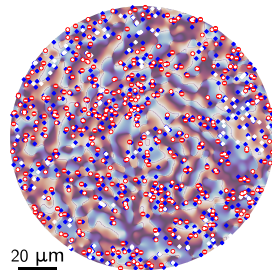
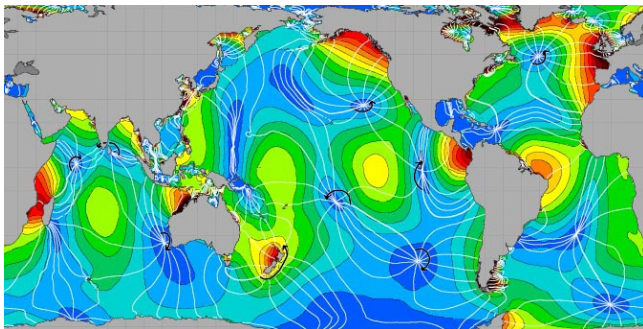


Revisiting Anderson's phase-slip mechanism from a topological perspective

Emerging Applications of Topological Textures, Heraklion, Sept. 2026

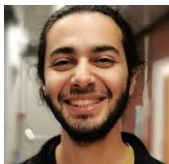
Nicolas Pavloff, LPTMS, Université Paris-Saclay



Revisiting Anderson's phase-slip mechanism from a topological perspective

Emerging Applications of Topological Textures, Heraklion, Sept. 2026

Nicolas Pavloff, LPTMS, Université Paris-Saclay



M. Baker-Rasooli
LKB, Paris



N. Du Toit
LPTMS, Paris-Saclay



Q. Glorieux
LKB, Paris

$$\psi(\mathbf{r}, t) = A \exp(i \chi)$$

→ ground state, stationary and uniform: $A(\mathbf{r}, t) = A_0$, $\chi(\mathbf{r}, t) = -\mu_0 t / \hbar$

→ Galilean boost, velocity $\mathbf{v}_0$: $\chi(\mathbf{r}, t) = (-\mu t + m \mathbf{v}_0 \cdot \mathbf{r}) / \hbar$ ($\mu = \mu_0 + \frac{1}{2} m v_0^2$)

→ non uniform velocity: $\chi(\mathbf{r}, t)$ with $\mathbf{v}(\mathbf{r}, t) = \frac{\hbar}{m} \nabla \chi$

$$\psi(\mathbf{r}, t) = A \exp(i\chi)$$

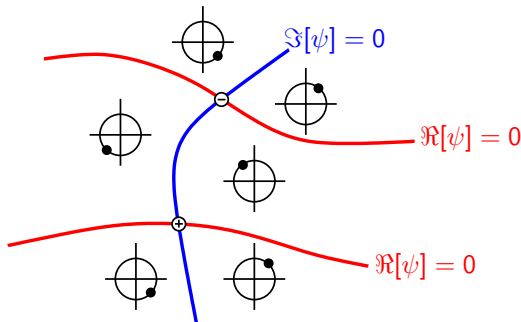
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This imposes $\nabla \times \mathbf{v} = 0$: irrotational flow.

Except if there are points $\mathbf{r}_i$ where χ is ill defined, i.e. where $A(\mathbf{r}_i) = 0$.



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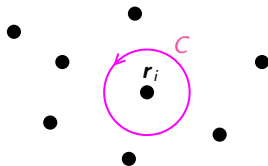
Except if there are points $\mathbf{r}_i$ where χ is ill defined, i.e. where $A(\mathbf{r}_i) = 0$.

Then, in 2D $\nabla \times \mathbf{v} = \omega(\mathbf{r}, t) \mathbf{e}_z$ with $\omega(\mathbf{r}, t) = \sum_i \Gamma_i \delta(\mathbf{r} - \mathbf{r}_i(t))$.

Onsager-Feynman quantization condition

$$\oint_C \mathbf{v} \cdot d\mathbf{r} = \Gamma_i = \frac{\hbar}{m} \oint_C d\chi = \frac{\hbar}{m} 2\pi n_i$$

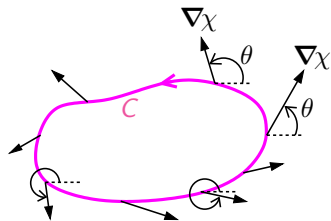
$$n_i \in \mathbb{Z}, \quad \text{typically } n_i = \pm 1$$



$$\mathbf{r} = x\mathbf{e}_x + y\mathbf{e}_y$$

$$\Psi(\mathbf{r}) = A(\mathbf{r}) \exp\{i\chi(\mathbf{r})\}$$

$$\mathbf{v} = \frac{\hbar}{m} \nabla \chi$$



$$I_V(C) = \frac{1}{2\pi} \oint_C \nabla \chi \cdot d\ell = \oint_C \frac{d\chi}{2\pi}$$

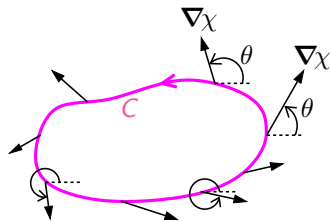
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I_V : vorticity, I_P : Poincaré index

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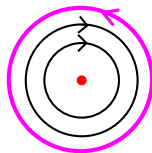


$$l_V(C) = \frac{1}{2\pi} \oint_C \nabla \chi \cdot d\ell = \oint_C \frac{d\chi}{2\pi}$$

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vortex



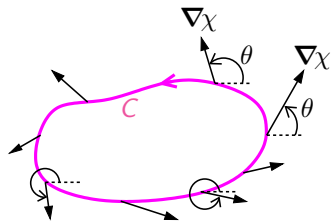
$$l_V \in \mathbb{Z}$$

$$l_P = +1$$

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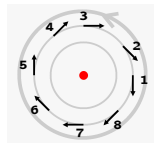


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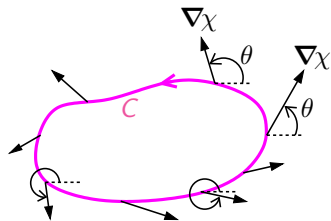
$$l_V < 0$$

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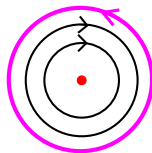


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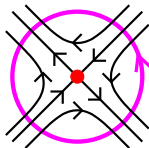
vortex



$$l_V \in \mathbb{Z}$$

$$l_P = +1$$

saddle



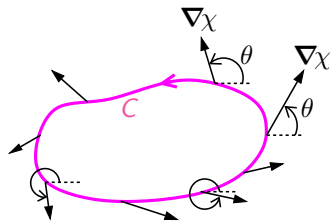
$$l_V = 0$$

$$l_P = -1$$

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$$\Psi(\mathbf{r}) = A(\mathbf{r}) \exp\{i\chi(\mathbf{r})\}$$

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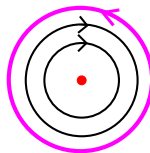


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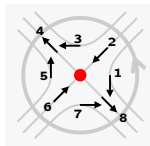
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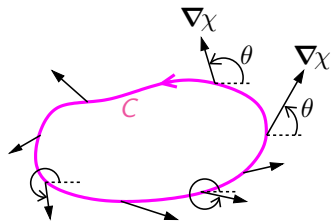
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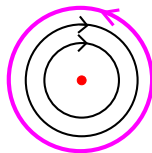


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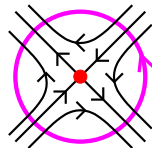
vortex



