Orsay, 13-14March 2014
Meeting in memory of Oriol Bohigas

SUM RULES: FROM ATOMIC NUCLEI TO ULTRACOLD GASES

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SUM RULES FOR NUCLEAR COLLECTIVE EXCITATIONS

O. BOHIGAS, A.M. LANE and J. MARTORELL

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Sum rules and collective oscillations in ultracold atomic gases: a few examples

Breathing mode of BECs in **1D configurations**: from mean field to impenetrable bosons

$$\omega^2_{breathing} = \frac{< z^2 >}{-2\partial < z^2 > / \partial \omega_z^2}$$

1

Measuring the angular Momentum of a **quantized vortex**:

$$(\omega_{+2} - \omega_{-2}) = \frac{2l_z}{m < r_\perp^2 >}$$

2

Quenching of the frequency of center of mass motion due to **spin orbit** coupling:

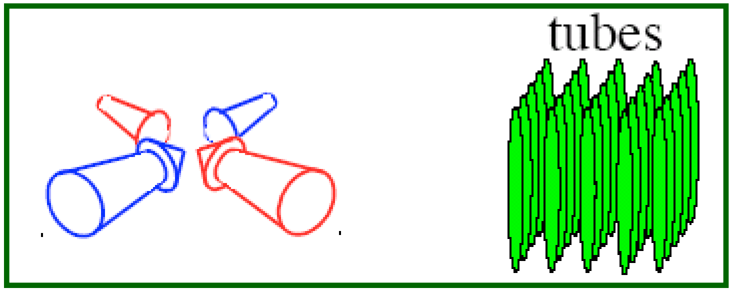
$$\omega_D = \omega_x \sqrt{\frac{1}{1 + k_0^2 \chi(\sigma_z)}}$$

3

1

Breathing mode of BEC's in **1D configurations**: from mean field to impenetrable bosons

1D configurations are currently realized with 2 pairs of counterpropagating lasers



1D regime $\mu << \hbar\omega_\perp$

(corresponding to freezing of radial degrees of freedom) is achieved if condition $a n_1 << 1$ is satisfied.

(a is 3D scattering length; n_1 is 1D density)

In opposite regime system behaves like a 3D configuration

In the presence of harmonic trapping a useful and accurate estimate of axial breathing frequency is provided by the sum rule formula

$$\omega^2_{breathing} = \frac{m_1(z^2)}{m_{-1}(z^2)} = \frac{< z^2 >}{-2\partial < z^2 > / \partial \omega_z^2}$$

Based on energy weighted and inverse energy weighted sum rules.

Value of $< z^2 >$ and its dependence on harmonic oscillator frequency depends on density profile.
The density profile is fixed by equation of state (1D chemical potential) in local density approximation according to:

$$\mu_1(n_1(z)) = \mu_0 - \frac{1}{2} m \omega_z^2 z^2$$

Different regimes are achievable in 1D. They are fixed by the value of the dimensionless parameter $n_1 a_{1D}$
Where $a_{1D} = a_\perp^2 / a$ is effective 1D scattering length

