Fluctuations in Many-Body Quantum Systems

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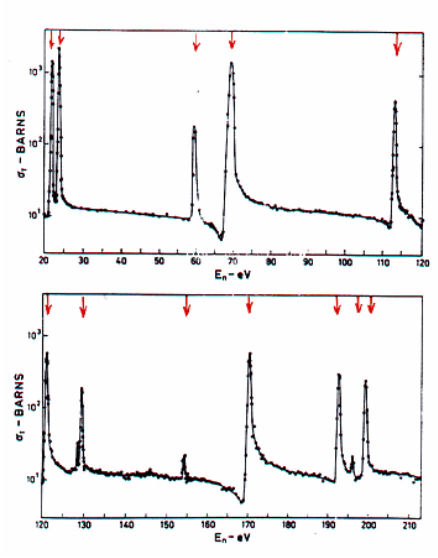
Colloquium in memory of
Orio Bohigas

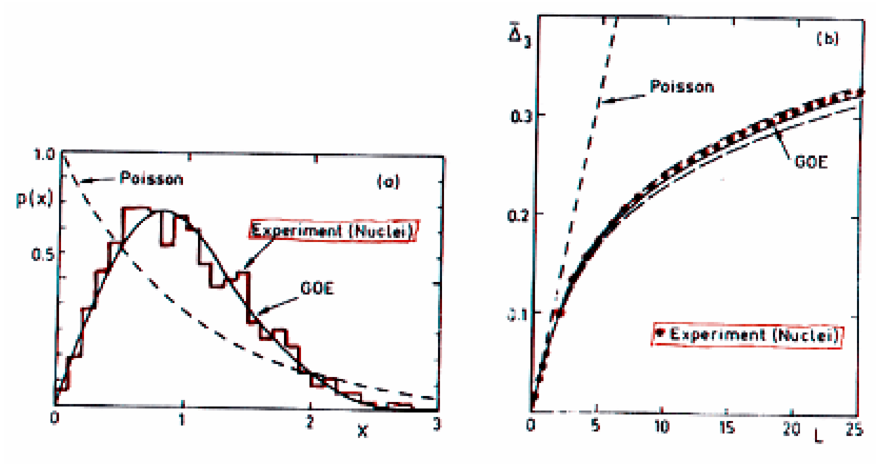
Orsay, March 2014

Slow neutron resonances



Total cross section n+Th^{232}

Haq, Pandey, Bohigas, PRL **48** (1982) 1086



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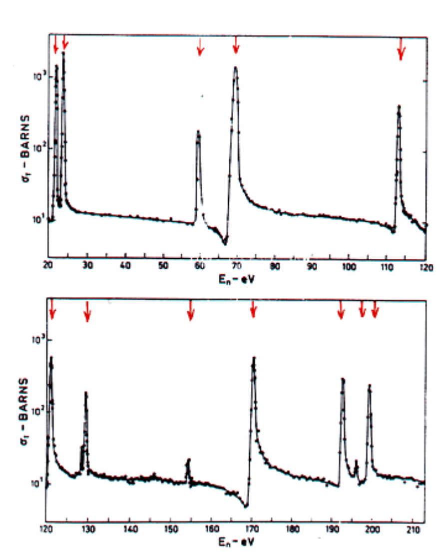

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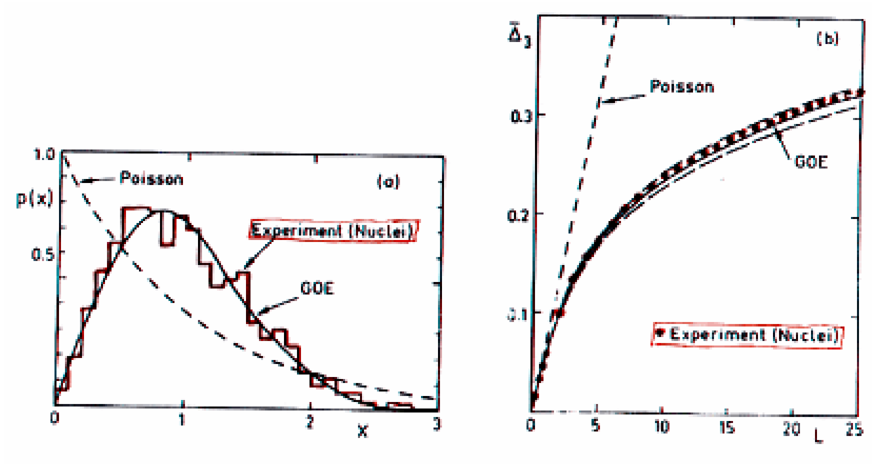
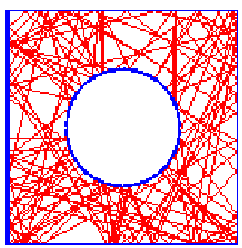
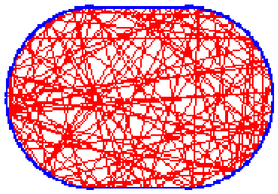


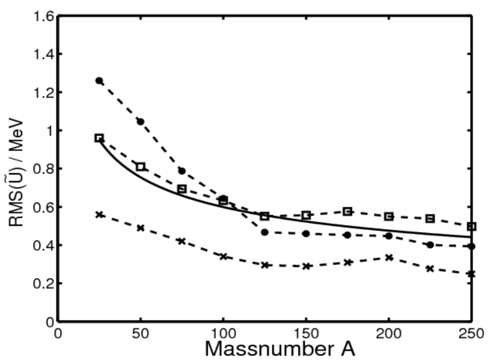




Chaotic Motion

MB Fluctuations

Nuclear Masses

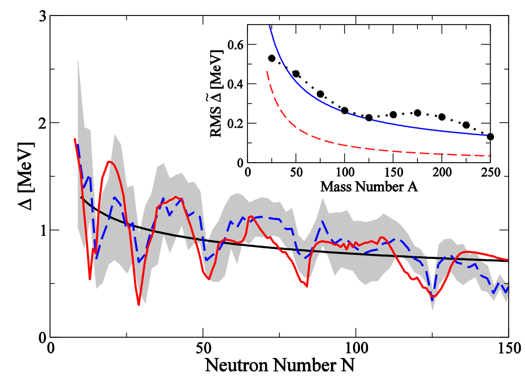


$$\tilde{\mathcal{B}}(\mu,T) \approx 2\hbar^2 \sum_p \sum_{r=1}^{\infty} \frac{A_{p,r}(\mu)}{r^2 \tau_p^2} \cos[rS_p/\hbar + \nu_{p,r}]$$

$$c(N) = \left\langle \tilde{\mathcal{B}}(N_0 + N/2)\, \tilde{\mathcal{B}}(N_0 - N/2) \right\rangle$$

S. Åberg, O. Bohigas, H. Olofsson

Pairing Gap

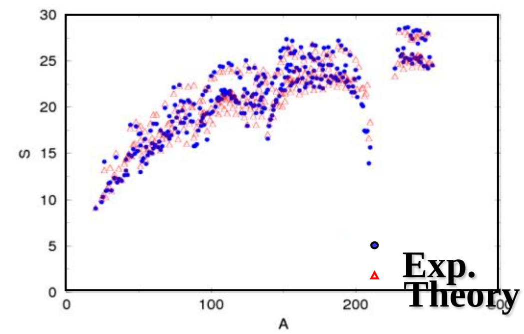


$$\tilde{\Delta} = \frac{\bar{\Delta}}{\bar{\rho}} \sum_p \sum_{r=1}^{\infty} A_{p,r}(\varepsilon_F)\, Y(r\tau_p) \cos[rS_p(\varepsilon_F)/\hbar + \nu_{p,r}]$$

$$Y(\tau) = \int_{-L}^{L} d\varepsilon \frac{\cos(\tau\varepsilon/\hbar)}{\sqrt{\varepsilon^2 + \bar{\Delta}^2}} \approx 2K_0(\tau/\tau_\Delta)$$

S. Åberg, H. Olofsson,

MB Level Density



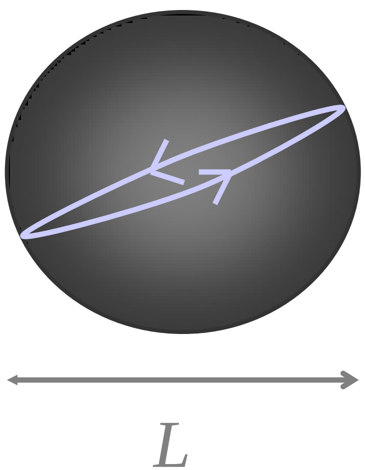
$$S(Q) = 2\sqrt{aQ} - \frac{135 + 4\pi^2}{144\sqrt{2}} \frac{1}{\sqrt{aQ}} + \ldots + \frac{1}{T}\left[\tilde{\mathcal{B}}(\mu,0) - \tilde{\mathcal{B}}(\mu,T)\right]$$

