{\phi}, g^0_{\mu\nu}) = 0$. This should give us a hint on how the backreaction of the quantum fields starts to destabilise the classical solution.

From the information we have on $\langle T_{\mu\nu}(\hat{\phi}, g^0_{\mu\nu}) \rangle$ near the horizon for the Schwarzschild metric one can show that, as expected, as a consequence of the quantum emission the BH mass decreases and the horizon shrinks [5–7]. This shrinking is caused by the absorption by the hole of negative energy-density from the vacuum polarization surrounding the horizon. This is the backreaction in the near horizon region [3].

To overcome the problem of the lack of knowledge of the quantum source term in the backreaction equations (1), many efforts have been devoted to the study of two-dimensional (one space and one time dimension) BH models. In 2D, restricting ourselves to conformal invariant quantum fields, the expectation value $\langle T_{\mu\nu}(\hat{\phi}, g_{\mu\nu})\rangle$ for an arbitrary 2D spacetime metric is known [8]. However in 2D the lhs of Eq. (1) (the Einstein tensor) vanishes identically. So one has to modify the original gravitational theory (general relativity) by introducing extra classical fields (dilatons) besides the metric to obtain a nontrivial dynamics [9–11]. In these models one can solve the modified backreaction equations self consistently for the evaporating BH metric. However it is not clear how these 2D results are significant for the real 4D world.

It is by now well established that Hawking BH radiation is not peculiar to gravitational physics. As shown by Unruh [12], phonons in a fluid with a transonic flow experience an effective BH metric leading to an acoustic analog of Hawking radiation. This radiation has been (indirectly) detected in a series of experiments performed by J. Steinhauer [13–15] and his group using Bose-Einstein condensates (BECs).

Following this, the most natural question which comes to mind is how these BEC acoustic BHs evolve taking into account Hawking radiation, i.e., the backreaction [16–20]. Although we have some experimental insight on how this backreaction proceeds [15], we still lack a theoretical description of this process. The starting point seems promising: unlike gravity we have in this case a solid underlying quantum theory described by the Heisenberg equation for the fundamental Bose quantum operator $\hat{\Psi}(\vec{x}, t)$ [21]

$$i\hbar \frac{\partial \hat{\Psi}}{\partial t} = -\frac{\hbar^2}{2m}\vec{\nabla}^2\hat{\Psi} + [U - \mu + g\hat{\Psi}^{+}\hat{\Psi}]\hat{\Psi}, \tag{3}$$

which obeys the standard Bose equal time commutation rules

$$[\hat{\Psi}(\vec{x}, t), \hat{\Psi}^{\dagger}(\vec{x}', t)] = \delta^3(\vec{x} - \vec{x}'), \tag{4}$$

$$[\hat{\Psi}(\vec{x}, t), \hat{\Psi}(\vec{x}', t)] = 0 = [\hat{\Psi}^{\dagger}(\vec{x}, t), \hat{\Psi}^{\dagger}(\vec{x}', t)]. \tag{5}$$

In Eq. (3) μ is the chemical potential, $g\,(>0)$ the interatomic coupling, $U(\vec{x})$ the external potential and m is the mass of the bosonic particle.

Following the Bogoliubov approach [21] one splits the field operator $\hat{\Psi}$ in a classical field $\phi(\vec{x}, t)$ describing the condensate and a quantum field $\hat{\psi}$ describing the quantum fluctuation on top of the condensate, i.e.,

$$\hat{\Psi}(\vec{x}, t) = \phi(\vec{x}, t) + \hat{\psi}(\vec{x}, t), \tag{6}$$

with $\phi = \langle\hat{\Psi}\rangle$, hence $\langle\hat{\psi}\rangle = 0$. The expectation value of Eq. (3) reads [22]

$$i\hbar\partial_t\phi = \left[-\frac{\hbar^2}{2m}\vec{\nabla}^2 + U - \mu + g|\phi|^2 + 2g\tilde{n}\right]\phi + g\tilde{m}\phi^* + g\langle\hat{\psi}^{\dagger}\hat{\psi}\,\hat{\psi}\rangle, \tag{7}$$

where $\tilde{n} = \langle\hat{\psi}^{\dagger}\hat{\psi}\rangle$ and $\tilde{m} = \langle\hat{\psi}\,\hat{\psi}\rangle$ are denoted as normal and anomalous averages respectively. Equation (7) is the backreaction equation, while the quantum field $\hat{\psi}$ satisfies the quantum operator equation [22]