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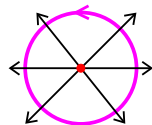


$$l_V = 0$$

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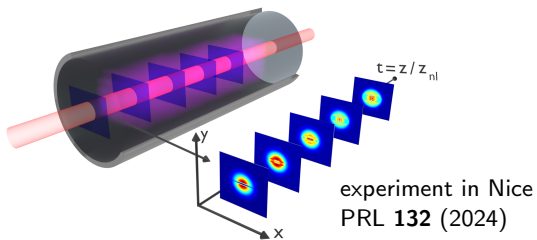
node

min/max of
the phase

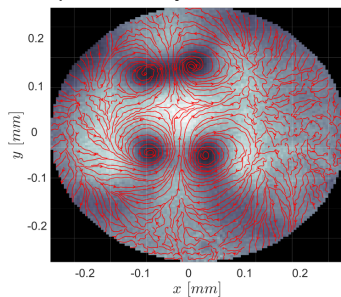


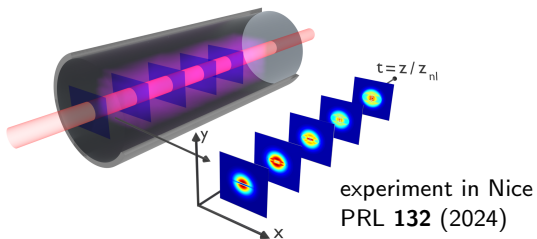
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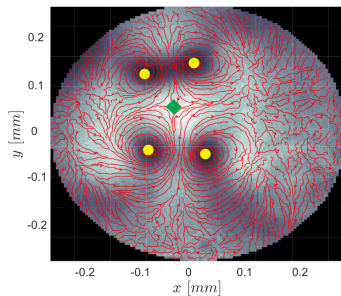


output intensity and streamlines





output intensity and streamlines



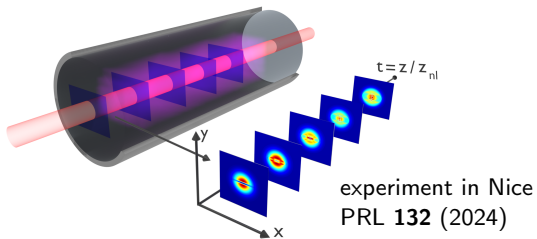
$$\mathbf{E}(z, \mathbf{r}_\perp, t) = \underbrace{\mathcal{A}(z, \mathbf{r}_\perp)}_{\text{complex amplitude}} \times \exp\{i(k_0 z - \omega t)\} \mathbf{e}_z$$

complex amplitude $\times$ carrier wave

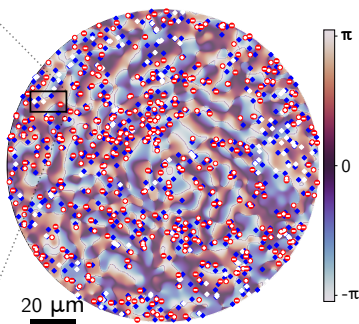
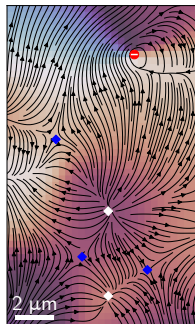
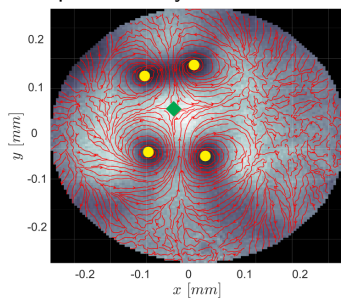
paraxial approximation:

$$i\partial_z \mathcal{A} = -\frac{1}{2n_0 k_0} \nabla_\perp^2 \mathcal{A} + k_0 n_2 |\mathcal{A}|^2 \mathcal{A} - \frac{i}{\Lambda_{\text{abs}}} \mathcal{A}$$

$$t_{\text{eff}} = z/z_{\text{nl}} \text{ with } z_{\text{nl}} = (k_0 n_2 |\mathcal{A}|^2)^{-1}$$



output intensity and streamlines



experiment in Lecce
PRR 7 (2025)

direct measure of the (large)
number of vortices, saddles and
nodes as a function of time

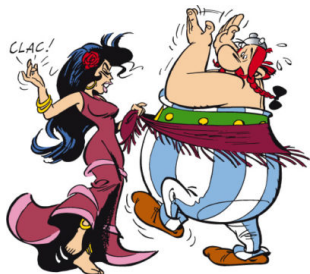
2D : vorticity, $\nabla \times \mathbf{v} = \omega(\mathbf{r}, t) \mathbf{e}_z$

N point vortices

vortex i , position $\mathbf{r}_i(t)$, circulation Γ_i : $\omega(\mathbf{r}, t) = \sum_i \Gamma_i \delta(\mathbf{r} - \mathbf{r}_i(t))$

Kirchhoff 1876:

$$H = -\frac{1}{2\pi} \sum_{i>j} \Gamma_i \Gamma_j \ln |\mathbf{r}_i - \mathbf{r}_j|, \quad \text{writing } \mathbf{r}_i(t) = (x_i, y_i) \text{ yields } \begin{cases} \Gamma_i \dot{x}_i = \partial H / \partial y_i \\ \Gamma_i \dot{y}_i = -\partial H / \partial x_i \end{cases}$$



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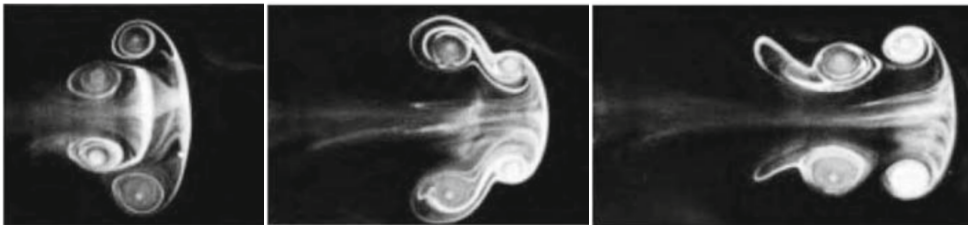
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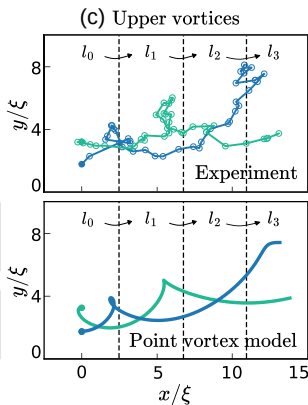
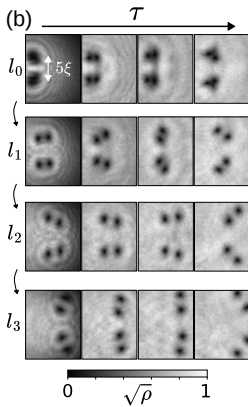
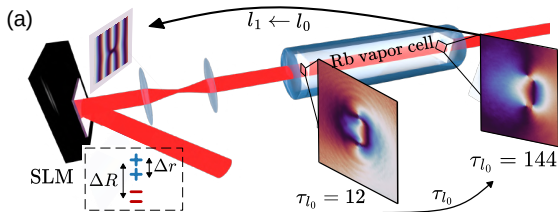
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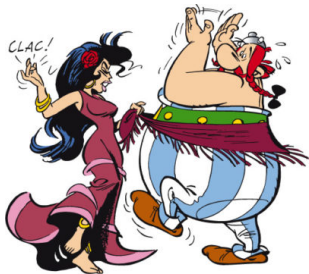
Kirchhoff 1876:

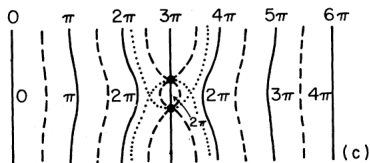
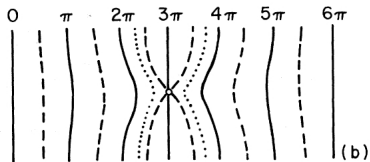
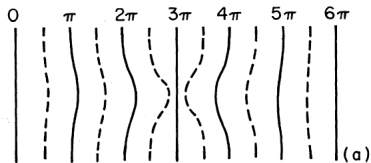
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Yamada, Matsui, Phys. Fluids 1978



