From an Experimental Test of the BGS Conjecture to Modeling Relativistic Effects in Microwave Billards

Oriol Bohigas Memorial Orsay 2014

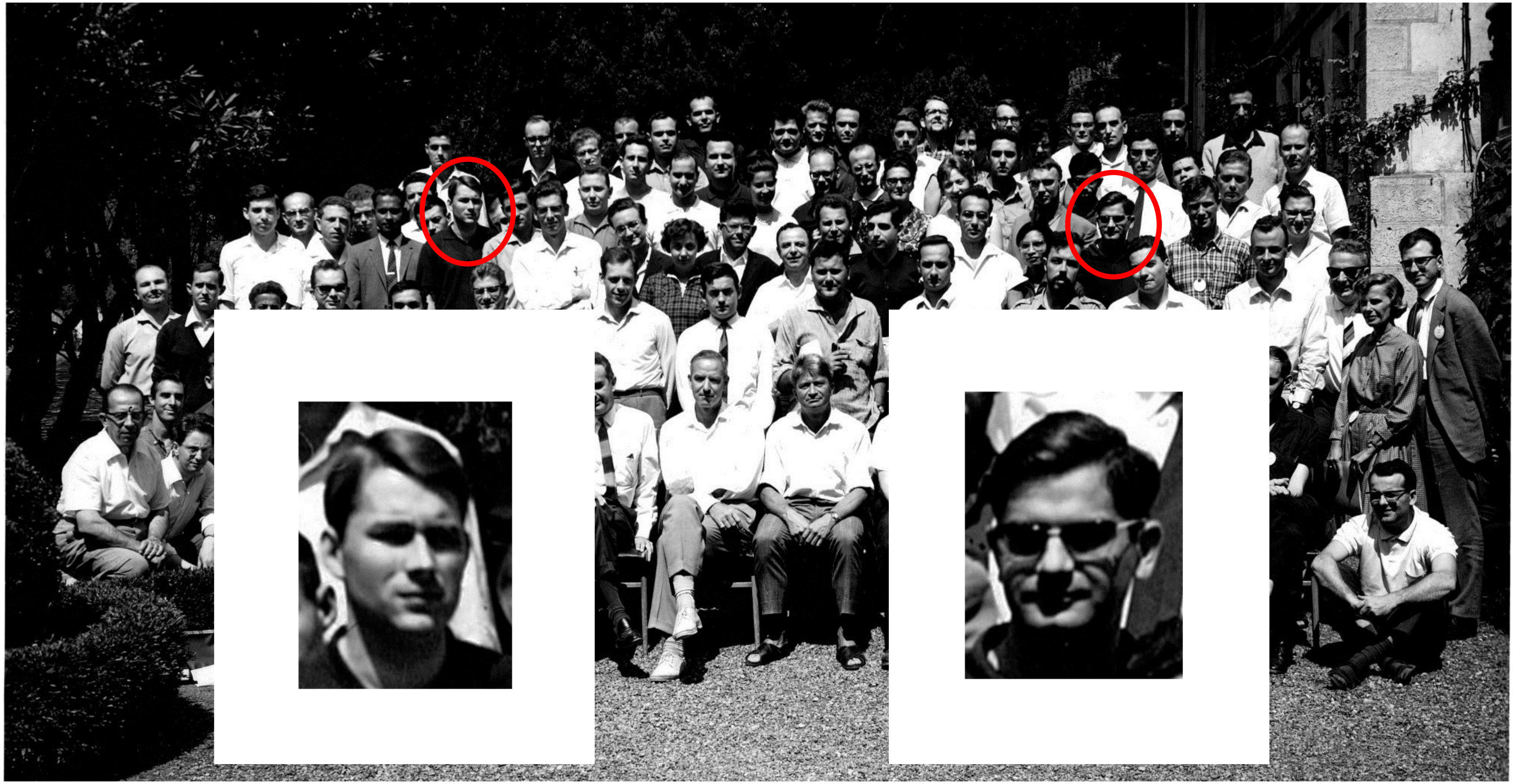
- Some personal recollections
- Some experimental tests of the BGS conjecture
- Dirac billiards and Graphene
- Outlook

Supported by DFG within SFB 634

C. Bouazza, B. Dietz, T. Klaus, M. Miski-Oglu, A. R. , T. Skipa, M. Wunderle

W. Ackermann, F. Iachello, N. Pietralla, L. von Smekal, J. Wambach

SCUOLA INTERNAZIONALE DI FISICA "ENRICO FERMI"
Varenna sul lago di Como - 1965



INSTITUTE FOR NUCLEAR THEORY (INT) Seattle 1994

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Bestowal of an Honarary Doctorate Degree Upon Oriol Bohigas by the TU Darmstadt in 2001

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Some Inspired by and Jointly Published Works with Oriol Bohigas and his Colleagues at the LPTMS and the Quantum Chaos Group at the TUD

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DISTRIBUTION OF EIGENMODES IN A SUPERCONDUCTING STADIUM BILLIARD WITH CHAOTIC DYNAMICS ← inspired

H.D. Gräf, H.L. Harney, H. Lengeler, C.H. Lewenkopf, C. Rangarcharyulu, A. Richter, P. Schardt and H.A. Weidenmüller

Phys. Rev. Letters <u>69</u>, (1992) 1296

EXPERIMENTAL VS. NUMERICAL EIGENVALUES OF A BUNIMOVICH STADIUM BILLIARD ← common

H. Alt, C. Dembowski, H.-D. Gräf, R. Hofferbert, H. Rehfeld, A. Richter, R. Schuhmann and <u>C. Schmit</u>

Phys. Rev. <u>E60</u>, (1999) 2851

FIRST EXPERIMENTAL EVIDENCE FOR CHAOS-ASSISTED TUNNELING IN A MICROWAVE ANNULAR BILLIARD ← inspired

C. Dembowski, H.-D. Gräf, A. Heine, R. Hofferbert, H. Rehfeld and A. Richter

Phys. Rev. Letters <u>84</u>, (2000) 867

GAUSSIAN UNITARY ENSEMBLE STATISTICS IN A TIME-REVERSAL INVARIANT MICROWAVE TRIANGULAR BILLIARD ← common

C. Dembowski, H.-D. Gräf, A. Heine, H. Rehfeld, A. Richter and <u>C. Schmit</u>

Phys. Rev. <u>E62</u>, (2000) R4516

SPECTRAL STATISTICS IN AN OPEN PARAMETRIC BILLIARD SYSTEM ← common

B. Dietz, A. Heine, A. Richter, <u>O. Bohigas</u> and <u>P. Leboeuf</u>

Phys. Rev. <u>E73</u>, (2006) 035201(R)

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FIRST EXPERIMENTAL OBSERVATION OF SUPERSCARS IN A PSEUDOINTEGRABLE BARRIER BILLIARD ← common

E. Bogomolny, B. Dietz, T. Friedrich, M. Miski-Oglu, A. Richter, F. Schäfer and C. Schmit

Phys. Rev. Letters 97, (2006) 254102

PROPERTIES OF NODAL DOMAINS IN A PSEUDOINTEGRABLE BARRIER BILLIARD ← inspired

B. Dietz, T. Friedrich, M. Miski-Oglu, A. Richter and F. Schäfer

Phys. Rev. E78, (2008) 045201

APPLICATION OF A TRACE FORMULA TO THE SPECTRA OF FLAT THREE-DIMENSIONAL DIELECTRIC RESONATORS ← common

S. Bittner, E. Bogomolny, B. Dietz, M. Miski-Oglu and A. Richter

Phys. Rev. E85, (2012) 026203

TRACE FORMULA FOR CHAOTIC DIELECTRIC RESONATORS TESTED WITH MICROWAVE EXPERIMENTS ← common

S. Bittner, B. Dietz, R. Dubertrand, J. Isensee, M. Miski-Oglu and A. Richter

Phys. Rev. E85, (2012) 056203

EXPERIMENTAL OBSERVATION OF LOCALIZED MODES IN A DIELECTRIC SQUARE RESONATOR ← common

S. Bittner, E. Bogomolny, B. Dietz, M. Miski-Oglu, and A. Richter

Phys. Rev. E88, (2013) 062906

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PHYSICAL REVIEW LETTERS

VOLUME 52 | 2 JANUARY 1984 | NUMBER 1

Characterization of Chaotic Quantum Spectra and Universality of Level Fluctuation Laws

O. Bohigas, M. J. Giannoni, and C. Schmit

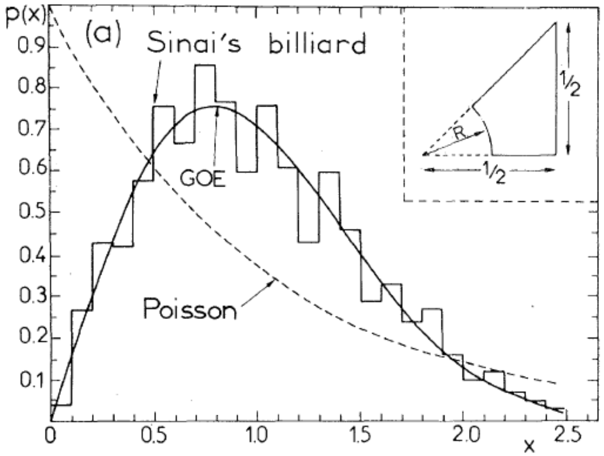
Division de Physique Théorique, Institut de Physique Nucléaire, F-91406 Orsay Cedex, France

(Received 2 August 1983)

It is found that the level fluctuations of the quantum Sinai's billiard are consistent with the predictions of the Gaussian orthogonal ensemble of random matrices. This reinforces the belief that level fluctuation laws are universal.