$$\begin{aligned} i\hbar\partial_t\hat{\psi} = {}& \left[-\frac{\hbar^2}{2m}\vec{\nabla}^2 + U - \mu + 2g|\phi|^2\right]\hat{\psi} + g\phi^2\hat{\psi}^{\dagger} \\ & + 2g\phi(\hat{\psi}^{\dagger}\hat{\psi} - \tilde{n}) + g\phi^*(\hat{\psi}\,\hat{\psi} - \tilde{m}) \\ & + g(\hat{\psi}^{\dagger}\hat{\psi}\,\hat{\psi} - \langle\hat{\psi}^{\dagger}\hat{\psi}\,\hat{\psi}\rangle). \end{aligned} \tag{8}$$

So far there is no approximation: Eqs. (7) and (8) describe self consistently the reciprocal effects of the condensate ϕ and the fluctuation $\hat{\psi}$. The expectation values entering the backreaction equation depend on ϕ which is then in principle determined self consistently solving Eq. (7). Unfortunately for a three (even two) spatial dimensional condensate the explicit form of these expectation values for a sonic black hole profile is out of reach of present calculations.

However, unlike the gravitational setting, with a BEC it is physically consistent to consider configurations with one spatial dimension. These are experimentally realized by freezing out the transverse degrees of freedom, and are the ones used by Steinhauer in his famous experiments [13–15]. So from now on we restrict ourselves to the one dimensional case which should in principle simplify computations. However the calculations of the expectation values entering Eq. (7) are much more complicated than in gravity since in the BEC the dispersion relation is a fourth order equation while in gravity it is second order. Not having even in one dimension the explicit form of the expectation values, one has to proceed perturbatively. The idea is similar to the one used in gravity [see Eq. (2)]. If one neglects the contributions of the quantum fields in Eq. (7) one obtains the Gross-Pitaevskii equation whose solution we denote as ϕ_{GP}

$$i\hbar\frac{\partial_t\phi_{GP}}{\partial t} = \left[-\frac{\hbar^2}{2m}\partial_x^2 + U - \mu + g|\phi_{GP}|^2\right]\phi_{GP}. \tag{9}$$

Similarly, in the quantum equation (8) we neglect the second and third order terms and the expectation values and replace ϕ by ϕ_{GP} obtaining the Bogoliubov-de Gennes equation for the quantum field $\hat{\psi}$ describing the fluctuations above the condensate background ϕ_{GP}, namely

$$i\hbar\frac{\partial\hat{\psi}}{\partial t} = \left[-\frac{\hbar^2}{2m}\partial_x^2 + U - \mu + 2g|\phi_{GP}|^2\right]\hat{\psi} + g\phi_{GP}^2\hat{\psi}^{\dagger}. \tag{10}$$

Our aim is to calculate the deviation from the leading order classical background ϕ_{GP} induced by quantum fluctuations. We set $\phi = \phi_{GP} + \delta\phi$ and rewrite the backreaction equation (7) linearizing in $\delta\phi$ and including quantum contributions up to second order in $\hat{\psi}$

$$i\hbar\partial_t\delta\phi = \left[-\frac{\hbar^2}{2m}\partial_x^2 + U - \mu + 2g|\phi_{GP}|^2\right]\delta\phi + g\phi_{GP}^2\delta\phi^* + 2g\tilde{n}_{GP}\phi_{GP} + g\tilde{m}_{GP}\phi_{GP}^*, \tag{11}$$

where the expectation values $\tilde{n}_{GP}$ and $\tilde{m}_{GP}$ are calculated on the Gross-Pitaevskii background ϕ_{GP} using the Bogoliubov-de Gennes Eq. (10) for the quantum field $\hat{\psi}$.

Unfortunately this scheme does not work as we face immediately a problem: $\tilde{n}_{GP}$ and $\tilde{m}_{GP}$ are infrared divergent in one dimension, preventing us to calculate the backreaction using Eq. (11).

