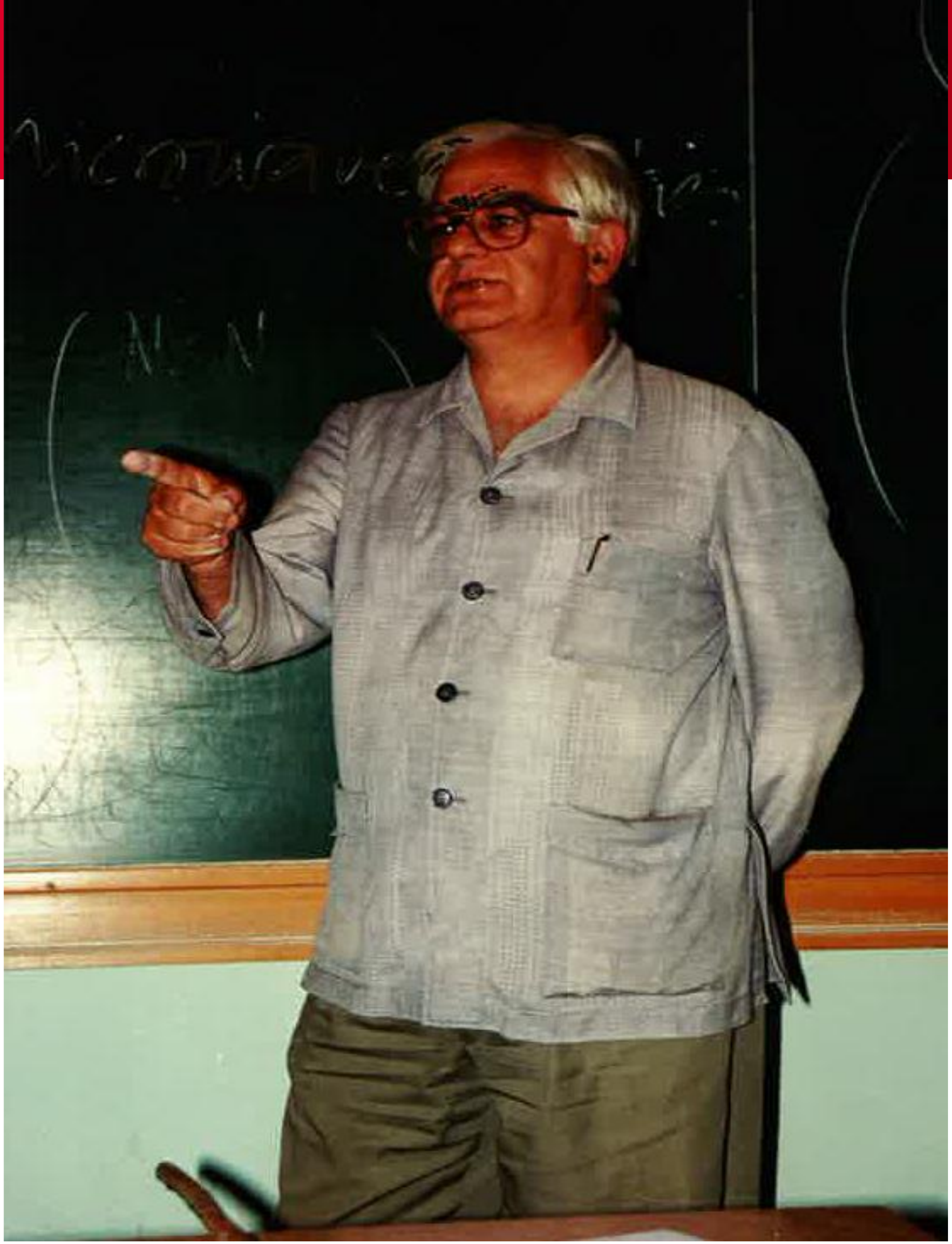
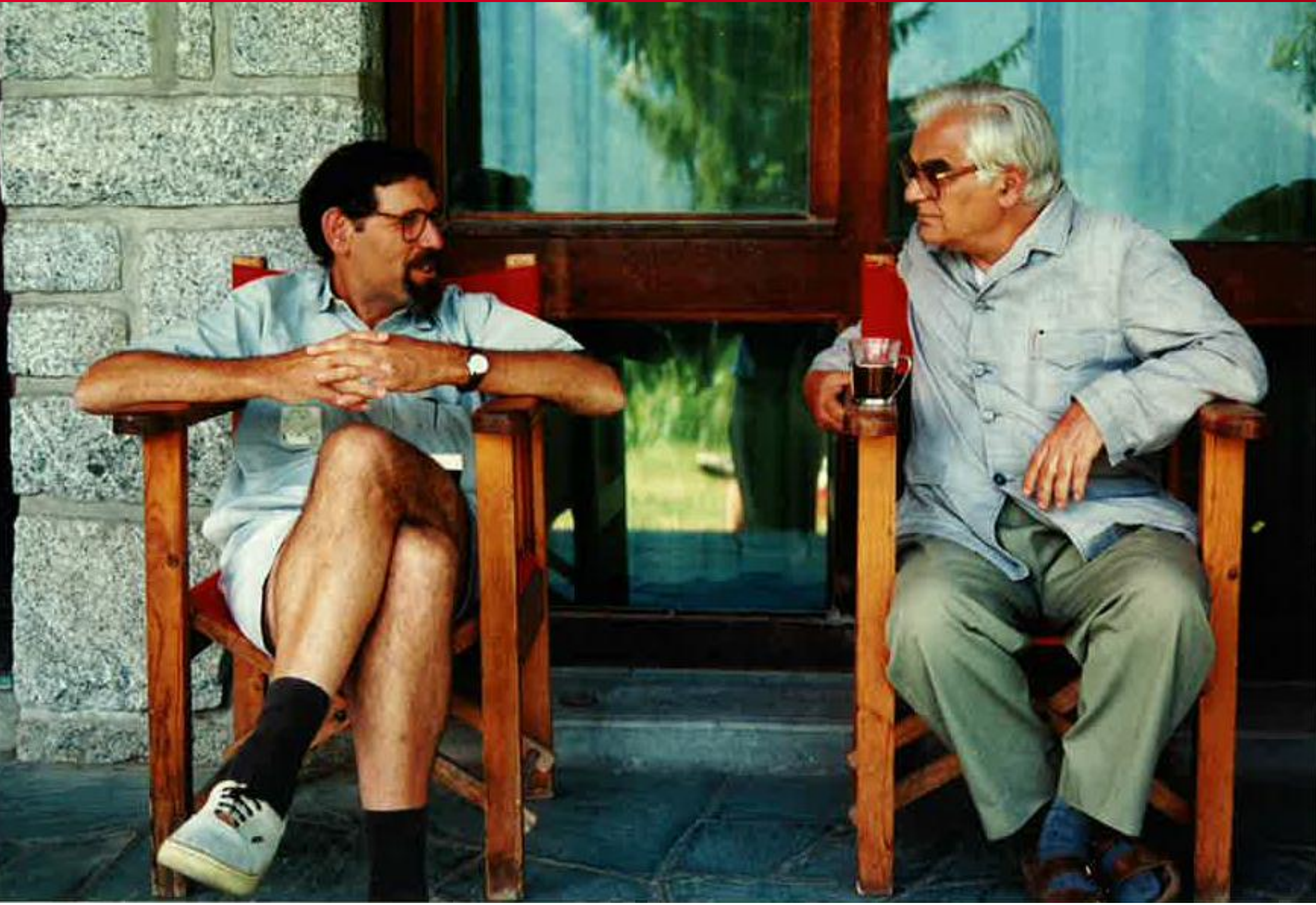
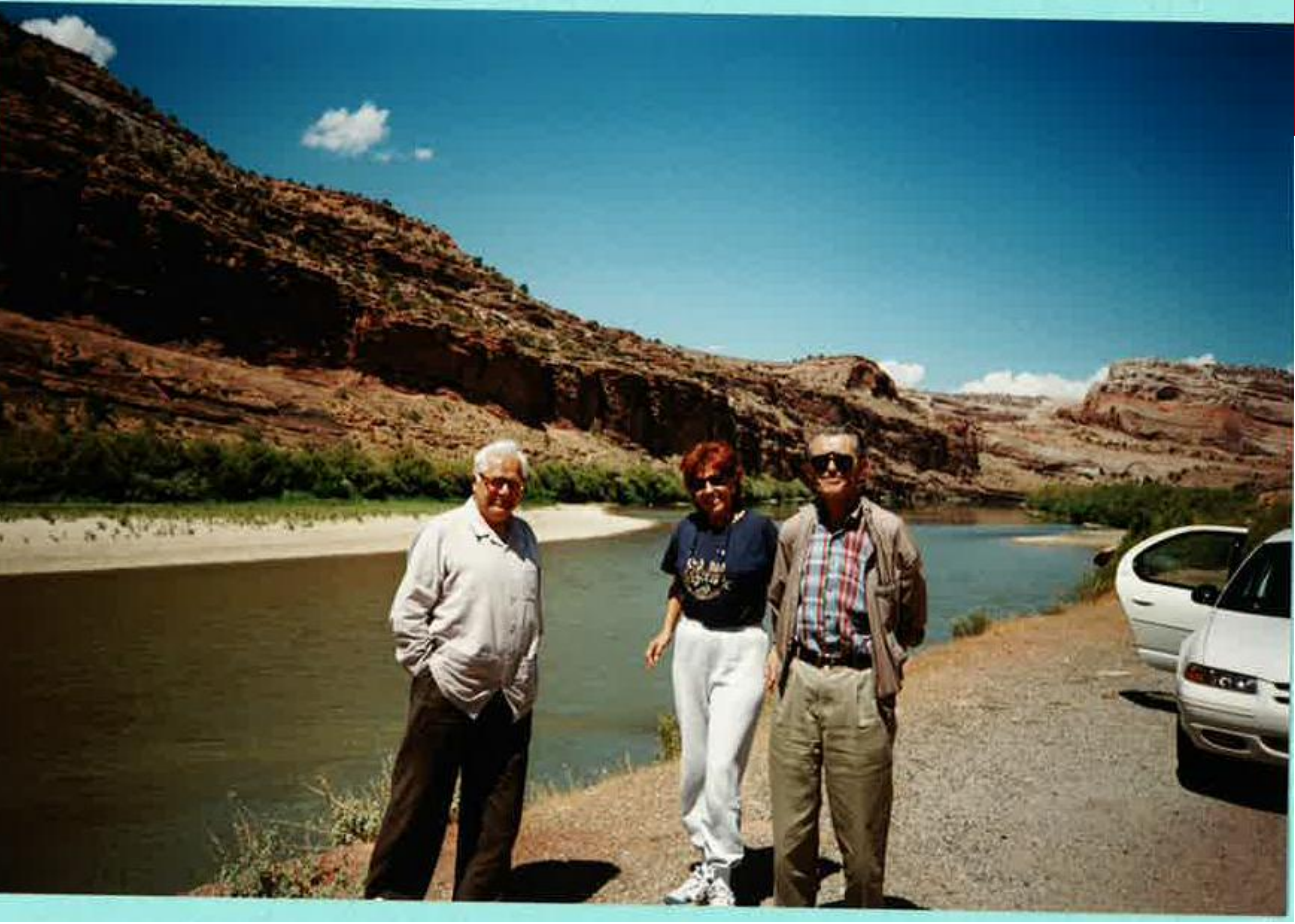


Les Houches

1994





Between Aspen and Utah near the Colorado river

Near Trento (Laguo di Garda)

Copenhagen (after Nobel symposium on Quantum Chos)

Sailing at Concarneau

DE LA RECHERCHE À L'INDUSTRIE

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Edge Thermopower Distributions

in Chaotic Quantum Dots and Disordered Nanowires

Low temperature elastic coherent regime

JEAN-LOUIS PICHARD

DSM/IRAMIS/SPEC

Edge = Spectrum edge of a chaotic scatterer
Edge of a nanowire impurity band

OUTLINE

1/ Delay-Time and Thermopower Distributions at the Spectrum Edges of a Chaotic Scatterer

A scattering approach :

Energy independent distibution of the scattering matrix S

Adel Abbout, Geneviève Fleury, and Jean-Louis Pichard
DSM/IRAMIS, Service de Physique de l'Etat Condensé, CEA Saclay

Khandker Muttalib
Department of Physics, University of Florida, Gainesville

Phys. Rev. B. 87, 115147 (2013)

2/ Gate-modulated thermopower in disordered nanowires: I. Low temperature coherent regime

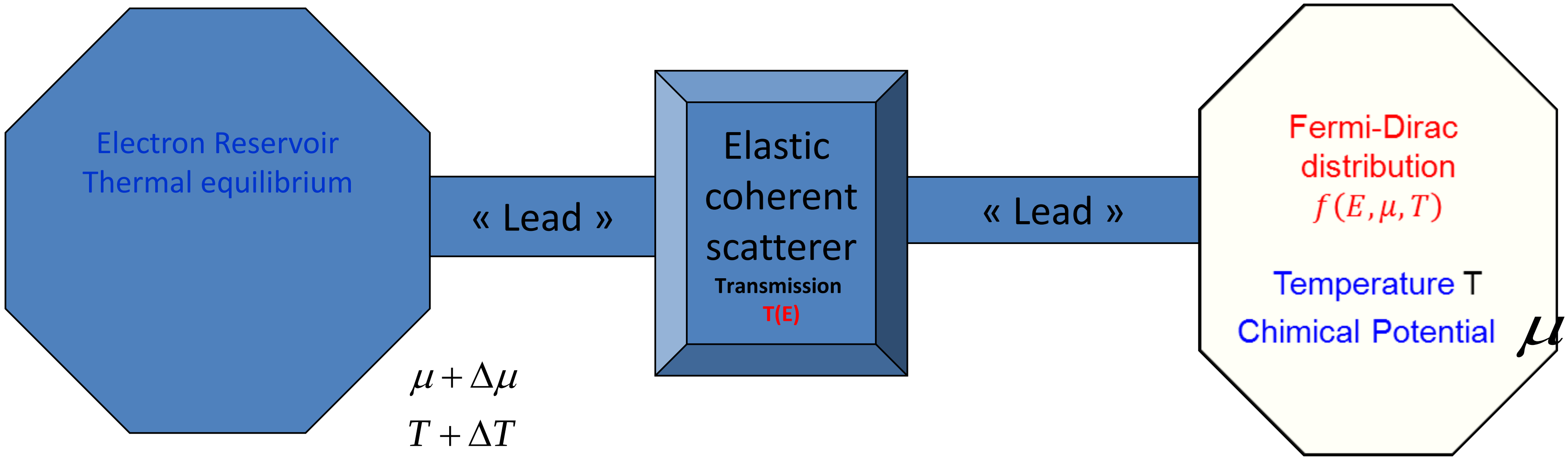
An Hamiltonian approach:

Energy independent distribution of the Hamiltonian H

Riccardo Bosisio, Geneviève Fleury and Jean-Louis Pichard

To appear in New Journal of Physics

Linear Response (mesoscopic regime) Imry and Sivan
Charge and heat currents induced by generalized forces



$$\begin{pmatrix} J_e \\ J_Q \end{pmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{pmatrix} \Delta\mu/_{eT} \\ \Delta T/_{T^2} \end{pmatrix}$$

$$L_{ij} = \frac{e^2 T}{h} \int dE \begin{bmatrix} 1 & E-\mu/_e \\ E-\mu/_e & \left(E-\mu/_e\right)^2 \end{bmatrix} T(E) \left(\partial f/_{\partial E}\right)$$

Conductances
(electrical and thermal)

$$G_e = \frac{L_{11}}{T}$$

$$G_Q = \frac{\det L}{T^2 L_{11}}$$

Thermo-electric coefficients
Seebeck and Peltier

$$S = \frac{L_{12}}{T L_{11}}$$

P=ST

(Kelvin-Onsager)

Importance to break particle-hole symmetry

Cutler-Mott Formula (à la Landauer)

(Sommerfeld Expansion)

- **Thermopower** in terms of the **energy derivative of the system transmission** $\boldsymbol{\tau(E)}$

$$\sigma_k = -\frac{(\pi k_B)^2 T}{3e} \frac{\partial \log(\tau(E))}{\partial E}\bigg]_{E=E_F}$$

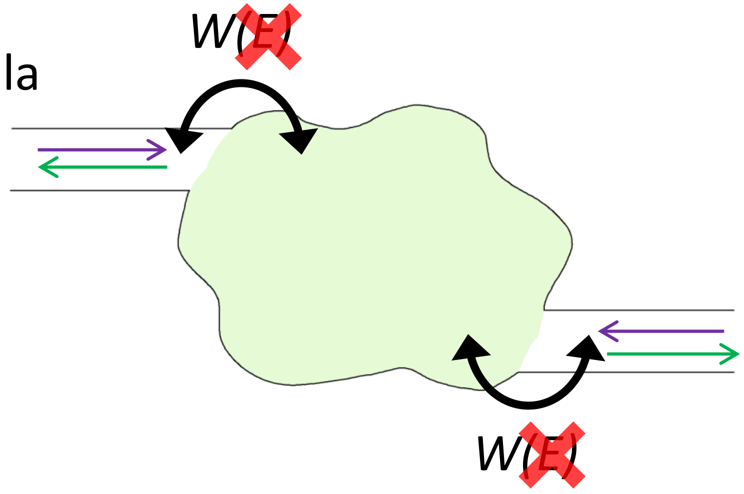
Importance of the Wigner-Smith time-delay matrix

$$\mathbf{Q(E)=-i}\, \hbar\, \boldsymbol{S(E)^{-1}} \,.\, \frac{\boldsymbol{\partial S(E)}}{\boldsymbol{\partial E}}$$

- Validity of Wiedemann-Franz law in the mesoscopic regime? Vavilov – Stone 2005

THE WIDE-BAND LIMIT

Two leads with N modes Weidenmuller Formula



$$S(E) = I - 2i\pi W^\dagger \frac{1}{E - H_M + i\pi W W^\dagger} W$$

Hypothesis : W is energy-independent
Correct in the wide-band limit (ie in the bulk)