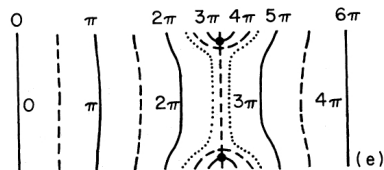
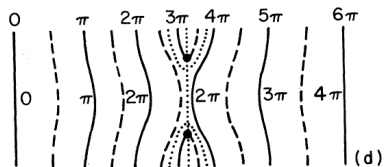


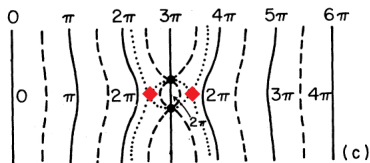
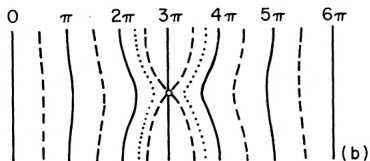
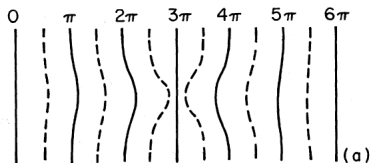




Anderson, RMP (1966)

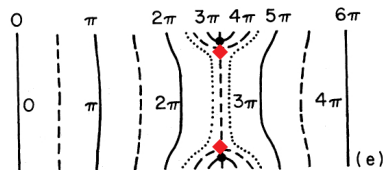
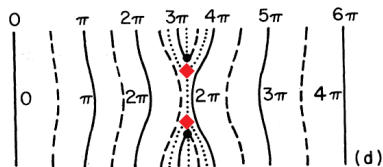
Langer, Fisher, PRL (1967)

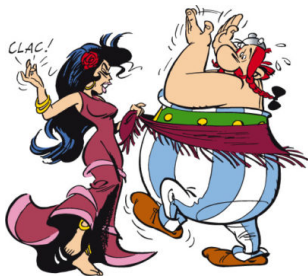
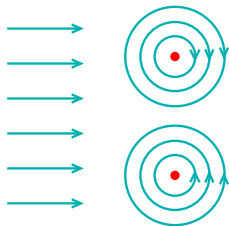


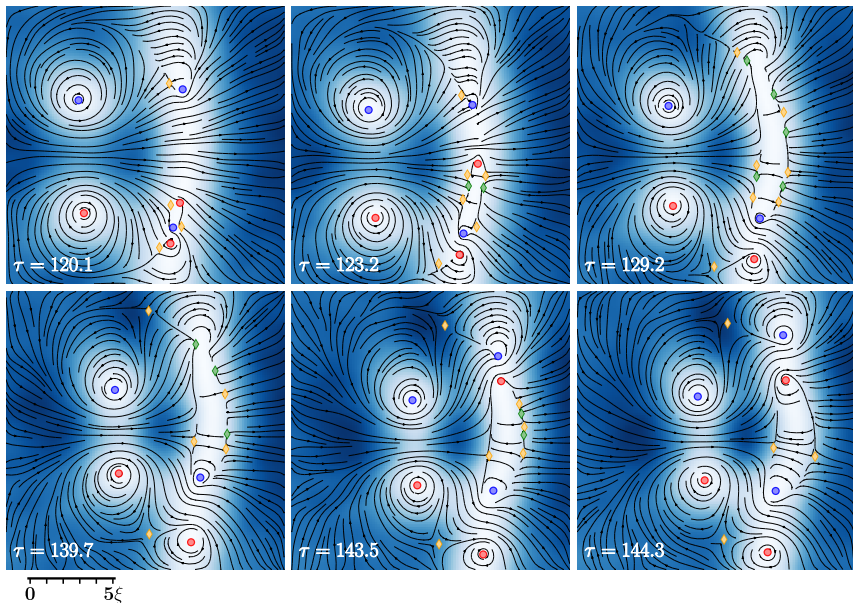


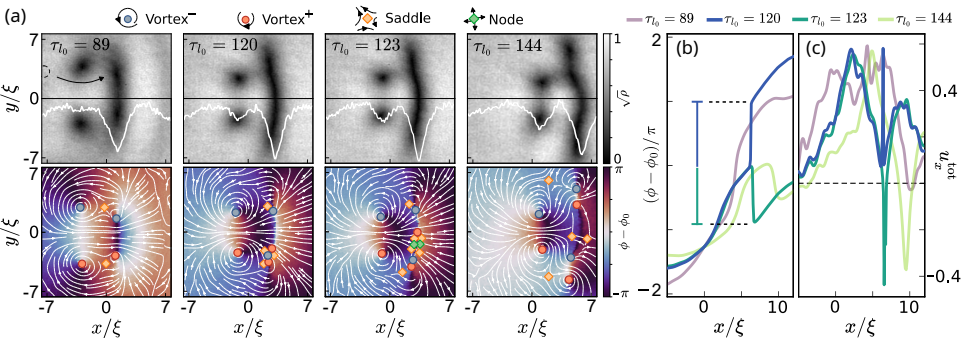
Anderson, RMP (1966)

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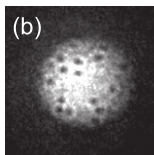
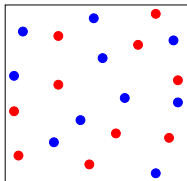
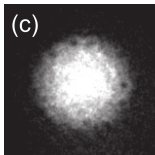
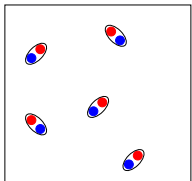
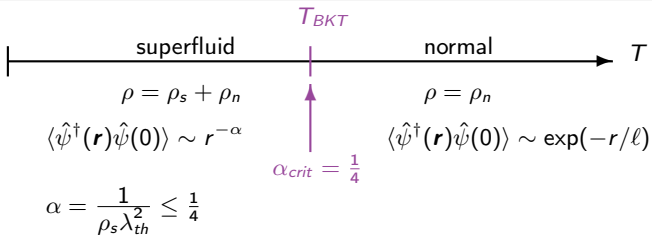








Berezinskii-Kosterlitz-Thouless transition



Shin group (Seoul)
PRL 2013

below T_{BKT}

$$\langle \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(0) \rangle \sim r^{-\alpha} \times fct(r/\ell_c(t))$$

dynamical scaling: $\ell_c(t) \propto t^{1/2}$

vortex-antivortex annihilation:

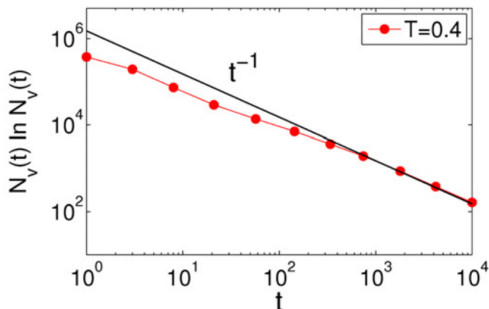
$$\ell_c(t) \propto (t/\ln t)^{1/2}$$

ℓ_c : vortex-antivortex distance
relaxation dynamics:

$$v = \text{mobility} \times F$$

$\frac{d\ell_c}{dt}$ (blue arrow from mobility)
 $1/\ln(\ell_c/\xi)$ (red arrow from F)
 $1/\ell_c$ (green arrow from F)

$$\# \text{vortices} = (L/\ell_c)^2 \propto \frac{\ln t}{t}$$



Jelić & Cugliandolo, J. Stat. Mech. 2011
see also Chu & Williams, PRL 2001

below T_{BKT}

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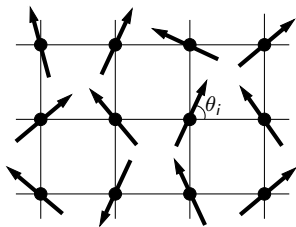
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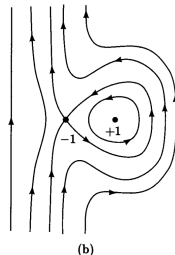
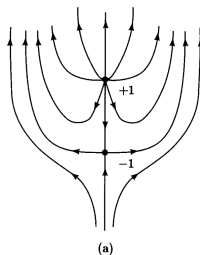
$$v = \text{mobility} \times F$$