$$\rho_{MB}(Q) = \frac{\sqrt{\pi}}{12 a^{1/4} Q^{5/4}} e^{S(Q)}$$

A. Comtet, S. Majumdar,
A. Monastra, A. Relaño,
J. Roccia

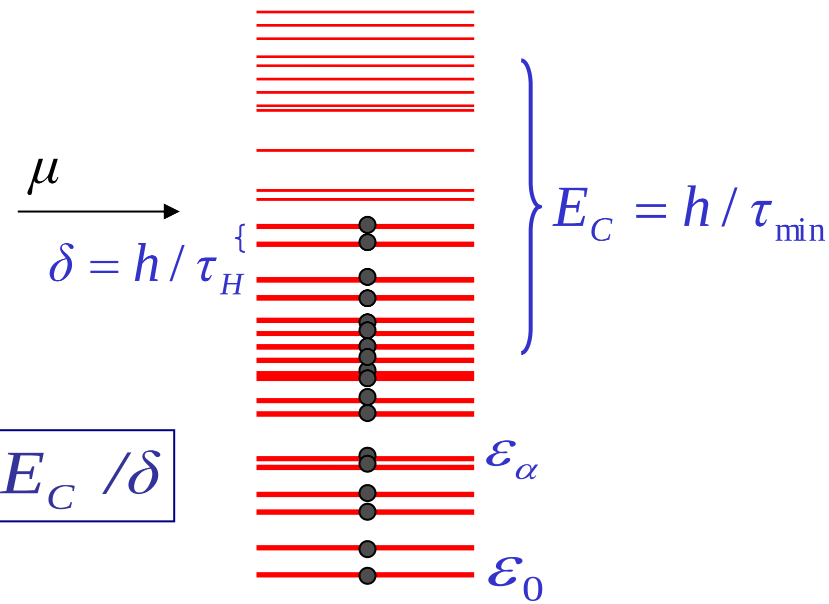
Mean Field Approximation

Confined Fermi gas



- Metallic clusters
- Nuclei
- Atomic Fermi gases

Ground state



$$g = E_C / \delta$$

Some basic formulae

- Trace Formula for the single-particle level density

$$\rho(\varepsilon) = \sum_k \delta(\varepsilon - \varepsilon_k) = \bar{\rho} + \tilde{\rho} \approx \bar{\rho} + 2\sum_p \sum_{r=1}^{\infty} A_{p,r}(\varepsilon) \cos [rS_p(\varepsilon)/\hbar + \nu_{p,r}]$$

- p = periodic orbits
- $A_{p,r}$ = stability amplitudes
- S_p= action = $\oint \vec{p}.d\vec{q}$
- $\nu_{p,r}$ = Maslov index

- Energy of a Fermi gas

$$\mathcal{B}(\mu,T) = \int d\varepsilon\, \varepsilon\, \rho(\varepsilon) f(\varepsilon,\mu,T) = \overline{\mathcal{B}}(\mu,T) + \widetilde{\mathcal{B}}(\mu,T)$$

Semiclassical theory (at *T*=0):

$$\widetilde{\mathcal{B}}(\mu,T) \approx 2\hbar^2 \sum_p \sum_{r=1}^{\infty} \frac{A_{p,r}(\mu)\, \kappa_T(r\,\tau_p)}{r^2\, \tau_p^2} \cos [rS_p/\hbar + \nu_{p,r}]$$

$$\kappa_T(\tau) = \frac{\tau/\tau_T}{\sinh(\tau/\tau_T)} \quad , \quad \tau_T = \frac{h}{2\pi^2 T}$$

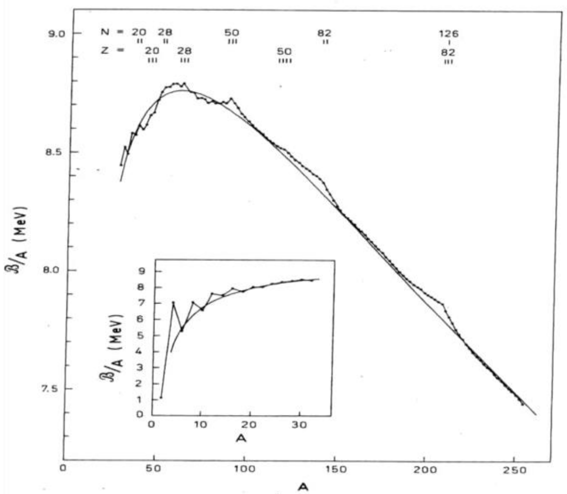
Nuclear Masses

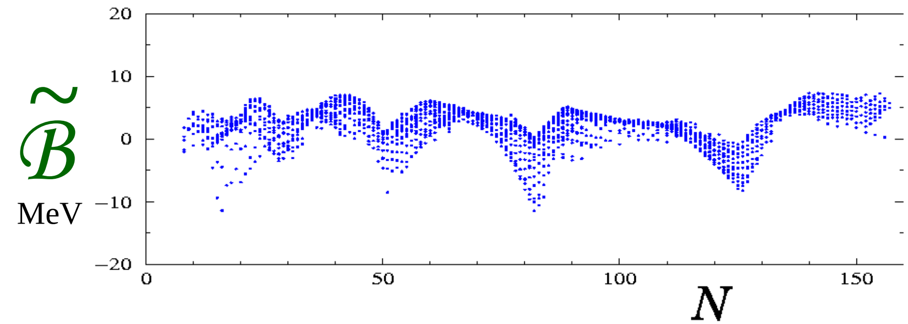
Average and fluctuations

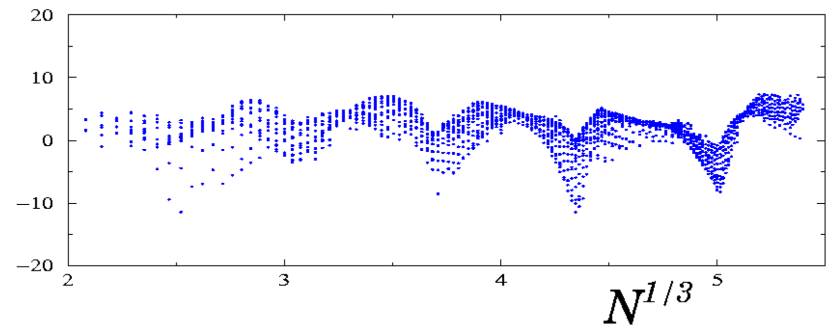
$$M = Z*M_P + N*M_N - \mathcal{B}(Z,N)/c^2 \;\Rightarrow\; \tilde{\mathcal{B}} = \overline{\mathcal{B}} - \mathcal{B}$$

$$\overline{\mathcal{B}} = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_A \frac{(N-Z)^2}{A} - a_p \frac{t_1}{A^{1/2}}$$

$$a_v = 15.67,\ a_s = 17.26,\ a_c = 0.714,\ a_A = 23.29,\ a_p = 11.2,\ t_1 = +1,0,-1$$

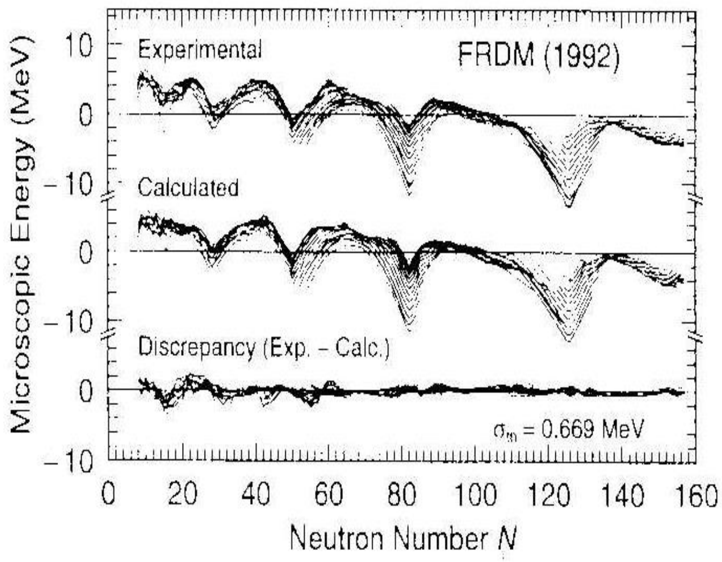




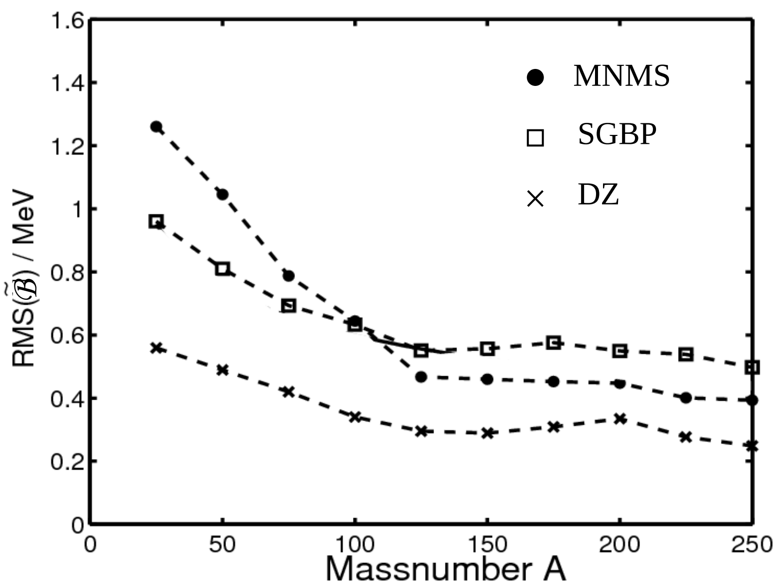


$$\tilde{\mathcal{B}}(\mu) \approx 2\hbar^2 \sum_p \sum_{r=1}^{\infty} \frac{A_{p,r}(\mu)}{r^2\,\tau_p^2} \cos\left[rS_p/\hbar + \nu_{p,r}\right]$$