Scattering approach
$S(E_F)$ in the circular ensemble

⟺

Hamiltonian approach
H_M in the gaussian ensemble

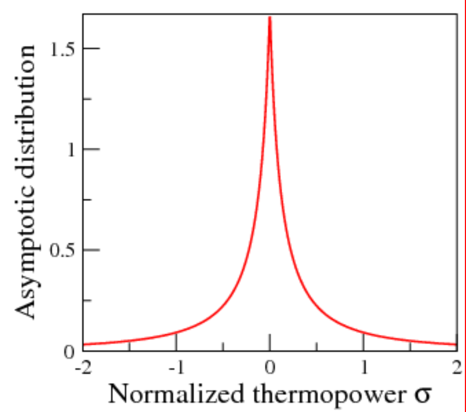
$W(E)$
$M \to \infty$

N modes

Derivation of delay-time distrib. $P(\tau_1, \tau_2, \ldots)$
where τ_i = eigenvalues of $Q = -i\hbar S^\dagger \frac{dS}{dE}$
$\Rightarrow$ Laguerre ensemble for $1/\tau_i$

$N = 1$

Thermopower distribution



Time-delay distribution : Brouwer, Frahm and Beenakker, *PRL* ***78*** *4737 (1997)*
Seebeck distrib. for N=1 : Van Langen, Silvestrov & Beenakker, *Superlatt. MicroStr.* ***23*** *691 (1998)*

Thermopower of a **chaotic** conductor connected to 2 single channel leads

S. A. van Langen, P. G. Silvestrov and C. W. J. Beenakker (**S is a 2x2 COE matrix**)

$$\sigma_k = \frac{\Delta}{2\pi} \frac{\partial \log \tau(E)}{\partial E} \Big]_{E=E_F} \quad (\textit{where}\ \Delta \text{ is the level spacing})$$

$$P(\sigma_k) = \int dc\, P(c) \iint d\tau_1 d\tau_2 P(\tau_1, \tau_2) \int dT p(T)\, (\tau_1 + \tau_2)\delta\big(\sigma_k - a(c, \tau_1, \tau_2, T)\big),$$

$$a(c, \tau_1, \tau_2, T) = \frac{\Delta}{2\pi\hbar} c(\tau_1 - \tau_2)\sqrt{1/T - 1}$$

- p (T)=1/(2√T) and $P(c) \propto \frac{1}{\sqrt{1-c^2}}$
- $P(\tau_1, \tau_2) \propto |\tau_1 - \tau_2| \frac{1}{\tau_1 \tau_2} e^{-\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)\frac{\pi\hbar}{\Delta}}$ **RMT Laguerre ensemble**

Hartree Correction Factor $\tau_1 + \tau_2$

VOLUME 78, NUMBER 25 PHY S I CAL REV I EW LETTERS 23 JUNE 1997

Quantum Mechanical Time-Delay Matrix in Chaotic Scattering

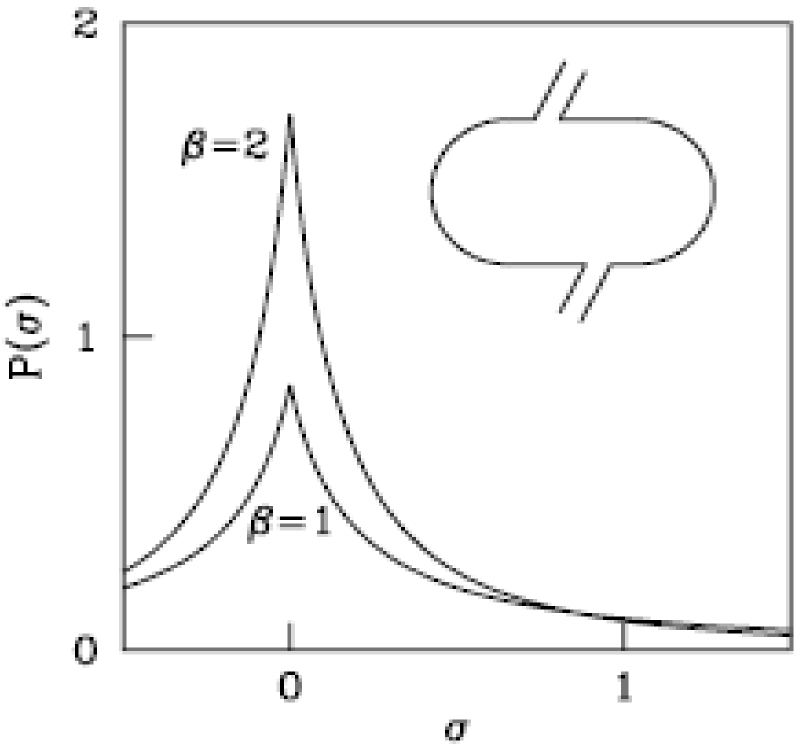
P. W. Brouwer, K. M. Frahm,* and C. W. J. Beenakker

Instituut-Lorentz, Leiden University, P.O. Box 9506, 2300 RA Leiden, The Netherlands

(Received 21 March 1997)

We calculate the probability distribution of the matrix *Q* 2*i* ¯ *hS21SyE* for a chaotic system with scattering matrix *S* at energy *E*. The eigenvalues t*j* of *Q* are the so-called proper delay times, introduced by Wigner and Smith to describe the time dependence of a scattering process. The distribution of the inverse delay times turns out to be given by the Laguerre ensemble from random matrix theory.

$P(\sigma_k)$ has a cusp at the origin and asymtotes as $\frac{log\ \sigma_k}{|{\sigma_k}^2|}$



Chaotic dot coupled to leads via 2 quantum point contacts having **1 opened channel**

Chaotic quantum dot

(Experiment)

Thermopower of a Chaotic Quantum Dot

S. F. Godijn, S. Möller, H. Buhmann, and L. W. Molenkamp
II. Physikalisches Institut, RWTH-Aachen, Templergraben 55, D-52056 Aachen, Germany

S. A. van Langen*
Instituut-Lorentz, Leiden University, P.O. Box 9506, 2300 RA Leiden, The Netherlands
(Received 10 November 1998)

The thermovoltage of a chaotic quantum dot is measured using a current heating technique. The fluctuations in the thermopower as a function of magnetic field and dot shape display a non-Gaussian distribution, in agreement with simulations using random matrix theory. We observe no contributions from weak localization or short trajectories in the thermopower. [S0031-9007(99)08804-3]

PACS numbers: 73.20.Dx, 05.45.−a, 72.20.Pa

trical conductance of small—characteristic size
ler than the electron mean free path—confined
stems (usually denoted as quantum dots) shows
ctuations. These fluctuations display correla-
inction of an external parameter such as shape or
ield, which can be described in a statistical man-
lectrons can, in fact, be viewed as billiard balls
a classically chaotic system where many random
at the system walls occur. Because of the wave-
of the electrons, quantum mechanics is needed
these systems fully. Chaos in quantum dots has
tigated [1–3] in conductance measurements but
s turns out to be difficult. So-called short tra-
] and weak localization effects [1,5] add up to
re of chaotic motion. Moreover, current heat-
lectrons in the dot appears to be unavoidable in
e measurements. Electron heating effects in the

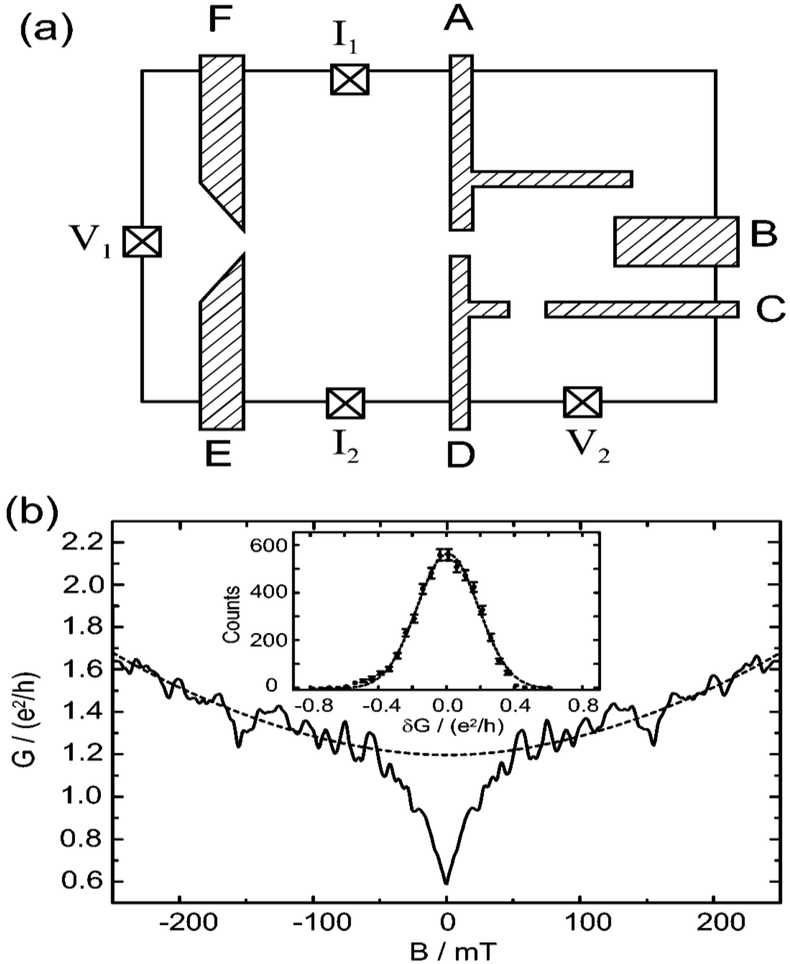
tion of parametric derivatives ($X = E, B$, shape, . .
conductance of a QD is the subject of recent F
vestigations [12,13]. The probability distributio
thermopower is again expected to be non-Gaus
chaotic conductors, exhibiting cusps at zero ampli
nonexponential tails [13,14].

In this paper, we present magnetothermopow
surements of a statistical ensemble of chaotic QI
observed thermopower fluctuations show a non-C
distribution. We present a numerical fit based
which describes the experimental data. We den
that effects such as short trajectories, weak loca
and dephasing are absent in thermopower measur

In Fig. 1a the measured device is shown schem
A QD (lithographic size 800 nm × 700 nm) is
statically defined (gates A, B, C, and D) in a s
high-mobility 2-dimensional electron gas (2DE

Quantum dot with **2 opened channels for each QPCs (S is a 4x4 matrix)**

Measure at 50 mK with a temperature difference of the same order
typical size 800 nm

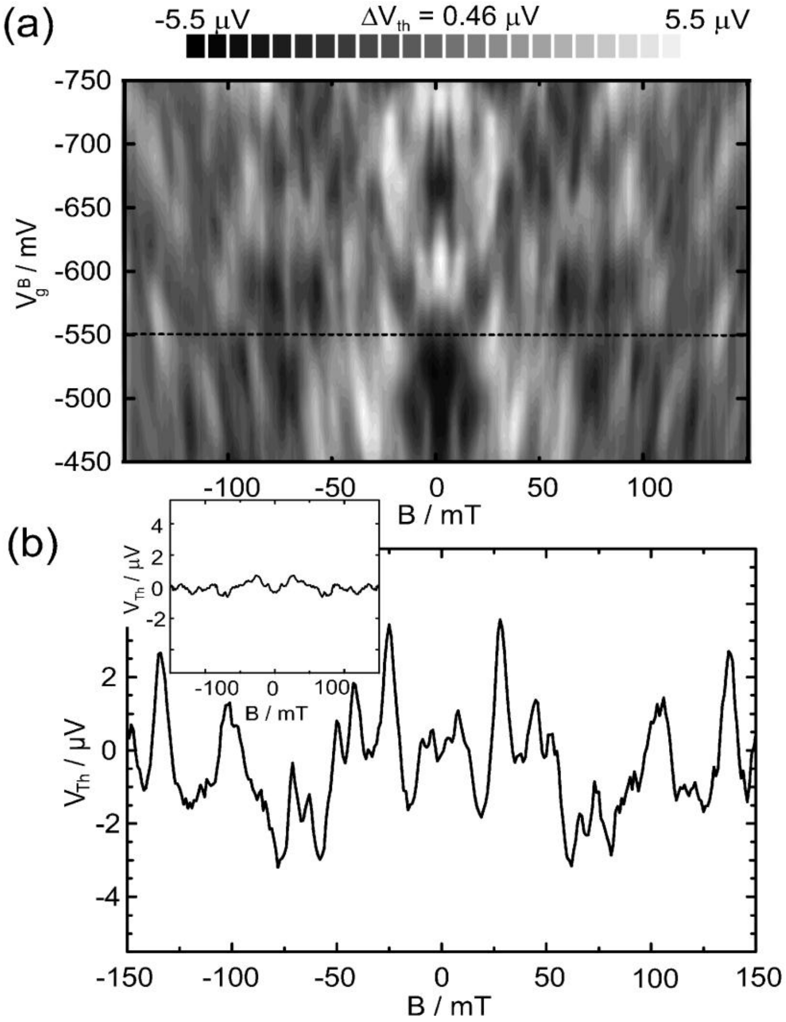


- (a) Schematic top view of the measured sample. The crosses denote the Ohmic contacts to the 2DEG; the hatched areas denote the gates. The heating current is applied between I1 and I2. The thermovoltage is measured between V1 and V2,

- The QD is defined by applying a negative voltage to gates A, B, C, and D.

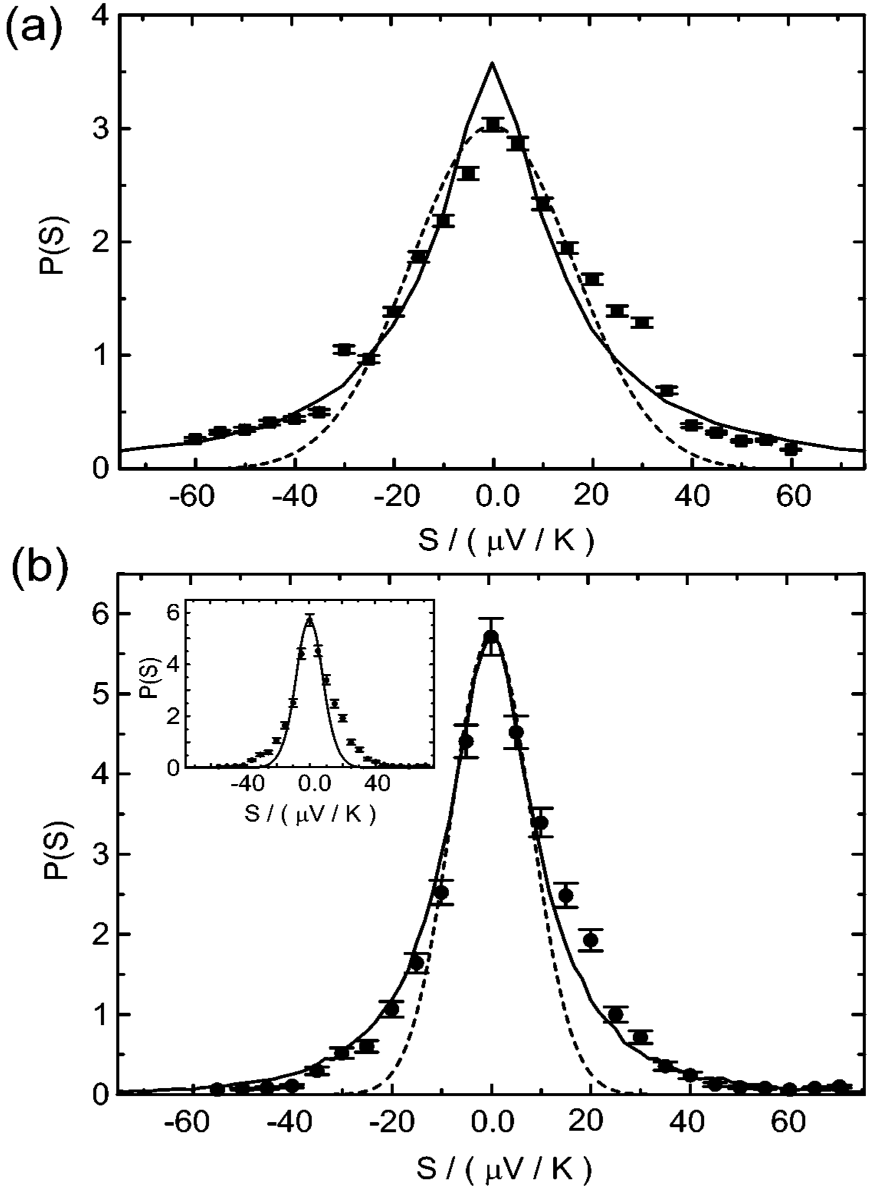
- (b) Magnetoconductance of the QD, averaged over
- a large number of different QD configurations. The influence
- of short trajectories is characterized by the dashed line. Inset:
- Conductance distribution for jBj $ 50 mT

Thermovoltage



(a) The gate voltage is changed by a constant small amount of d*V*B 10 mV for each magnetic field sweep. The light areas denote a large (maximum 5.5 mV) positive thermovoltage, and the dark areas a large (maximum 25.5 mV)negative thermovoltage.

(b) Individual thermovoltage trace

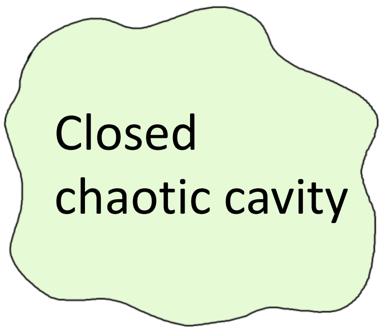


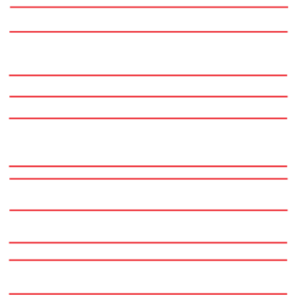
- **(a) Thermopower distribution in the presence of TRS (*B* < 40 mT).**
  Experimental results (dots),
  simulation results(solid line),
  Gaussian fit (dashed line).