↓ $\frac{d\ell_c}{dt}$
 ↓ $1/\ln(\ell_c/\xi)$
 ↓ $1/\ell_c$

$$\# \text{vortices} = (L/\ell_c)^2 \propto \frac{\ln t}{t}$$



$$H = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j)$$



XY vortex pairs (Chaikin-Lubensky)

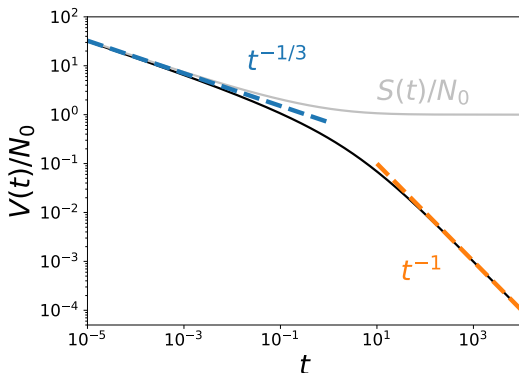
Bristol NHH mechanism: $\text{vortex}_{(+)} + \text{vortex}_{(-)} + \text{saddle} + \text{saddle} \xrightarrow{e} \emptyset$

$V_{(+)} = V_{(-)} = \frac{1}{2}V$, also $S = V + N$ and $N = N_0 \ll V(t=0)$

$$\frac{dV}{dt} = -\frac{e}{2}V^2S^2 = -\frac{e}{2}V^2(N_0 + V)^2$$

Initially, $V \gg N_0$: annihilation ≈ 4 vortices process $\sim V \propto t^{-1/3}$

At late time, $V \ll N_0$: annihilation ≈ 2 vortices process $\sim V \propto t^{-1}$



quantum vortices

carry

2 *topological indices*

quantum vortices

carry

2 *topological indices*

mechanisms of vortex formation/annihilation:

- fulfill **topological** and **quantum** constraints
- Experimentally relevant

Involve critical points $\neq$ vortices:

candidate observables for studying the transition to turbulence and its decay

quantum vortices

carry

2 *topological indices*

mechanisms of vortex formation/annihilation:

- fulfill **topological** and **quantum** constraints
- Experimentally relevant

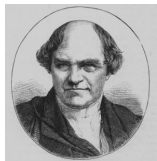
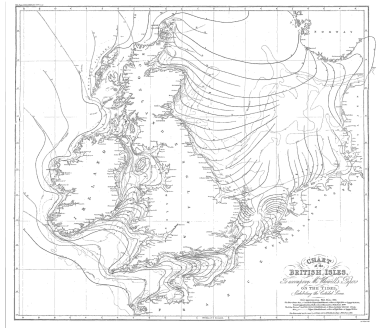
Involve critical points $\neq$ vortices:

candidate observables for studying the transition to turbulence and its decay

 concepts of singular optics relevant for quantum fluids

 Thermodynamic counterpart in BKT transition ?
Relevant for quench dynamics !

 Can one think of other nonlinear phenomena that are topologically constrained?

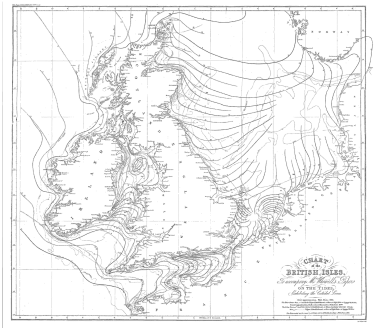


W. Whewell
Phil. Trans. R. Soc. 1836

Principal tidal constituent (M2: semi-diurnal)



cotidal lines and amphidromic points. Whewell 1836

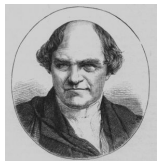
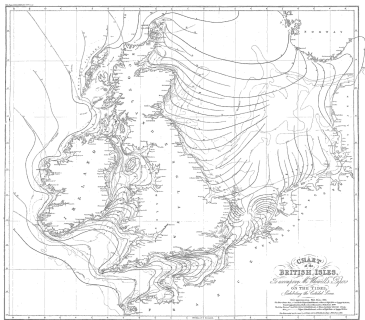


W. Whewell
Phil. Trans. R. Soc. 1836

Principal tidal constituent (M2: semi-diurnal)



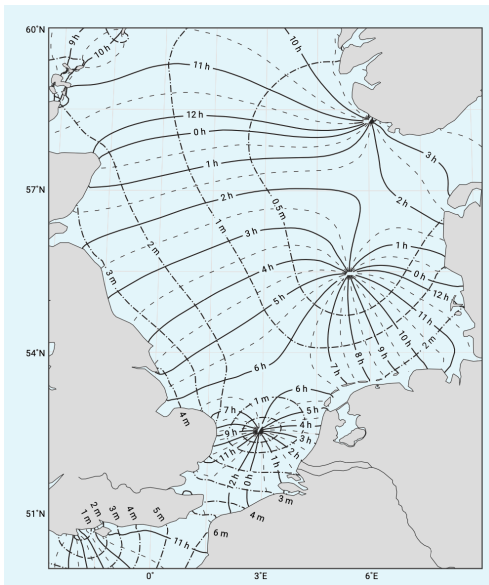
cotidal lines and amphidromic points. Whewell 1836



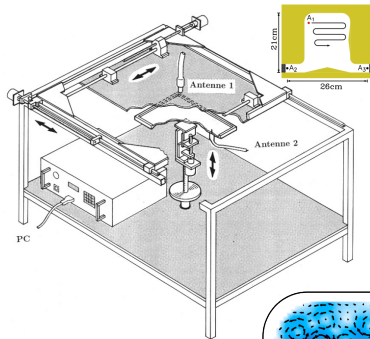
W. Whewell
Phil. Trans. R. Soc. 1836

$$\psi(\mathbf{r}, t) = h(\mathbf{r}) \exp\{i(\chi(\mathbf{r}) - \omega t)\}$$

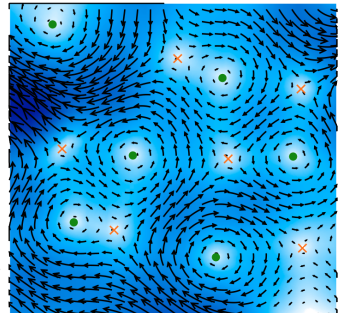
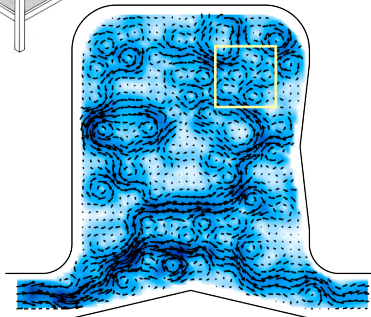
where $\omega = 2\pi/T$, $T = 12 \text{ h } 25 \text{ mn}$



co-range and cotidal lines



U. Kuhl, Eur. Phys. J. (2007)



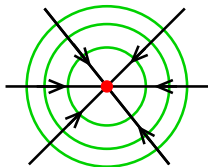
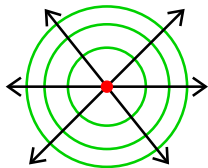
In a quantum fluid

$$\Psi(\mathbf{r}) = A(\mathbf{r}) \exp\{i\chi(\mathbf{r})\}$$

$$\mathbf{v} = \nabla\chi$$

phase χ : velocity potential

min or max of χ :



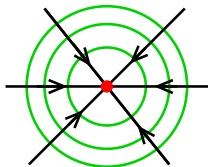
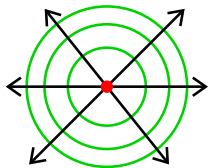
In a quantum fluid

$$\Psi(\mathbf{r}) = A(\mathbf{r}) \exp\{i\chi(\mathbf{r})\}$$

$$\mathbf{v} = \nabla\chi$$

phase χ : velocity potential

min or max of χ :



Nodes are forbidden:

- in an incompressible fluid:

$$\nabla \cdot \mathbf{v} = 0 \implies \nabla^2 \chi = 0$$

Hence not seen, e.g., in a Coulomb gas model

- in a stationary configuration:

No sink nor source !