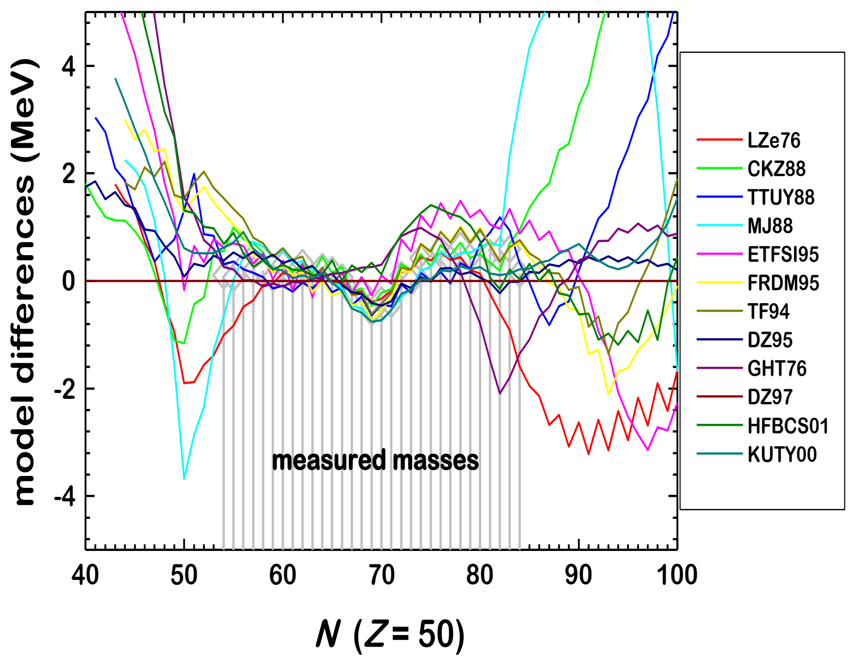
Accuracy of (global) nuclear mass models



- **P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, 1995**
- **M. Samyn, S. Goriely, M. Bender, and J. M. Pearson, 2004**
- **J. Duflo and A. P. Zuker, 1995**
- **Compilation: G. Audi, A. H. Wapstra, and C. Thibault, 2003**



(Courtesy of D. Lunney)



Are there fundamental limits to our predictive abilities?

News and Views

Nature **417**, 499-501 (30 May 2002)

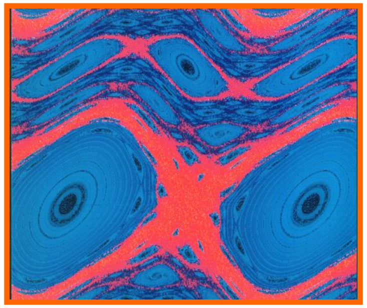
Nuclear physics: Weighing up nuclear masses

Sven Åberg

Abstract

It is difficult to match theoretically calculated masses of atomic nuclei to their experimentally measured values. It seems that chaotic motion inside the nucleus may be the reason for this discrepancy. The atomic nucleus is central to many physical and astrophysical processes, including the nuclear processes that power the Sun and synthesize the elements. Which atoms exist depends on the stability of different types of nuclei, and the stability of an individual nucleus depends on its mass.

Basic Working Hypotheses and its consequences



- H1: The dynamical motion of nucleons corresponds to a mixed dynamics

- Accordingly, the fluctuating part of the energy is splitted in two parts:

$$\tilde{\mathcal{B}} = \tilde{\mathcal{B}}_{reg} + \tilde{\mathcal{B}}_{ch}$$

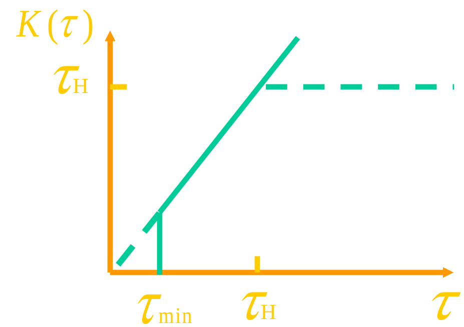
- It can easily be shown, from the semiclassical theory, that $\left\langle \tilde{\mathcal{B}}_{reg} \tilde{\mathcal{B}}_{ch} \right\rangle = 0$

- H2: Estimate the typical size of the contribution of the chaotic component, and compare it to the typical error in global nuclear mass models

(Phase-space volume of chaotic component?)

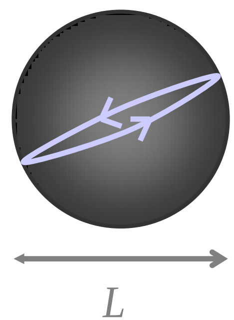
Typical size of the contribution of a Chaotic motion of the nucleons to the mass of a nucleus:

M.V. Berry'85

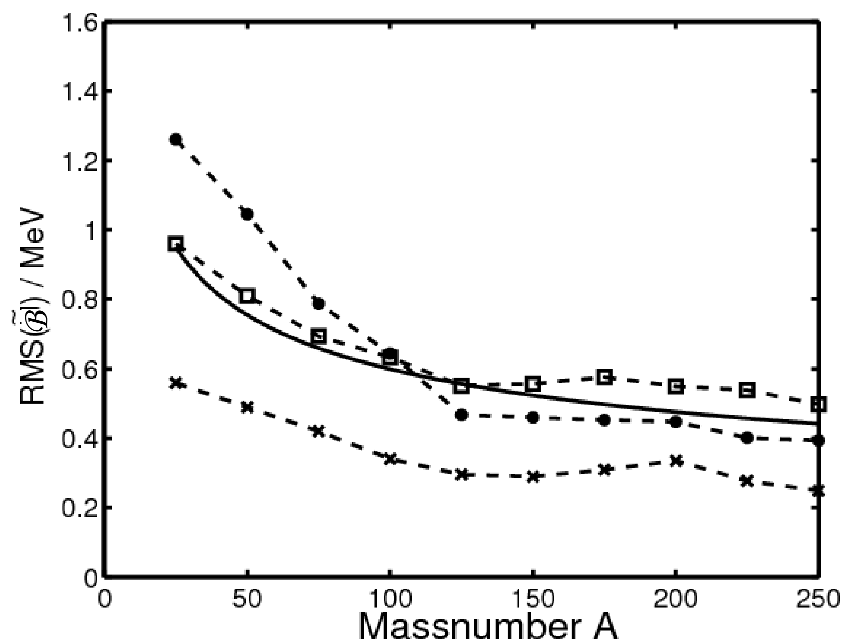


$$\left\langle \widetilde{\mathcal{B}}^2 \right\rangle = \frac{\hbar^2}{2\pi^2} \int_0^\infty \frac{d\tau}{\tau^4} K(\tau)$$

$$\left\langle \widetilde{\mathcal{B}}^2 \right\rangle_{\rm ch} = \frac{1}{8\pi^4} E_C^2 \;, \quad \beta = 1$$



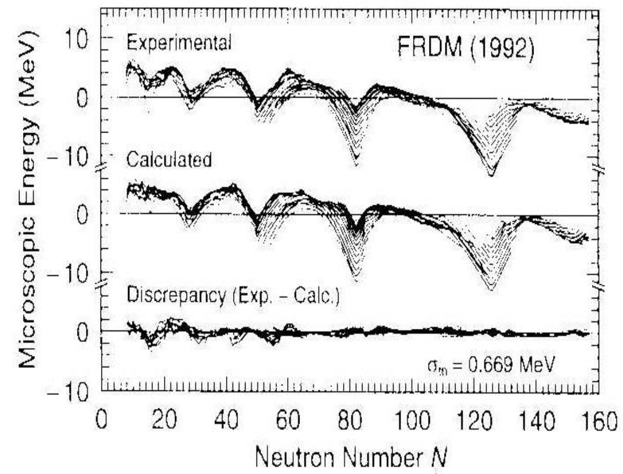
$$\sigma_{ch} = \sqrt{\left\langle \widetilde{\mathcal{B}}^2 \right\rangle_{ch}} = \frac{2.8}{A^{1/3}} \; \text{MeV}$$



O. Bohigas and P. Leboeuf
PRL 88 (2002) 092502

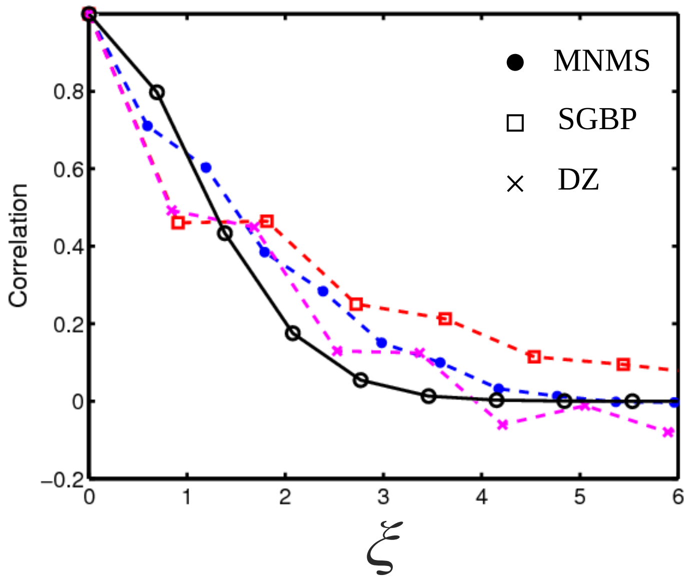
Correlations of the residual mass vs Chaotic motion

$$c(N) = \left\langle \tilde{\mathcal{B}}(N_0 + N/2)\,\tilde{\mathcal{B}}(N_0 - N/2) \right\rangle$$



$$c(\xi) = c(N)/c(0); \qquad \xi = \sqrt{\left\langle \left(\partial_N \tilde{\mathcal{B}}\right)^2 \right\rangle / \left\langle \tilde{\mathcal{B}}^2 \right\rangle}$$

$$c(\xi) = \left(1 - \frac{\xi^2}{4}\right) e^{-\xi^2/4} + \frac{\xi^4}{16}\,\Gamma\left(0, \frac{\xi^2}{4}\right)$$