• • • • • • The purpose of this Letter is to use some of the systematic tools developed in RMT to make a detailed comparison of the level fluctuations of the quantum Sinai's billiard (SB) with GOE predictions. The choice of a two-dimensional billiard is convenient for our aim for several reasons: (i) Billiards have the lowest possible number of degrees of freedom allowing for chaotic motion; (ii) for billiards, it is possible to make a precise separation between global and local properties [cf. the Weyl formula, Eq. (1)]; (iii) billiards have a discrete spectrum with an infinite number of eigenvalues and by computing a large number of them one can reach a high statistical significance of the results. Finally, SB is known to be strongly chaotic (K system) and there exists an efficient method to compute its eigenvalues.• • • • • •

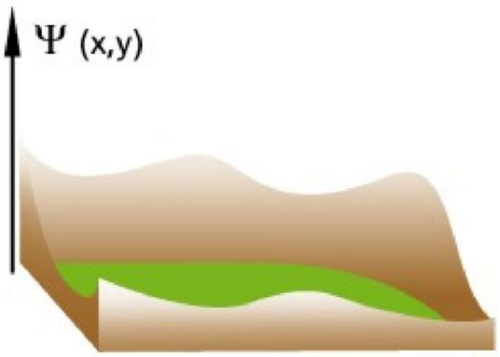
The BGS Conjecture



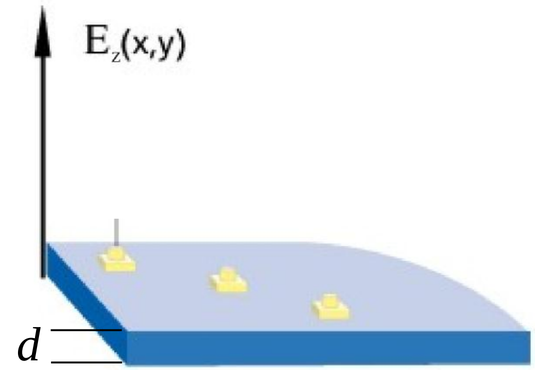
• • • • • • In summary, the question at issue is to prove or disprove the following conjecture: Spectra of time-reversal–invariant systems whose classical analogs are K systems show the same fluctuation properties as predicted by GOE (alternative stronger conjectures that cannot be excluded would apply to less chaotic systems, provided that they are ergodic). If the conjecture happens to be true, it will then have been established the *universality of the laws of level fluctuations* in quantal spectra already found in nuclei and to a lesser extent in atoms. Then, they should also be found in other quantal systems, such as molecules, hadrons, etc.

Quantum Billiards and Microwave Billiards

Quantum billiard



Microwave billiard



Analogy $\lambda > 2d$

$$\left(\frac{\hbar^2}{2m}\Delta + E\right)\Psi = 0,\ \Psi|_{\partial\Omega} = 0 \quad \longleftrightarrow \quad \left(\Delta + k^2\right) E_z = 0,\ E_z|_{\partial\Omega} = 0$$

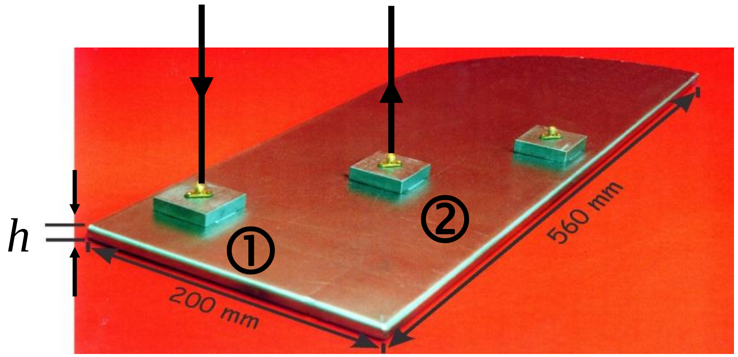
eigenvalue E ↔ wave number $k = \frac{2\pi f}{c}$

eigenfunction Ψ ↔ electric field strength E_z

Measurement Principle

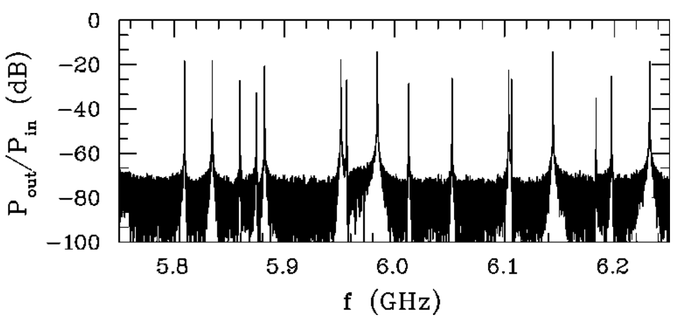
- Measurement of scattering matrix element S_{21}

rf power in   rf power out



$$\longrightarrow \quad \frac{P_{out,2}}{P_{in,1}} = \left|S_{21}\right|^2$$

Resonance spectrum   Resonance density   Length spectrum (Gutzwiller)



$$\longrightarrow \quad \rho(f) = \rho_{Weyl}(f) + \rho_{fluc}(f) \quad \xrightarrow{FT} \quad \tilde{\rho}(l)$$

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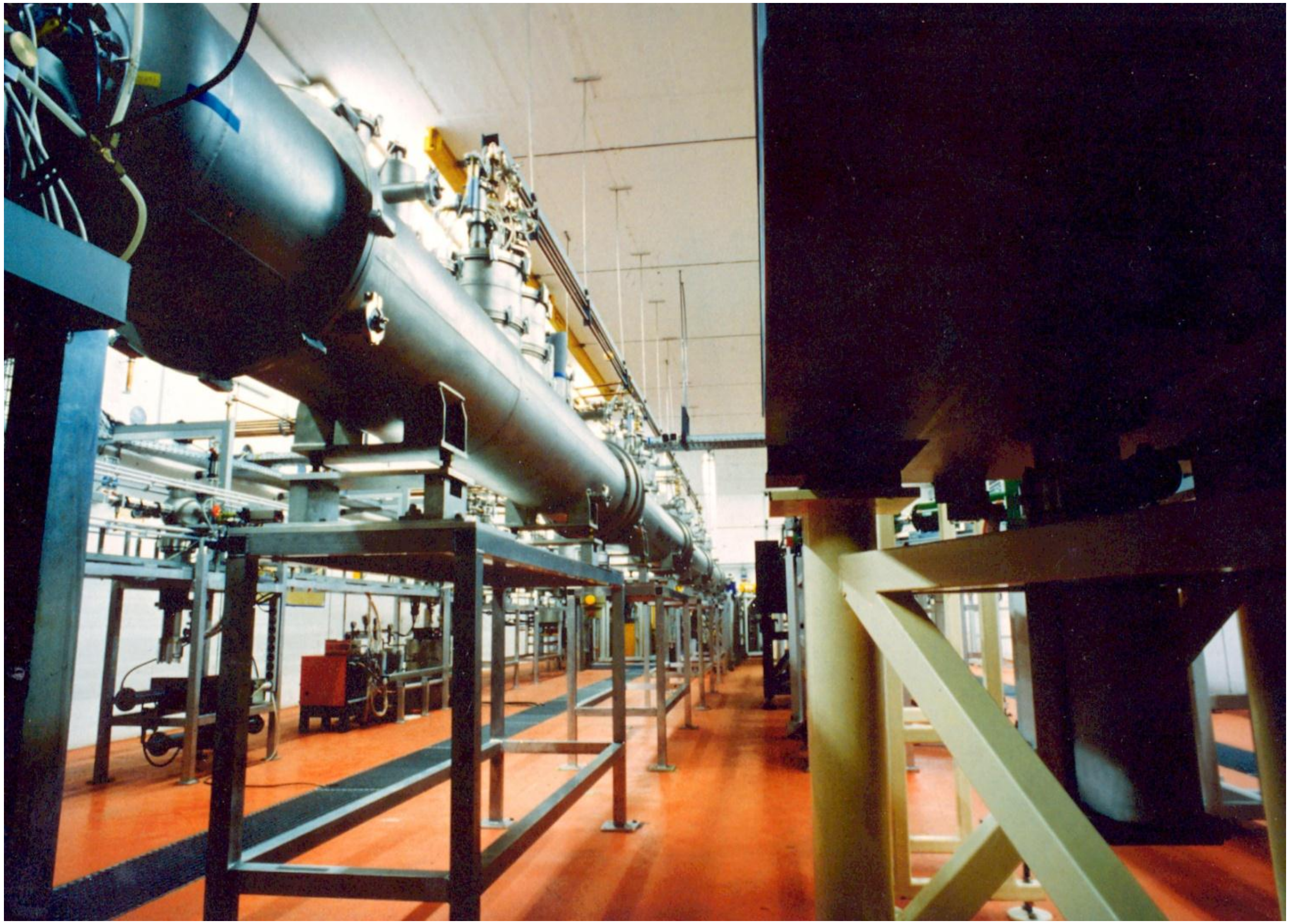
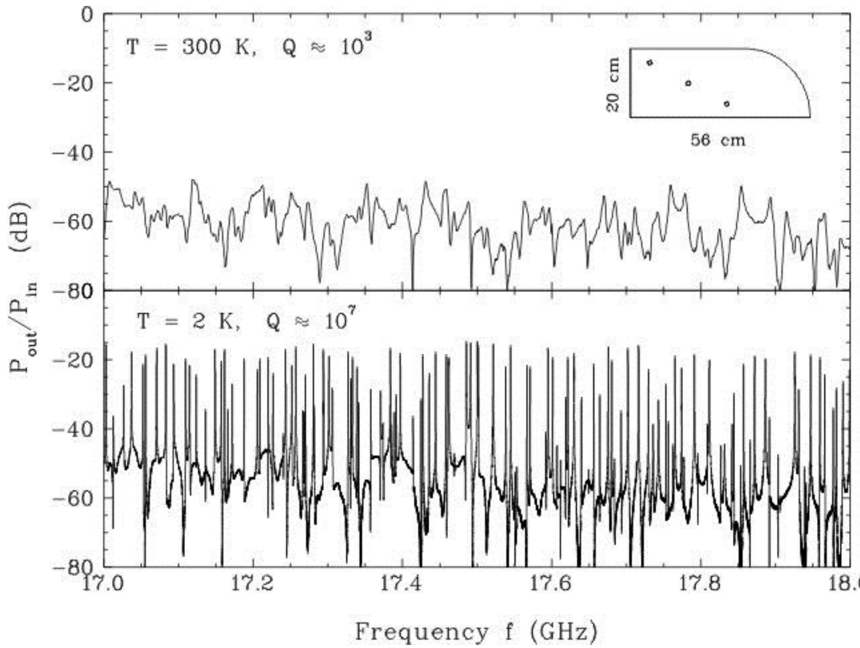


Distribution of Eigenmodes in a Superconducting Stadium Billiard with Chaotic Dynamics