To overcome this problem we will use for the fundamental quantum field $\hat{\Psi}$ an amplitude-phase formalism proposed first by Popov [23,24]. We rewrite $\hat{\Psi}$ as

$$\hat{\Psi} = e^{i(\theta+\hat{\theta})}\sqrt{\rho(1+\hat{\eta})}, \tag{12}$$

where θ and ρ are respectively the classical phase and density of the condensate field and $\hat{\theta}$ and $\hat{\eta}$ the related phase and density fluctuations. These operators satisfy the commutation rule

$$[\hat{\eta}(x,t), \hat{\theta}(x',t)] = \frac{i}{\rho}\delta(x-x') \tag{13}$$

and $\langle\hat{\eta}\rangle = 0 = \langle\hat{\theta}\rangle$.

Expanding the field $\hat{\Psi}$ as given in (12) in terms of $\hat{\theta}$ and $\hat{\eta}$ and comparing the resulting expectation value with that of (6) yields

$$\phi = \varphi\left(1 + \frac{i}{2}\langle\hat{\theta}\,\hat{\eta}\rangle - \frac{1}{2}\langle\hat{\theta}^2\rangle - \frac{1}{8}\langle\hat{\eta}^2\rangle + ..\right), \tag{14}$$

where $\varphi \equiv \sqrt{\rho}e^{i\theta}$. Expression (14) shows that, whereas the classical fields ϕ and φ are identical at leading (Gross-Pitaevskii) order, they differ at higher order. More importantly, at the order of our treatment, φ is a well defined quantity which obeys a regular equation [see Eqs. (15) and (16) below] while ϕ does not.

As done before we consider a small deviation from the Gross-Pitaevskii solution $\varphi_{GP} = \sqrt{\rho_{GP}}e^{i\theta_{GP}}$, i.e. $\varphi = \varphi_{GP} + \delta\varphi$. Setting for the density and phase backreaction corrections $\rho = \rho_{GP} + \delta\rho$, $\theta = \theta_{GP} + \delta\theta$, we have $\frac{\delta\varphi}{\rho_{GP}} = \frac{\delta\rho}{2\rho_{GP}} + i\delta\theta$. Separating real and imaginary parts in Eq. (11) we obtain [25]

$$\hbar(\partial_t + v_{GP}\partial_x)\delta\theta - \frac{\hbar^2}{4m\rho_{GP}}\partial_x\left(\rho_{GP}\partial_x\left(\frac{\delta\rho}{\rho_{GP}}\right)\right) + g\delta\rho = -\frac{g\rho_{GP}}{2}g^{(2)} - \frac{\hbar}{2}(\partial_t + v_{GP}\partial_x)\mathrm{Re}\langle\hat{\eta}\,\hat{\theta}\rangle - \frac{\hbar^2}{4m\rho_{GP}}\partial_x\left[\rho_{GP}\partial_x\left(\langle\hat{\theta}^2\rangle - \frac{\delta(0)}{4\rho_{GP}} + \frac{g^{(2)}}{4}\right)\right], \tag{15}$$

$$\partial_t\delta\rho + \partial_x(v_{GP}\delta\rho + \rho_{GP}\delta v + \mathrm{Re}\langle\rho_{GP}\hat{\eta}\,\hat{v}\rangle) = 0, \tag{16}$$

where $\delta v = \frac{\hbar}{m}\partial_x\delta\theta$, $\hat{v} = \frac{\hbar}{m}\partial_x\hat{\theta}$, $v_{GP} = \frac{\hbar}{m}\partial_x\theta_{GP}$.

All expectation values have to be taken on the Gross-Pitaevskii background φ_{GP} using the Bogoliubov-de Gennes equation in the density-phase representation

$$\hbar(\partial_t + v\partial_x)\hat{\theta} - \frac{\hbar^2}{4m\rho_{GP}}\partial_x(\rho_{GP}\partial_x\hat{\eta}) + g\rho_{GP}\hat{\eta} = 0, \tag{17}$$

$$(\partial_t + v_{GP}\partial_x)\hat{\eta} + \frac{\hbar}{m\rho_{GP}}\partial_x(\rho_{GP}\partial_x\hat{\theta}) = 0. \tag{18}$$

In Eq. (15)

$$g^{(2)} = \langle\hat{\eta}^2\rangle - \frac{\delta(0)}{\rho_{GP}}, \tag{19}$$

which because of the Dirac $\delta(0)$ contribution is ultraviolet regular in one dimension.

Note that $\langle\hat{\theta}^2\rangle$ is infrared divergent in one dimension, but this divergence is killed by acting on it with the spatial derivative in Eq. (15) [26]. This is analogous to the situation in two-dimensional gravity, where the two-point function is infrared divergent, but contributes to the field equation only through its appearance in $\langle\hat{T}_{\mu\nu}\rangle$, which involves derivatives and is therefore infrared regular. So our one dimensional backreaction equations (15), (16) are both ultraviolet and infrared finite.