H. Olofsson, S. Aberg, O. Bohigas and P. Leboeuf
PRL 96 (2006) 042502

Concluding remarks

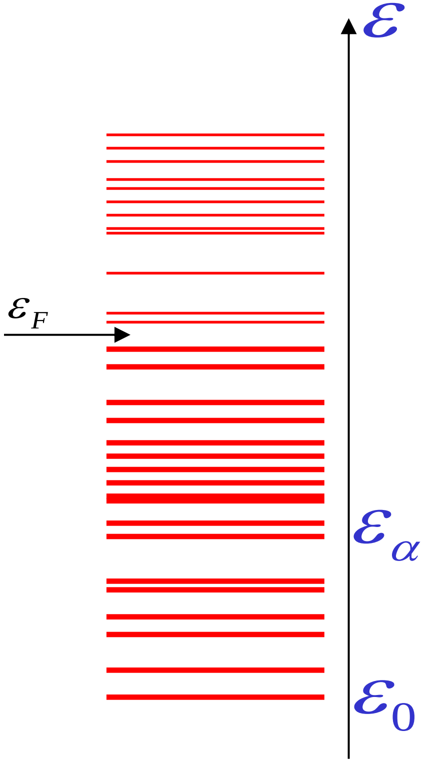
- Evidence of a new « regime » in the ground states properties of nuclei
- No adjustable parameters
- Chaos does not set an « a priori » bound or fundamental limit to the accuracy of theoretical mass models
- It suggests, however, that it may be difficult to improve significantly global mass models (improved sensitivity to details of the Hamiltonian)
- Local models (with sufficient number of parameters) may go well below
- Arguments given may be valid beyond mean field (periodic orbits of the full phase space of interacting particles?)

Pairing Gap – Mean-field BCS theory

Pairing Hamiltonian: $$H = \sum_k \varepsilon_k a_k^+ a_k - G \sum_{j,k} a_j^+ a_{\bar{j}}^+ a_{\bar{k}} a_k$$

Gap is a solution of

$$\frac{2}{G} = \int_{-L}^{L} \frac{d\varepsilon \, \rho(\varepsilon)}{\sqrt{\varepsilon^2 + \Delta^2}}$$



- $\rho(\varepsilon) = \sum_\alpha \delta(\varepsilon - \varepsilon_\alpha)$ Single-particle level density
- L: cutoff parameter
- G: interaction strength

Fluctuations

$$\rho(\varepsilon) \Rightarrow \tilde{\rho} = 2\sum_{p}\sum_{r=1}^{\infty} A_{p,r}(\varepsilon)\cos\left[rS_p(\varepsilon)/\hbar + \nu_{p,r}\right]$$

$\Delta << L$:

$$\tilde{\Delta} = \frac{\bar{\Delta}}{\bar{\rho}}\sum_{p}\sum_{r=1}^{\infty} A_{p,r}(\varepsilon_F)\; Y(r\,\tau_p)\cos\left[rS_p(\varepsilon_F)/\hbar + \nu_{p,r}\right]$$

$\varepsilon_F = Fermi\ energy$

where
$$Y(\tau) = \int_{-L}^{L} d\varepsilon \frac{\cos(\tau\varepsilon/\hbar)}{\sqrt{\varepsilon^2 + \bar{\Delta}^2}} \approx 2K_0(\tau/\tau_\Delta)$$

Modified Bessel function of 2nd kind

New time scale

$$\tau_\Delta = \frac{\hbar}{\bar{\Delta}}$$

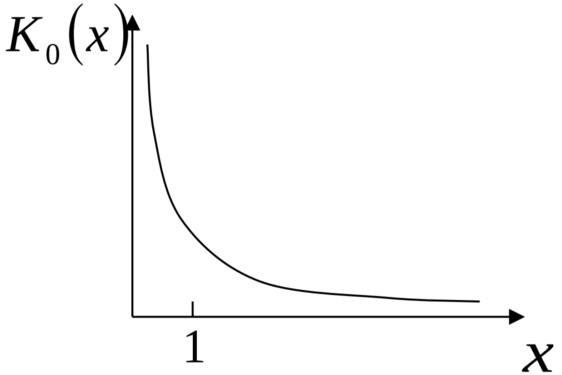
$$\bar{\Delta} = 2L\exp\left(-1/\bar{\rho}G\right)$$

Note: $\tilde{\Delta}$ is independent of L, G (no further renormalization needed, aside $\bar{\Delta}$)

→ parameter free

Qualitative properties of the gap fluctuations:

$$K_0(x) = \begin{cases} -\gamma - \log(x/2) + O(x^2 \log x) & x \to 0 \\ \pi e^{-x} / 2\sqrt{x} & x \to \infty \end{cases}$$



$$\tilde{\Delta} = 2 \frac{\overline{\Delta}}{\overline{\rho}} \sum_p \sum_{r=1}^{\infty} A_{p,r}(\varepsilon_F)\, K_0(r\,\tau_p) \cos[rS_p(\varepsilon_F)/\hbar + \nu_{p,r}]$$

- Generic fluctuations of the pairing gap as one varies shape, particle number, etc
- Due to interferences between different orbits having different actions
- Symmetries of the potential and regularity of the classical dynamics are important
- Orbits with $\tau_p >> \tau_\Delta$ are exponentially damped
- In contrast, orbits with $\tau_p < \tau_\Delta$ are logarithmically enhanced
- Detailed description of the pairing fluctuations requires a specific model

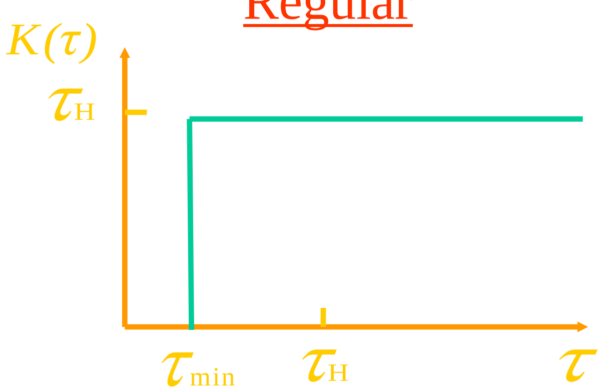
Typical size of the gap fluctuations

$$\left\langle \tilde{\Delta}^2 \right\rangle = \frac{\overline{\Delta}^2}{2\tau_H^{\;2}} \int_0^\infty d\tau\, Y^2(\tau) K(\tau)$$

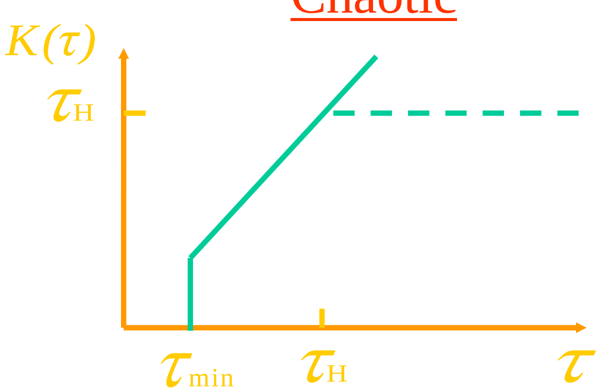
$\tau_H = h/\delta$ Heinsenberg time

$K(\tau)$: Form factor (Fourier transform of the spectral two-point correlation fonction)

Regular



Chaotic



$K(\tau) \approx \tau_H \quad \tau \geq \tau_{\min}$

$K(\tau) \approx \tau \quad \tau \geq \tau_{\min}$

$$\overline{\Delta} > \delta \Rightarrow \tau_\Delta / \tau_H = \delta / 2\pi\overline{\Delta} << 1$$

In the limit $D = \frac{\tau_{\min}}{\tau_{\Delta}} = \frac{2\pi}{g}\frac{\bar{\Delta}}{\delta} \to 0$

$$\sigma^2 \to \begin{cases} \frac{\pi}{4}\frac{\bar{\Delta}}{\delta} & \text{Regular dynamics} \\ \frac{1}{2\pi^2} & \text{Chaotic dynamics: univers.} \end{cases}$$