H.-D. Gräf,[1] H. L. Harney,[2] H. Lengeler,[3] C. H. Lewenkopf,[2] C. Rangacharyulu,[4] A. Richter,[1] P. Schardt,[1] and H. A. Weidenmüller[2]

[1]*Institut für Kernphysik, Technische Hochschule Darmstadt, W-6100 Darmstadt, Germany*
[2]*Max-Planck-Institut für Kernphysik, W-6900 Heidelberg, Germany*
[3]*AT-Division, CERN, CH-1211 Geneva 23, Switzerland*
[4]*Department of Physics, University of Saskatchewan, Saskatoon, Saskatchewan, Canada S7N 0W0*
(Received 8 June 1992)

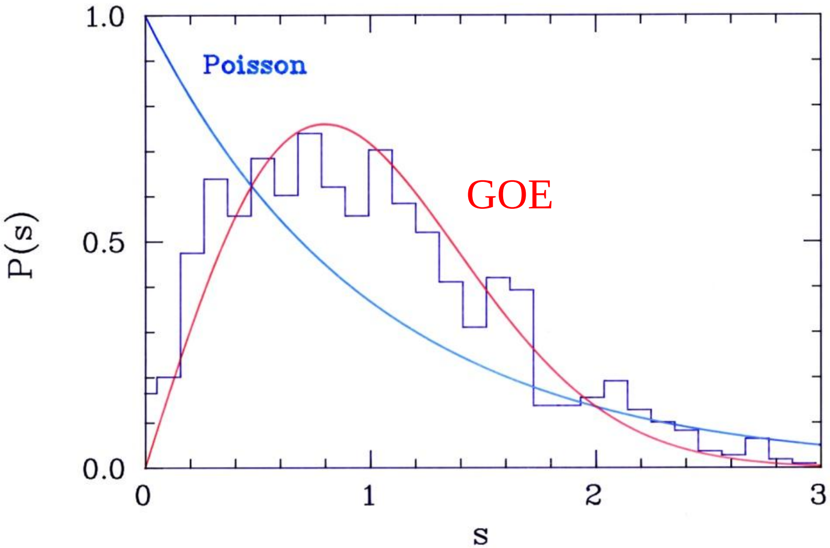
The complete sequence of 1060 eigenmodes with frequencies between 0.75 and 17.5 GHz of a quasi-two-dimensional superconducting microwave resonator shaped like a quarter of a stadium billiard with a Q value of $Q \approx 10^5$–10^7 was measured for the first time. The semiclassical analysis is in good agreement with the experimental data, and provides a new scheme for the statistical analysis and comparison with predictions based on the Gaussian orthogonal ensemble.



Nearest Neighbor Spacing Distribution

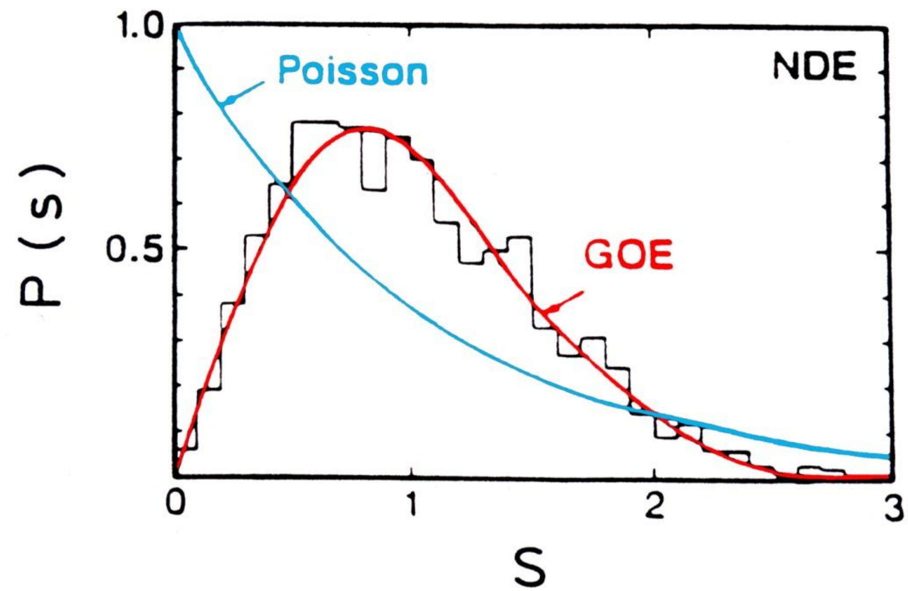
Stadium billiard

- Scale: 10^{-1} m



Nuclear Data Ensemble
Bohigas, Haq + Pandey (1983)

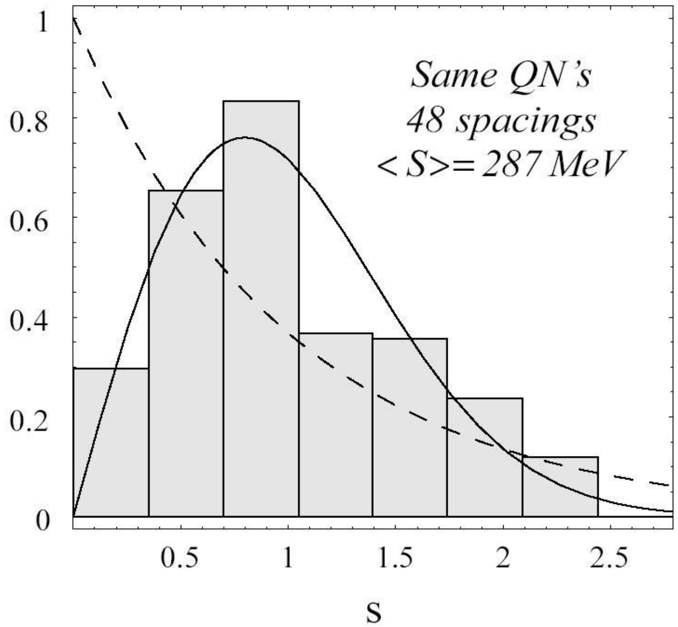
- Scale: 10^{-15}m



- Universal (generic) behaviour of the two systems

Universality in Microscopic and Mesoscopic Systems: Quantum Chaos in Hadrons

- Combined data from measured baryon and meson mass spectra up to 2.5 GeV (from PDG)
- Spectra can be organized into multiplets characterized by a set of definite quantum numbers: isospin, spin, parity, strangeness, baryon number, ...

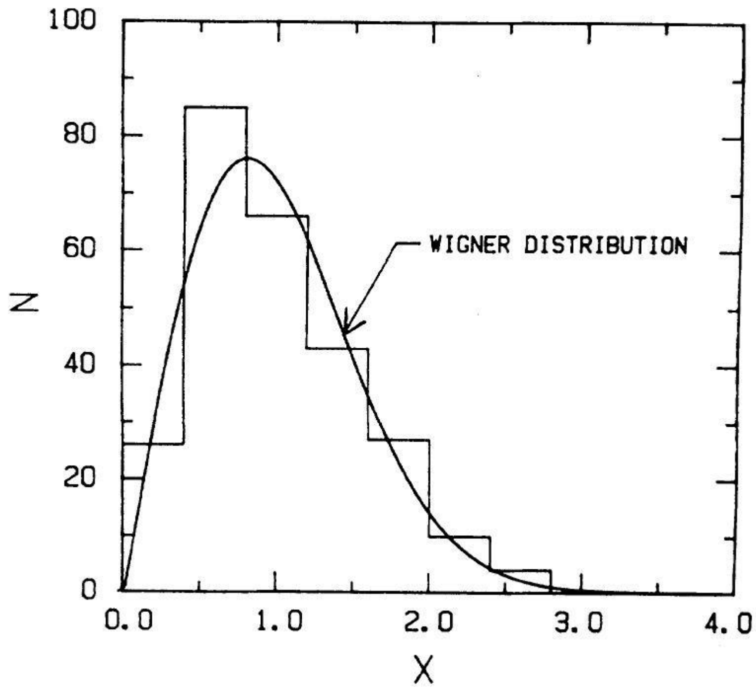


- Scale: 10^{-16} m

Pascalutsa (2003)

Universality in Microscopic and Mesoscopic Systems: Quantum Chaos in Atoms

- 8 sets of atomic spectra of highly excited neutral and ionized rare earth atoms combined into a data ensemble
- States of same total angular momentum and parity



- Scale: 10^{-10} m

Camarda + Georgopoulos (1983)

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Universality in Microscopic and Mesoscopic Systems: Quantum Chaos in Molecules

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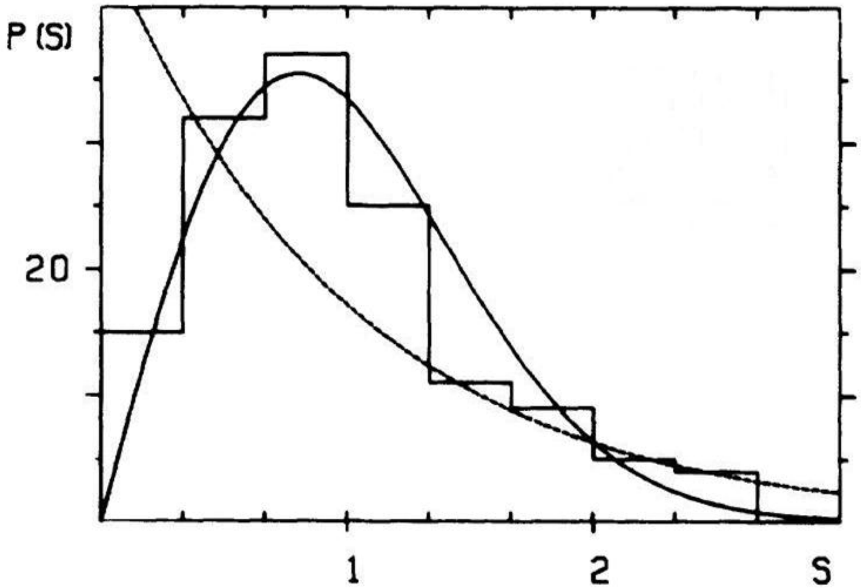
- Vibronic levels of NO_2
- States of same quantum numbers



- Scale: 10^{-9} m

Zimmermann et al. (1988)