We note that for $v_{GP} = 0$ our backreaction equations reduce to those obtained by Mora and Castin [27] with a more rigorous approach by discretizing space. To solve the backreaction equations one needs to determine the quantum source terms in the whole physical space. As shown in [28] this can be accurately done only by taking into account also the zero modes solutions of the Bogoliubov-de Gennes equation and this is particularly critical near the horizon of a sonic BH. The discussion of this delicate point is postponed to a future work.

As a simple example of the use of the backreaction equations we consider the case of a uniform condensate for which there is no contribution of the zero modes. For a homogeneous condensate the field operators can be expanded in terms of plane waves as

$$\hat{\eta} = \frac{1}{\sqrt{\rho_{GP}}}\int_0^{+\infty} d\omega(u_\omega + v_\omega)\hat{b}_\omega e^{-i\omega t + ik(\omega)x} + \mathrm{H.c.}, \tag{20}$$

$$\hat{\theta} = \frac{1}{2i\sqrt{\rho_{GP}}}\int_0^{+\infty} d\omega(u_\omega - v_\omega)\hat{b}_\omega e^{-i\omega t + ik(\omega)x} + \text{H.c.}, \quad (21)$$

where the dispersion relation reads

$$(\omega - v_{GP}k)^2 = c_{GP}^2\left(k^2 + \xi_{GP}^2\frac{k^4}{4}\right), \quad (22)$$

$c_{GP} = \sqrt{\frac{g\rho_{GP}}{m}}$ is the Gross-Pitaevskii speed of sound and $\xi_{GP}(= \hbar(mg\rho_{GP})^{-1/2})$ the corresponding healing length. The Bogoliubov modes are [29–31]

$$u_\omega = \frac{\omega - v_{GP}k + \frac{c_{GP}\xi_{GP}k^2}{2}}{\sqrt{4\pi c_{GP}\xi_{GP}k^2|(\omega - v_{GP}k)(\frac{dk}{d\omega})^{-1}|}}, \quad (23)$$

$$v_\omega = -\frac{\omega - v_{GP}k - \frac{c_{GP}\xi_{GP}k^2}{2}}{\sqrt{4\pi c_{GP}\xi_{GP}k^2|(\omega - v_{GP}k)(\frac{dk}{d\omega})^{-1}|}}. \quad (24)$$

The operators $\hat{b}_\omega, \hat{b}_\omega^\dagger$ satisfy standard Bose commutation rules. The backreaction equations simplify to $(\delta\mu = -\hbar\partial_t\delta\theta)$

$$g\delta\rho + mv_{GP}\delta v = \delta\mu - \frac{1}{2}g\rho_{GP}g^{(2)}, \quad (25)$$

$$v_{GP}\delta\rho + \rho_{GP}\delta v + \text{Re}\langle\rho_{GP}\hat{\eta}\,\hat{v}\rangle = \text{const} = \delta J. \quad (26)$$

Using the expansions (20), (21) one gets

$$g^{(2)} = -\frac{2}{\pi\rho_{GP}\xi_{GP}}, \quad (27)$$

$$\text{Re}\langle\rho_{GP}\hat{\eta}\,\hat{v}\rangle = 0, \quad (28)$$

which we note are independent of v_{GP}. So for a condensate at rest ($v_{GP} = 0$) the result reads [27]

$$\delta\rho = -\frac{1}{2}\rho_{GP}g^{(2)} = \frac{1}{\pi\xi_{GP}}, \quad (29)$$

$$\delta v = 0 = \delta\mu. \quad (30)$$

From $\mu = g\rho_{GP}$ and $\rho_{GP} = \rho - \delta\rho$ we obtain the equation of state in the form

$$\mu = g\rho - \frac{g}{\pi\xi}. \quad (31)$$

Using the thermodynamical relation $mc^2 = \rho(\frac{\partial\mu}{\partial\rho})$ we obtain, for the speed of sound, [32]

$$c = \sqrt{\frac{g\rho}{m}}\sqrt{1 - \frac{1}{2\pi\rho\xi}}, \quad (32)$$

which corrects the Gross-Pitaevskii result c_{GP}. Galilean invariance imposes that this result remains unaltered in the presence of a finite flow velocity $v_{GP} \neq 0$. Eqs. (31) and (32) indicate that the small parameter of our approach is the quantity $1/\rho\xi$, which is of order 2×10^{-2} in Steinhauer's experiments [13].