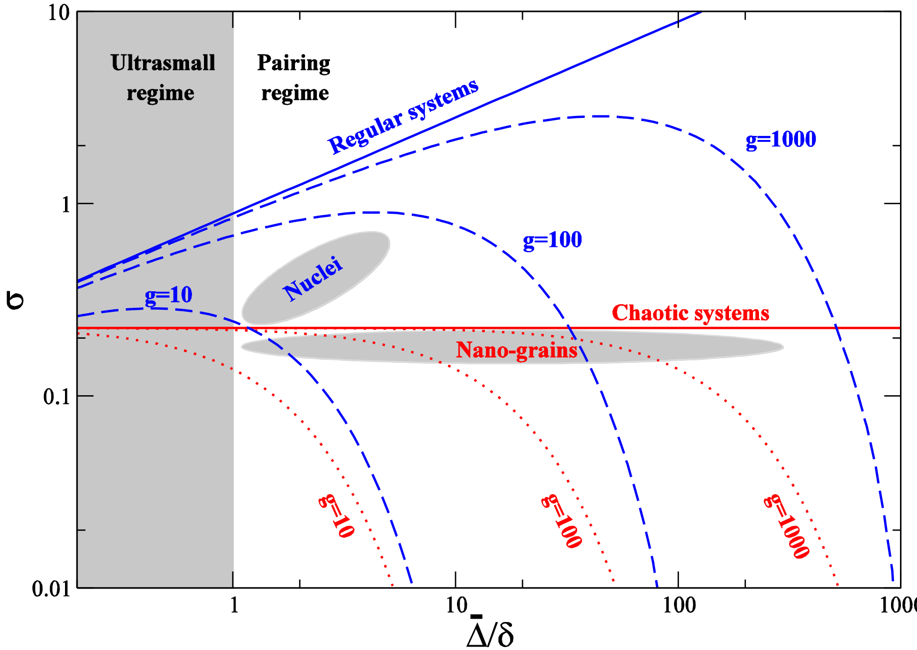
Regular dynamics

Chaotic dynamics: univers.

(Matveev & Larkin, PRL **78**, 3749 (1997))

In contrast, when $D = \frac{\tau_{\min}}{\tau_{\Delta}} = \frac{2\pi}{g}\frac{\bar{\Delta}}{\delta} >> 1$

$\sigma^2 \to 0$ Exponentially fast



Application: ground states of atomic nuclei

$$M = Z*M_P + N*M_N \; c^2 - \mathcal{B}(Z,N)/c^2$$

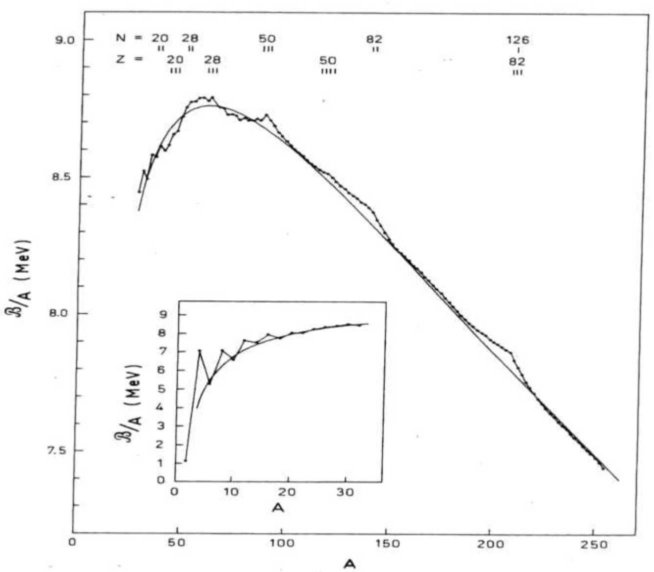
Ground state superfluidity

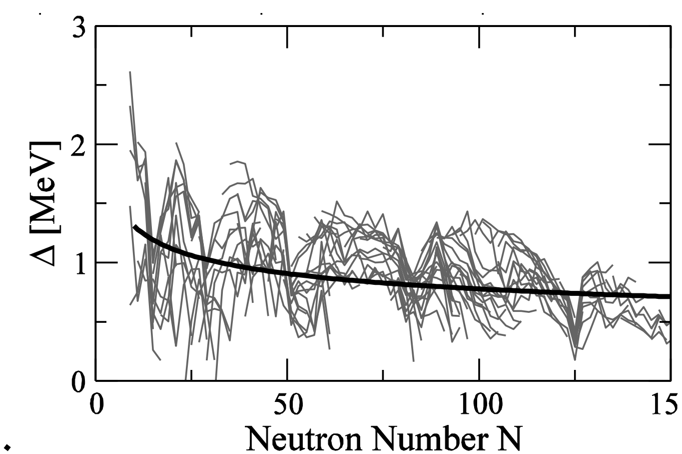
mass difference between even & odd number of part.

$$\Delta_3(N) = B(N) - \left[B(N+1) + B(N-1)\right]/2$$

N: odd neutron or proton number

(J. Dobaczewski et al, PRC **63**, 024308 (2001))





$$\overline{\Delta} \approx \frac{2.7}{A^{1/4}}\,\text{MeV}$$

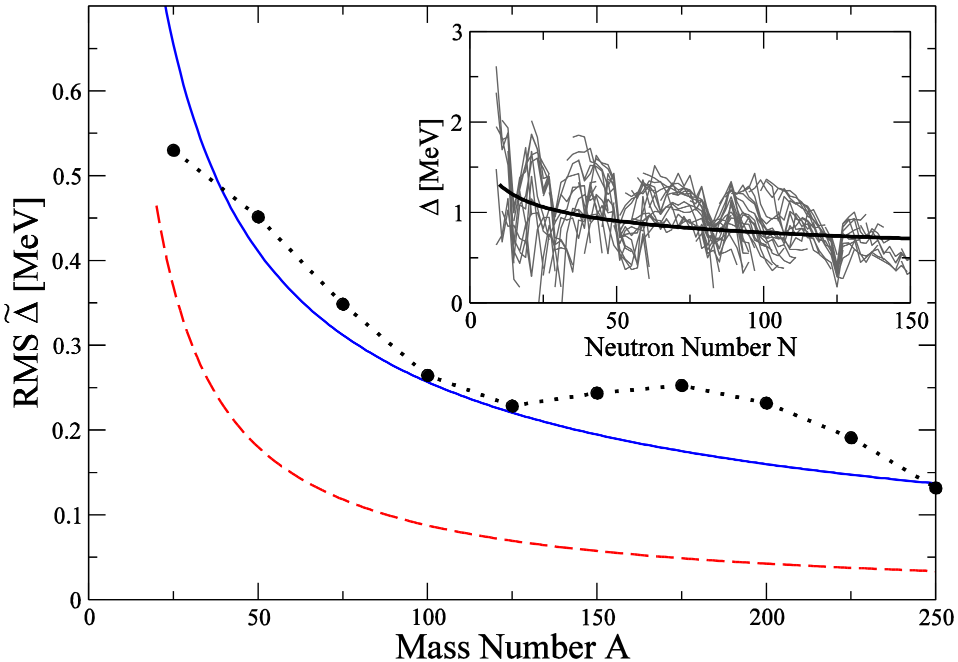
Estimate of the RMS gap fluctuations for atomic nuclei

- Mean level spacing $\delta = \frac{50}{A} MeV$

- Dimensionless conductance $g = 1.6\ A^{2/3}$

$$\bar{\Delta} \approx \frac{2.7}{A^{1/4}} \text{MeV}$$

$$\Rightarrow \quad D = \frac{2\pi}{g}\frac{\bar{\Delta}}{\delta} \approx 0.21 A^{1/12} = \begin{cases} 0.27 & A = 25 \\ 0.33 & A = 250 \end{cases}$$



$$\sigma^2_{reg} = \frac{\pi}{4}\frac{\bar{\Delta}}{\delta} F_0(D)$$

$$\sigma^2_{ch} = \frac{1}{2\pi^2} F_1(D)$$

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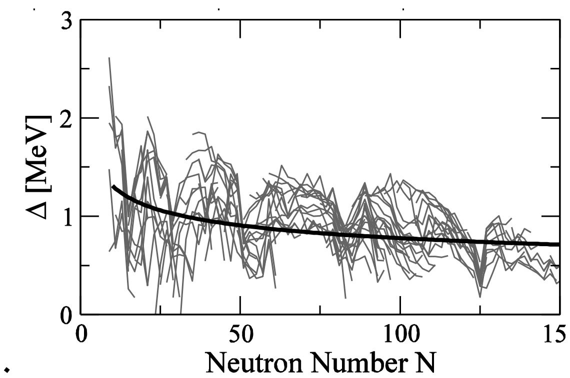
PRL **100,** 037005

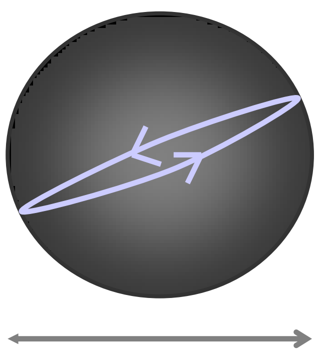
Detailed description of the fluctuations



Spherical Box+deformations

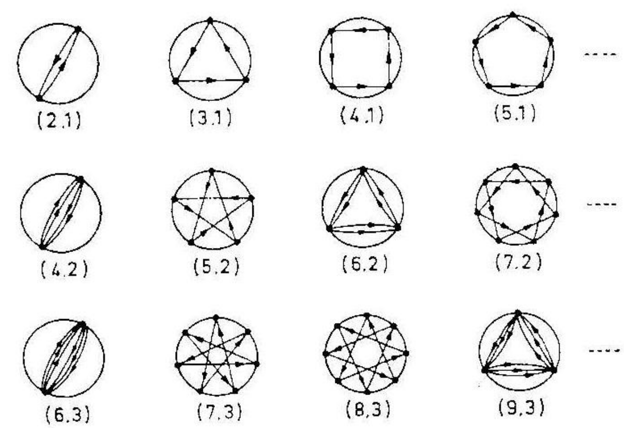
Energy minimization at fixed *N* determines deformation