- **(b) Thermopower distribution for broken TRS (*B* > 50 mT).**

  Experimental results (dots),
  simulation results (solid line),
  and Gaussian fit (dashed line).

BULK STATISTICS vs EDGE STATISTICS





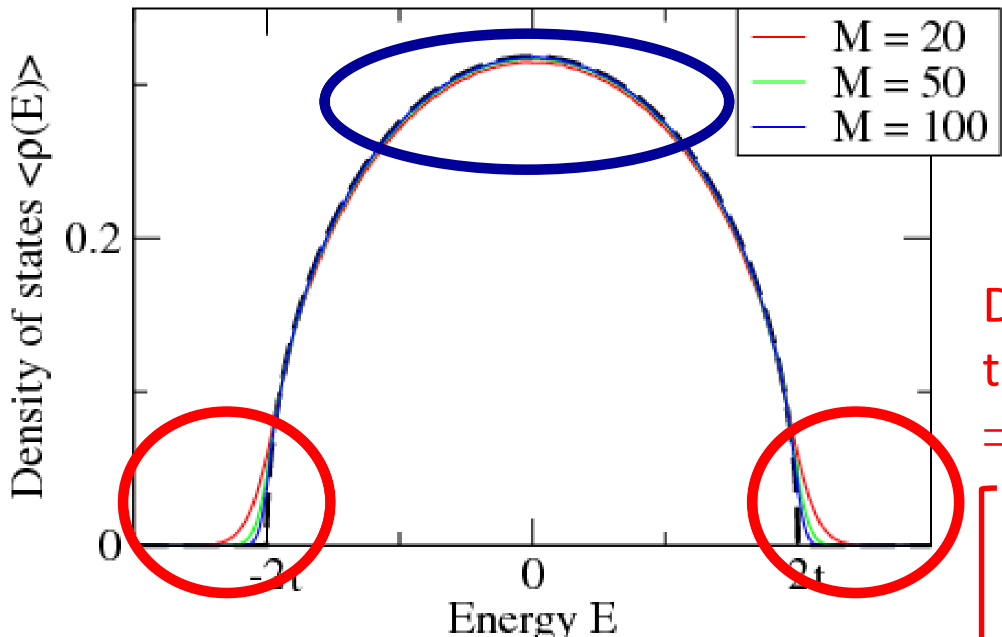
Hamiltonian H_M in Gaussian ensemble :

$$P(H_M) \propto \exp(-tr\ \frac{M H_M^2}{4t^2})$$

Oriol Bohigas et al, 1983

Universal statistics of eigenvalues in the **bulk**

- Wigner nearest-neighbor spacing distrib.
- Sine-Kernel for the 2-points correlation fct



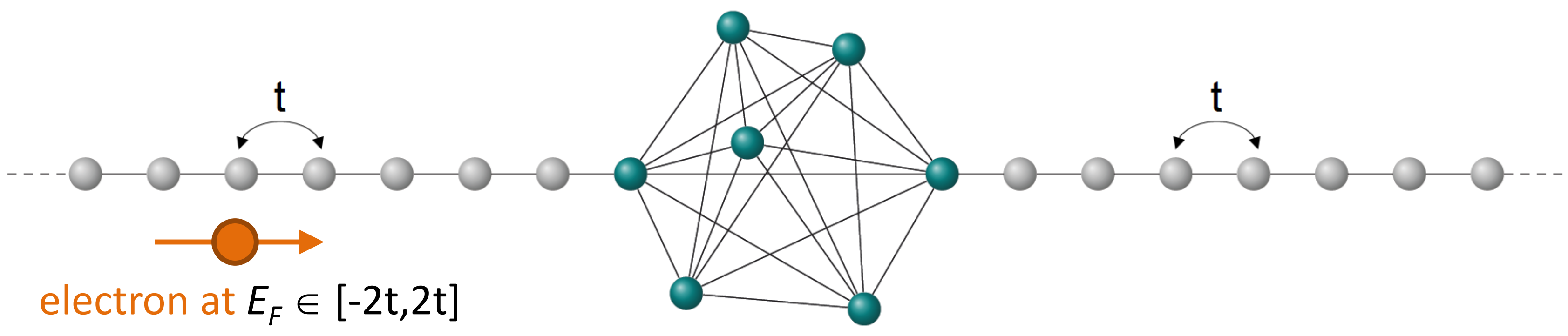
Different universality class at the **edges** of the spectrum

⇒ Tracy-Widom

- TW level spacing distribution
- Airy-Kernel for the 2-points correlation fct

Tracy and Widom, *Comm. Math. Phys* ***159****, 151 (1994)*; Forrester, *Nucl. Phys. B* ***402****, 709 (1993)*

MODEL FOR CHAOTIC SCATTERING BEYOND THE WIDE-BAND LIMIT



Left lead
(described by self-energy Σ_L)

Chaotic cavity
made of M sites
(Hamiltonian H_M)

Right lead
(described by self-energy Σ_R)

Relation between scattering matrix S and Hamiltonian H_M (see Datta) :

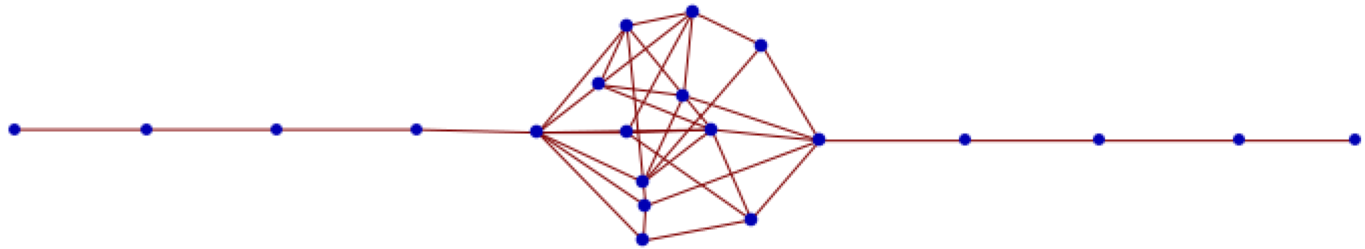
$$S(E) = -\mathbb{I}_2 + i\tau^\dagger \sqrt{\Gamma} \frac{1}{E - H_M - \Sigma_L - \Sigma_R} \sqrt{\Gamma}\, \tau \quad \text{with} \quad \Gamma = Im(\Sigma_L) = Im(\Sigma_R)$$

With energy-dependent self-energies $\Sigma_L(E)$ and $\Sigma_R(E)$

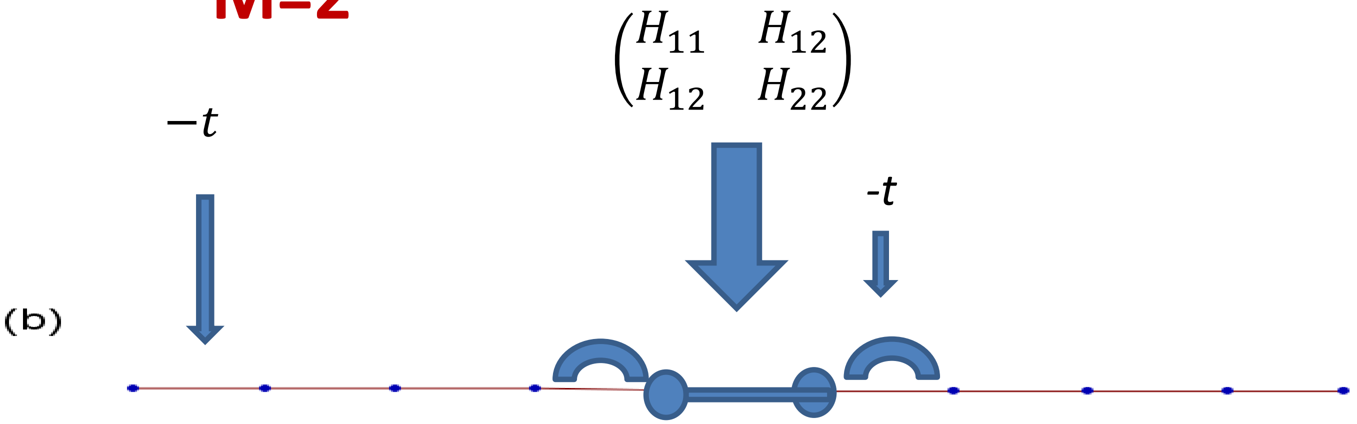
Beenakker and co-workers (PRL 1997) neglected this energy dependency

Chaotic scattering at E_F $\Leftrightarrow$ Hamiltonian H_M sampled from the **Cauchy** ensemble $\mathscr{C}(E_F)$
P(S)=constant

1d-Lattice model embedding a chaotic scatterer with M sites



M=2



S-Matrix $\boldsymbol{t = 1}$

- 2x2 Green's function $\quad$ **G**= $\dfrac{\mathbf{1}}{\mathbf{E-H-\Sigma}}$ $\quad$ **E**= $\begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}$

- Self Energy of the leads

$$E = -2\cos k; \qquad \boldsymbol{\Sigma} = \begin{pmatrix} -e^{ik} & 0 \\ 0 & -e^{ik} \end{pmatrix}$$

- 2x2 Scattering Matrix

S=-**1**+2isin k $\boldsymbol{G}$ $\qquad$ **1**= $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

H and **S** can be diagonized by the **same energy independent** orthogonal transformation R_θ
(Big simplification!)

Distribution for H which yields COE scattering

Cauchy Ensemble with adjusted center and width

- $\begin{pmatrix} e^{i\theta_1} & 0 \\ 0 & e^{i\theta_2} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} + 2isink \begin{pmatrix} \frac{1}{E-E_1+e^{ik}} & 0 \\ 0 & \frac{1}{E-E_2+e^{ik}} \end{pmatrix}$
- $\mathrm{P}(E_1, E_2) dE_1 dE_2 \propto \left| e^{i\theta_1} - e^{i\theta_2} \right| d\theta_1 d\theta_2$
- $\mathrm{P}(E_1, E_2) \propto |E_1 - E_2| \prod_{i=1}^{2} \left(\frac{1}{1+\left(\frac{E'-E_i}{\Gamma}\right)^2} \right)^{3/2}$
- $E' = E - \mathcal{R}(\Sigma)$=$E$+cos k $\Gamma = |\Im(\Sigma)| = \sin k$
- E=E_F ; $k = k_F$ (scattering at the Fermi energy)

Gaussian versus Cauchy ensemble:
Coulomb Gas Analogy

- $p(E_1, E_2)$=exp-$\beta H(E_1, E_2\,)$

- $\beta = 1$
- H $(E_1, E_2\,)$=-log$|E_1 - E_2| + \sum_{i=1}^{2} V(E_i)$

- **Gaussian** $V(E_i) = \frac{3}{2}\left(\frac{E' - E_i}{\Gamma}\right)^2$

  $\boldsymbol{H_{ij}}$
  Independent matrix elements

- **Cauchy** $V(E_i) = \frac{3}{2}\log\left(1 + \left(\frac{E' - E_i}{\Gamma}\right)^2\right)$

  Correlated matrix elements

Previous works by P. A. Mello (Poisson Kernel) and P. Brouwer (Ph-D thesis)

Cauchy Ensemble

Notation $H_M \in C(M, \varepsilon, \Gamma)$

- $P(H_M) \propto \det\left([H_M - \epsilon 1_M]^2 - [\Gamma^2 1_M]\right)^{-\frac{M+1}{2}}$

  Center ϵ and Width Γ

- **For $M = 2$**

$$S \in COE \leftrightarrow H_2 \in C(2, \frac{E_F}{2}, \Gamma_F)$$

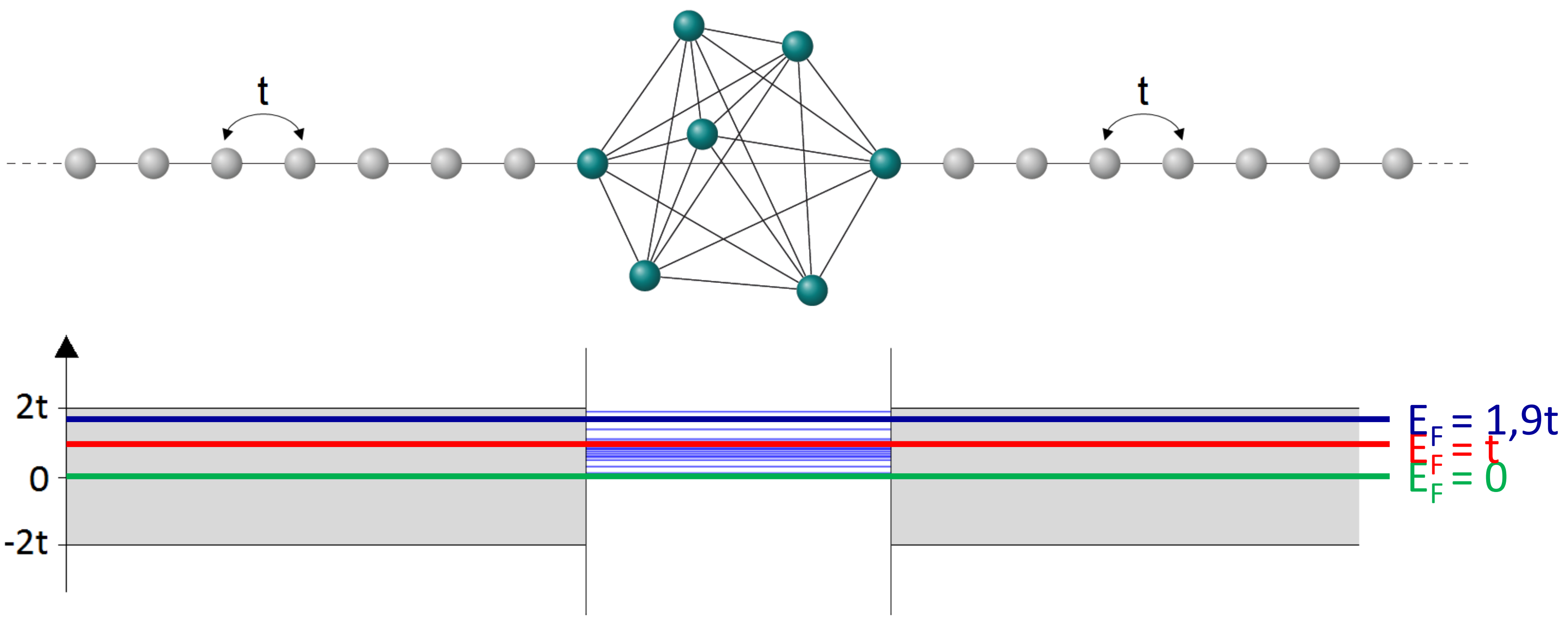
More Theorems

- $H_M \in C(\mathrm{M}, \frac{E_F}{2}, \Gamma_F) \rightarrow \mathrm{S}(E_F) \in \mathrm{COE}$

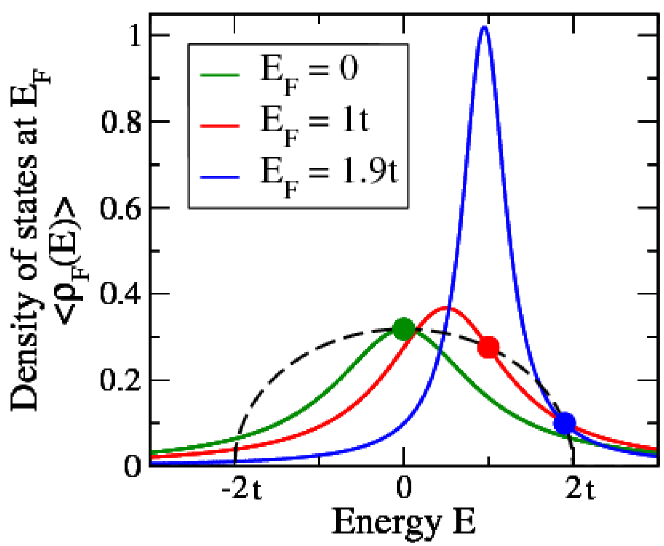
- $\frac{\partial S(E)}{\partial E} = 2i \frac{\partial(\Gamma(E) A_2(E))}{\partial E}$

The time delay matrix has the **M=2** distribution if $\Gamma(E) \rightarrow 0$. This occurs at the **edges of the conduction band of the leads.**

PROBING THE SPECTRUM TAILS



When Ef goes from the band center to the band edge (+-2t) , the center of the **density of states per site** is shifted and its width is reduced



When Ef goes to +-2t, we explore the spectrum tails

Relevant dimensionless scale

$$\alpha(E_F, M) = \frac{\Gamma_F^2}{\Delta_F t}?$$

Analytical results obtained when M=2

1. **Thermopower Distribution**

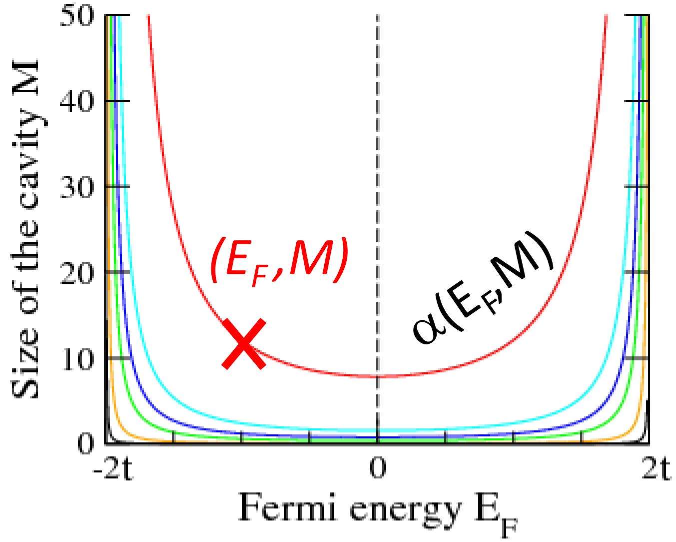
$$P(\sigma) = \frac{\alpha}{2} \ln \frac{1 + \sqrt{1 - (\pi\alpha\sigma/2)}}{\pi\alpha|\sigma|/2}$$