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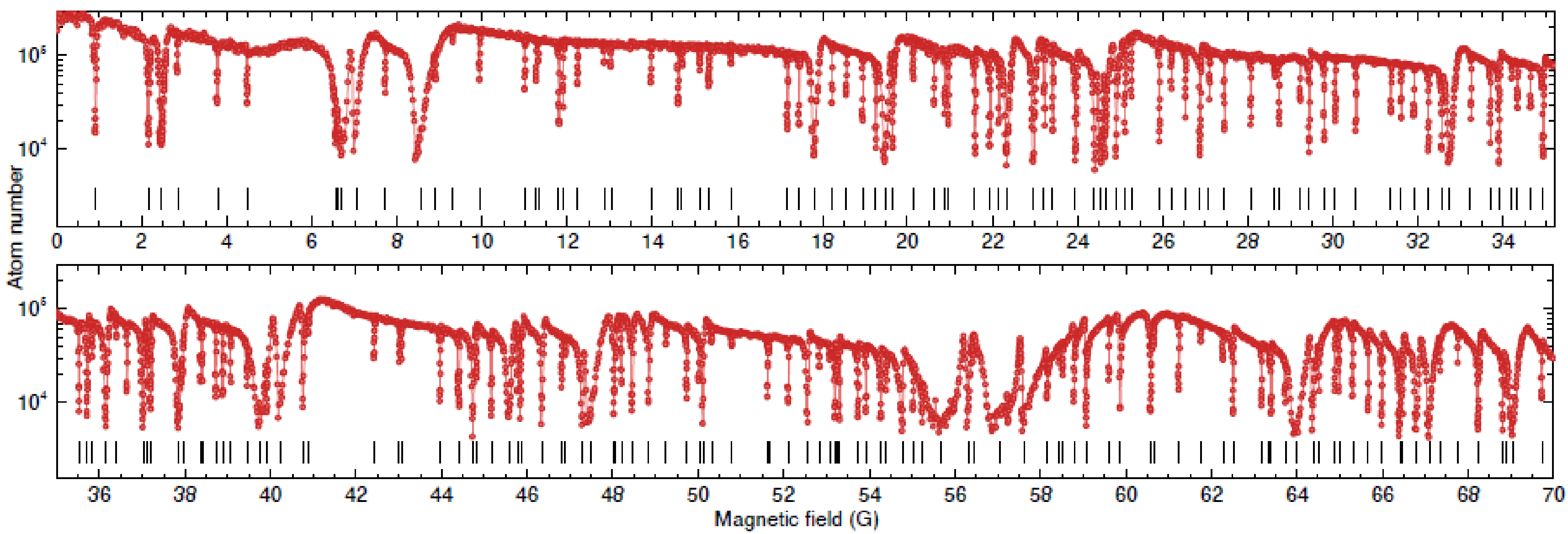
Universality in Microscopic and Mesoscopic Systems: Quantum Chaos in Ultracold Collisions of ^{168}Er Atoms

- High-resolution trap-loss spectroscopy of Fano-Feshbach resonances in an optically-trapped ultracold sample of ^{168}Er atoms
- Scattered atoms as a function of an external magnetic field



A. Frisch et al. (2013)

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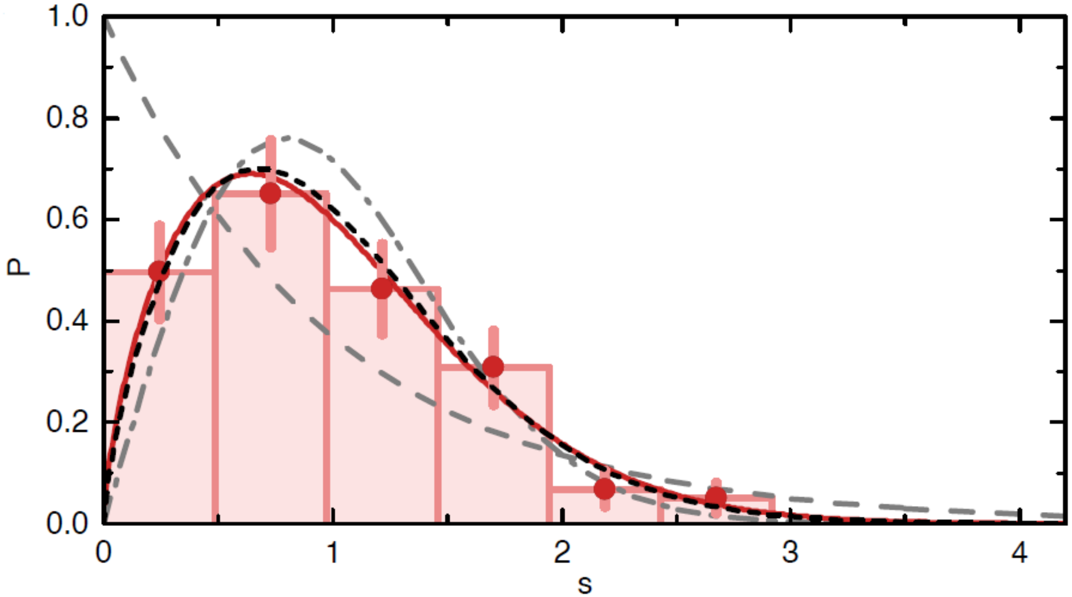
- 190 Fano-Feshbach resonances
- GOE – · – · ·
- Brody – – – –



- Scale: 10^{-6} m

A. Frisch et al. (2013)

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How is the Behavior of the Classical System Transferred to the Quantum System?

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- Non-relativistic Schrödinger billiards:

- There is a one-to-one correspondence between billiards, microscopic and mesoscopic systems.

  $\rightarrow$ BGS conjecture:

  ”The spectral properties of a generic chaotic system coincide with those of random matrices from the GOE“.

- Semiclassical proof of this conjecture has been given by S.Heusler et al. PRL **98**, 044103 (2007) + New J. Phys. **11**, 103025 (2009)

- Next: Relativistic Dirac billiards $\rightarrow$ Graphene and photonic crystals

Nobel Prize in Physics 2010

Photo: Sergeom, Wikimedia Commons

Andre Geim

Photo: University of Manchester, UK

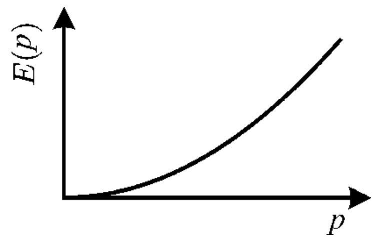
Konstantin Novoselov

The Nobel Prize in Physics 2010 was awarded jointly to Andre Geim and Konstantin Novoselov *"for groundbreaking experiments regarding the two-dimensional material graphene"*

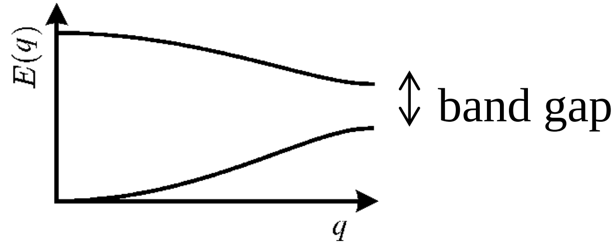
Reminder: Dispersion Relation of a Free Electron and of an Electron in a Solid

- Free electron: wavelength $\lambda = h/p$ and energy $E = p^2/2m$

- $\mathrm{E}(p)$ is a continuous function of momentum p

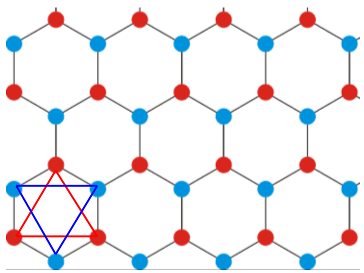


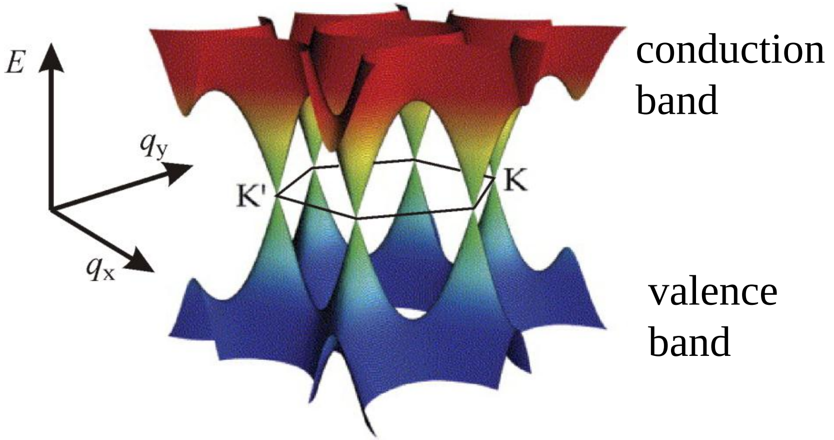
- Solid: electron moves within the periodic potential of the crystal lattice
  $\rightarrow$ $\mathrm{E}(q)$ is not anymore a continuous function of quasimomentum q
  $\rightarrow$ band structure



Graphene

- “What makes graphene so attractive for research is that the spectrum closely resembles the Dirac spectrum for massless fermions.”
  (M. Katsnelson, Materials Today, (2007))