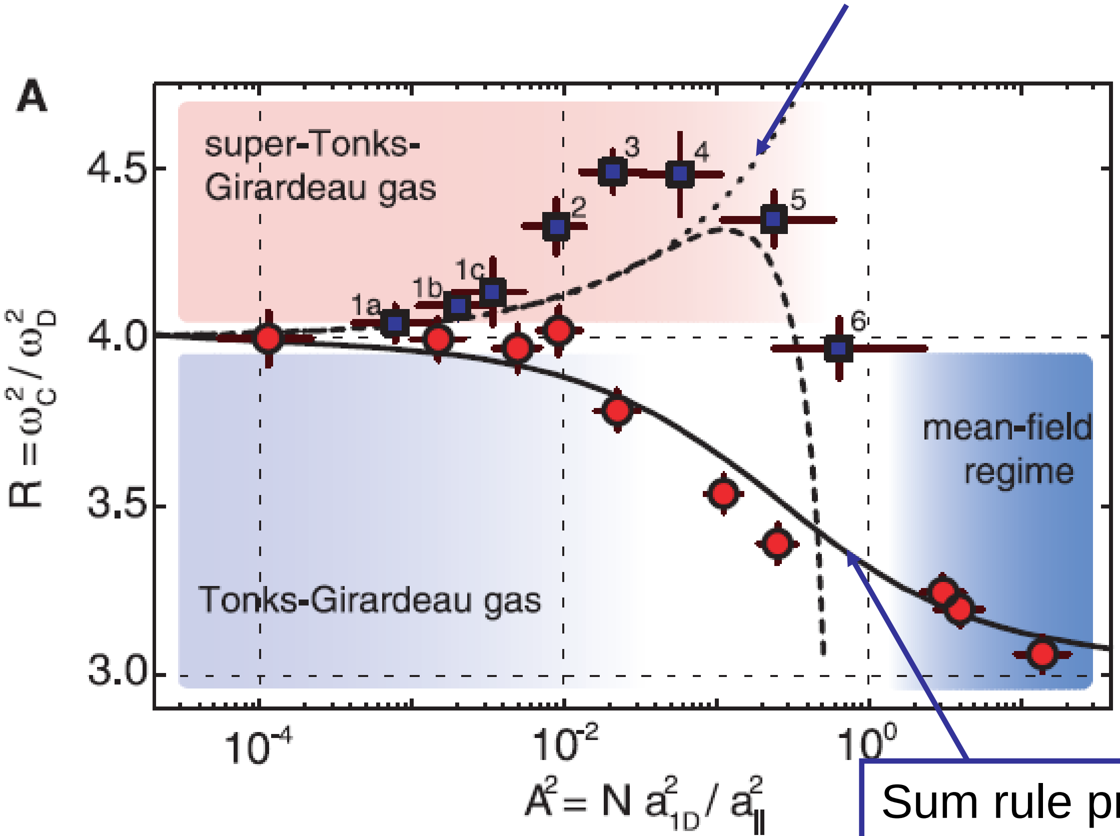
- 1D mean field $n_1 a_{1D} >> 1$ $\mu = 2\hbar\omega_\perp a n_1$ $\omega = \sqrt{3}\omega_z$

- Tonks-Girardeau (impenetrable bosons) $n_1 a_{1D} << 1$ $\mu = \pi^2 \frac{\hbar^2}{2m} n_1^2$ $\omega = 2\omega_z$

For intermediate value of $n_1 a_{1D}$
Lieb-Liniger theory (1963) provides exact value for equation of state that can be used to evaluate density profiles and the breathing frequency (Menotti and Stringari (2002)

$$\mu = \frac{\hbar^2}{2m} n_1^2 f(n_1 a_{1D})$$

Beyond the mean field regime in 1D:
Innsbruck 2009



Sum rule prediction
(Menotti and S.S., 2002)

2

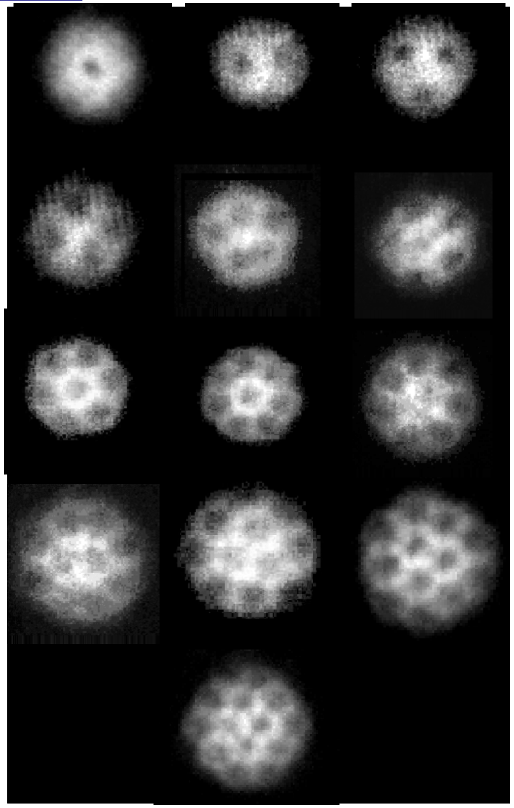
Measuring the angular momentum of a **quantized vortex**:

In the presence of a vortical configuration quadrupole oscillations with opposite angular momentum $m = \pm 2$ are no longer degenerate.