Shortest periodic orbits



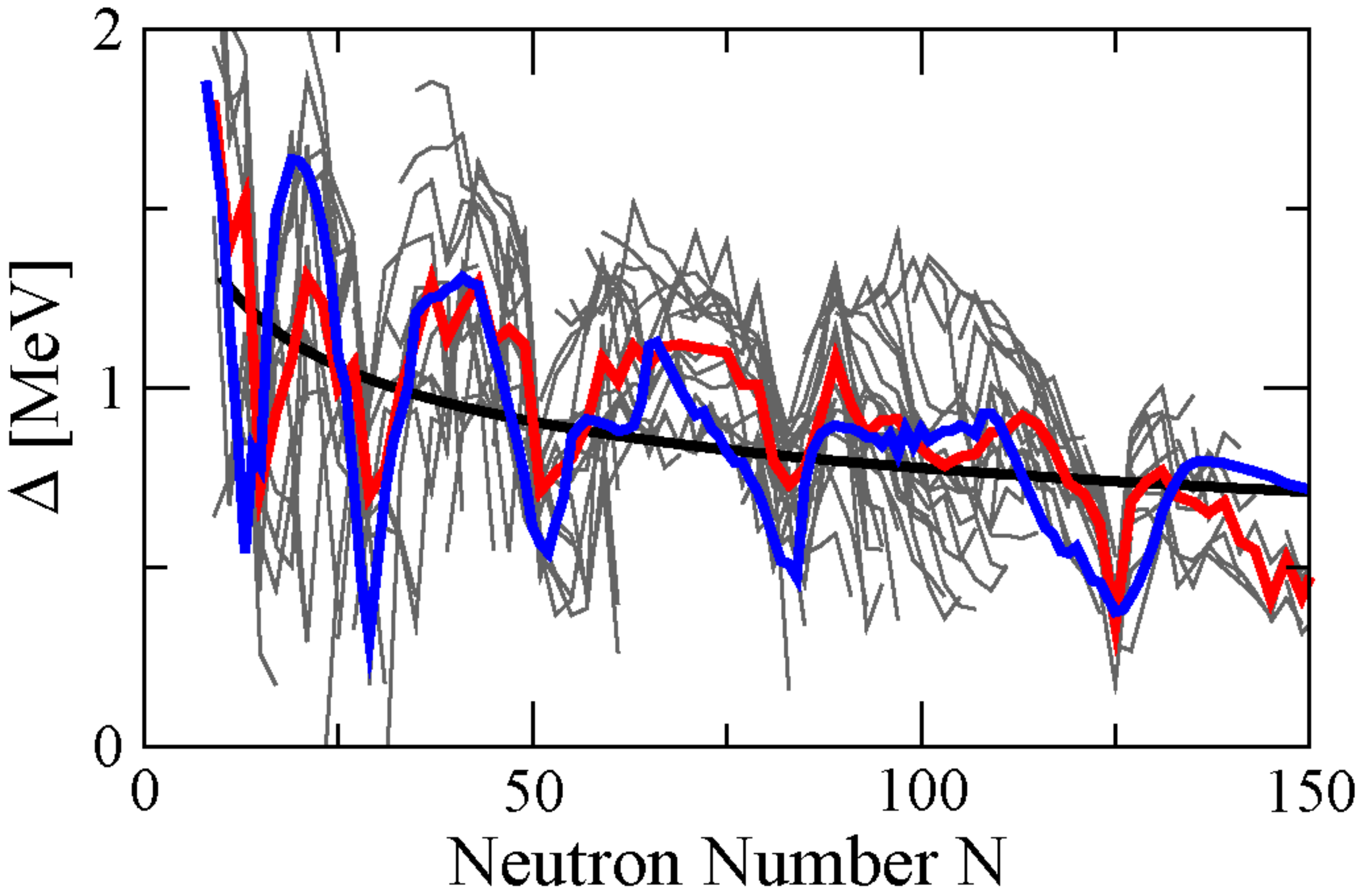
$$\mathrm{L} = 2\,\mathrm{r} = 2\,\mathrm{r}_0\ \mathrm{A}^{1/3}$$

$$S_p = \hbar\, k_F L_p \propto A^{1/3}$$



$$\tilde{\Delta} = 2\frac{\bar{\Delta}}{\bar{\rho}} \sum_p \sum_{r=1}^{\infty} A_{p,r}(\varepsilon_F)\, K_0(r\,\tau_p)\, f_p(def)\cos\left[rS_p(\varepsilon_F)/\hbar + \nu_{p,r}\right]$$

Nuclear Pairing Gap Fluctuations



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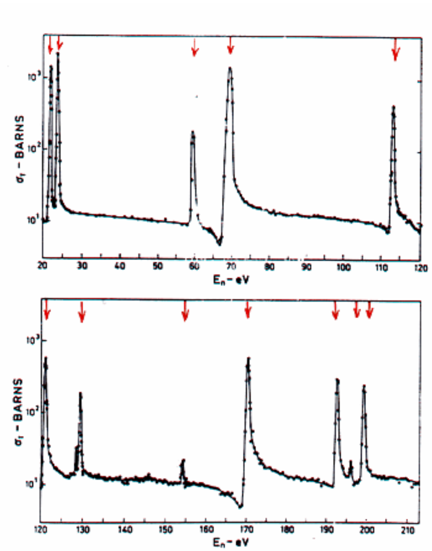
PRL **100,** 037005

Concluding remarks

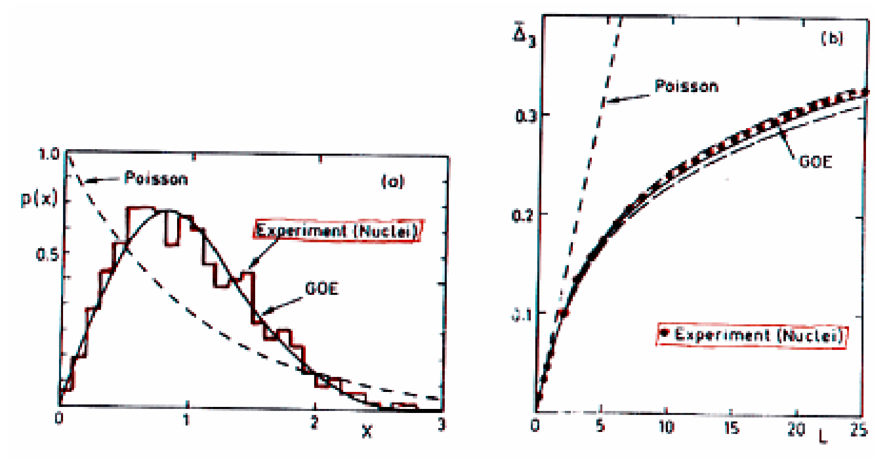
- General theory that describes the pairing gap fluctuations in finite Fermi systems
- Semiclassical basis
- Importance of the nature of the underlying classical dynamics
- Fluctuations are more important in regular systems
- General description of their typical size
- Excellent agreement with nuclear data
- Detailed description of the fluctuations is possible
- Cold atoms – Metallic nanoparticles – Experiments
- Theory of the smooth part is missing

Slow neutron resonances

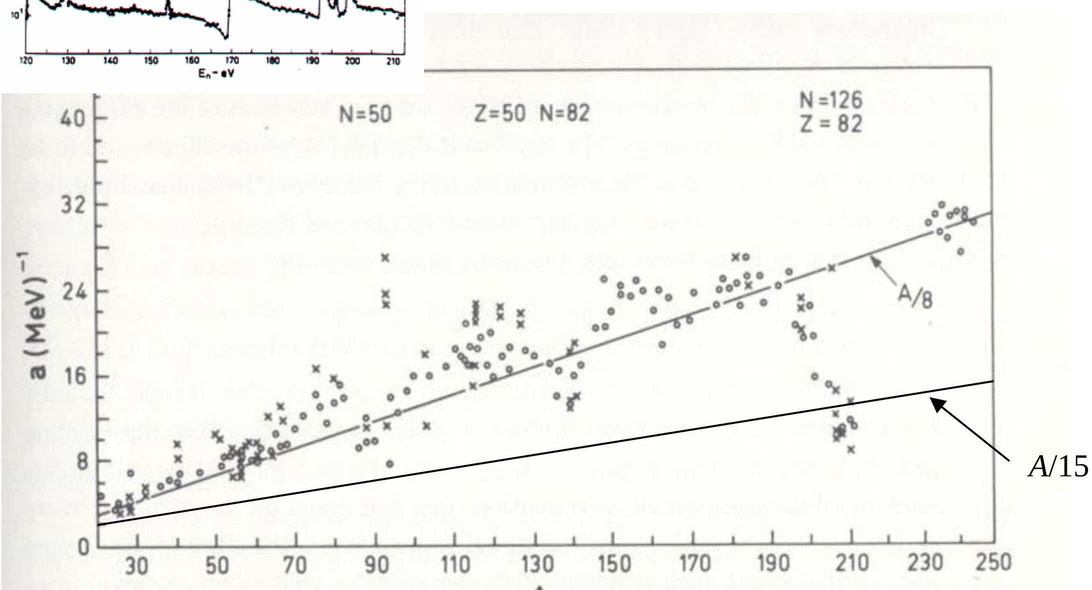
Total cross section $n + {}^{232}\mathrm{Th}$



Haq, Pandey, Bohigas, PRL **48** (1982) 1086



Bohr and Mottelson, Volume I



The parameter a appearing in the Fermi gas level density formula

$$\rho_{MB}(A,Q) \sim \frac{e^{2\sqrt{aQ}}}{Q}$$

Bethe (1936)

$$Q = \mathcal{B} - \mathcal{B}_{GS}$$

$$a = \frac{\pi^2}{6}\bar{\rho} \approx \frac{A}{15}\,\mathrm{MeV}^{-1}$$

MB Level Density → Mean Field app.