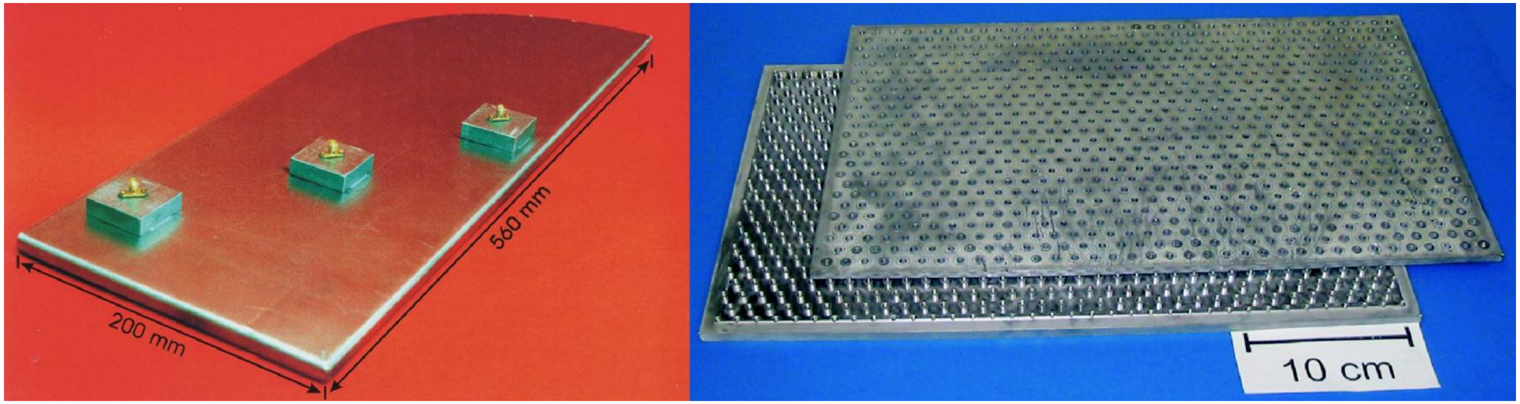




- Two triangular sublattices of carbon atoms
- Near each corner of the first hexagonal Brillouin zone the electron energy E has a conical dependence on the quasimomentum and the bands touch each other at the Dirac points
- $E = \hbar\, v_F q$ with $v_F = c/300 \rightarrow$ at the Dirac point electrons behave like relativistic fermions $E = cp$
- $\alpha = e^2/\hbar c \rightarrow \alpha_{Graphene} = 300 \cdot 1/137 \approx 2 \rightarrow$ strongly interacting system (QCD)
- Experimental realization of graphene in analog experiments of microwave photonic crystals

Photonic Crystal in an Open Flat Microwave Billiard

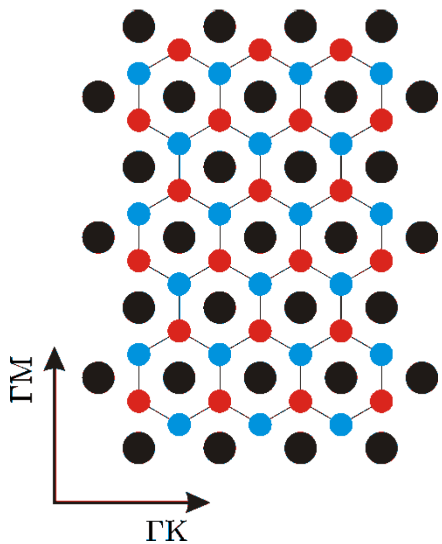
- A photonic crystal is a structure, whose electromagnetic properties vary periodically in space, e.g. an array of metallic cylinders

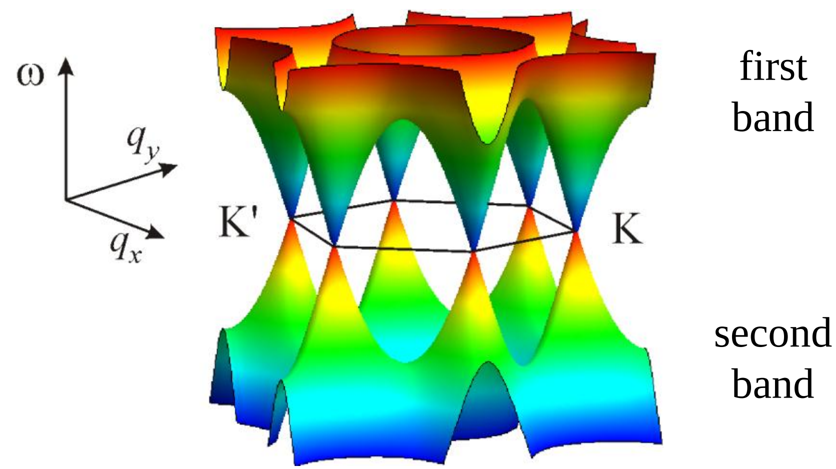


- Flat "crystal" (resonator) → E-field is perpendicular to the plates (TM_0 mode)
- Propagating modes are solutions of the scalar Helmholtz equation
  → Schrödinger equation for a quantum multiple-scattering problem
  → Numerical solution yields the band structure

Calculated Photonic Band Structure

- Dispersion relation $\omega(q)$ of a photonic crystal exhibits a band structure analogous to the electronic band structure in a solid





- The triangular photonic crystal possesses a conical dispersion relation $\rightarrow$ Dirac spectrum with a Dirac point where bands touch each other
- The voids form a honeycomb lattice like atoms in graphene

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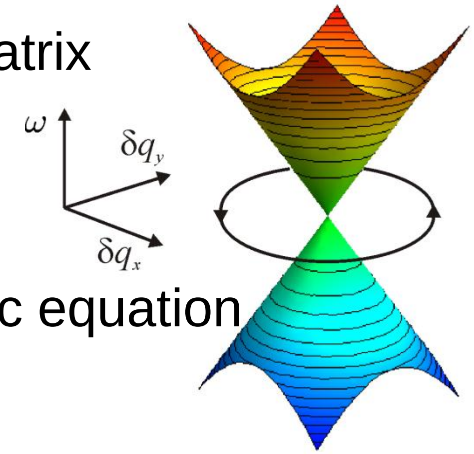
Effective Hamiltonian around the Dirac Point

- Close to Dirac point the effective Hamiltonian is a 2x2 matrix



$$\hat{H}_{\text{eff}} = \omega_D \mathbb{1} + v_D \left(\delta q_x \hat{\sigma}_x + \delta q_y \hat{\sigma}_y \right)$$

- Substitution $\delta q_x \rightarrow -i\partial_x$ and $\delta q_y \rightarrow -i\partial_y$ leads to the Dirac equation

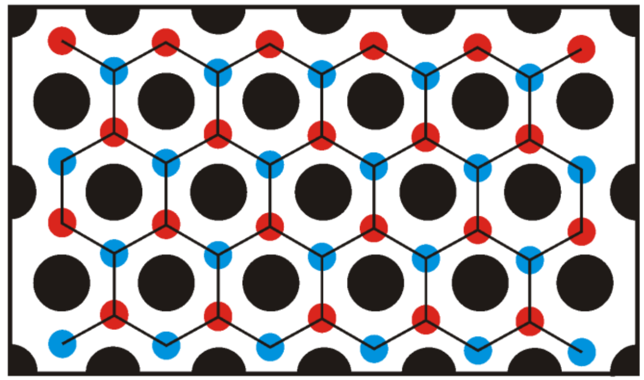
$$\begin{pmatrix} 0 & \partial_x - i\partial_y \\ \partial_x + i\partial_y & 0 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = i \, \frac{\omega - \omega_D}{v_D} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

- Experimental observation of a Dirac spectrum in open photonic crystal

  (S. Bittner *et al.*, PRB **82**, 014301 (2010))

- Microwave Dirac billiards

Microwave Dirac Billard:
Photonic Crystal in a Box→ “Artificial Graphene“



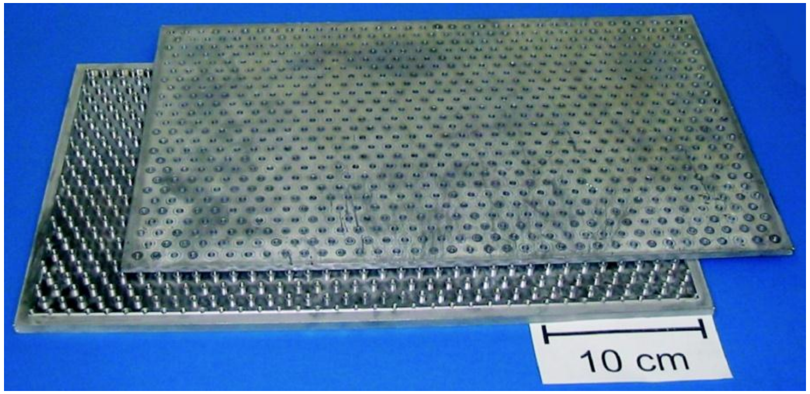
- Graphene flake: the electron cannot escape → Graphene Dirac billiard
- Photonic crystal: electromagnetic waves can escape from it

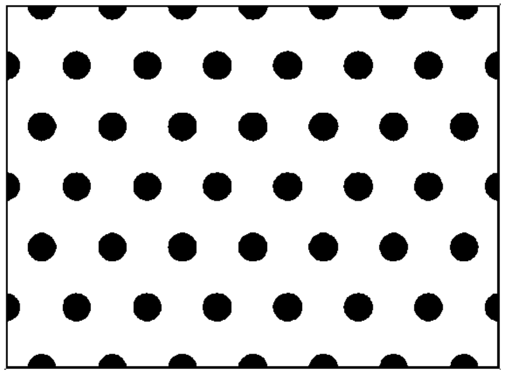
  → microwave Dirac billiard: “Artificial Graphene“
- Serves as a model system for graphene Dirac billiards

  (J. Wurm *et al.*, PRB **84**, 075468 (2011))
- Boundaries sustain the translational symmetry