2. **Eigenvalue Density of the Time-Delay matrix Q**

$$P(\tau') = \frac{4\alpha}{\sqrt{1 - (4\pi\alpha\tau')^2}}$$

$$\tau' = \frac{\tau - \frac{\hbar}{\Gamma_F}}{\tau_H}$$

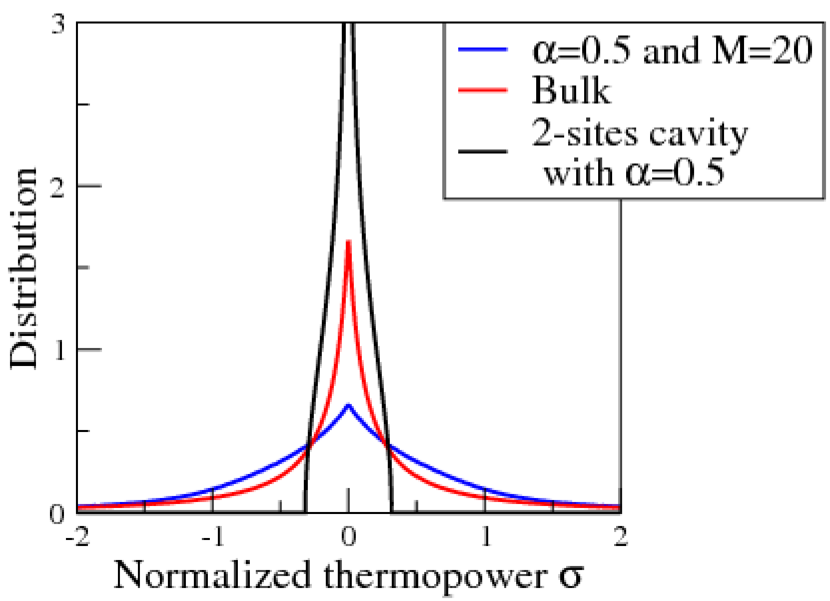
USING THE PARAMETER α FOR EXPLORING THE (E_F-M) DIAGRAM



$$\alpha(E_F, M) = \frac{\Gamma_F^2}{\Delta_F t} = \frac{1}{8\pi}\left|\frac{E_F - 2t}{tM^{-1/3}}\right|^{2/3}\left|\frac{E_F + 2t}{tM^{-1/3}}\right|^{2/3}$$

Measures the energy distance to the band edge in unit of $tM^{-1/3}$

TRACY-WIDOM SCALING ?



$$\eta = \frac{\int d\sigma\, |P(\sigma) - P_{bulk}(\sigma)|}{\int d\sigma\, |P_{M=2}(\sigma,\alpha) - P_{bulk}(\sigma)|}$$

WHY α ? CONNEXION TO TRACY-WIDOM

Lorentzian ensemble [H_M in $\mathscr{L}(E_F)$] ⟷ **Circular** ensemble [S chaotic at E_F] ⟷ **Gaussian** ensemble [H_M in $\mathcal{G}$]

Bul…

(if … $\Delta_F \to 0$)

$\alpha \to \infty$

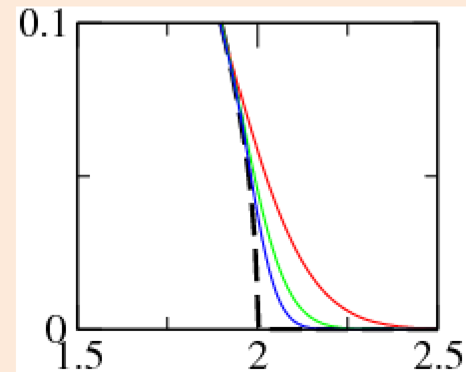
If $M \to \infty$ wit… same Δ_F

for t… …nsembles

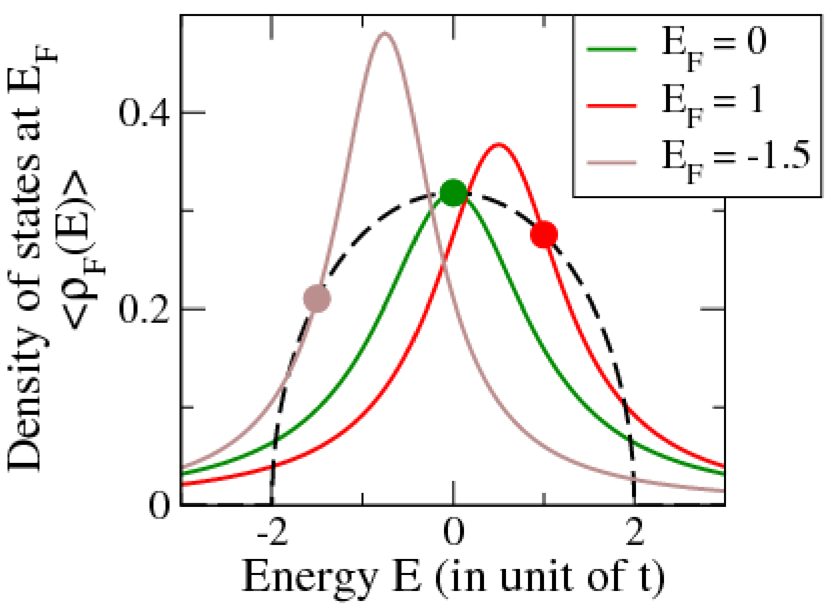
$\alpha \to \infty$

Tracy-Widom rescaling with the parameter α

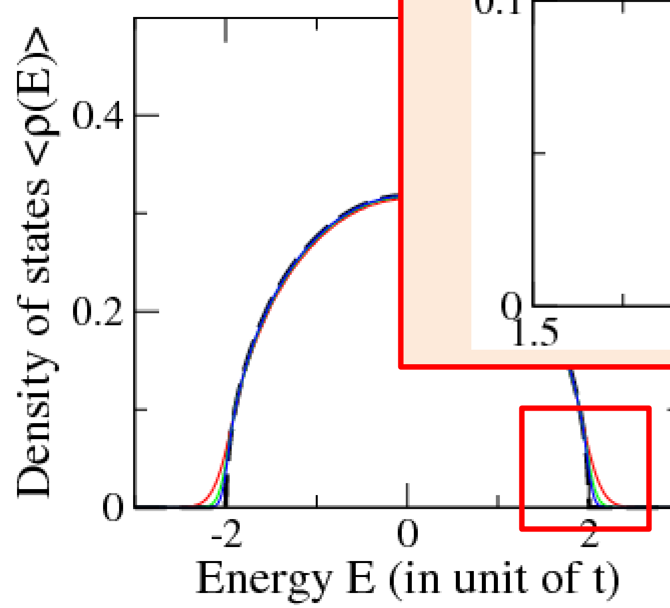
$$\langle \rho(\alpha) \rangle \propto \frac{1}{\alpha^{1/6}} \exp(-K\alpha)$$



Lorentzian ensemble

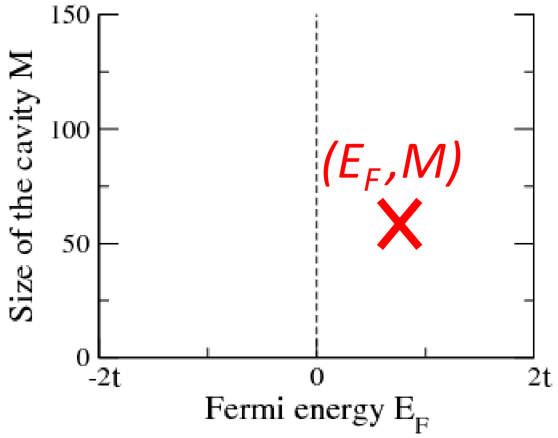


Gaussian e…



Brouwer, PhD Thesis (1997); Vavilov et al, PRL ***86****, 874 (2001)*

DO THE CALCULATION ...



Hamiltonian H_M in Cauchy ensemble $\mathscr{C}(E_F)$

↓

Scattering matrix S (chaotic at E_F)

↓

Time-delay matrix $Q = -i\hbar S^{\dagger} \frac{dS}{dE}$

↙ ↘

Distribution of delay times τ_i

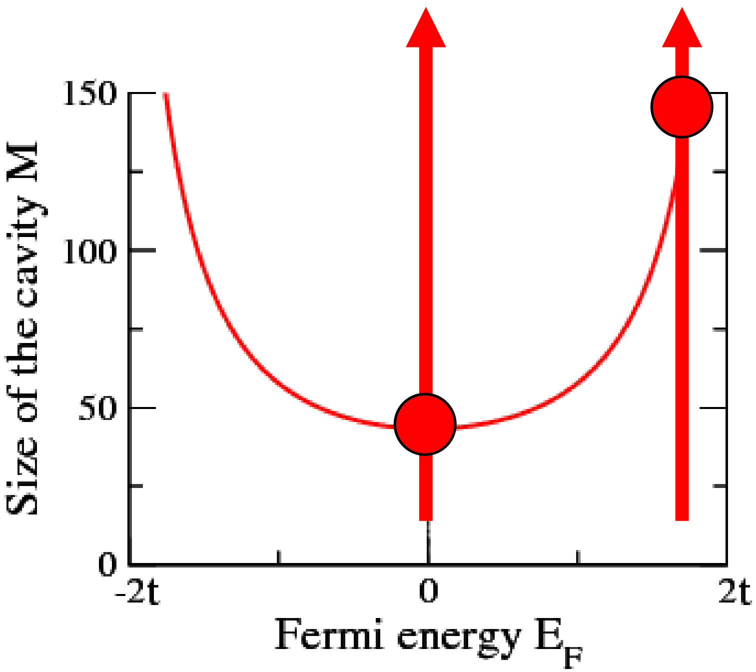
Distribution of thermopower $\mathscr{S}_i$

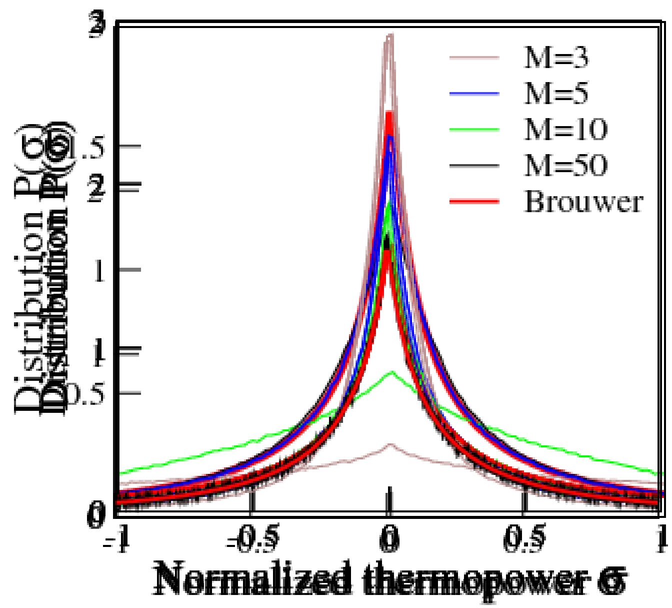
↓

Distribution of rescaled thermopower

$$\sigma_i = \frac{\Delta_F}{2\pi} \mathscr{S}_i$$

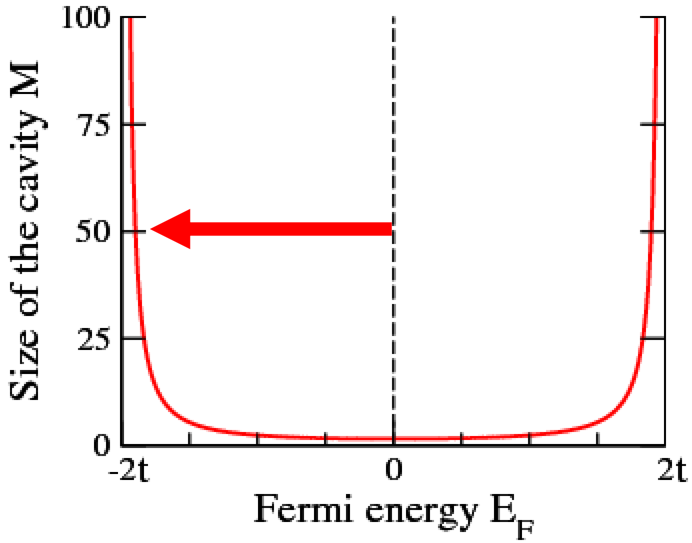
Limit $\alpha \to \infty$

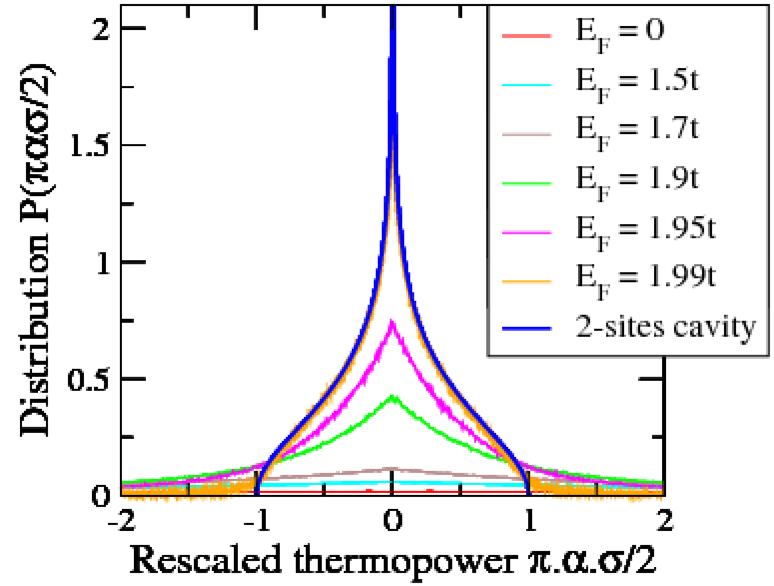




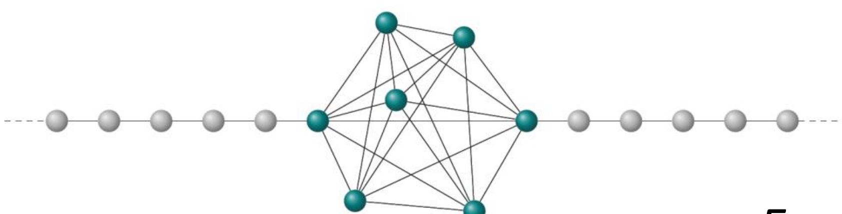
At fixed energy E_F and large size M, we recover previous result valid in the bulk

AND CLOSE TO THE BAND EDGE ? Lim $\alpha \to 0$





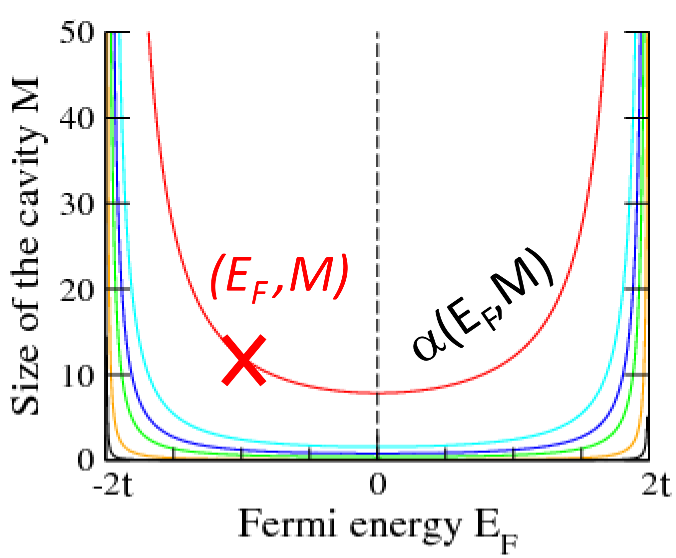
$$P(\sigma) = \frac{\alpha}{2} \ln \frac{1 + \sqrt{1 - (\pi\alpha\sigma/2)}}{\pi\alpha|\sigma|/2}$$



$E_F \to \pm 2t$ (band edge)
at fixed M

Close to the band edge, the thermopower distribution is **very different** from the one in the bulk. It is given by the analytical result derived for a 2-sites cavity

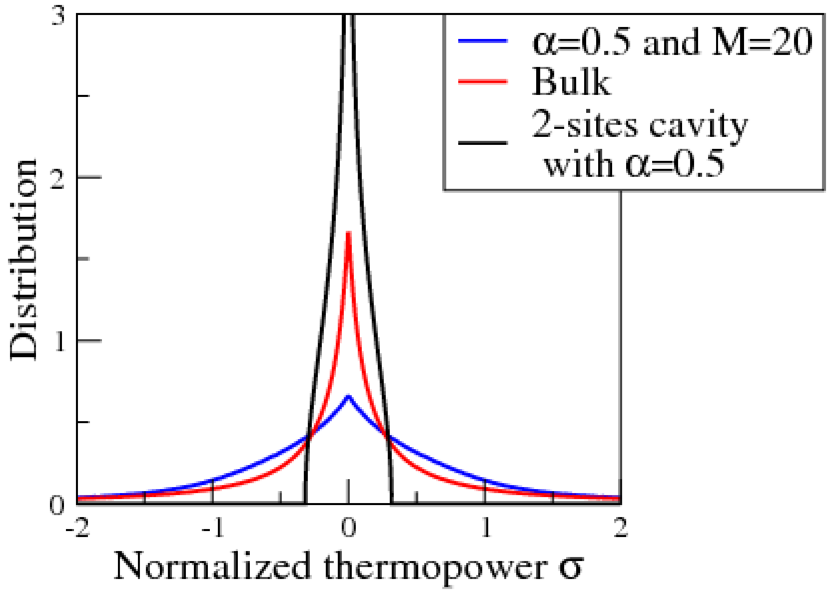
USING THE PARAMETER α FOR EXPLORING THE (E_F-M) DIAGRAM



$$\alpha(E_F, M) = \frac{\Gamma_F^2}{\Delta_F t} = \frac{1}{8\pi}\left|\frac{E_F - 2t}{tM^{-1/3}}\right|^{2/3}\left|\frac{E_F + 2t}{tM^{-1/3}}\right|^{2/3}$$