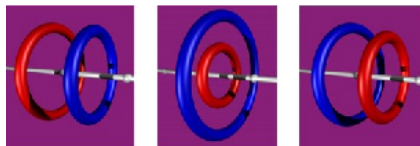
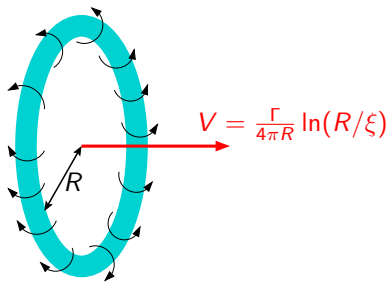
$$\nabla^2 \psi + [f(A) + U(\mathbf{r}) - \mu]\psi = 0 \quad (1)$$

where $\psi = A \exp(i\chi)$:

$$\nabla^2 \psi = \{ \nabla^2 A - A |\nabla \chi|^2 + i(A \nabla^2 \chi + 2 \nabla A \cdot \nabla \chi) \} e^{i\chi}$$

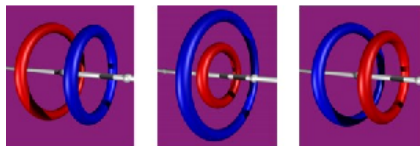
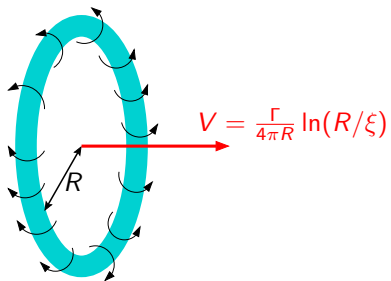
at a stagnation point where $\nabla \chi = 0$,
the **imaginary part** of (1) yields $\nabla^2 \chi = 0$

dynamics of vortex filaments/rings



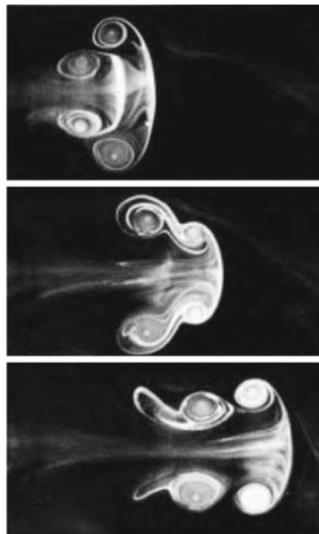
Niemi, PRL 2005

dynamics of vortex filaments/rings



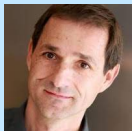
Niemi, PRL 2005

Yamada, Matsui, Phys. Fluids 1978





P. Azam
DGA Toulouse



R. Kaiser
Inst. Phys. Nice



T. Congy
Northumbria Univ.



G. Ciliberto
LIP6, Paris



R. Panico
Univ. Bonn



G. Martone
Nanotec Lecce



D. Ballarini
Nanotec Lecce



A. Lanotte
Nanotec Lecce

Refs:

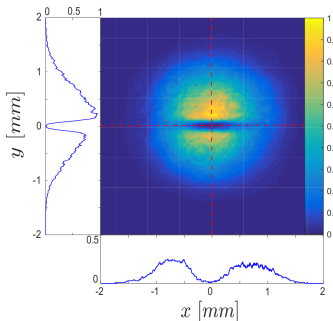
Congy, Azam, Kaiser, Pavloff, Phys. Rev. Lett. **132**, 033804 (2024),

Panico et al., Phys. Rev. Res. **7**, L022063 (2025).

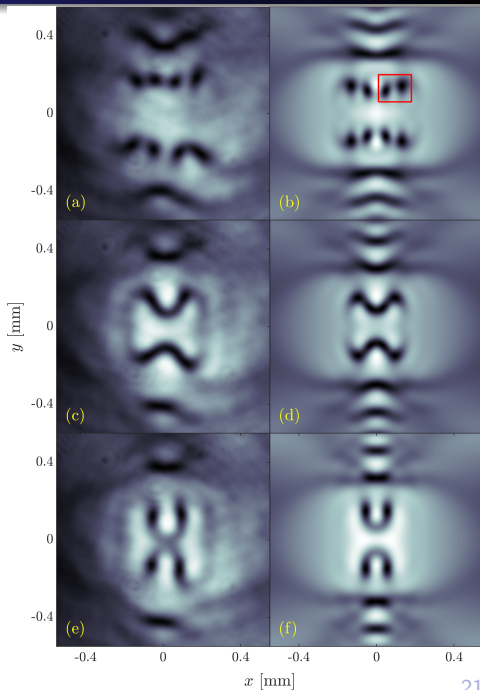
Reproducing the experimental results in Nice

$$\psi(\vec{r}, 0) = \sqrt{I_1} \exp\left(-\frac{r^2}{w_G^2}\right) + \sqrt{I_2} \exp\left(-\frac{x^2}{w_x^2} - \frac{y^2}{w_y^2}\right) e^{i\Phi_2}$$

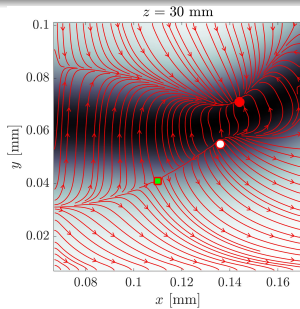
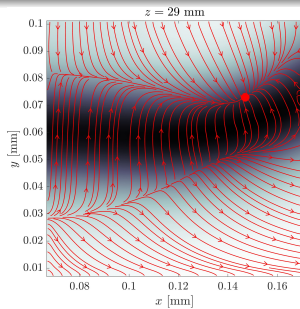
$$I_1 = I_2 \quad \Phi_2 \simeq \pi$$



- (a,b) : $\Phi_2 = 0.96 \pi$
 (c,d) : $\Phi_2 = \pi$
 (e,f) : $\Phi_2 = 1.05 \pi$

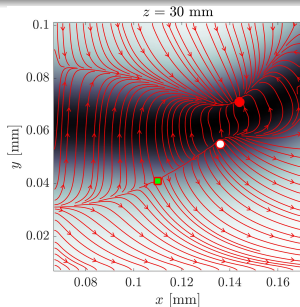
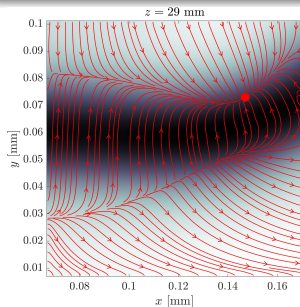