  → the whole plane can be covered with a perfect crystal lattice

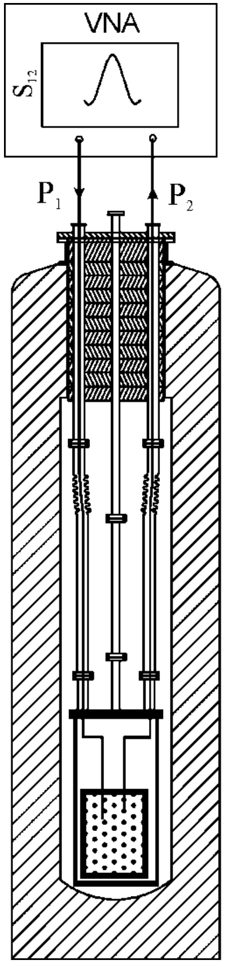
Superconducting Dirac Billiard

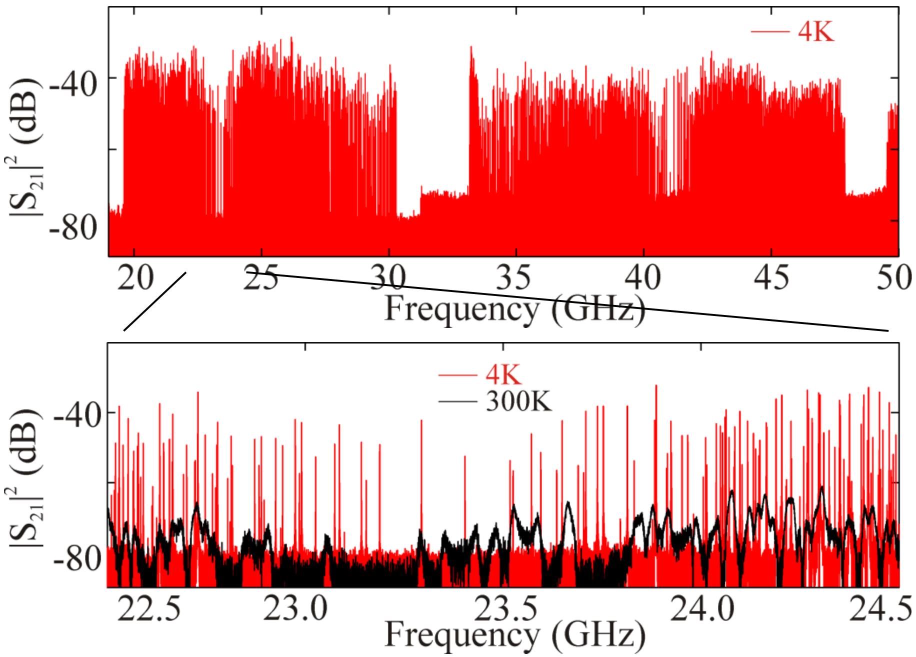




- 888 cylinders (scatterers) milled out of a brass plate
- Height *d* = 3 mm $\rightarrow f_{max}^{2D}$ = 50 GHz for 2D system
- Lead plated $\rightarrow$ superconducting below 7.2 K $\rightarrow$ high Q value
- Boundary does not violate the translation symmetry $\rightarrow$ no edge states

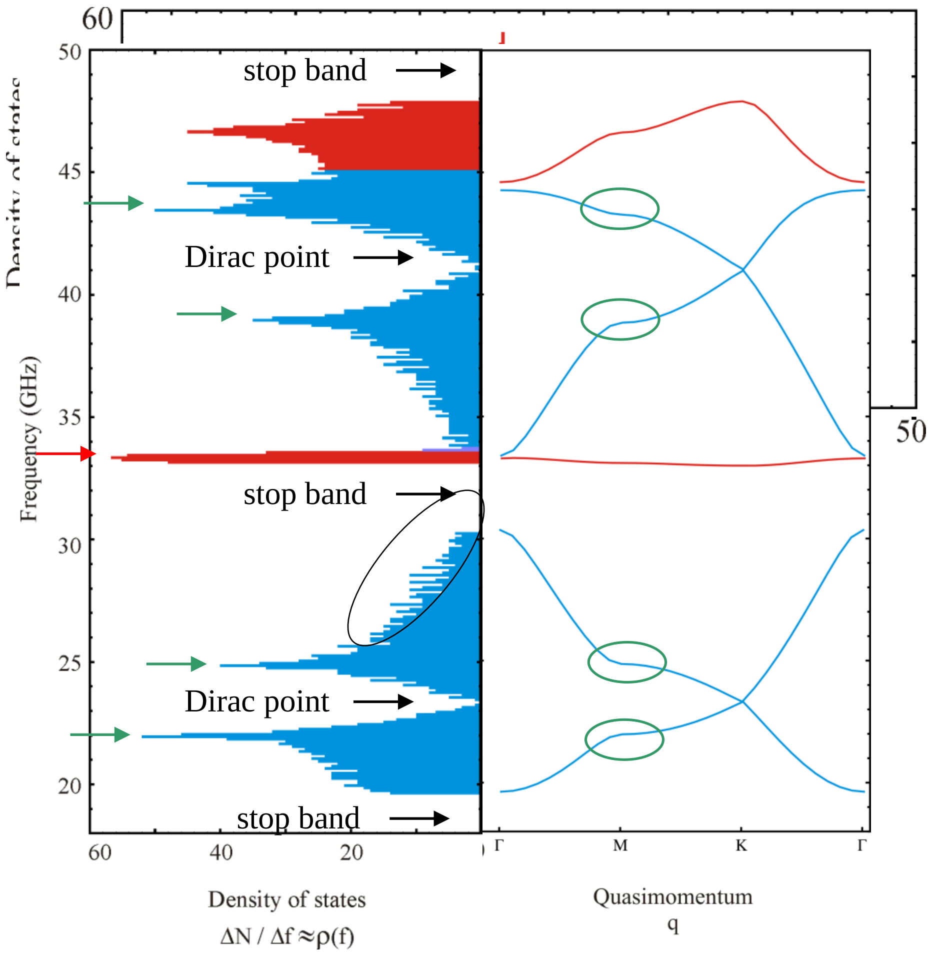
Transmission Spectrum at 4 K





- Pronounced stop bands and Dirac points
- Quality factors > $5 \cdot 10^5$
- $\langle \Gamma \rangle / \langle D \rangle = 10^{-3} \rightarrow$ complete spectrum
- Altogether 5000 resonances observed

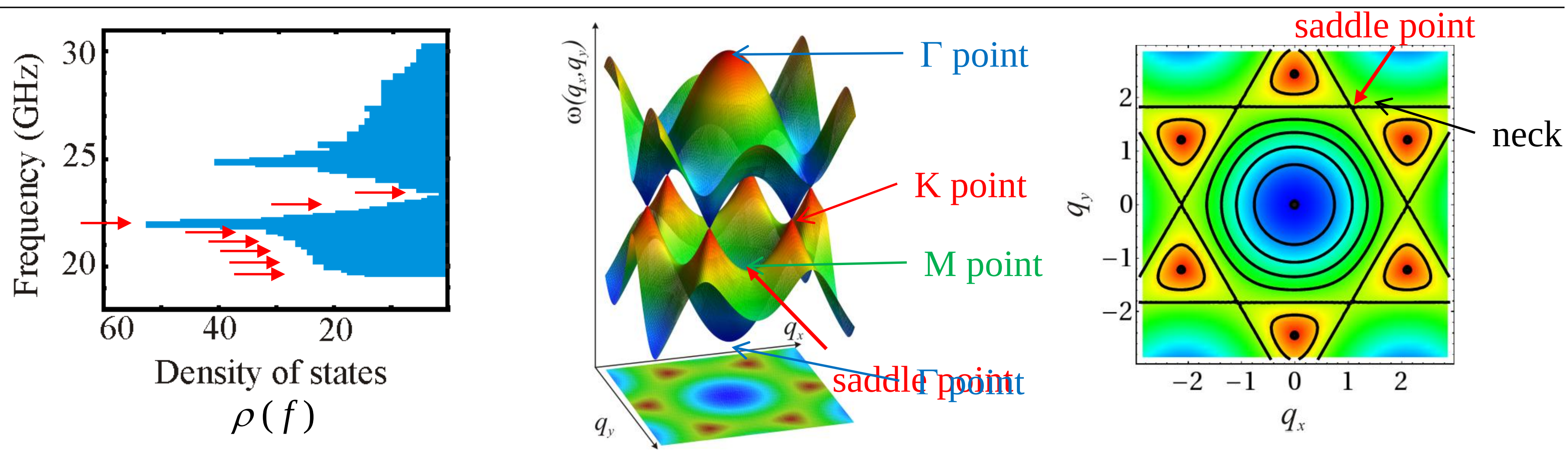
Density of States of the Measured Spectrum and the Band Structure



- Positions of stop bands are in agreement with calculation
- DOS related to slope of a band
- Dips correspond to Dirac points
- Flat band has very high DOS
- High DOS at van Hove singularities → ESQPT?
- Qualitatively in good agreement with prediction for graphene

  (Castro Neto *et al.*, RMP **81**,109 (2009))
- Oscilations around the mean density → finite size effect

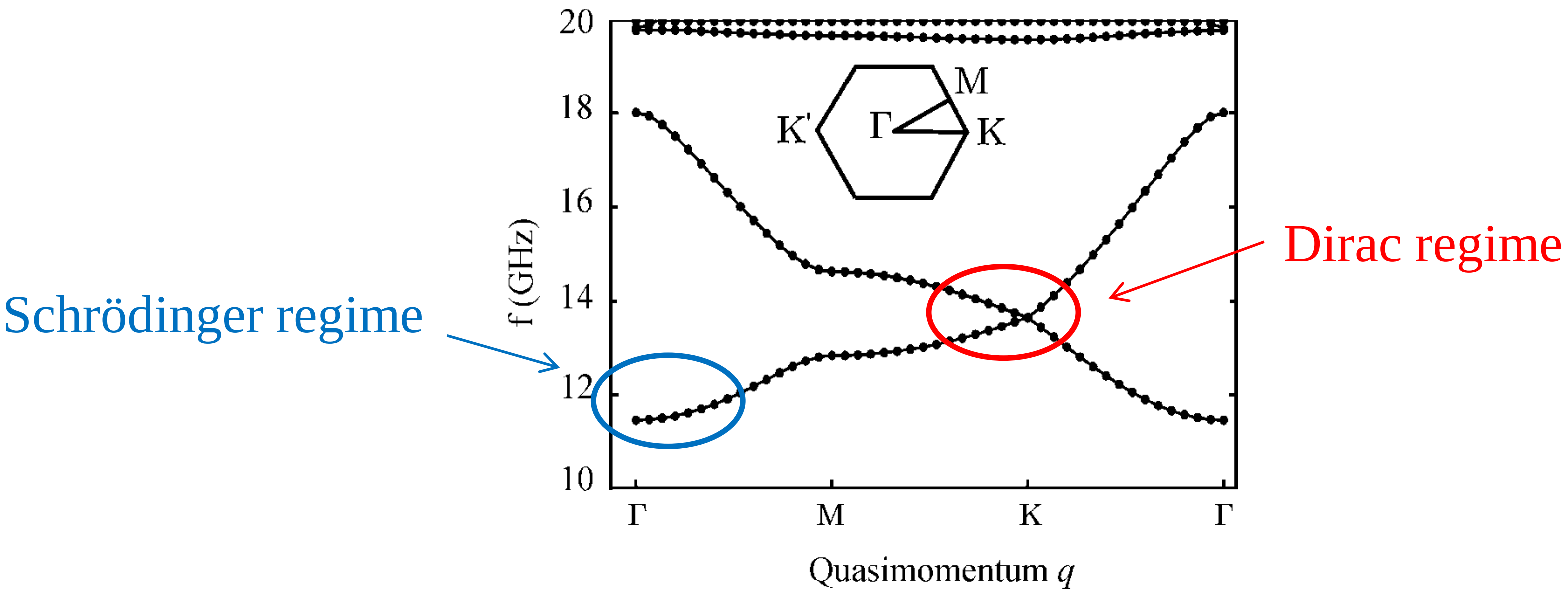
Experimental DOS and Topology of Band Structure