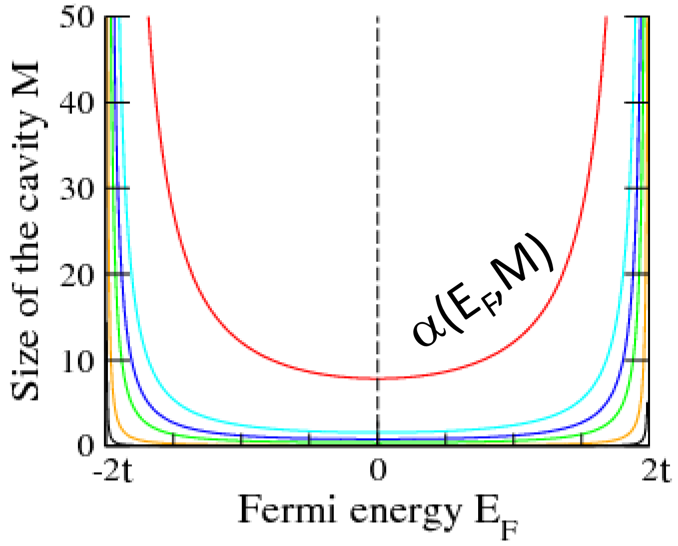
Measures the energy distance to the band edge in unit of $tM^{-1/3}$

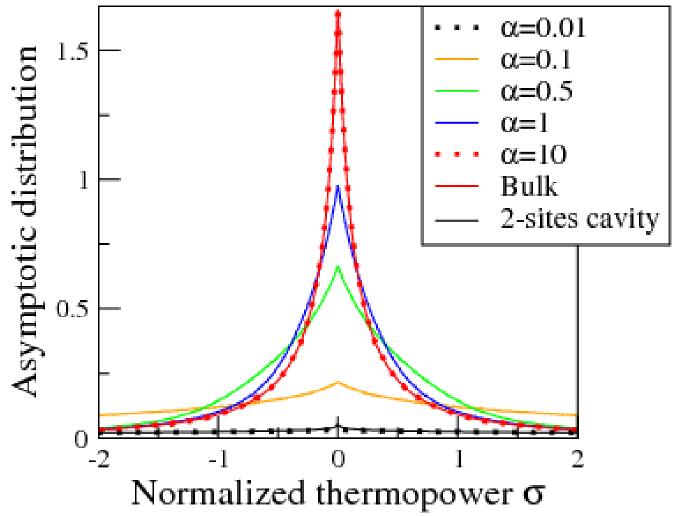
TRACY-WIDOM SCALING

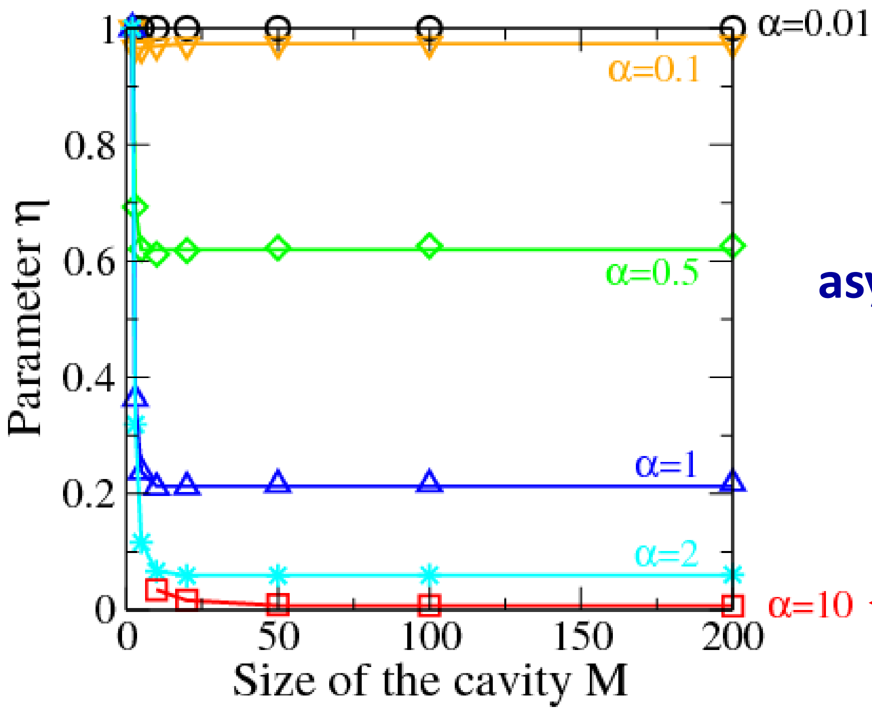


$$\eta = \frac{\int d\sigma\, |P(\sigma) - P_{bulk}(\sigma)|}{\int d\sigma\, |P_{M=2}(\sigma,\alpha) - P_{bulk}(\sigma)|}$$

NEW ASYMPTOTIC THERMOPOWER DISTRIBUTIONS





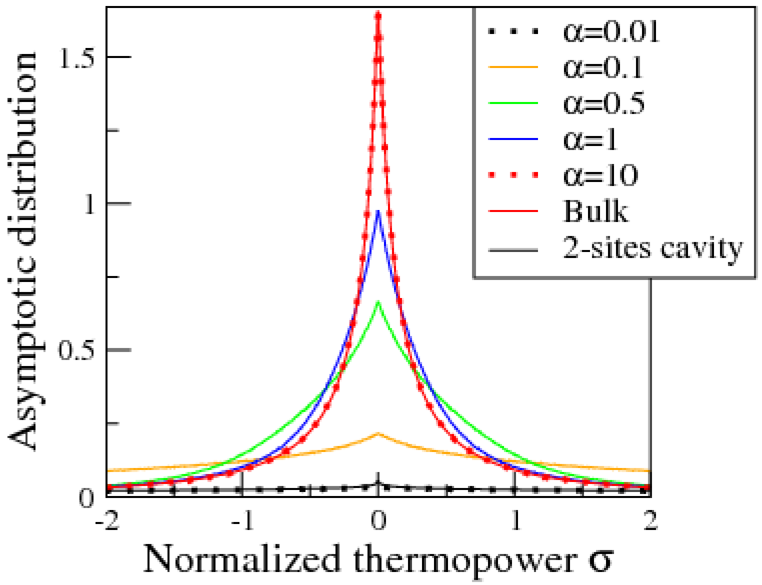


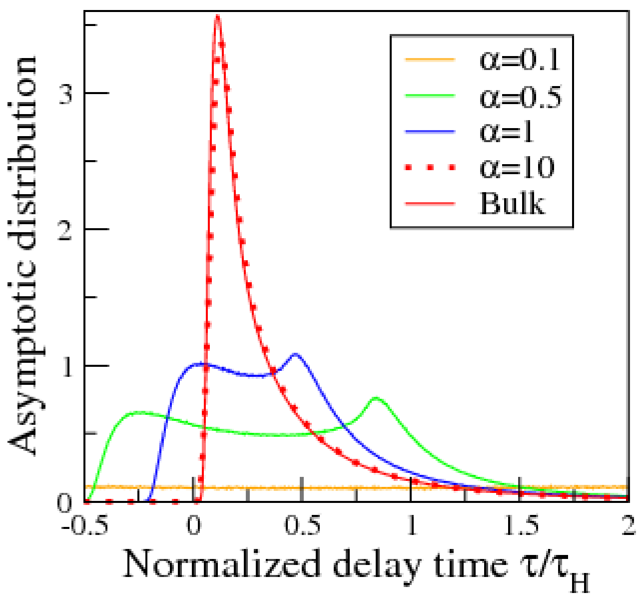
Edge distribution

Infinite number of **asymptotic** thermopower distributions in-between

Bulk distribution

DESCRIPTION OF THE BULK-EDGE CROSSOVER





- Infinity of asympotic thermopower distributions, indexed by $\alpha \in [0,\infty]$
- The two extreme limits are analytically understood : $\alpha \to \infty$ (bulk)
  $\alpha \to 0$ (2-sites cavity)
- The parameter α appears to be the Tracy-Widom parameter
- Same conclusions for the delay-time distribution

Thermopower distribution of a disordered nanowire in the field effect transistor device configuration:

R. Bosisio, G. Fleury and JLP

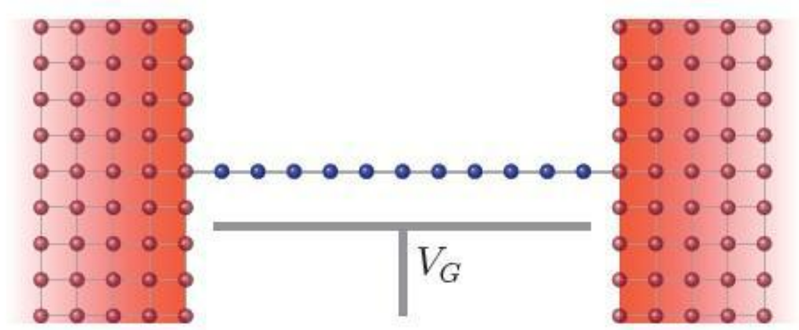
New Journal of Physics, arXiv:1310.4923v2 [cond-mat.mes-hall]

Related work for the delay time by C. Texier and A. Comtet, Phys. Rev. Lett. 82, 4220 (1999)

1d lattice of length L (***N*** sites) with nearest hopping terms t, random on-site potentials and gate potential V_G

Anderson Localization with **localization length ξ(E)**

Left lead (self-energy Σ_L)



Right lead (self-energy Σ_R)

$$H_{1d} = -\boldsymbol{t}.\Sigma_{ij}\left(c_j{}^{+}c_i + H.C\right) + \Sigma_i \ \boldsymbol{\varepsilon}_i n_i$$

$$H_{gate} = \Sigma_i \boldsymbol{V_G}\, n_i$$

$\boldsymbol{\varepsilon_i}$ Box distribution of **width W** and **center 0**

Study of the localized limit $N > \xi$

Elastic coherent transport, Linear Response, Sommerfeld expansions

→ **"Mott Formula "**

$$S = t \left. \frac{\mathrm{d} \ln \mathcal{T}}{\mathrm{d}E} \right|_{E_F}$$

Typical thermopower

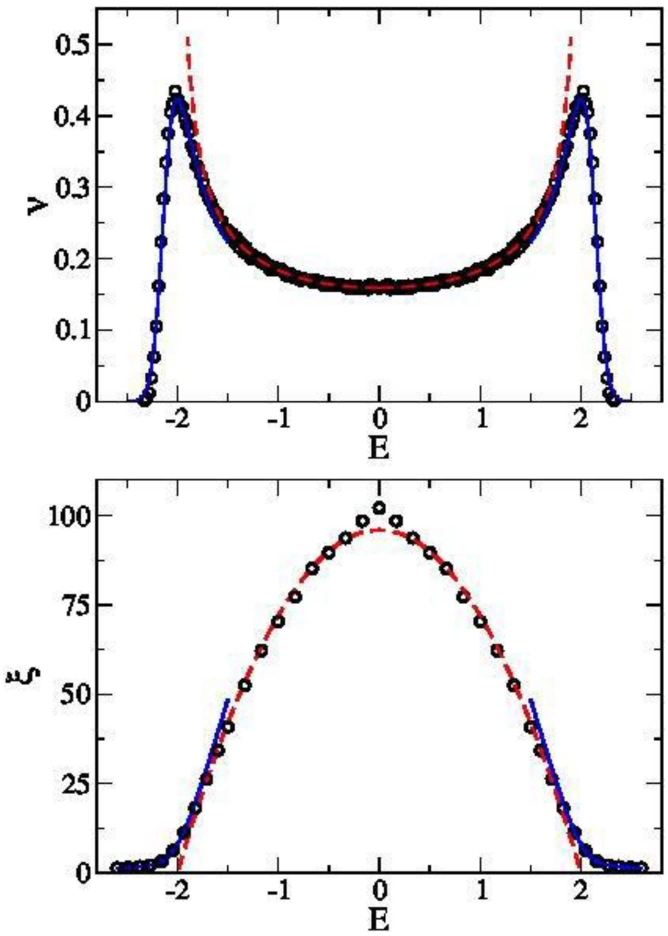
$$[\ln \mathcal{T}]_0(E) = -\frac{2N}{\xi(E)}$$

$$\mathcal{S} = \frac{\pi^2}{3} \left(\frac{k_B}{e} \right) \left(\frac{k_B T}{t} \right) S$$

← In physical units

To predict the typical behavior of S, one just need to know how the localization length ξ depends on the energy E.

Weak Disorder expansions of the 1d density of states ν=ρ/N and of the localization length ξ (assuming $V_G = 0$)



Numerical check with $W = 1$

BULK

$$\rho_b(E)/N = \frac{1}{2\pi t\sqrt{1-(E/2t)^2}}$$

$$\xi_b(E) = \frac{24}{W^2}\left(4t^2 - E^2\right)$$

B. Derrida & E. Gardner, J. Physique 45, 1283 (1984)

$$\rho_e(E)/N = \sqrt{\frac{2}{\pi}}\left(\frac{12}{tW^2}\right)^{1/3}\frac{\mathcal{I}_1(X)}{[\mathcal{I}_{-1}(X)]^2}$$

EDGE

$$\xi_e(E) = 2\left(\frac{12t^2}{W^2}\right)^{1/3}\frac{\mathcal{I}_{-1}(X)}{\mathcal{I}_1(X)}$$

$$X = (|E| - 2t)t^{1/3}(12/W^2)^{2/3}$$

$$\mathcal{I}_n(X) = \int_0^\infty y^{n/2}\, e^{-\frac{1}{6}y^3 + 2Xy}\, dy$$

TYPICAL THERMOPOWER AT LOW T: WEAK DISORDER THEORY & NUMERICAL CHECK WITH W=1

Using Sommerfeld expansions for having Mott formula

Bulk:

$$S_0^b = -N \frac{(E_F - V_g) W^2}{96t^3[1 - ((E_F - V_g)/2t)^2]^2}$$

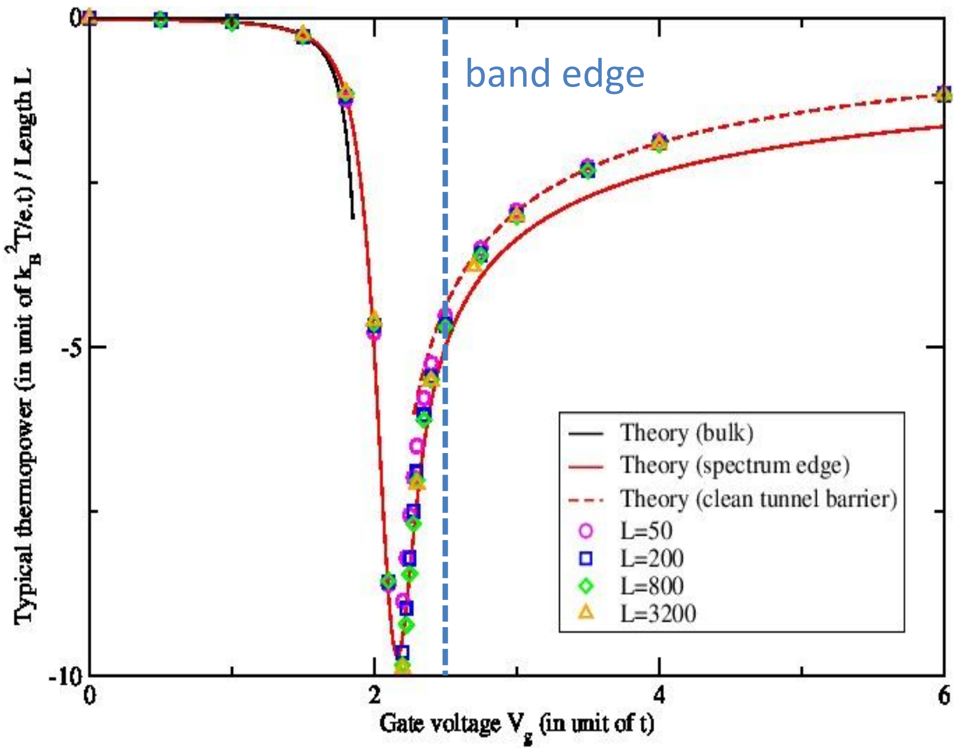
Edge:

$$S_0^e = -2N \left(\frac{12t^2}{W^2}\right)^{1/3} \left\{ \frac{\mathcal{I}_3(X)}{\mathcal{I}_{-1}(X)} - \left[\frac{\mathcal{I}_1(X)}{\mathcal{I}_{-1}(X)}\right]^2 \right\}$$

$$X = (|E_F - V_g| - 2t) t^{1/3} (12/W^2)^{2/3}$$

Tunnel Barrier:

$$\frac{S_0^{TB}}{N} \underset{N\to\infty}{\approx} \frac{1}{N} \frac{2t}{\Gamma(E_F)} \left.\frac{d\Gamma}{dE}\right|_{E_F} \pm \frac{1}{\sqrt{\left(\frac{E_F - V_g}{2t}\right)^2 - 1}}$$



Large Enhancement of the Thermopower near the band edge of the nanowire

R. Bosisio, G. Fleury and J-L. Pichard, (2013)

MESOSCOPIC FLUCTUATIONS: THERMOPOWER DISTRIBUTIONS

In the Bulk → Cauchy distribution (demonstrated in [1] for the case S_0=0)

$$P(S) = \frac{1}{\pi}\frac{\Lambda}{\Lambda^2 + (S - S_0)^2} \qquad \Lambda = \frac{2\pi t}{\Delta_F}$$