Formation of saddles and nodes: saddle-node bifurcation



$$\Phi_2 = 0.96 \pi$$

Formation of vortices and nodes: saddle-node bifurcation

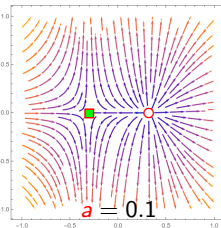
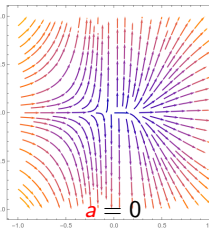
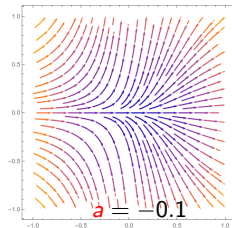


$$\Phi_2 = 0.96 \pi$$

Saddle-node:

$$\begin{cases} v_x = x^2 - a \\ v_y = y \end{cases}$$

$$a \in \mathbb{R}$$

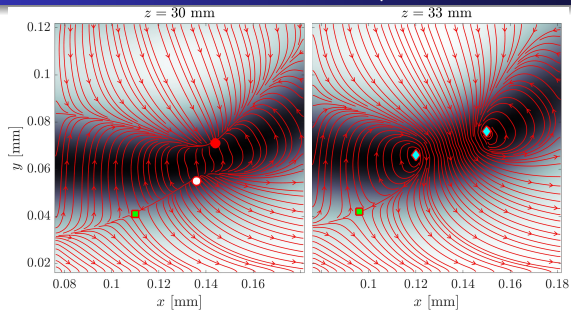


$$\mathbf{v} = \nabla\left(\frac{1}{3}x^3 - ax + \frac{1}{2}y^2\right)$$

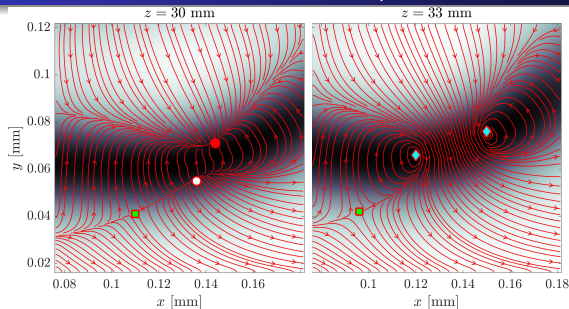
orbitally equivalently: $\mathbf{v} = \nabla\chi$ where

$\chi(\mathbf{r}) = (\text{almost})\text{any fct of } Z = \frac{1}{3}x^3 - ax + \frac{1}{2}y^2$

Mechanism of vortex formation/annihilation: fold-Hopf bifurcation



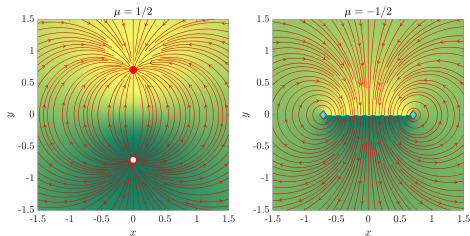
Mechanism of vortex formation/annihilation: fold-Hopf bifurcation



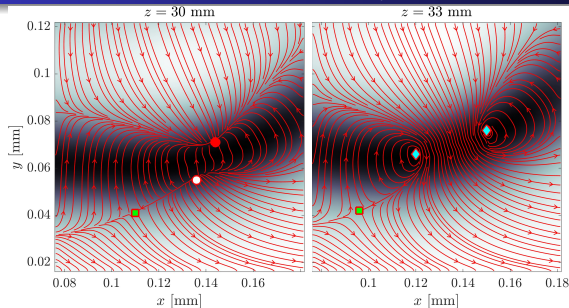
fold-Hopf bifurcation:

$$\begin{cases} v_x = -2\sigma xy \\ v_y = \mu + \sigma x^2 - y^2 \end{cases}$$

$$\sigma = 1, \mu \in \mathbb{R}$$



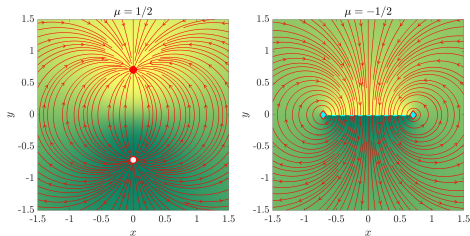
Mechanism of vortex formation/fold-Hopf bifurcation



fold-Hopf bifurcation:

$$\begin{cases} v_x = -2\sigma xy \\ v_y = \mu + \sigma x^2 - y^2 \end{cases}$$

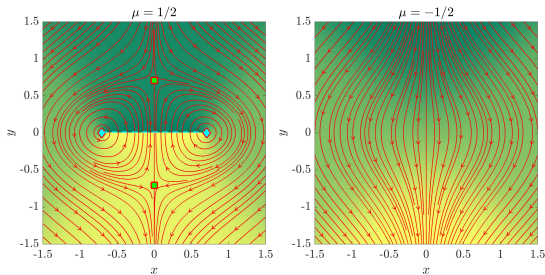
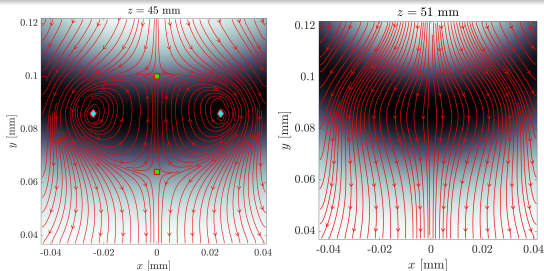
$\sigma = 1, \mu \in \mathbb{R}$



orbitally equivalent system: $\vec{v} = \vec{\nabla} \chi_{\text{fH}}, \chi_{\text{fH}}(\vec{r}) \equiv \arg [x^2 + \sigma(y^2 + \mu) + i\sigma y]$.

- ✓ gradient system
- ✓ verifies Onsager-Feynman quantization condition

vortex formation/annihilation: Bristol NHH mechanism ... also fold-Hopf



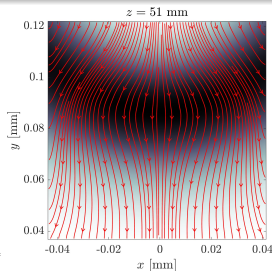
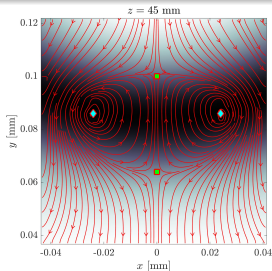
$$\vec{v} = \vec{\nabla} \chi_{\text{fH}}$$

where
with

$$\chi_{\text{fH}}(\vec{r}) \equiv \arg [x^2 + \sigma(y^2 + \mu) + i\sigma y]$$

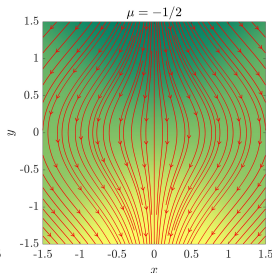
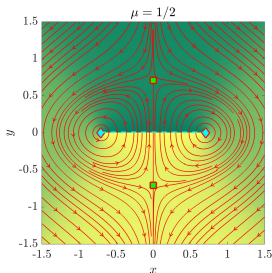
$$\sigma = -1 \quad \text{and} \quad \mu \in \mathbb{R}$$

vortex formation/annihilation: Bristol NHH mechanism ... also fold-Hopf



close to the bifurcation point

the phase of a model wave function (exact solution of Helmholtz equation) matches $\chi_{fH}(\vec{r})$



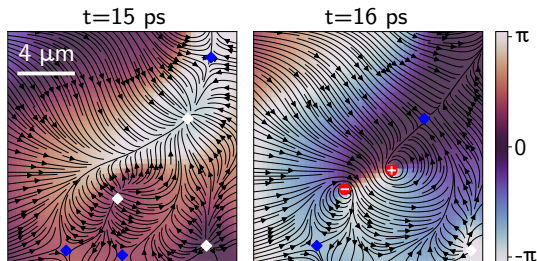
$$\vec{v} = \vec{\nabla} \chi_{fH}$$

where
with

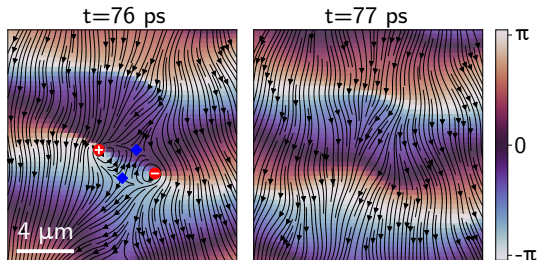
$$\chi_{fH}(\vec{r}) \equiv \arg [x^2 + \sigma(y^2 + \mu) + i\sigma y]$$