Single Particle Spectrum

$$-\frac{\hbar^2}{2m}\nabla^2\psi_\alpha = \varepsilon_\alpha \psi_\alpha$$

$$\psi_\alpha\big|_{\mathcal{D}} = 0$$

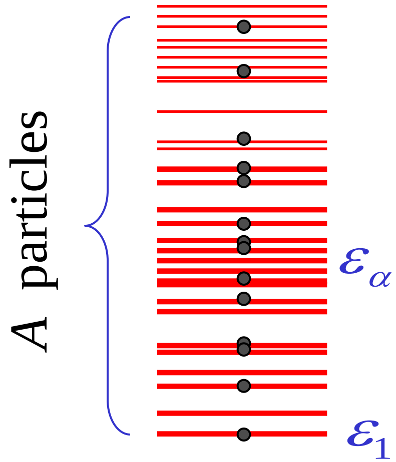
<u>Single Particle Level Density</u>

$$\rho(\varepsilon) = \sum_\alpha \delta(\varepsilon - \varepsilon_\alpha) = \bar{\rho}(\varepsilon) + \tilde{\rho}(\varepsilon)$$

$\rho_{MB}(A,\mathcal{B})\,d\mathcal{B} =$ # of states with energy between $\mathcal{B}$ and $\mathcal{B}+d\mathcal{B}$

$$= \sum_\nu \delta(A - A_\nu)\,\delta(\mathcal{B} - \mathcal{B}_\nu)\,d\mathcal{B}$$

$$= \frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty} d\alpha \int_{-b-i\infty}^{-b+i\infty} d\beta\, e^{-\beta\Omega(\alpha,\beta) - \alpha A + \beta\mathcal{B}}\, d\mathcal{B}$$



$$\begin{cases} A_\nu = \sum_\alpha n_\alpha^{(\nu)} \\ \mathcal{B}_\nu = \sum_\alpha n_\alpha^{(\nu)} \varepsilon_\alpha \end{cases}$$

Fermions: $n_\alpha^{(\nu)} = 0,1$

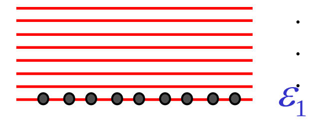
Bosons: $n_\alpha^{(\nu)} = 0,1,2,\ldots$

$$\Omega(\mu,T) = -\frac{1}{\beta}\int d\varepsilon\, \rho(\varepsilon) \log\left[1 + e^{\beta(\mu-\varepsilon)}\right]$$

Equally spaced S.P. spectrum

Bosons

$$Q = n\delta = 4\delta = \delta \times \begin{cases} 1+1+1+1 \\ 1+1+2 \\ 1+3 \\ 2+2 \\ 4 \end{cases}$$



$p_A(n)$: Number of partitions of n in at most *A* terms

$$\Rightarrow \rho_{MB}(A,Q) = \sum_{n=1}^{\infty} p_A(n)\,\delta(Q - n\delta)$$

For **n<A**, $p_A(n) = p(n)$: partition function. Defining $\rho_{MB}(A,Q) = \bar{\rho}\, e^{S_{HR}(A,Q)}$

$$S_{HR}(A,Q) \sim 2\sqrt{\frac{\pi^2}{6}\bar{\rho}Q} - \log\left(\sqrt{48}\bar{\rho}Q\right) - \frac{\pi^2 + 72}{24\sqrt{6}\pi}\frac{1}{\sqrt{\bar{\rho}Q}} + \left(\frac{1}{24} - \frac{3}{4\pi^2}\right)\frac{1}{\bar{\rho}Q} + O\left((\bar{\rho}Q)^{-3/2}\right)$$

Hardy-Ramanujan 1918 – Rademacher 1937 $\bar{\rho} = \delta^{-1}$

Equally spaced S.P. spectrum

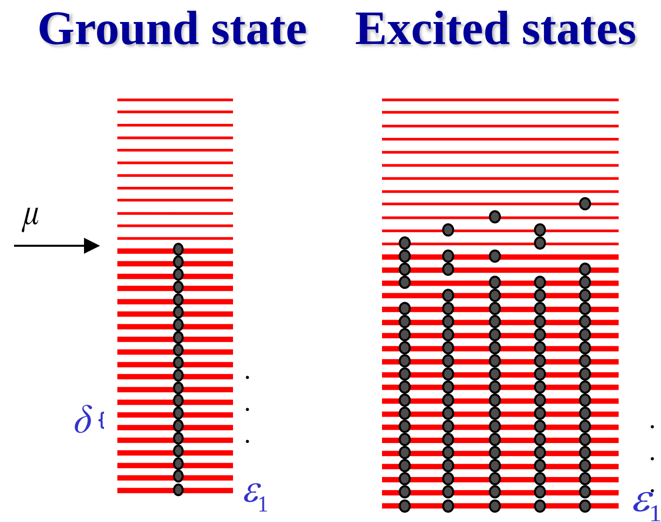
Fermions

$$Q = n\delta = 4\delta = \delta \times \begin{cases} 1+1+1+1 \\ 1+1+2 \\ 1+3 \\ 2+2 \\ 4 \end{cases}$$

Partition in exactly *A* distinct terms

$$q_A\left(\frac{A(A+1)}{2}+n\right) = p_A(n)$$

Partition in at most *A* terms



Young Diagrams

$$\Rightarrow \rho_{MB}(A,Q) = \sum_{n=1}^{\infty} p_A(n)\,\delta(Q - n\delta)$$

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Hardy-Ramanujan 1918 – Rademacher 1937 $\bar{\rho} = \delta^{-1}$

Arbitrary S.P. spectrum: smooth part

$$\rho_{MB}(A,Q) = \bar{\rho}\, e^{S(A,Q)} \qquad A \to \infty$$

Fermions:

$$S(A,Q) \sim 2\sqrt{\frac{\pi^2}{6}\bar{\rho}Q} - \log\left(\sqrt{48}\bar{\rho}Q\right) + O\left((\bar{\rho}Q)^{-1/2}\right)$$

(Bethe 1936)

$$\delta << Q < \mu$$

$$\bar{\rho} = \bar{\rho}(\mu(A))$$

Bosons:

$$\bar{\mathcal{N}}(\mu) = \int^{\mu} \bar{\rho}(\varepsilon) d\varepsilon \ \sim c_{\nu}\mu^{\nu}$$

$$\nu = \begin{cases} 1 & \text{Equid. Spectrum} \\ D/2 & D\text{-dim cavity} \\ D & D\text{-dim. HO} \end{cases}$$

$$S(A,Q) \sim a_{\nu} Q^{\nu/(\nu+1)} - \log\left(b_{\nu} Q^{(\nu+3)/2(\nu+1)}\right)$$

$$\delta << Q < A\delta$$

(Brack-Bhaduri 2005)
(Comtet, Leboeuf., Majumdar 2006)

Maxwell-Boltzman Regime

$Q \to \infty,$ A finite ➡ Classical behavior

$$\overline{\mathcal{N}}(\mu) = \int^{\mu} \bar{\rho}(\varepsilon) d\varepsilon \ \sim c_{\nu} \mu^{\nu}$$

Polynomial growth

$$\bar{\rho} Q >> A^{3}$$

$$\boxed{\rho_{MB}(A,Q) \sim \theta_{A,\nu} Q^{\nu A}}$$

(Sommermann, Weidenmüller 1993)
(Eckhardt 1998)
(Comtet, Leboeuf, Majumdar 2006)

Level density at energy Q and a fixed number of particles A: Departures from the asymptotic law

Bose Gas

$$F(Q,A)=\frac{\rho(A,Q)}{\rho(Q)}$$

$\nu=1$

$$F(A,Q)=Exp\left(-\frac{1}{c}Exp(-cx)\right),\qquad x=\frac{A}{\sqrt{\bar{\rho}Q}}-\frac{c}{2}\log(\bar{\rho}Q)$$

Gumbel

(Erdös-Lehner)

$$\bar{\rho}Q\sim(A/\log A)^2$$

$0<\nu<1$

$$F(A,Q)=Exp\left(-a_\nu\left(\frac{Q^{1/(1+\nu)}}{A}\right)^{\nu/(1-\nu)}\right)$$

Fréchet

$\nu>1$

$$F(A,Q)=Exp\left(-C_{Q,\nu}(A-A_c)^{\theta_\nu}\right),\qquad A_c=s_\nu/\beta^\nu,\ A<A_c$$

$$F(A,Q)=1,\qquad A>A_c$$

Weibull

(Bose condensation)

Extreme Statistics!