- Each frequency f in the experimental DOS $\rho(f)$ is related to an isofrequency line of band structure in q space
- Close to band edges isofrequency lines form circles around the Γ point
- At the saddle point the isofrequency lines become straight lines which cross each other and lead to the van Hove singularities
- Parabolically shaped surface merges into the 6 Dirac cones around Dirac frequency

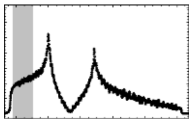
→ topological phase transition (“Neck Disrupting Lifschitz Transition“) from non-relativistic to relativistic regime (B. Dietz et al., PRB 88, 1098 (2013))

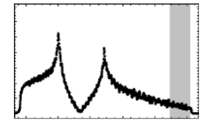
Schrödinger and Dirac Dispersion Relation in the Photonic Crystal



- Dispersion relation along irreducible Brillouin zone ΓMKΓ
- Quadratic dispersion around the Γ point → Schrödinger regime $f = q^2/2m_{eff}$
- Linear dispersion around the K point → Dirac regime $f = qv_D$

Computed Intensity Distributions $|E_z(x,y)|^2$ in the Schrödinger Regime





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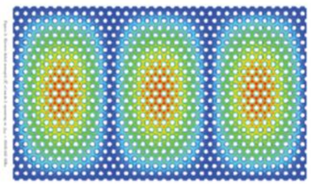

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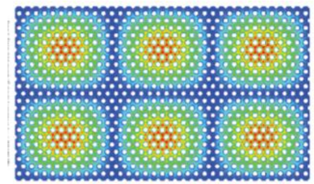

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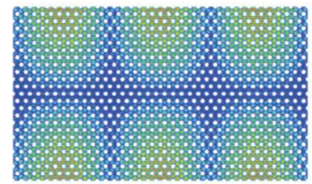

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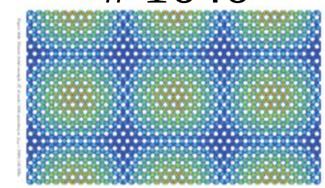

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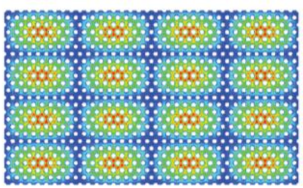

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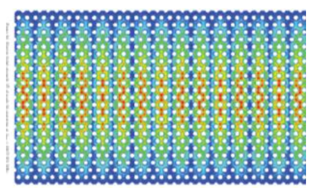

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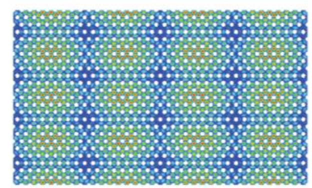

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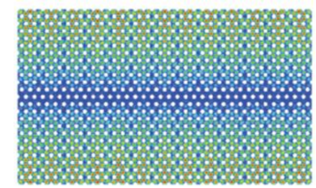


- First ~250 states counted from lower (upper) band edge coincide with those of empty rectangular billiard with Dirichlet / Dirichlet (Dirichlet / Neumann) boundary conditions

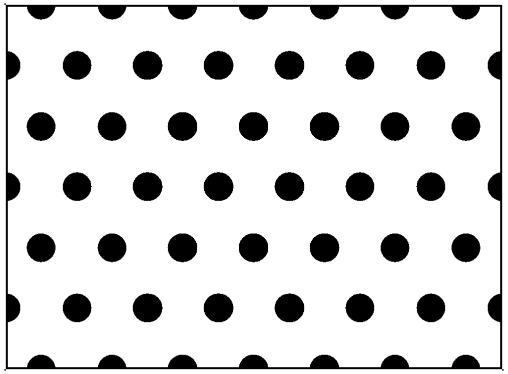
Periodic Orbit Theory (POT)
Gutzwiller‘s Trace Formula

- Description of quantum spectra in terms of classical periodic orbits

spectrum		wave numbers		spectral density	FT	length spectrum
$\{f_i\}$	$\longrightarrow$	$\{k_i\}$	$\longrightarrow$	$\rho(k) = \sum \delta(k - k_i)$	$\longrightarrow$	$\tilde{\rho}(l) = \int_0^{k_{max}} dk\, e^{ikl} \rho(k)$

Peaks at the lengths *l* of PO’s

Dirac billiard

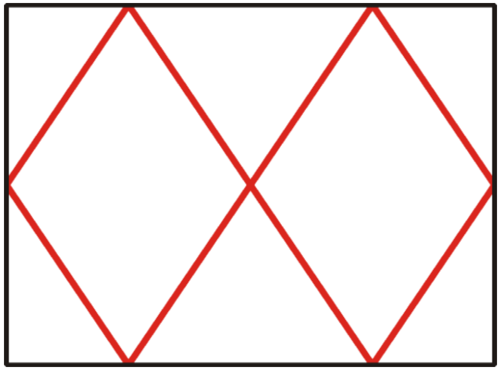


Effective description $\longrightarrow$

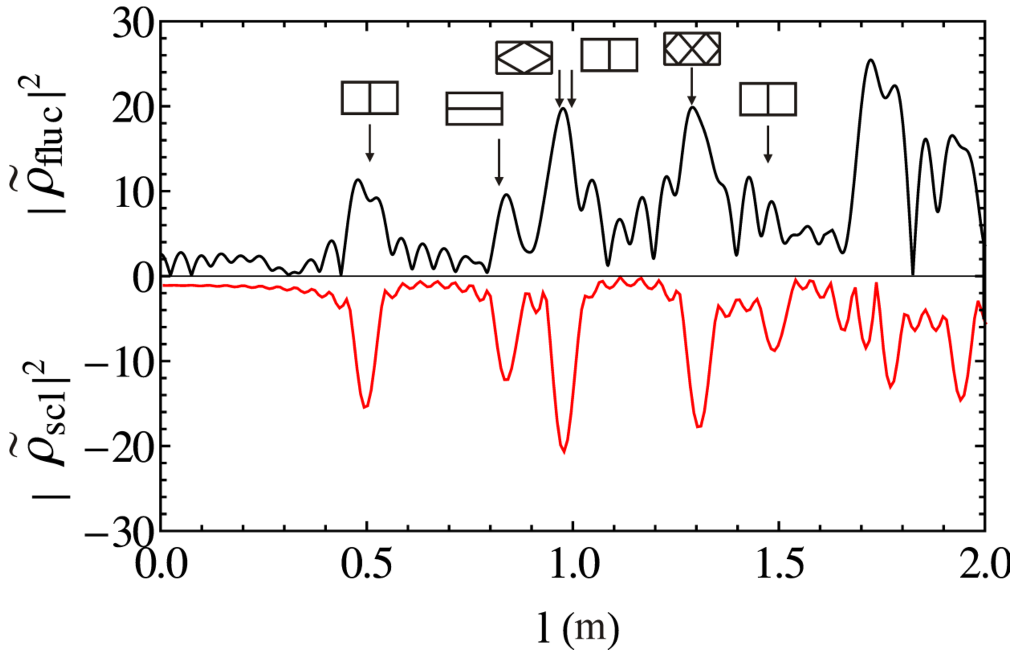
S: $f = q^2/2m_{eff}$

D: $f = q v_D$

Periodic orbits

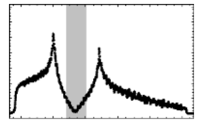


Experimental Length Spectrum: Schrödinger Regime

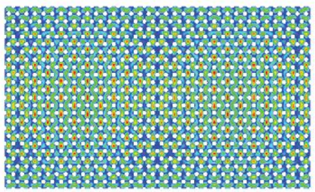


- Effective description ($f = q^2/2m_{eff}$) has a relative error of 5% at the frequency of the highest eigenvalue in the regime
- Very good agreement
- Next: Dirac regime

Computed Intensity Distributions around the Dirac Frequency



813



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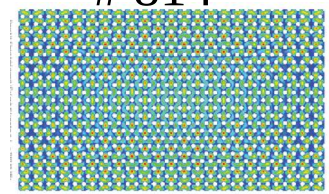

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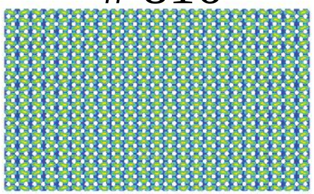

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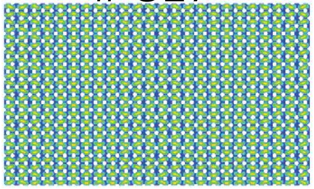

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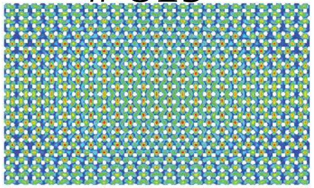

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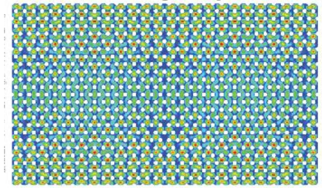
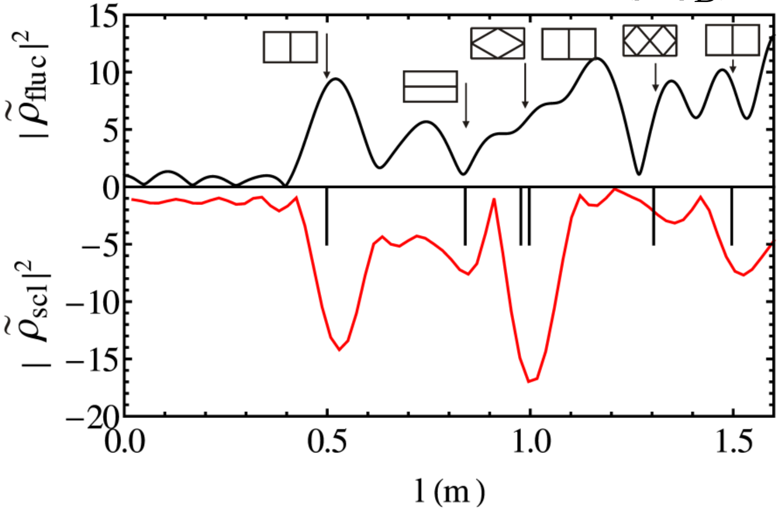