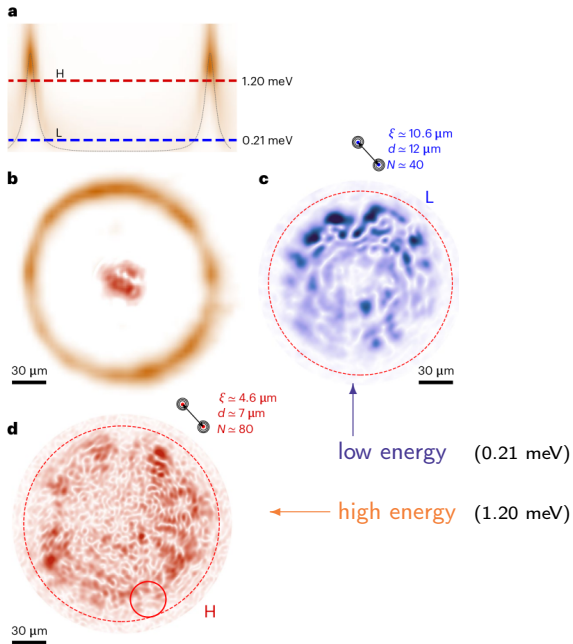
$$\sigma = -1 \quad \text{and} \quad \mu \in \mathbb{R}$$

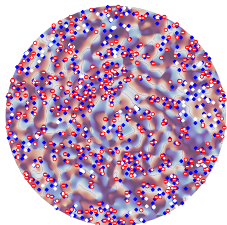
Node collision
(fold-Hopf with $\sigma = 1$)



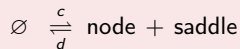
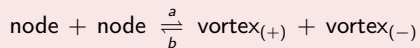
Bristol NHH mechanism
(fold-Hopf with $\sigma = -1$)



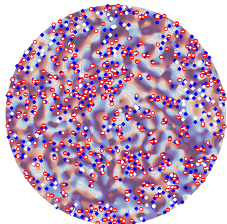




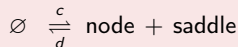
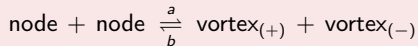
simplest possible processes



The (positive) quantities a , b , c , and d are the reaction rates.



simplest possible processes



The (positive) quantities a , b , c , and d are the reaction rates.

Rate equations:

$$\frac{dV_{\pm}}{dt} = aN^2 - bV_{+}V_{-},$$

$$\frac{dS}{dt} = c - dNS,$$

$$\frac{dN}{dt} = c - 2aN^2 + 2bV_{+}V_{-} - dNS,$$

where $N(t)$ denotes the number of nodes, $S(t)$ the number of saddles, and $V_{+}(t)$ [$V_{-}(t)$] the number of vortices with positive [negative] vorticity.

In the following $b = 0$

$$(b = 0) \quad \frac{dV_{\pm}}{dt} = aN^2, \quad \frac{dS}{dt} = c - dNS, \quad \frac{dN}{dt} = c - 2aN^2 - dNS,$$

No imparted angular momentum:

$$V_+(t) - V_-(t) = C^{st} \ll \text{typical} \left(V_+(t) + V_-(t) \right) \rightsquigarrow V_+(t) = V_-(t) \equiv \frac{V(t)}{2}$$

rescaled quantities

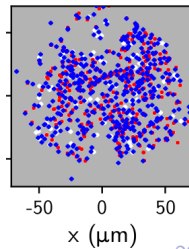
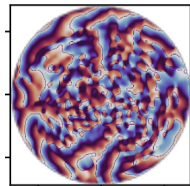
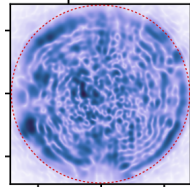
$$\tau = t/t_0, \quad n = N/N_0, \quad v = V/N_0, \quad s = S/N_0$$

$$\text{with } t_0 = 1/\sqrt{2ac} \text{ and } N_0 = \sqrt{c/2a}$$

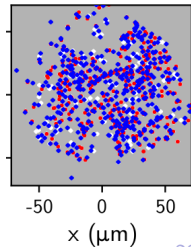
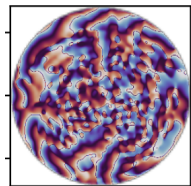
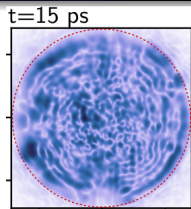
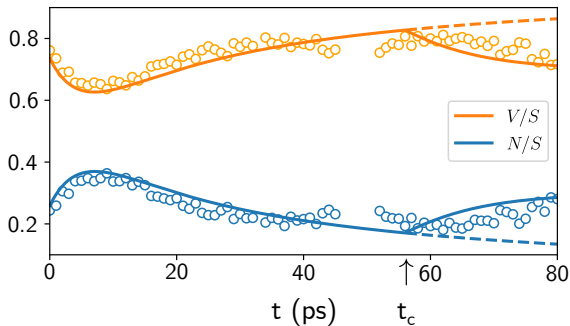
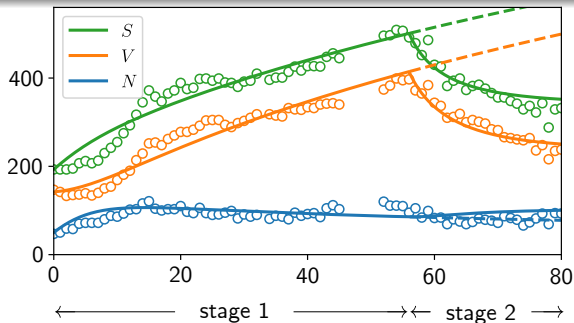
$$\boxed{\frac{dv}{d\tau} = n^2, \quad \frac{ds}{d\tau} = 1 - \gamma ns, \quad \frac{dn}{d\tau} = 1 - n^2 - \gamma ns,}$$

where the single parameter $\gamma \equiv d/2a$ governs the qualitative features of the dynamical system.

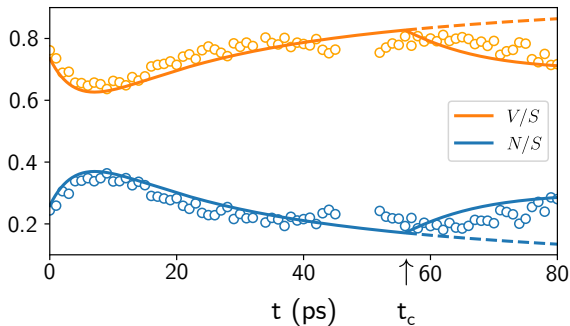
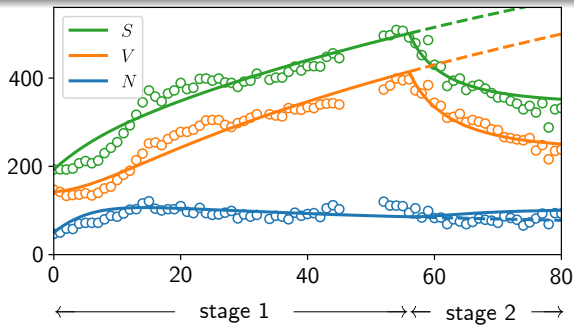
$t=15$ ps



experimental results



experimental results

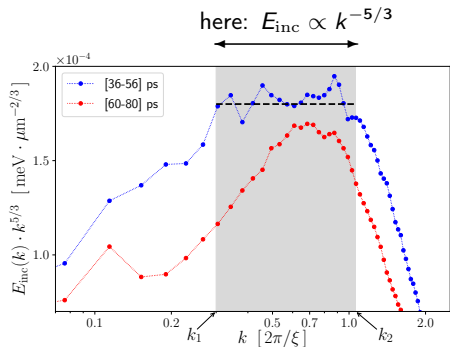
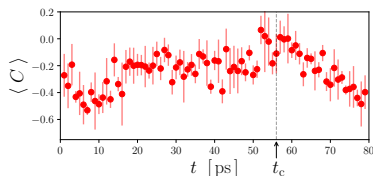


$$I_P = N(t) + V(t) - S(t) \\ = C^{st} \simeq 0$$

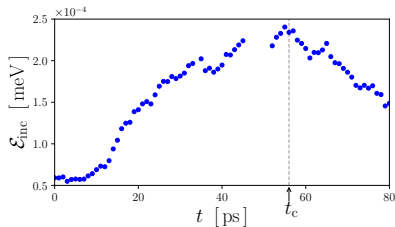
Quantitative evaluation of clustering:

$$C = \frac{1}{V} \sum_{i=1}^V c_i \quad (c_i = \pm 1)$$

White, Barenghi, Proukakis, PRA (2012)



$$\mathcal{E}_{\text{inc}}(t) = \int_{k_1}^{k_2} E_{\text{inc}}(k, t) dk$$



End of clustering

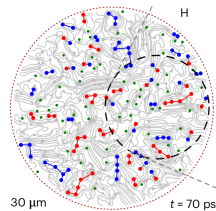
The vortices are no longer grouped in packs of the same vorticity.