S_0 = typical thermopower
Δ_F = mean level spacing near E_F

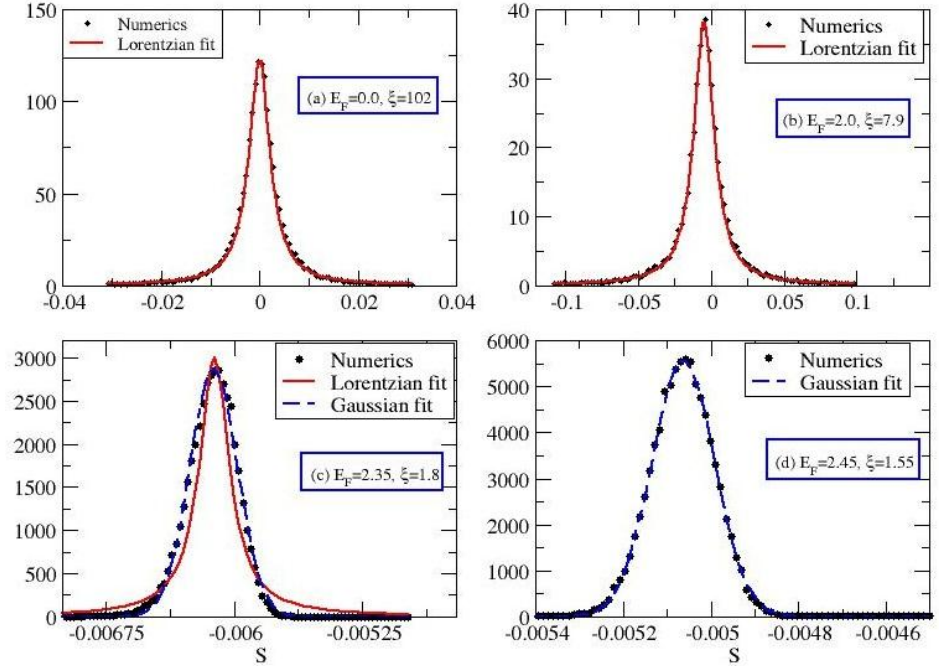
Near the Edge → Gauss distribution (characterized numerically)

$$P(S) = \frac{1}{\sqrt{2\pi}\lambda}\exp\left[-\frac{(S-S_0)^2}{2\lambda^2}\right] \qquad \lambda \approx 0.6\frac{Wt\sqrt{N}}{(E_F - V_g)^2 - (2t + W/4)^2}$$

1D nanowire with disorder W=1 → Spectrum edge at E=2.5t

V_g = 0.0 V_g = 2.0

V_g = 2.35 V_g = 2.45



[1] S. A. van Langen, P. G. Silvestrov, and C.W. J. Beenakker, Supperlattices Microstruct. 23, 691 (1998).
[2] R.Bosisio, G. Fleury and J-L. Pichard, (2013)

MESOSCOPIC FLUCTUATIONS: CHARACTERIZING THE TRANSITION

$$\eta = \frac{\int dS |P(S) - P_G(S)|}{\int dS |P_L(S) - P_G(S)|}$$

Parameter which measures the "distance" between the observed numerical distribution and the best Lorentzian (P_L) and Gaussian (P_G) fits

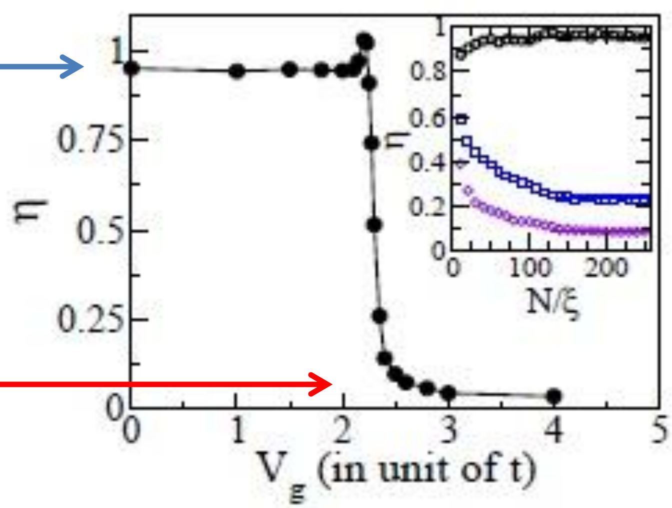
- $\eta = 1$ if Cauchy distribution
- $\eta = 0$ if Gauss distribution

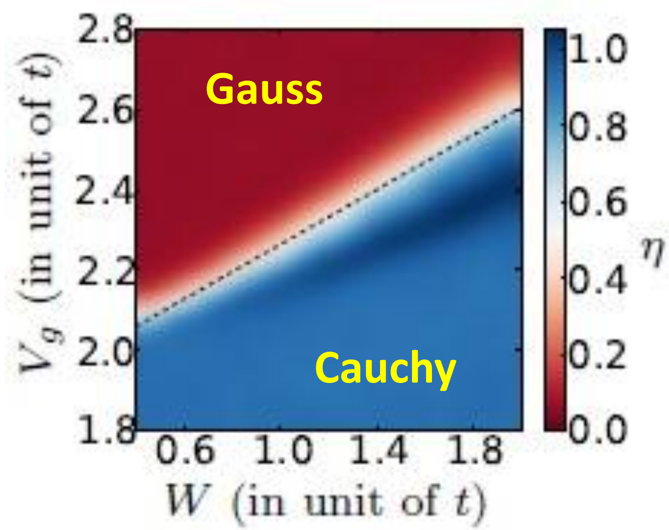
Edge: $V_G = 2{,}5$

W=1

Cauchy

Gauss





[1] R.Bosisio, G. Fleury and J-L. Pichard, arXiv:1310.4923v2 [cond-mat.mes-hall]

The thermopower (delay time) distribution of a disordered chain has a universal Cauchy distribution in the bulk of its spectrum which becomes Gaussian as the spectrum edges edge of the impurity band) are approached.

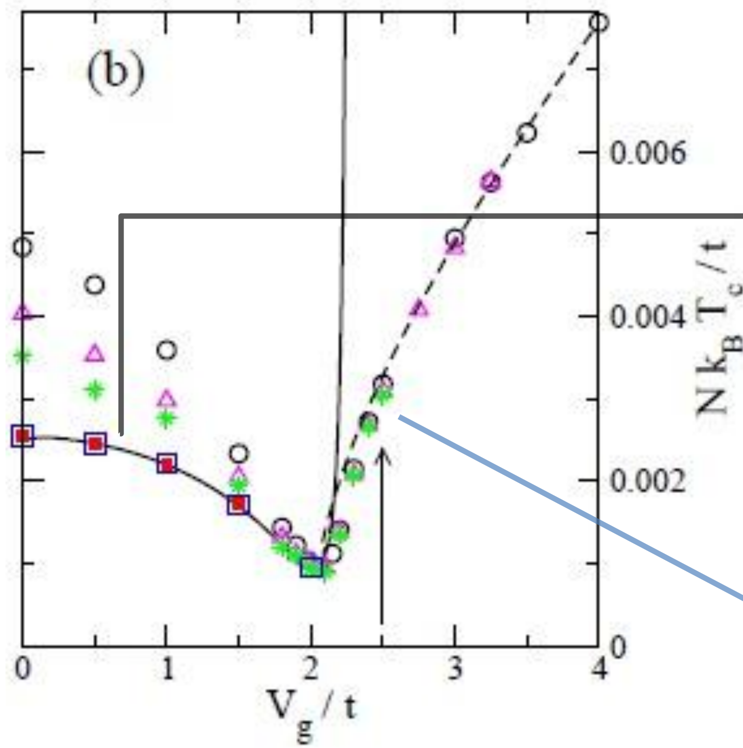
Both for the chaotic cavity and the disordered chain, the fluctuations at the edges differ from those in the bulk of the spectrum,

“SOMMERFELD" TEMPERATURE

Validity of the Sommerfeld expansion
leading to Mott formula for S

Validity of Sommerfeld Expansion → Wiedemann-Franz (WF) law, Mott formula

Range of validity of W-F law for $W = 1$ and $E_F = 0$ as a function of V_G



Sommerfeld temperature is proportional to the mean energy level spacing in the system:

$$k_B T_c \propto \Delta_F$$

Proportionality constant depends on required precision

Result for the tunnel barrier:

$$N k_B T_c \propto t\sqrt{[(E_F - V_g)/(2t)]^2 - 1}$$

Estimation for Si nanowire: ~ 100 mK

ENHANCED TEP NEAR THE BAND EDGE OF SEMICONDUCTING NWS AT ROOM TEMPERATURE

http://arxiv.org/pdf/1307.0249v1.pdf

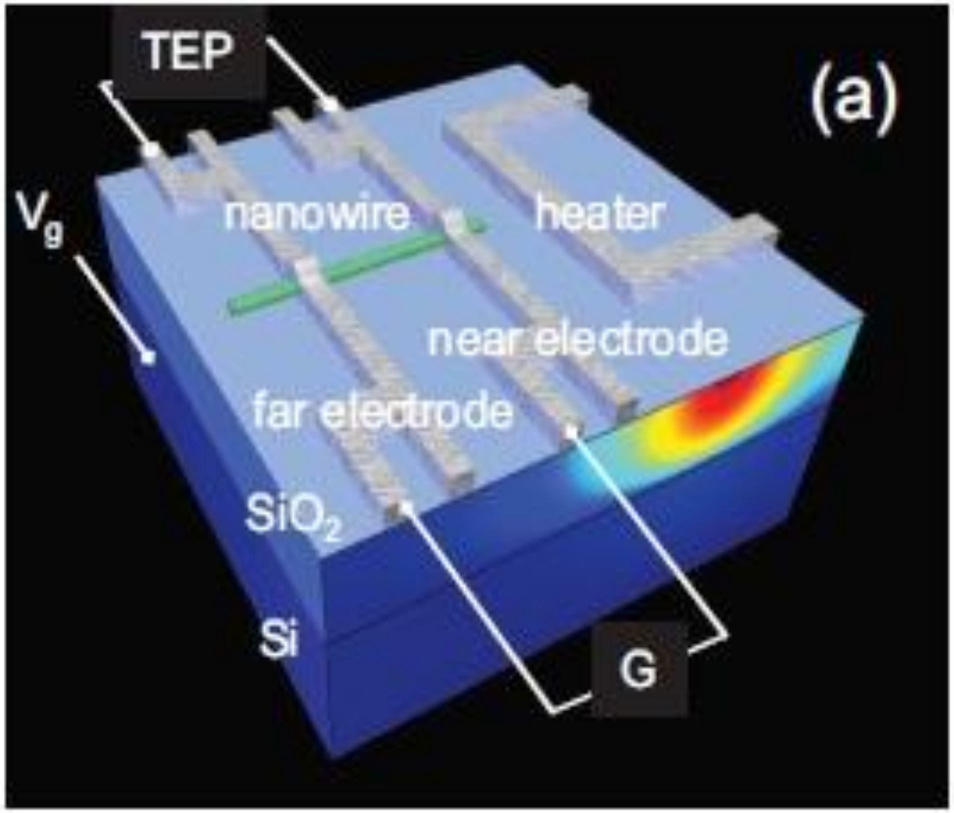
Electric Field Effect Thermoelectric Transport in Individual Silicon and Germanium/Silicon Nanowires

Yuri M. Brovman1, Joshua P. Small1, Yongjie Hu2, Ying Fang2, Charles M. Lieber2, and Philip Kim1
1 Department of Applied Physics and Applied Mathematics and Department of Physics,
Columbia University, New York, New York, 10027, USA and
2 Department of Chemistry and Chemical Biology,
Harvard University, Cambridge, MA 02139, USA

We have simultaneously measured conductance and thermoelectric power (TEP) of individual silicon and germanium/silicon core/shell nanowires in the field effect transistor device configuration. As the applied gate voltage changes, the TEP shows distinctly different behaviors while the electrical conductance exhibits the turn-off, subthreshold, and saturation regimes respectively. At room temperature, peak TEP value of ∼300μ V/K is observed in the subthreshold regime of the Si devices.

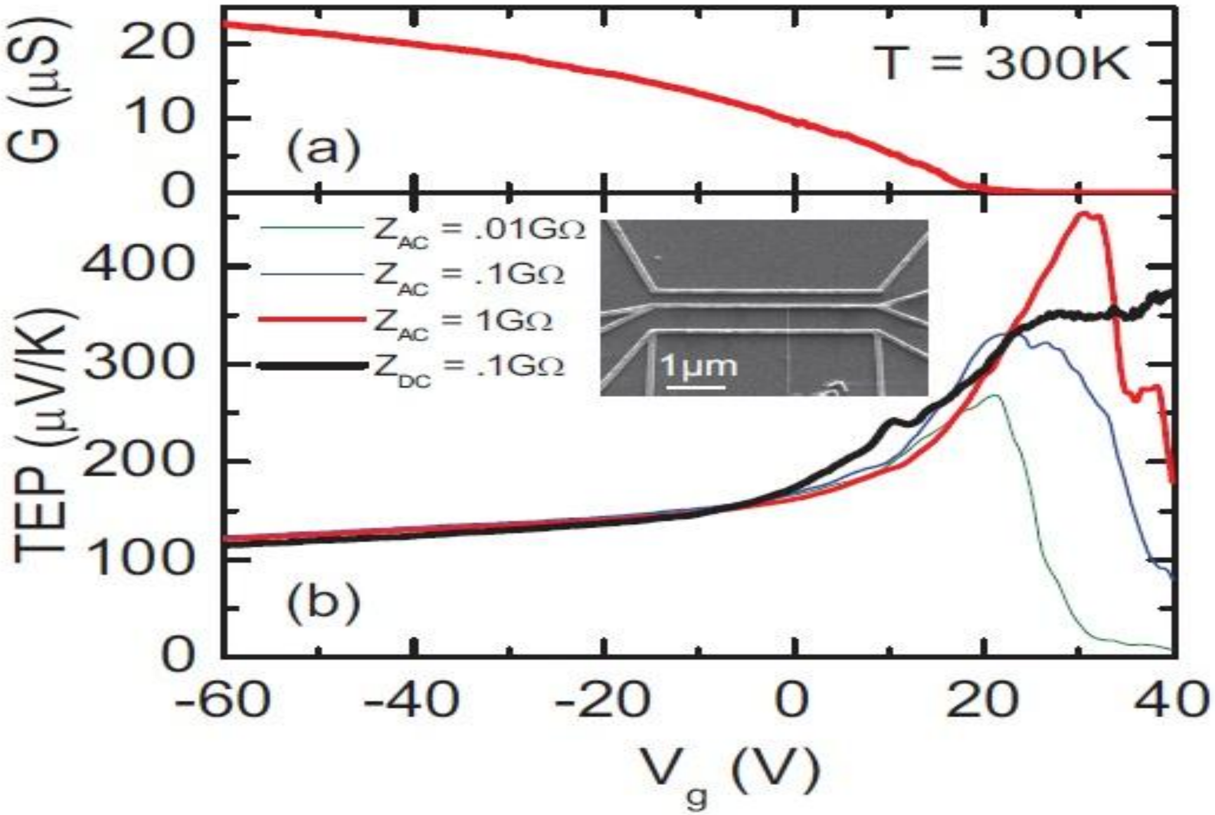
Substantially large peak TEP values are observed in the subthreshold regime of the Si and Ge/Si devices**, indicating largely enhanced TEP near the band edge of semiconducting NWs.**

FIELD EFFECT TRANSISTOR DEVICE CONFIGURATION



Schematic diagram of the simultaneous measurement technique of conductance and thermopower on individual nanowires. The finite element simulation shows a temperature profile, with red being the hottest and blue being the bath temperature, of the cross section of the substrate.

GE/SI NANOWIRE AT ROOM TEMPERATURE



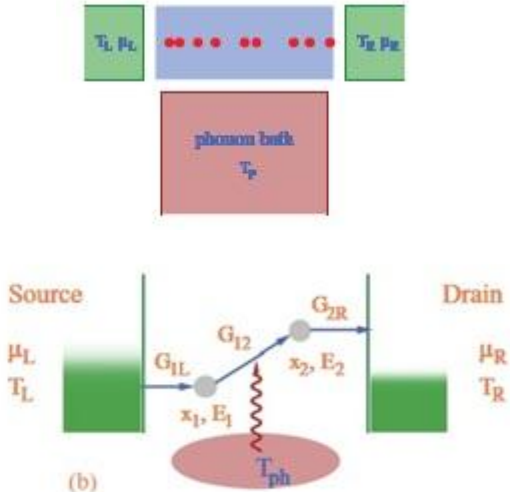
Conductance (a) and thermopower (b) of a Ge/Si nanowire as a function of gate voltage taken at T = 300 K. The inset in (b) shows a typical SEM image of a 12 nm Ge/Si device. Large input impedance becomes important when measuring TEP near the band edge of a semiconductor, as the FET device turns off.