Resulting **splitting** of quadrupole frequencies is at the origin of **precession** effect (Zambelli and S.S 1998)

$$\Omega_{prec} = \frac{1}{4}(\omega_{+2} - \omega_{-2}) = \frac{l_z}{2m < r_\perp^2 >}$$

(analog of **Foucault's pendulum**)



Vortices at ENS
Chevy, 2001

Result for frequency splitting $(\omega_{+2} - \omega_{-2})$
was derived using exact sum rules relative to the quadrupole choice $Q_{\pm 2} = (x \pm iy)^2$

for the excitation operators

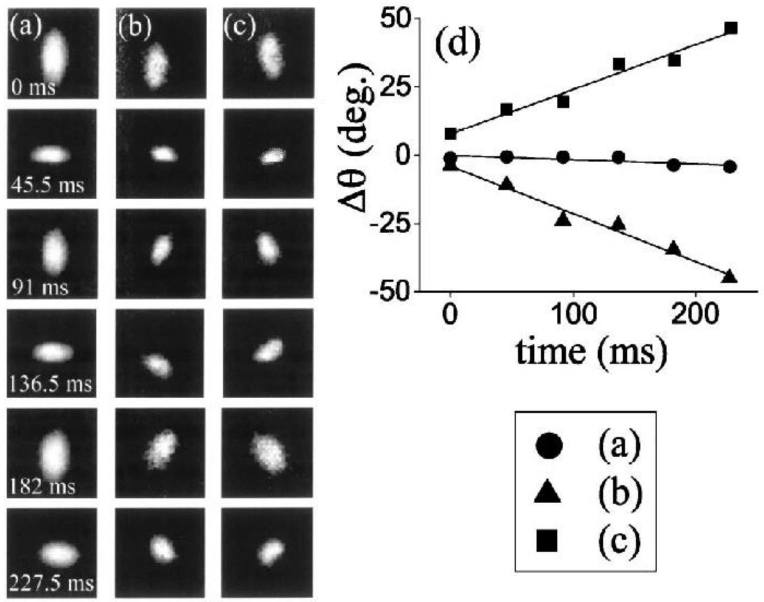
$$m_0^+ - m_0^- = <[Q_-, Q_+]> = 0$$

$$m_1^+ + m_1^- = <[Q_-, [H, Q_+]]> = 8N \frac{\hbar^2}{m} < r_\perp^2 >$$

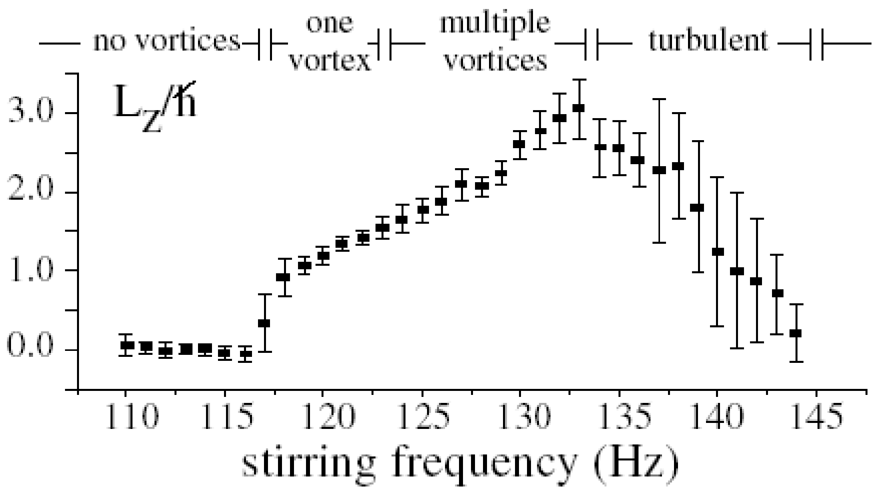
$$m_2^+ - m_2^- = <[[Q_-, H], [H, Q_+]]> = 16N \frac{\hbar^3}{m^2} l_z$$