Nodes don't seem to play a significant role.

→ Bristol NHH mechanism:

$$\text{vortex}_{(+)} + \text{vortex}_{(-)} + \text{saddle} + \text{saddle} \xrightleftharpoons[f]{e} \emptyset$$

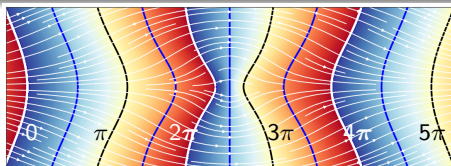
for simplicity, take $f = 0$.



$$\frac{dv}{d\tau} = n^2 - \epsilon v^2 s^2, \quad \frac{ds}{d\tau} = 1 - \gamma ns - \epsilon v^2 s^2, \quad \frac{dn}{d\tau} = 1 - n^2 - \gamma ns,$$

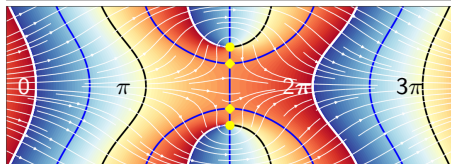
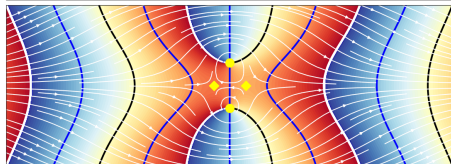
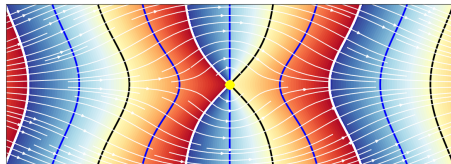
where $\epsilon = \frac{1}{2} e N_0^3 t_0 = ec / (8a^2)$ is the rescaled rate of annihilation of saddles and vortices through the NHH mechanism.

$S \approx V \rightsquigarrow$ NHH is effectively a 4 vortices mechanism (also 3 and 2 since $V \approx S + N$ with $N \approx \text{cst}$).



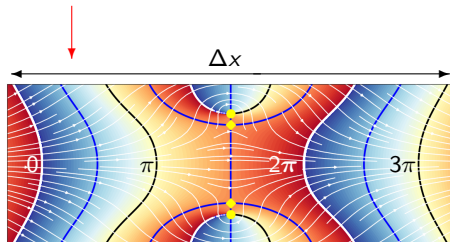
Initially:

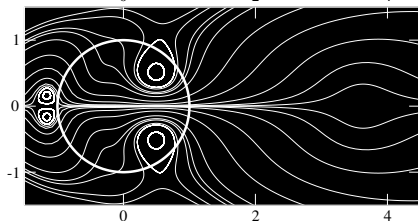
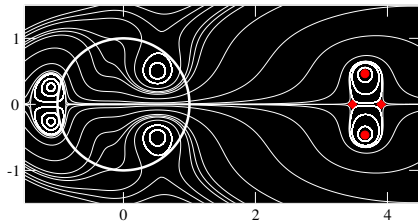
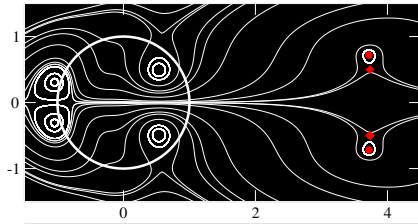
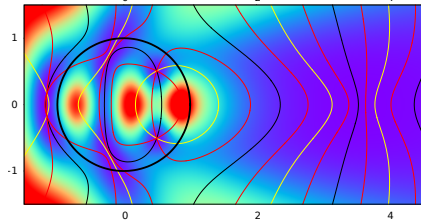
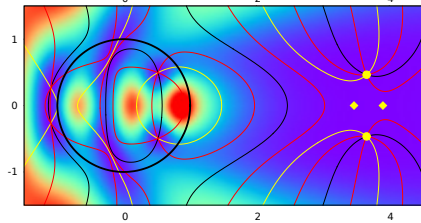
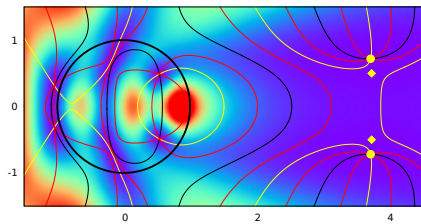
$\Delta\chi = 5\pi$ along $y = 0$ and fixed Δx

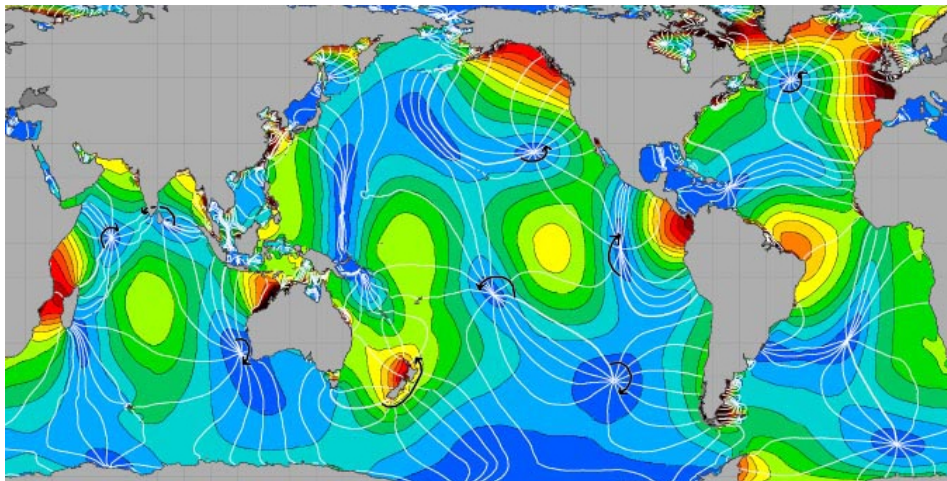


Eventually:

$\Delta\chi = 3\pi$ along $y = 0$, same Δx







$$\psi(\mathbf{r}, t) = h(\mathbf{r}) \exp\{i(\chi(\mathbf{r}) - \omega t)\} \quad 2\pi/\omega = T = 12 \text{ h } 25 \text{ min}$$

$$\Delta t_{\text{cotidal}} = 1 \text{ h } 2 \text{ min}, \quad \omega \times \Delta t_{\text{cotidal}} = \pi/6$$

⇒ quantized circulation of tidal current of the M2 component

$\mathbf{v}$: continuous vector field on a compact differentiable manifold $\mathcal{M}$

$$I_P = \chi$$

where χ is the Euler characteristic of $\mathcal{M}$.

For the theorem to apply to the disk: $\mathbf{v}$ should point in the outward normal direction along $\partial\mathcal{M}$

Name	Image	χ
Interval		1
Circle		0
Disk		1
Sphere		2
Torus (Product of two circles)		0