From Eq. (32) we can calculate the correction to the speed of sound and hence the correction to Gross-Pitaevskii Mach number $M_{GP} = \frac{v_{GP}}{c_{GP}}$, namely

$$\frac{\delta M}{M_{GP}} = \frac{\delta v}{v_{GP}} - \frac{\delta c}{c_{GP}} = -\frac{1}{2}\frac{\delta\rho}{\rho_{GP}} + \frac{1}{4\pi\xi_{GP}\rho_{GP}} = -\frac{1}{4\pi\xi_{GP}\rho_{GP}}. \quad (33)$$

Here the backreaction, by changing the density and hence the speed of sound, decreases the Mach number from the Gross-Pitaevskii value.

Coming to non homogeneous configurations, most of the sonic BH flow profiles discussed in the literature and also experimentally realized contain a subsonic asymptotic region (say $x \to +\infty$) where the condensate is homogeneous and, similarly, a homogeneous supersonic asymptotic region (say $x \to -\infty$ for a fluid flowing from right to left). Let us call the first asymptotic region u (for upstream) and the other d (downstream). In these regions the backreaction equations for a stationary asymptotically homogeneous flow read

$$g\delta\rho_\alpha + mv_{GP}^\alpha\delta v_\alpha = \delta\mu - \frac{1}{2}g\rho_{GP}^\alpha g_\alpha^{(2)}, \quad (34)$$

$$v_{GP}^\alpha\delta\rho_\alpha + \rho_{GP}^\alpha\delta v_\alpha = \delta J - J_\alpha, \quad (35)$$

where $\alpha = u,\ d$ and $J_\alpha = \text{Re}\langle\rho_{GP}\hat{\eta}\,\hat{v}\rangle_\alpha$. The quantities labeled with α in these equations are asymptotic quantities in the far upstream and downstream regions. The general structure of $g_\alpha^{(2)}$ for a sonic BEC BH is the following

$$g_\alpha^{(2)} = -\frac{2}{\pi\rho_{GP}^\alpha\xi_{GP}^\alpha} + g_\alpha^{(H)}, \quad (36)$$

where the first term reproduces Eq. (27) while $g_\alpha^{(H)}$ is induced by Hawking radiation and vanishes in the absence of a sonic horizon. This splitting is reminiscent of the analogous one present in the 2D gravitational $\langle T_{\mu\nu}\rangle$ in the Unruh state, which can be splitted in a vacuum polarization (Boulware) term plus the Hawking radiation contribution. On the other hand, the flux term J_α comes (like in gravity) only from Hawking radiation.

Simple manipulations of Eqs. (34), (35) lead to the asymptotic variations of the condensate density as

$$\delta\rho_\alpha = \frac{\delta\mu - \frac{1}{2} g\rho^\alpha_{GP} g^{(2)}_\alpha - \frac{m v^\alpha_{GP}}{\rho^\alpha_{GP}}(\delta J - J_\alpha)}{g\left(1 - \frac{(v^\alpha_{GP})^2}{(c^\alpha_{GP})^2}\right)}, \qquad (37)$$

and δv_α can be obtained inserting this in the continuity equations (35). For our perturbative approach to be valid (i.e., $\frac{\delta\rho_\alpha}{\rho^\alpha_{GP}} \ll 1$) even in the case when the asymptotic regions approach the sonic limit (i.e., $|v^\alpha_{GP}| \to c^\alpha_{GP}$) $\delta\mu$ and δJ have to be such that the numerator of Eq. (37) vanishes in this limit. Example of interesting sonic BH profiles [33] with the explicit calculations of the source terms $g^{(2)}$ and J_α and the corresponding $\delta\rho_\alpha$ and δv_α are given elsewhere [25].

As we have said the calculation of the quantum source terms in the backreaction equations for BEC black holes is much more complicated than in gravity because of the fourth order dispersion relation and the contribution of zero modes (which are not present in gravity) in particular in the horizon region. However we have now a consistent theoretical scheme to attack the backreaction problem for one dimensional condensates and moreover we should not forget that in the BEC case we have a significant advantage we do not have in gravity: we can have experimental insights to guide our theoretical investigations.

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