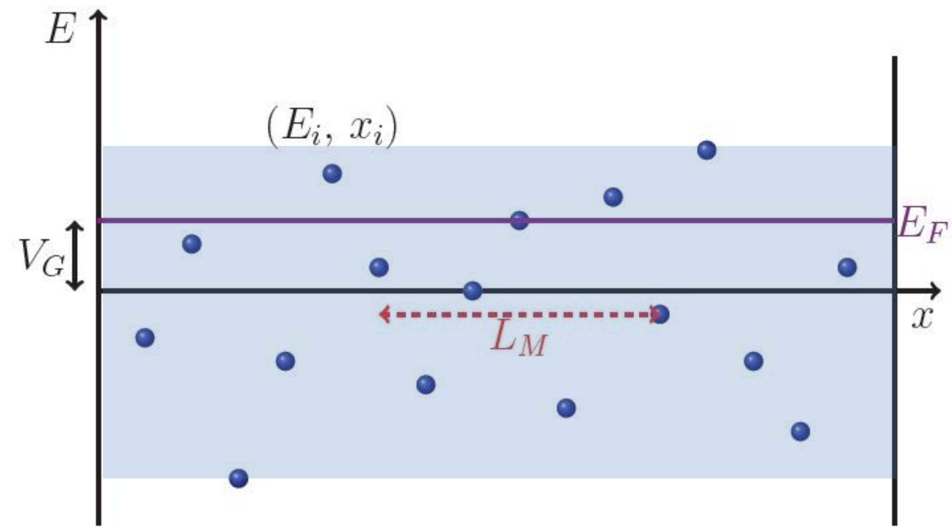
Thank You

Variable Range Hopping (VRH) Transport in gated disordered NWS

Hopping between pairs of localized states mediated by phonons
Conductance: competition between **tunneling** and activated processes

$$G_{ij} \sim e^{-2|x_i - x_j|/\xi} \, e^{-(|E_i-\mu| + |E_j - \mu| + |E_i - E_j|)/2k_B T}$$





Maximization of the conductance yields the scale of typical hop:

$$L_M \simeq \left(\frac{\xi}{2\nu T} \right)^{1/2}$$

Mott's Hopping length

ξ = localization length
ν = density of states / volume

[1] J-H. Jiang, O. Entin-Wohlman and Y. Imry, Phys. Rev. B 87, 205420 (2013).
[2] R.Bosisio, G. Fleury and J-L. Pichard, (2013)

TEMPERATURE SCALES

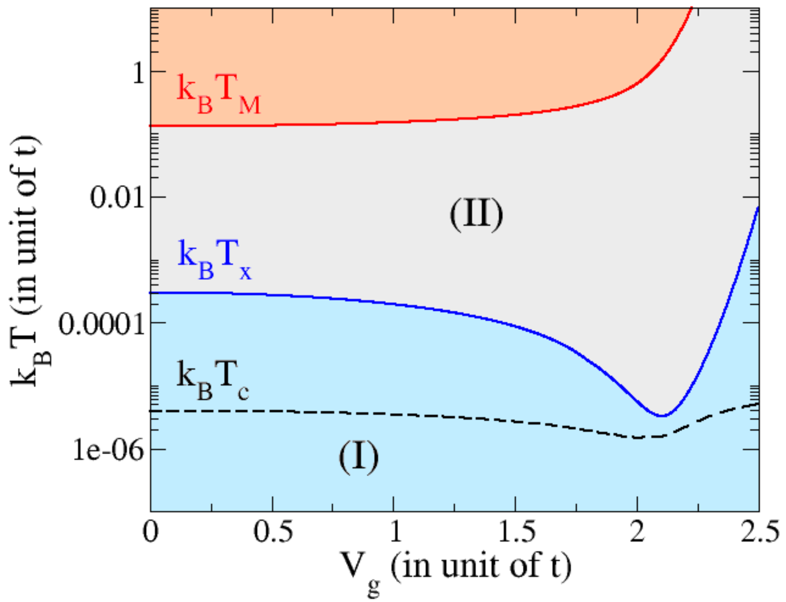
Low T: $L_M >> L$ → elastic transport

$L_M \simeq \left(\frac{\xi}{2\nu T}\right)^{1/2}$

Increasing T: $L_M \sim L$ → onset of inelastic processes → $T_x \sim \frac{\xi}{2\nu L^2}$

T ↓

Increasing T: $L_M \sim \xi$ → simple activated transport → $T_M \simeq (\xi^d \nu)^{-1}$



VRH Typical Conductance : (Mott's picture) $$G(T) \sim exp\left\{-\left(\frac{T_M}{T}\right)^{1/(d+1)}\right\}$$ d=1 (dimensionality)

What about the thermopower?

Conductance: Kurkijärvi (1973), Lee (1984), Fogler (2005)
Thermopower: Zvyagin (~80's)

MILLER-ABRAHAMS RESISTOR NETWORK

1. **Transition rates**

Between localized states

[Inelastic transition rates (Fermi Golden Rule)]

$$\Gamma_{ij} = \gamma_{ij}\, f_i\left(1 - f_j\right)\left[N_{BE}\left(\varepsilon_i - \varepsilon_j\right) + \theta\left(\epsilon_i - \epsilon_j\right)\right]$$

$$\gamma_{ij} = \alpha_{e-ph}.\, e^{-|x_i - x_j|/\xi}$$

Between lead and localized states

[Elastic tunneling rates]

$$\Gamma_{Li} = \gamma_{Li}\, f_i\left(1 - f_j\right) \qquad \gamma_{ij} = e^{-|x_i - x_j|/\xi}$$

2. **Conductances**

$$\boldsymbol{G_{ij} = \frac{e^2}{k_B T}\Gamma_{ij}}$$

3. **Local chemical potential (out of equilibrium transport)**

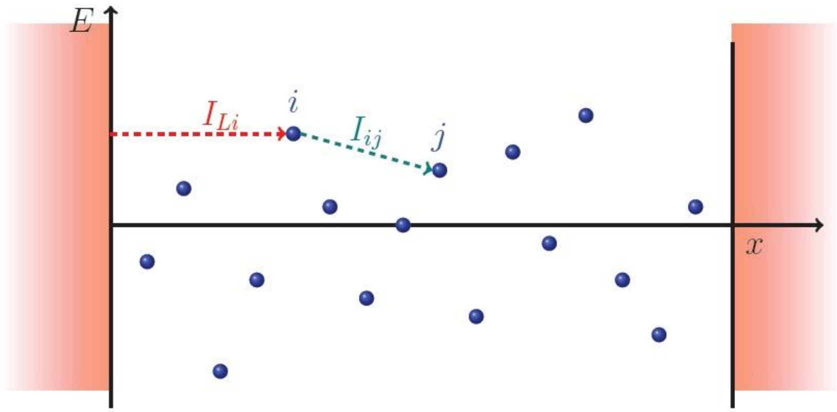
$$\boldsymbol{f_i(\mu) \rightarrow f_i(\mu + \delta\mu_i)}$$

4. **Current**

$$I_{ij} = G_{ij}\frac{\delta\mu_i - \delta\mu_j}{e}$$

RANDOM RESISTOR NETWORK [1,2]

energy levels localized at (random) positions $\mathbf{x}_i$



$\mathbf{I}_{ij}$: hopping current between sites i and j

$\mathbf{I}_{iL(R)}$: tunneling current between site i and leads

$f_i = f_i^0 + \delta f_i$ "local" FD distribution

Current conservation at node i : $$\left(\sum_{j\neq i} I_{ij}\right) + I_{iL} + I_{iR} = 0$$

Electric current: $$I_L^e = \sum_i I_{iL} = -\sum_i I_{iR}$$

Heat current: $$I_{L(R)}^Q = \sum_i \left(\frac{E_i - \mu_{L(R)}}{e}\right) I_{iL(R)}$$

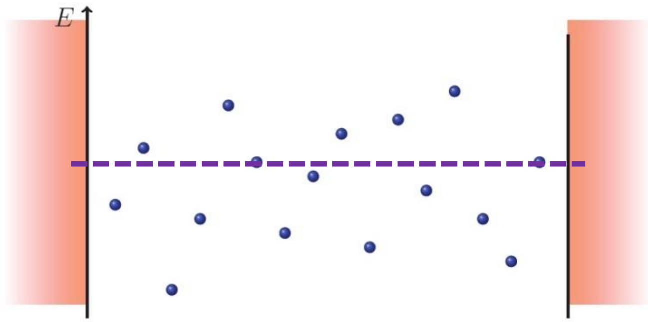
Peltier: $\Pi = \frac{I_L^Q}{I_L^e}$

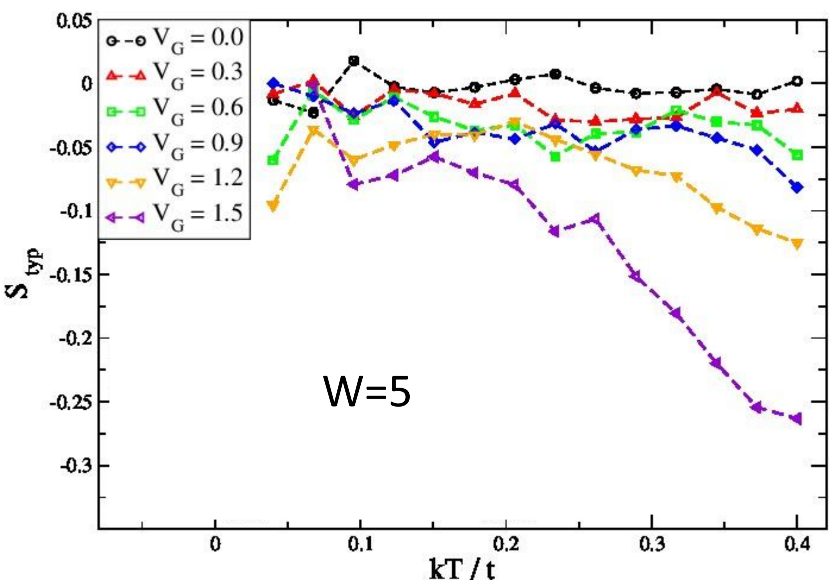
Thermopower: $S = \frac{\Pi}{T}$
(from Onsager relations)

[1] A. Miller and E. Abrahams, Phys. Rev. 120, 745 (1960)
[2] J-H. Jiang, O. Entin-Wohlman and Y. Imry, Phys. Rev. B 87, 205420 (2013).

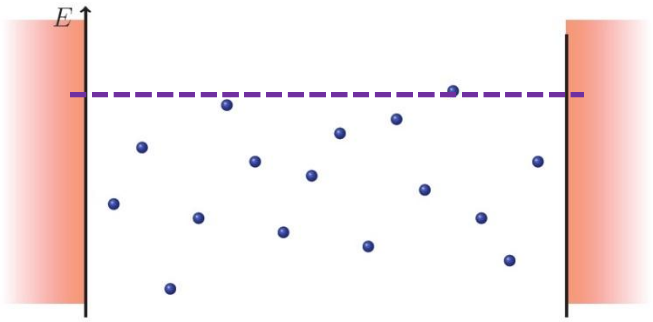
EFFECT OF V_g ON TYPICAL THERMOPOWER IN VRH

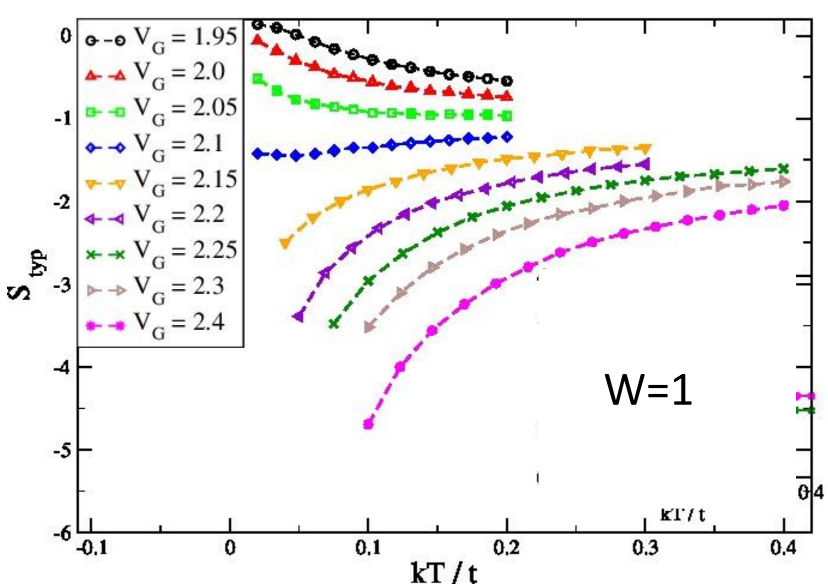
Bulk





Edge





[1] R.Bosisio, G. Fleury and J-L. Pichard, (2013)

THANK YOU

LOW TEMPERATURE COHERENT ELASTIC TRANSPORT VALIDITY OF MOTT FORMULA FOR THE THERMOPOWER

VOLUME 81, NUMBER 16 PHYSICAL REVIEW LETTERS 19 OCTOBER 1998

Thermometer for the 2D Electron Gas using 1D Thermopower

N. J. Appleyard, J. T. Nicholls, M. Y. Simmons, W. R. Tribe, and M. Pepper
Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, United Kingdom

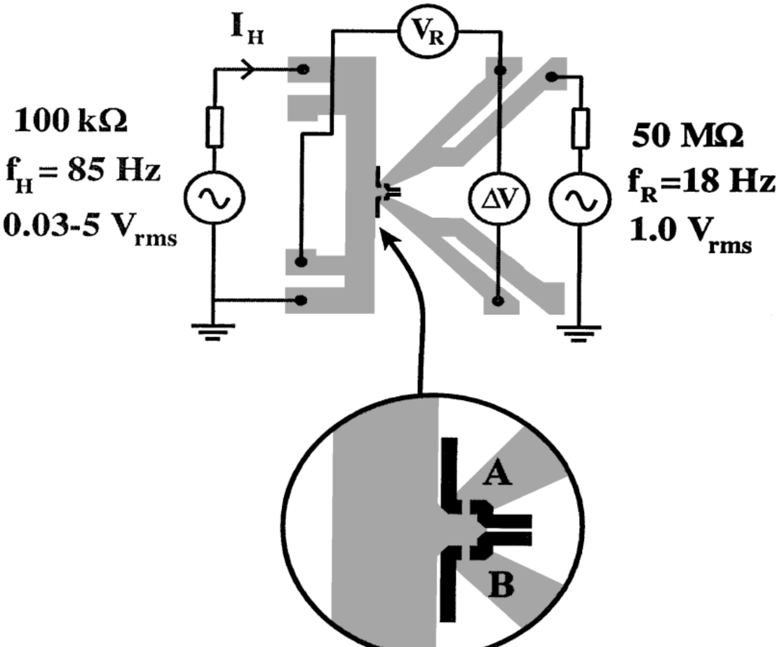


FIG. 1. Schematic of the device and measurement circuit. The etched mesa, shown in grey, consists of a heating channel and two voltage probes, where the two 1D constrictions are defined. The four-terminal resistance *R* is measured simultaneously with the thermopower *S*, but at a different frequency. Magnified view: The two pairs of split gatesdefining the constrictions *A* and *B* are shown in solid black.

$$S = \frac{\Delta V}{T_e - T_l}\Big]_{I=0} = \frac{(\pi k_B)^2}{3e}\left(T_e + T_l\right)\frac{\partial(\ln G)}{\partial \mu}$$

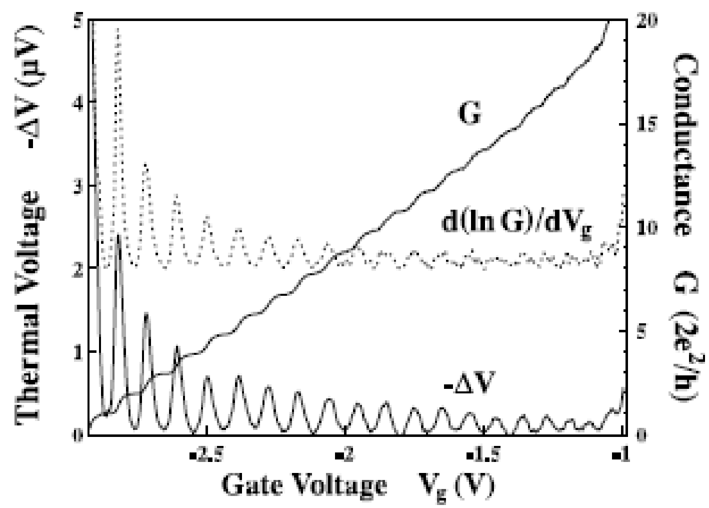


FIG. 2. Experimental traces of the conductance *G* and the thermopower voltage from constriction *A*, using a heating current of **1.5 mA** at a lattice temperature of **305 mK,** so that ***Te* ~ 600 mK**. The dashed line shows the predicted thermopower signal from the Mott relation [Eq. (1)].

OUTLINE

❑ I . Elastic Coherent Regime (Low temperatures)

A - 1d localized systems:

- **Measure of the conductance** of disordered nanowires in the field effect transistor device configuration **(Sanquer et al)** and the Mott formula for the thermopower.

- **Theory using an Anderson model for a 1d disordered chain** Thermoelectric transport (i) the bulk; (ii) the edge and (iii) eventually the outside of the impurity band - Typical behavior and fluctuations of the thermopower

B - 0d chaotic systems

- **Measure of the thermopower** of a chaotic cavity (Molenkamp et al).

- **Random matrix theory** of the thermopower of a chaotic cavity:

Universal thermopower distribution in the bulk (Wigner-Dyson) and at the edge (Tracy-Widom) of the spectrum.

❑ II. Inelastic Activated Regime (Intermediate temperatures) in 1d.

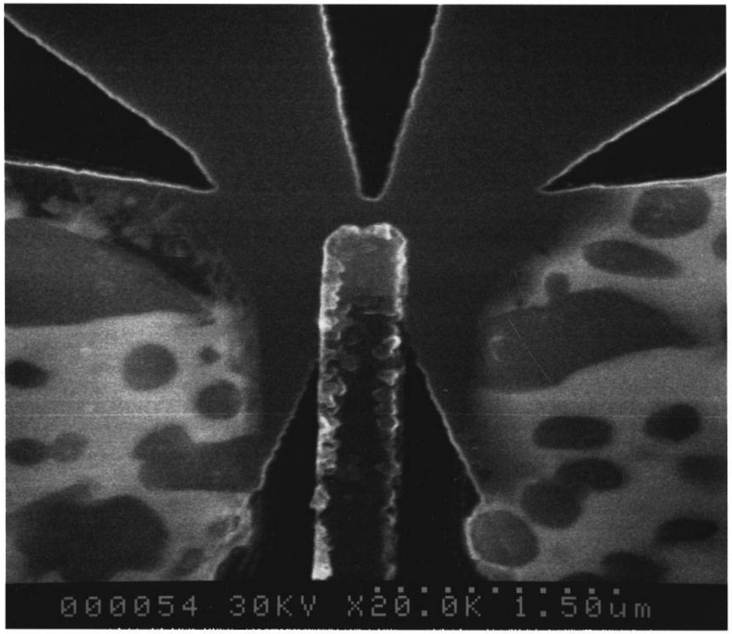
Mott variable range hopping and Miller-Abrahams resistor network, Seebeck and Peltier coefficients in the bulk and near the edges of the impurity band.

❑ III. Thermoelectric transport at room temperature (Kim et al).