Precession of quadrupole shape and quantization of angular momentum

$$\Omega_{prec} = \frac{1}{4}(\omega_{+2} - \omega_{-2}) = \frac{l_z}{2m < r_\perp^2 >}$$



Measurement of angular momentum in BEC's
(Chevy et al., 2000)

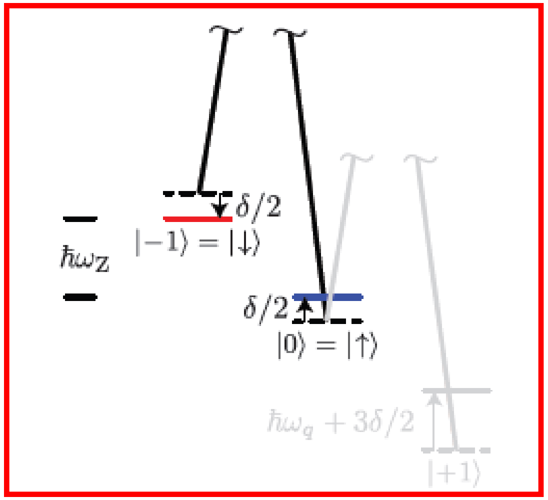


3

Quenching of center of mass frequency In the presence of spin orbit coupling

- **Experimental realization** of synthetic gauge fields is providing new challenging many-body configurations in ultra-cold atomic gases

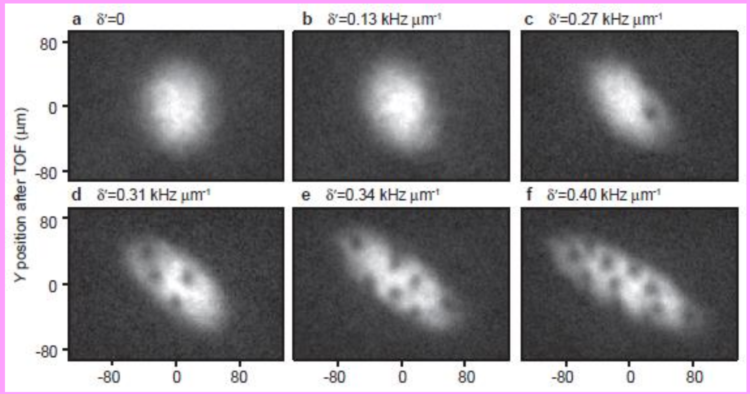
- **New quantum phases** (stripes, magnetism, vortices) and new phase transitions in both Bose and Fermi gases.



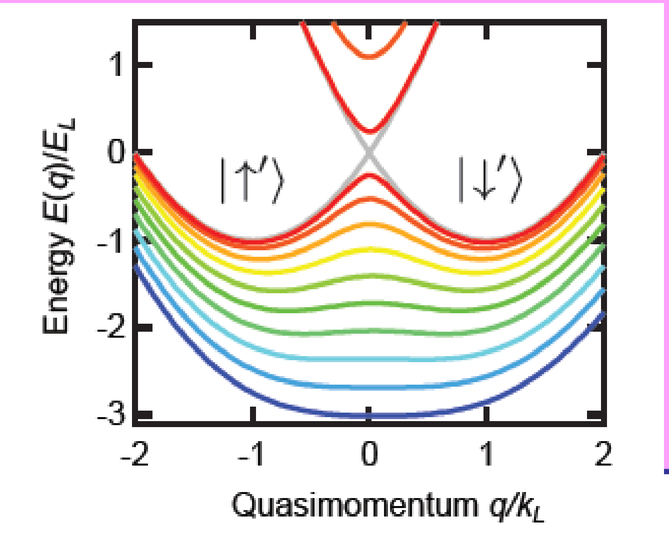
Two detuned and **polarized** laser beams + non linear Zeeman field provide Raman transitions between two spin states, giving rise to new gauge terms in the Hamitonian

Different strategies (both pursued by Ian Spielman's team at Nist)