(Comtet, Leboeuf., Majumdar 2006)

Arbitrary S.P. spectrum: fluctuations

Fermions

$$\rho_{MB}(A,Q) = \bar{\rho}\, e^{S(A,Q)}$$

$$\delta << Q < \mu$$

$$S(A,Q) \approx 2\sqrt{\frac{\pi^2}{6}\bar{\rho} Q} - \log\left(\sqrt{48}\bar{\rho} Q\right) + \frac{1}{T}\left[\tilde{\mathcal{B}}(\mu,0) - \tilde{\mathcal{B}}(\mu,T)\right]$$

Leboeuf, Monastra, Relaño, PRL 94 (2005) 102502

$$T = \sqrt{\frac{6}{\pi^2 \bar{\rho}} Q}$$

Fluctuating part of the energy of the gas:

$$\tilde{\mathcal{B}}(\mu,T) = \mathcal{B}(\mu,T) - \overline{\mathcal{B}}(\mu,T) = \int d\varepsilon\, \varepsilon\, \rho(\varepsilon) f(\varepsilon,\mu,T) - \int d\varepsilon\, \varepsilon\, \bar{\rho}(\varepsilon) f(\varepsilon,\mu,T)$$

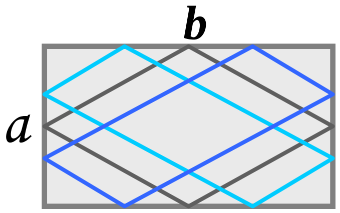
$$\approx 2\hbar^2 \sum_p \sum_{r=1}^{\infty} \frac{A_{p,r}(\mu)\, \kappa_T(r\,\tau_p)}{r^2\, \tau_p^2} \cos\left[r S_p/\hbar + \nu_{p,r}\right]$$

$$\kappa_T(\tau) = \frac{\tau/\tau_T}{\sinh(\tau/\tau_T)} \quad , \quad \tau_T = \frac{h}{2\pi^2 T}$$

Numerical test

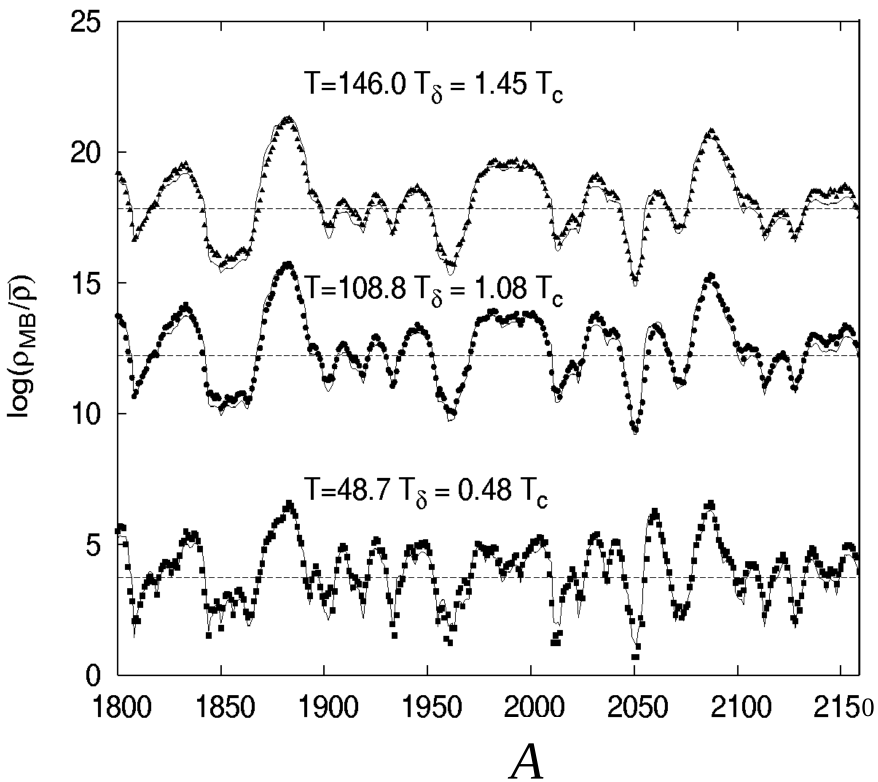
<u>2000 particles in a 2D rectangular box</u>

$$E_{\vec{n}} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{n_1^2}{a^2}+\frac{n_2^2}{b^2}\right); \quad n_i = 1,2,\ldots \ \ (d.b.c.)$$



$$a = \sqrt{\left(1+\sqrt{5}\right)/2}$$

$$b = 1/a$$



$$\begin{cases} T_\delta = \delta / 2\pi^2 = (2\pi^2 \bar{\rho})^{-1} \\ T_C = E_C / 2\pi^2 \end{cases}$$

Comparison with experimental data

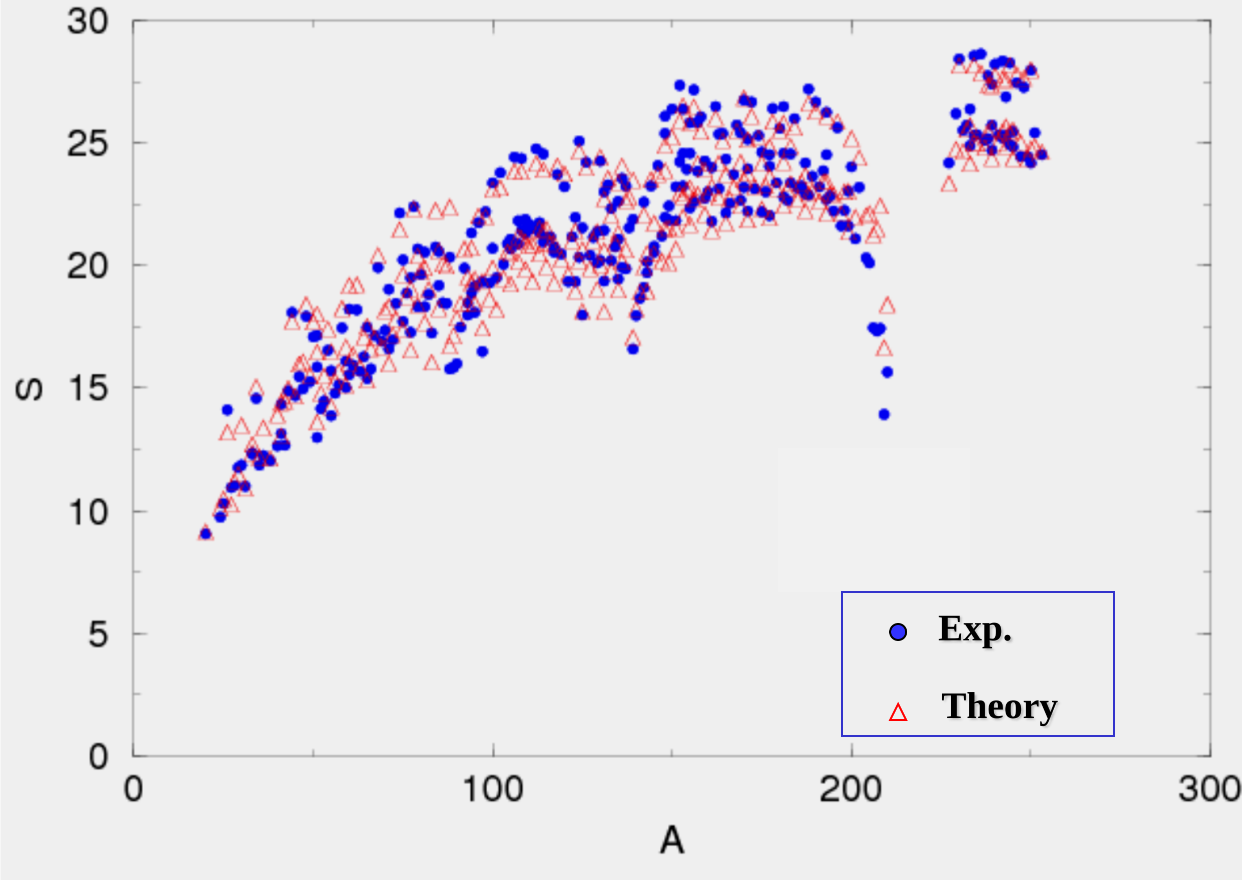
$$\rho_{MB}(Z,N,Q) = \rho_0\, e^{S(Z,N,Q)}$$

$$S(Z,N,Q) = 2\sqrt{aQ} + \frac{1}{T}\left[\tilde{\mathcal{B}}(Z,N,0) - \tilde{\mathcal{B}}(Z,N,T)\right]$$

- $a \approx \frac{A}{7.35} - \frac{A^2}{7692}\ \mathrm{MeV}^{-1}$
- $Q = S_n = \mathcal{B}(Z,N+1) - \mathcal{B}(Z,N)$
- $T = \sqrt{\frac{Q}{a}}$
- $\tilde{\mathcal{B}}(Z,N,T) \approx \kappa(A,T)\tilde{\mathcal{B}}(Z,N,0)$

$\kappa(A,T)$: Cut off parameter

Comparison with experiments II



P. Leboeuf and J. Roccia, PRL 97, 2006

Concluding remarks

- Two extreme behaviors for the average density: exponential growth in the degenerate gas limit, polynomial in the « classical » limit (→lower order corrections)
- In between, finite size corrections related to extreme statistics (→Fermions)
- Fluctuations in the many body density of states for fermions in the deg. gas limit, temperature dependence, *A* dependence, dynamics (→fermions in other regimes (MB), bosons)
- Connexion between RMT and shell corrections to the density (residual inter)
- In, e. g., nuclear physics, above neutron resonances the system is open (→ single particle, and many body, level densities for an open system)
- …