TUNNELING AND INTERFERENCES IN VERY SMALL GA AS METAL-SEMICONDUCTOR FIELD-EFFECT TRANSISTORS

PHYSICAL REVIEW B VOLUME 59, NUMBER 16 15 APRIL 1999-II

W. Poirier
CEA-DSM-DRECAM-SPEC, C.E. Saclay, 91191 Gif sur Yvette Cedex, France
D. Mailly
CNRS-LMM, 196 Avenue Henri Ravera, 92220 Bagneux, France
M. Sanquer
CEA-DSM-DRFMC-SPSMS, CEA-Grenoble, 17 Rue des Martyrs, 38054 Grenoble, France



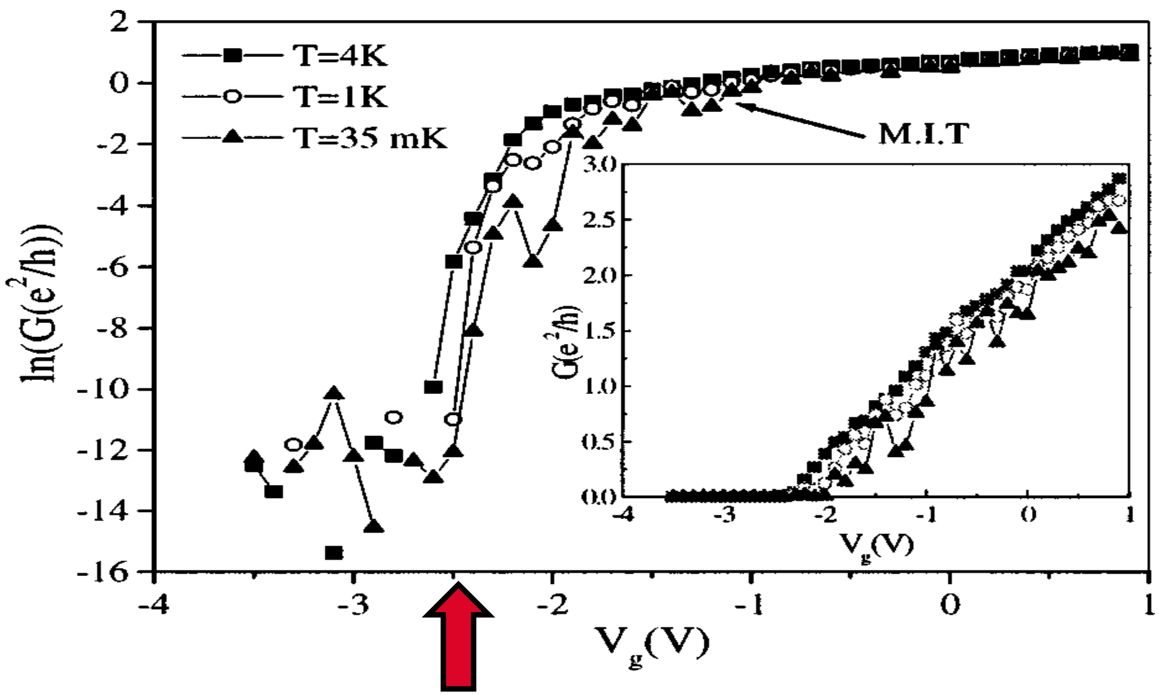
Quantum coherent elastic transport in the field effect transistor device configuration at very low temperature

We study the transport through gated GaAs:Si wires of 0.5 μm length in the insulating regime and observe transport via tunneling at very low temperature. We describe the mean positive magnetoconductance and the mesoscopic fluctuations of the conductance ~versus energy or magnetic field! purely within one-electron interference model.

FIG. 1. SEM picture of the GaAs:Si **submicronic** MESFET. The 0.5-mm-thick aluminum Schottky gate is visible on the bottom. **The gate** does not cover the whole constriction width, but **covers entirely the conducting channel if one considers the depletion width**. The GaAs is doped at $\mathbf{10^{23}}$ **Si** $\boldsymbol{m^{-3}}$) 300-nm-thick layer is etched toform four large contact pads to the active region under the gate.Au*x*Ge12*x*Ni Ohmic contacts are visible on the right and the left. The volume of the active region is estimated to be **0.2 x 0.2 x 0.5** $\boldsymbol{\mu m^3}$ (taking into account depletion layers for V_{gate}= 0 V, about 120 nm).

GATE MODULATED CARRIER DENSITY

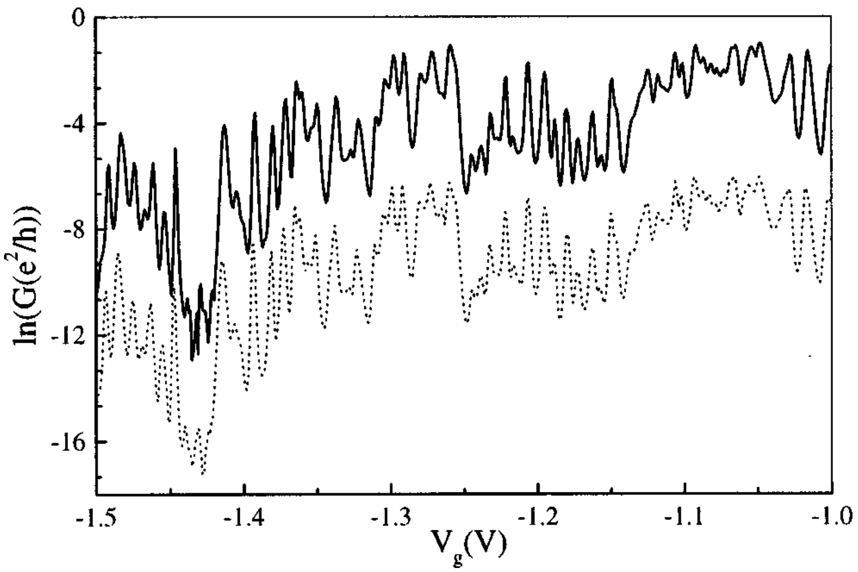
Anderson Insulator → ← **Conductor**



ln *G*(*V* gate) at three temperatures in a large gate voltage range
(details of the conductance pattern are not seen for this gate voltage sampling)
Inset: the same curve in a linear scale. Note the linearity at voltages above the transition.

Edge of the impurity band = -2,5 V
(Complete depletion of the disordered nanowire)

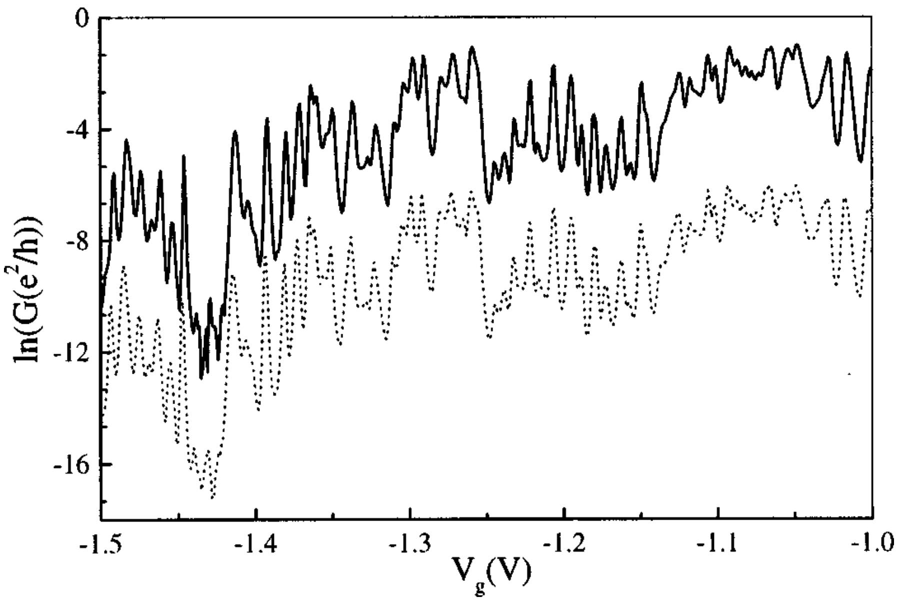
REPRODUCIBLE CONDUCTANCE FLUCTUATIONS INDUCED BY A VARIATION OF THE GATE VOLTAGE IN THE INSULATING REGIME



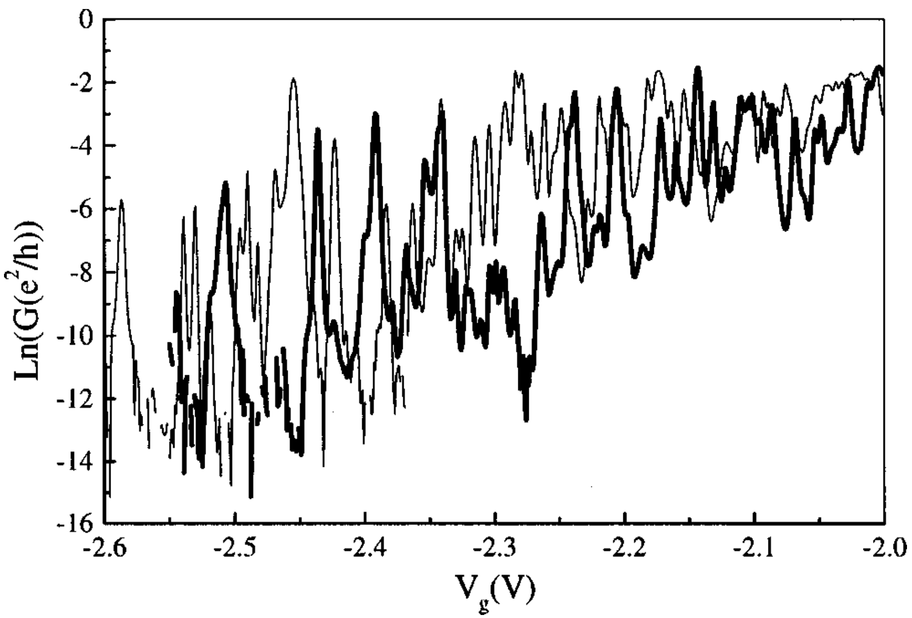
ln *G*(*V* gate) at ***T* =100 *m*K** in the **0.5-*μ*m-long** sample for two successive experiments without thermal cycling, showing the excellent reproducibility of the conductance pattern (curves are shifted for clarity).

CONDUCTANCE FLUCTUATION S INDUCED BY VARYING THE GATE VOLTAGE

Bulk of the impurity band



Edge of the impurity band

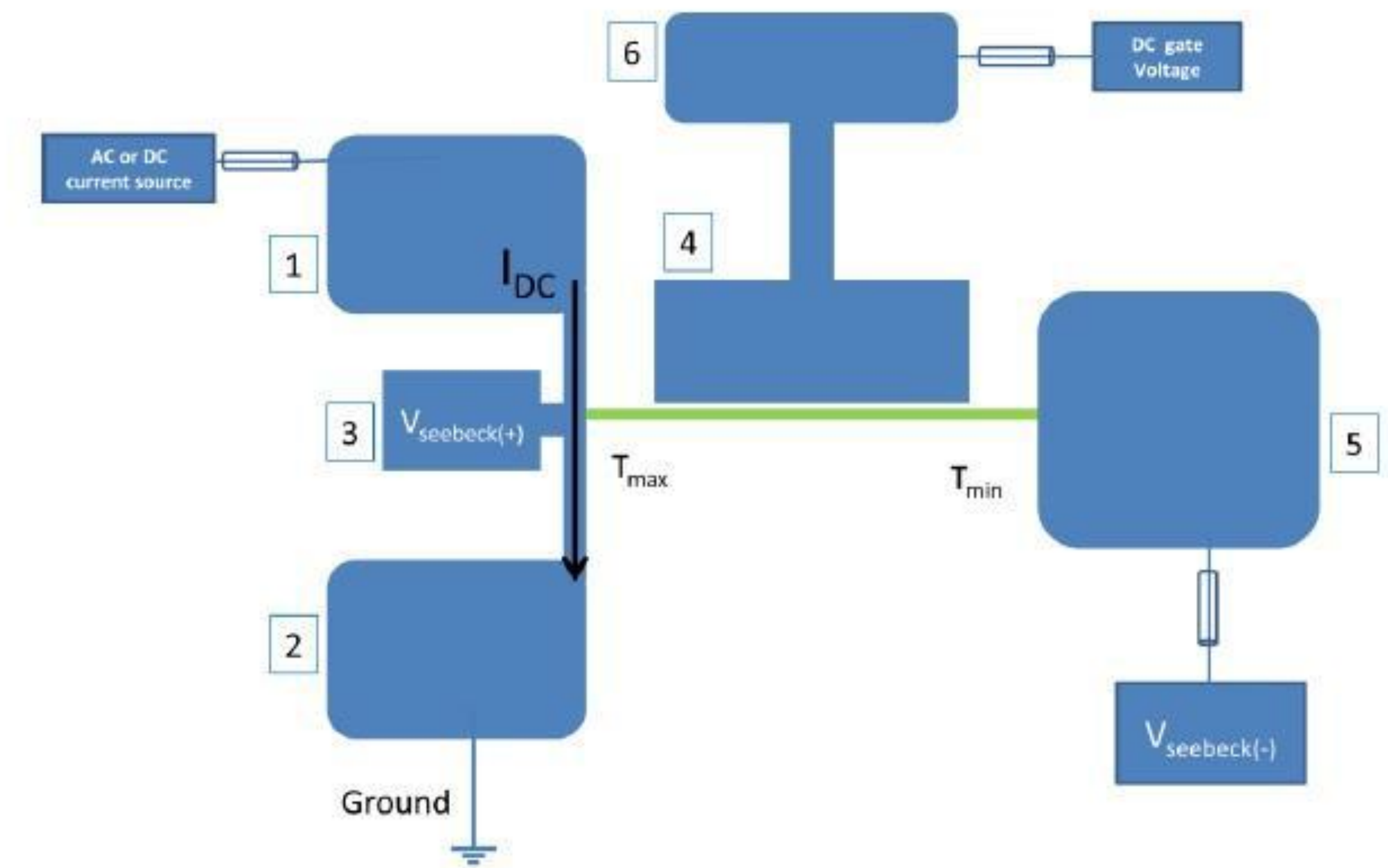


Mott Formula :

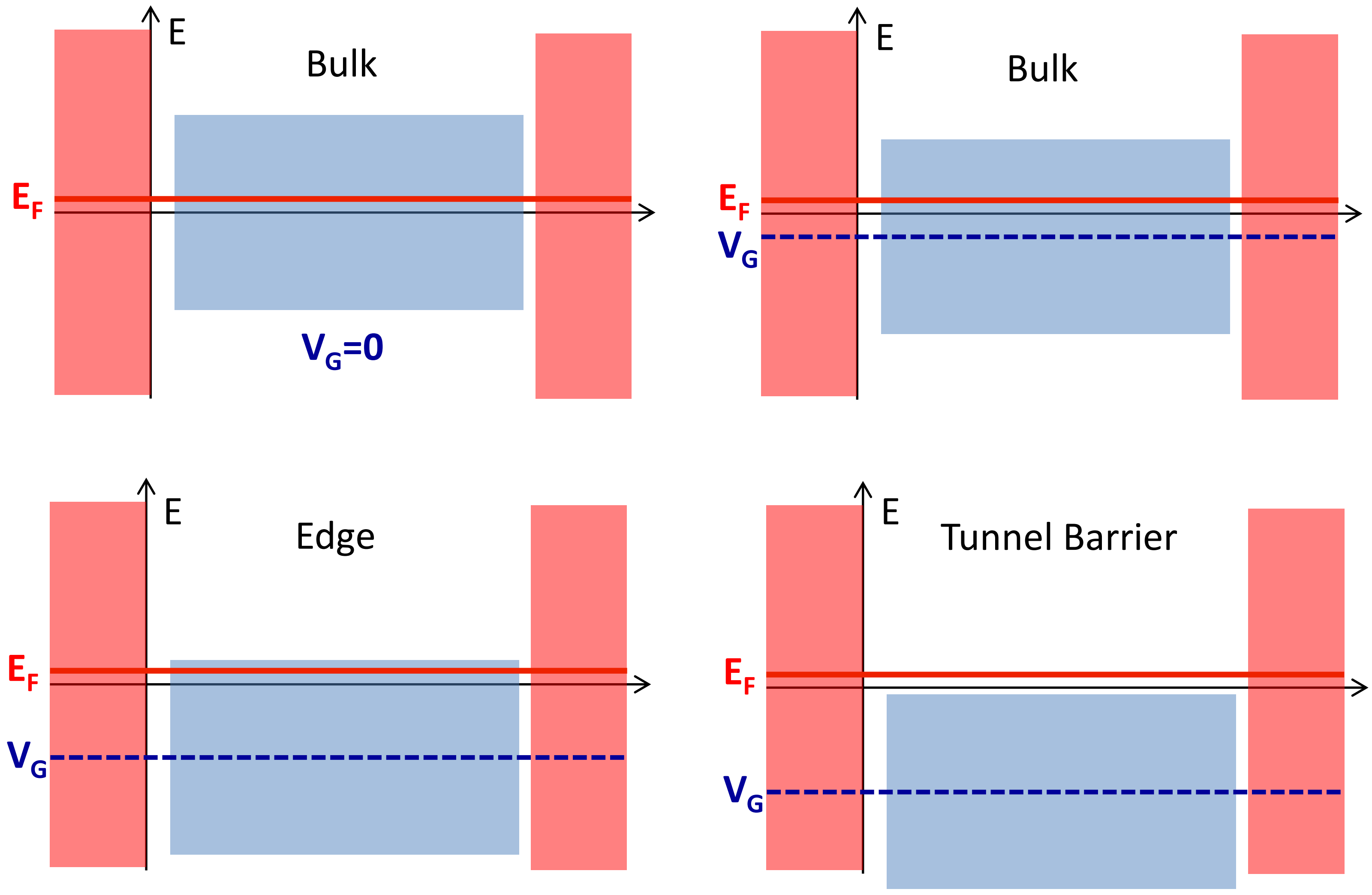
$$S \approx \frac{\partial \ln(G)}{\partial V_g}$$

➡ **Larger thermopower at the band edges**

PRINCIPLE OF A MEASURE OF THE THERMOPOWER (O. BOURGEOIS)



EFFECT OF GATE VOLTAGE ON THE IMPURITY BAND



What matters is the <u>relative</u> position of E_F inside the impurity band