- Spatially dependent laser detuning $\Delta\omega_L(z)$ with strong Raman coupling Ω (proportional to laser power) gives rise to effective **Lorentz force** in **neutral systems .** Generation of vortices and possible route to quantum Hall regime.
  (Lin et al., Nature 2009)



- Working with vanishing effective Zeeman field and small Raman coupling Ω gives rise to the appearence of **two minima** which can host a Bose-Einstein condensate.
  (Lin et al., Nature 2011)



The unitary transformation $e^{i(2k_0 x - \Delta\omega_L t)\sigma_Z}$

brings the system into spin rotated frame where the Hamiltonian takes the translationally invariant spin-orbit form

$$H = \sum_i h_0(i) + \sum_{\alpha,\beta} \frac{1}{2} \int d\vec{r}\, g_{\alpha\beta} n_\alpha n_\beta$$

Spin up-up, up-down and down-down coupling constants

with

$$h_0 = \frac{1}{2}[\left(p_x - k_0\sigma_z\right)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$

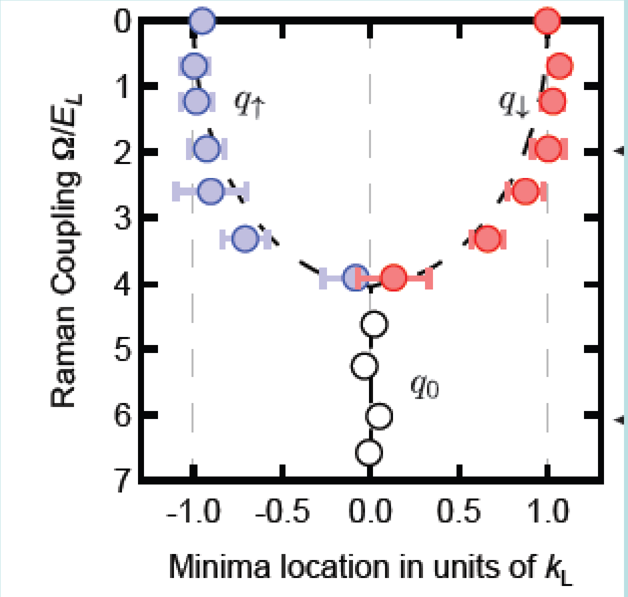
New ingredients of the Hamiltonian:

$2k_0$ is wave vector difference between the two laser fields

Ω is Raman coupling (fixed by laser intensity)

δ is effective Zeeman field

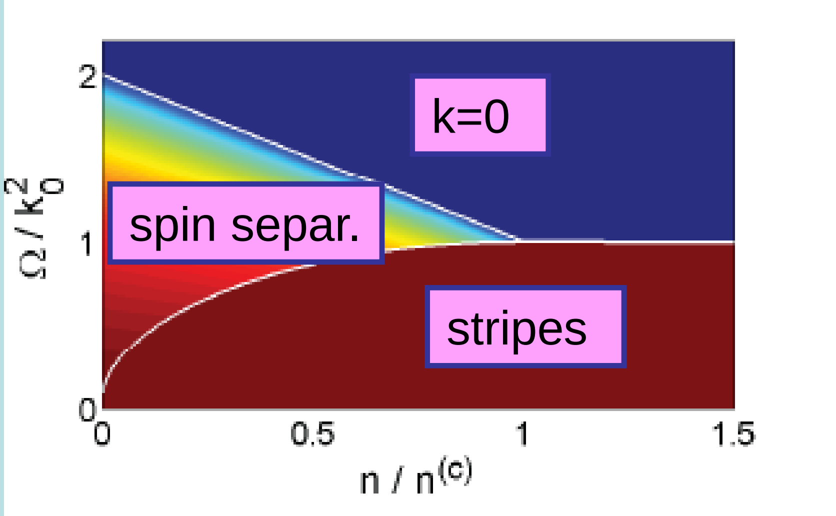
Experimental implementation of the SO hamiltonian with BECs by the Spielman team at NIST (Nature 2011)



Theory of the new quantum phases:

Ho and Zhang (PRL 2011), ……

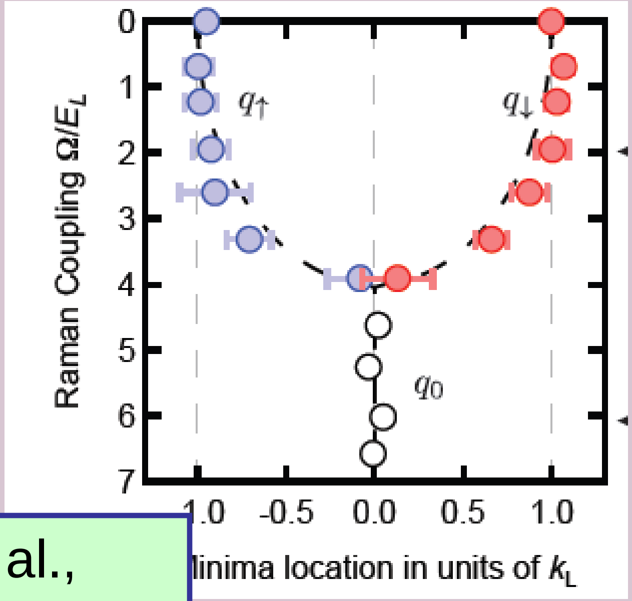
Yun Li, Pitaevskii, S., PRL 2012



Nature of phase transitions

The transition between the **plane wave** and **the k=0** phase is second order.
It has been observed at the predicted value of the Raman coupling

$$\Omega = 2k_0^2$$



Lin et al., Nature 2011

$E_L = k_0^2/2$

Near the transition the spin polarizability $\chi(\sigma_z)$ exhibits divergent behavior

Plane wave phase

$$\chi(\sigma_z) = \frac{\Omega^2}{k_0^2(4k_0^2 - \Omega^2)}$$

Zero momentum phase

$$\chi(\sigma_z) = \frac{2}{\Omega - 2k_0^2}$$

Center of mass oscillation

Dipole mode in the presence of spin-orbit coupling

Yun Li, G. Martone and S.S, EPL 2012)

Coupling between center of mass and spin degrees of freedom is explicitly revealed by **commutation rule**

$$[H, X] = -i\left(P_x - k_0\sigma_z\right)$$

- Reflects change in **equation of continuity** and **violation of Galilean invariance**

- Implies **new dynamic** behavior of center of mass motion

- Commutation rule $[H, P_x] = i\omega_x^2 X$

- is instead unaffected by spin-orbit coupling

Sum rules for the dipole operator

Energy weigthed sum rule

$$m_1(X) = \frac{1}{2} < [X,[H,X]] >= \frac{N}{2}$$

and **inverse energy weighted sum rule**

$$m_{-1}(X) = \frac{N}{2\omega_x^2}$$

are unaffected by spin-orbit coupling.

The inverse cubic sum rule instead depends on spin polarizability $\chi(\sigma_z)$

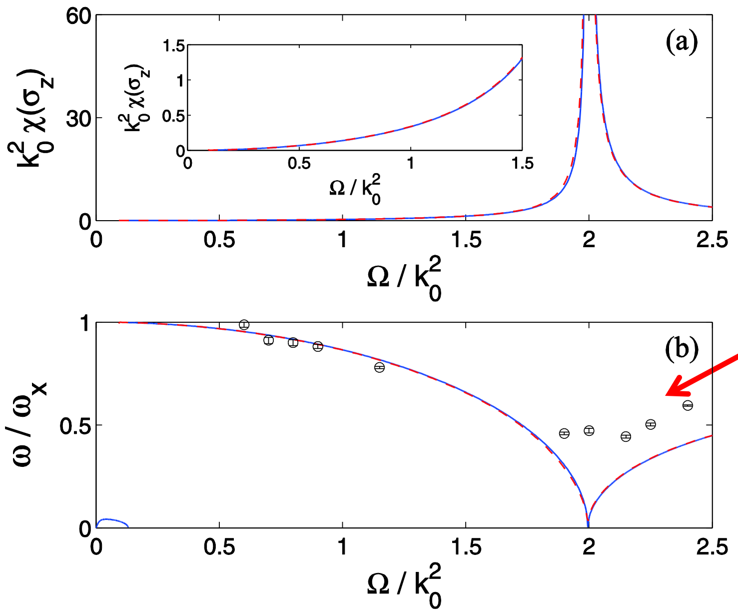
$$m_{-3}(X) = \frac{N}{2\omega_x^4}[1 + k_0^2 \chi(\sigma_z)]$$

Ratio between the lowest energy weighted sum rules then provides the result

$$\omega_D = \omega_x \sqrt{\frac{1}{1 + k_0^2 \chi(\sigma_z)}}$$

(Yun Li et al., EPL 2012)

Spin polarizability and dipole frequency
theory vs exp (Zhang et al. PRL 2012)



Finite values caused by nonlinear effect

- Divergent behavior of spin polarizability results in **strong quenching** of center of mass oscillation !

Main conclusion

Sum rule approach continues to prove an efficient tool to understand the dynamic behavior of many-body systems.

Efficient alternative to model calculations and to exact numerical approaches (very difficult to implement for dynamics)

Another precious lesson from Oriol !

arXiv:1312.1972

Quantum Chaos in Ultracold Collisions of Erbium

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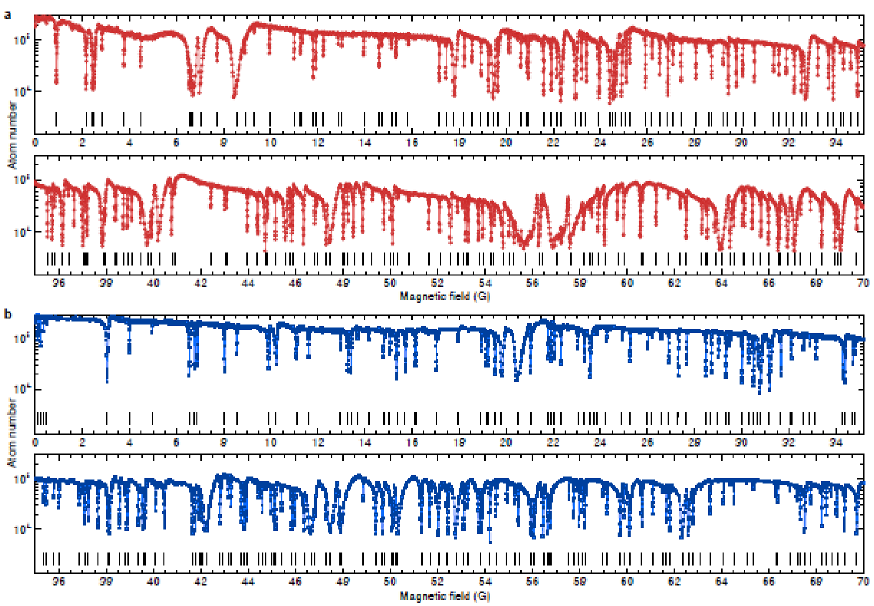
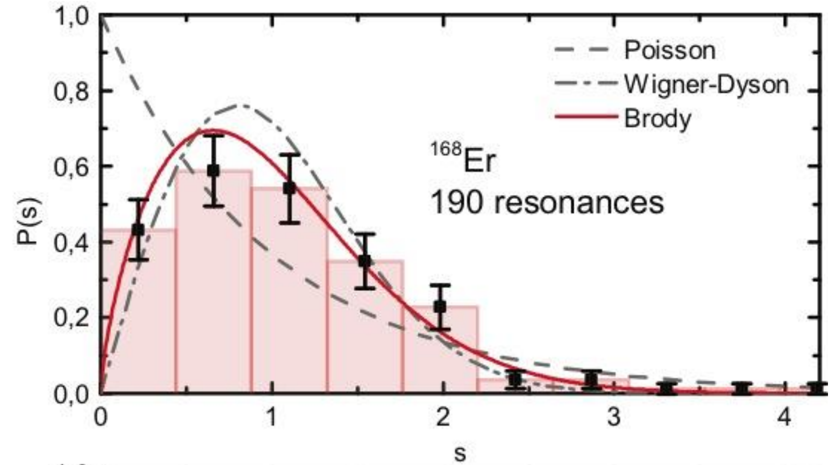
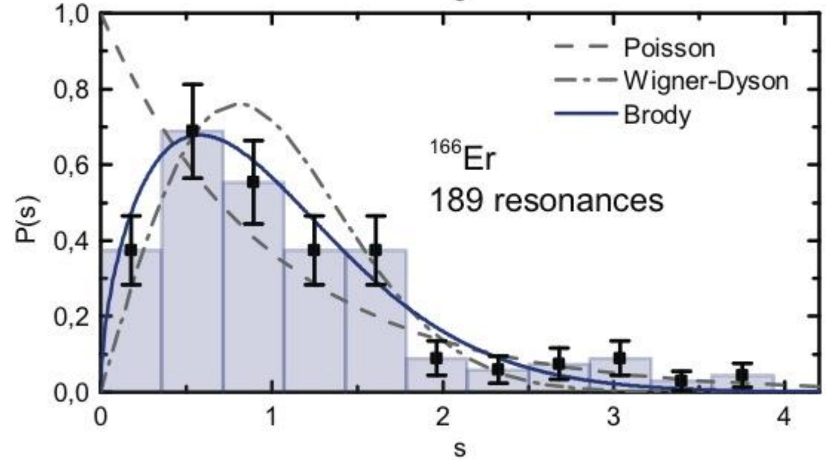
(Dated: December 9, 2013)

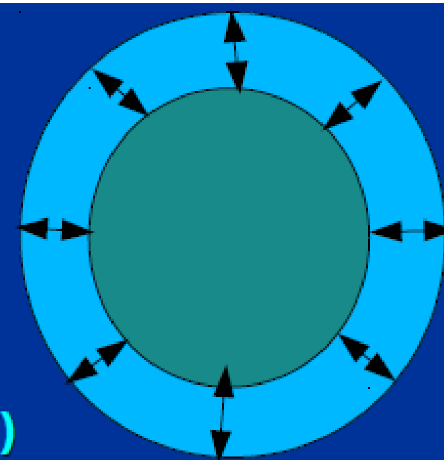
Applying RMT to Er Feshbach resonances







Historically, **G**iant **M**onopole **R**esonances have been observed In **large systems** and interpreted as an harmonic **collective motion** of compression (**“breathing mode”**).



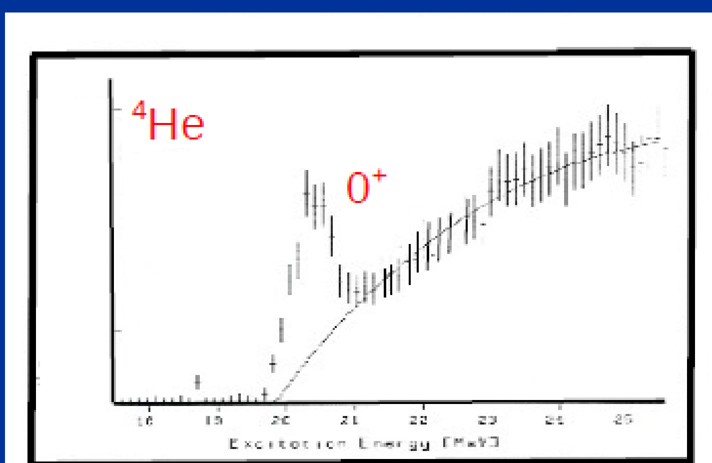
The degree of collectivity of a **GMR** is judged from the percent of exhaustion of **sum rules** (purely collective: 100%):

e.g. EWSR: $m_1 = 2N \langle r^2 \rangle / m$ **(Ferrel 1957)**

“Sum rules for nuclear collective excitations”

O. Bohigas, A.M Lane, J. Martorell, Phys. Rep. 51 (1979) 267

$m_{-1} = 2N \langle r^2 \rangle^2 / K$ **(defines the compressibility K)**



Recent surprise in a small nucleus:

Results of an ab initio calculation of the 0+ state of ^{4}He with modern realistic 2+3-body potential: the resonance exhausts only 34% of m_1 , but

53% of m_0 and **64%** of m_{-1} (**K=30** MeV)

Is the 0+ resonance of ^{4}He a “breathing mode”?

Slide: courtesy of G. Orlandini