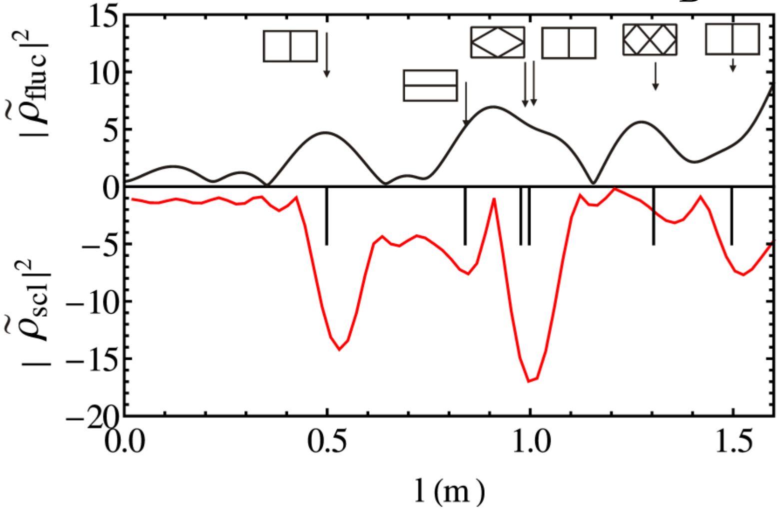
- Intensity is spread over the whole billiard area
- A long wave structure is barely seen

Experimental Length Spectrum: Dirac Regime

Above Dirac point (f>f_D)

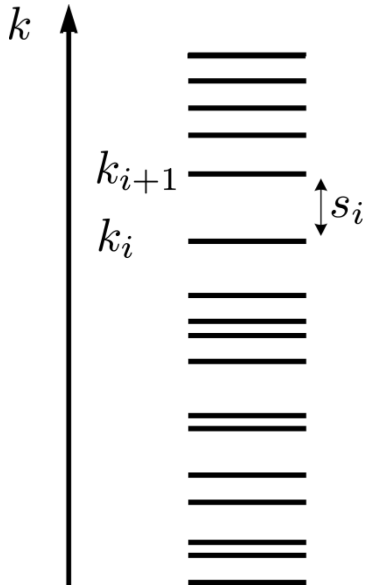


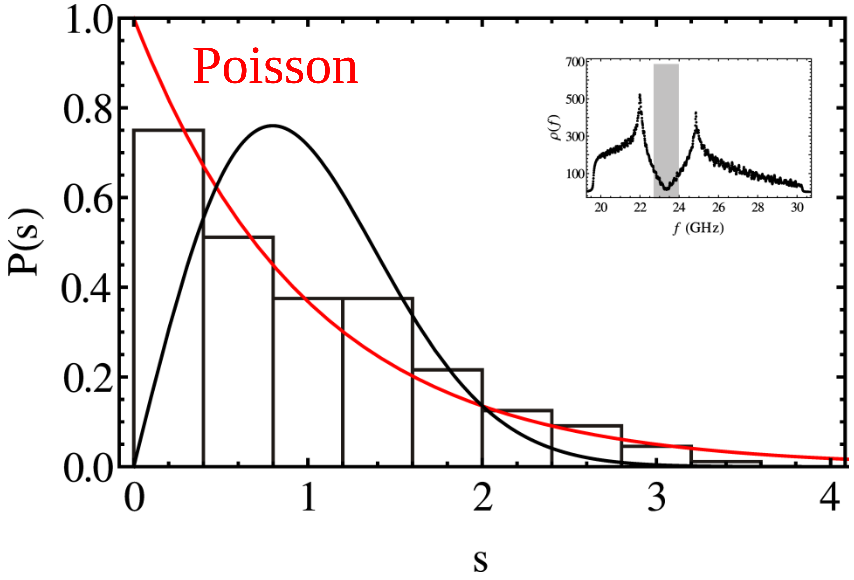
Below Dirac point (f<f_D)



- Some peak positions deviate from the lengths of POs
- Comparison with semiclassical predictions for a Dirac billiard (J. Wurm *et al.*, PRB **84**, 075468 (2011))
- Effective description ($f = qv_D$) has a relative error of 20% at the frequency of the highest eigenvalue in the regime

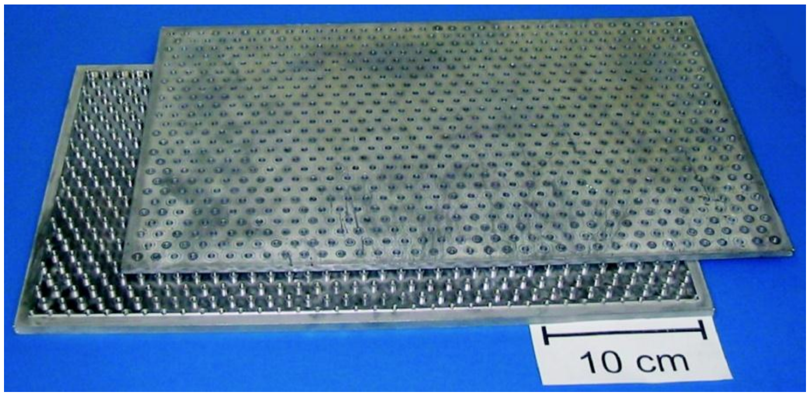
Spectral Properties of a Rectangular Dirac Billiard: Nearest Neighbour Spacing Distribution



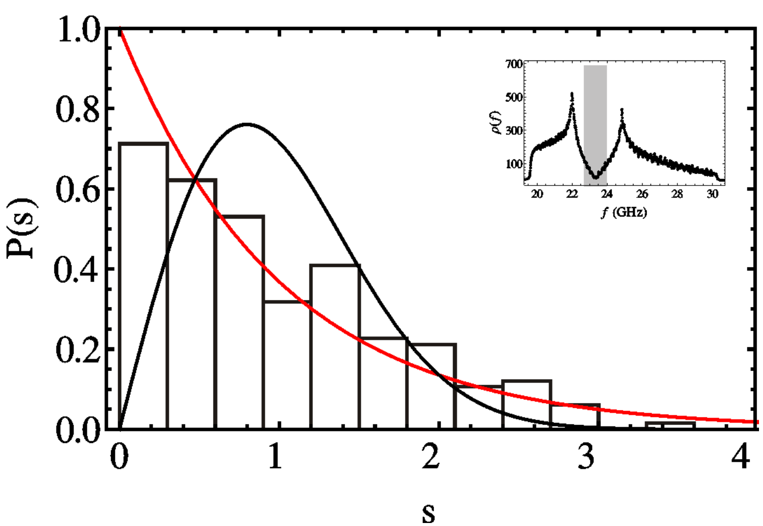


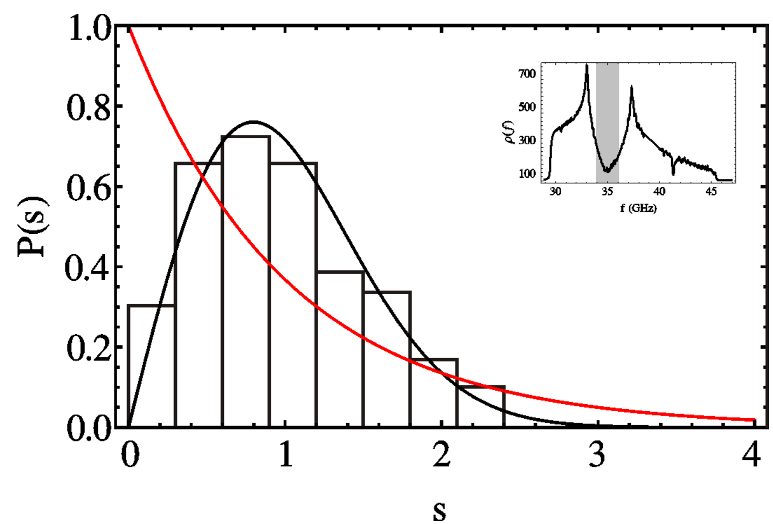
- Spacing between adjacent levels depends on DOS $\rho(k)$
- 130 levels in the Schrödinger regime
- 159 levels in the Dirac regime

Regular and Chaotic Dirac Billards

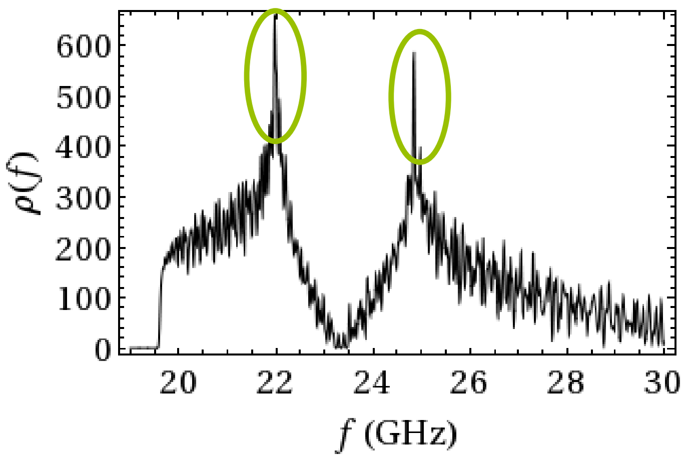


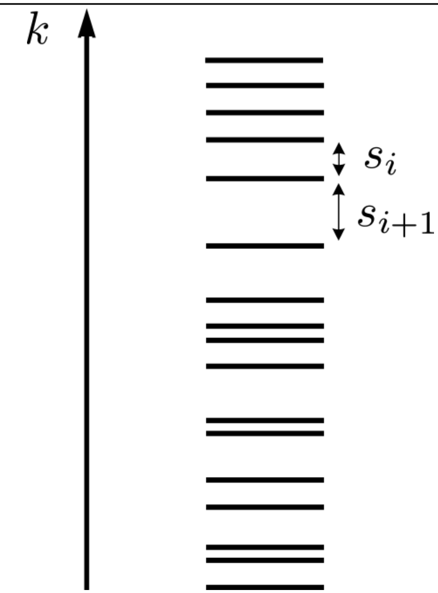
Berry and Mondragon (1987)





Spectral Properties Around the van Hove Singularities
Ratio Distribution of Adjacent Spacings

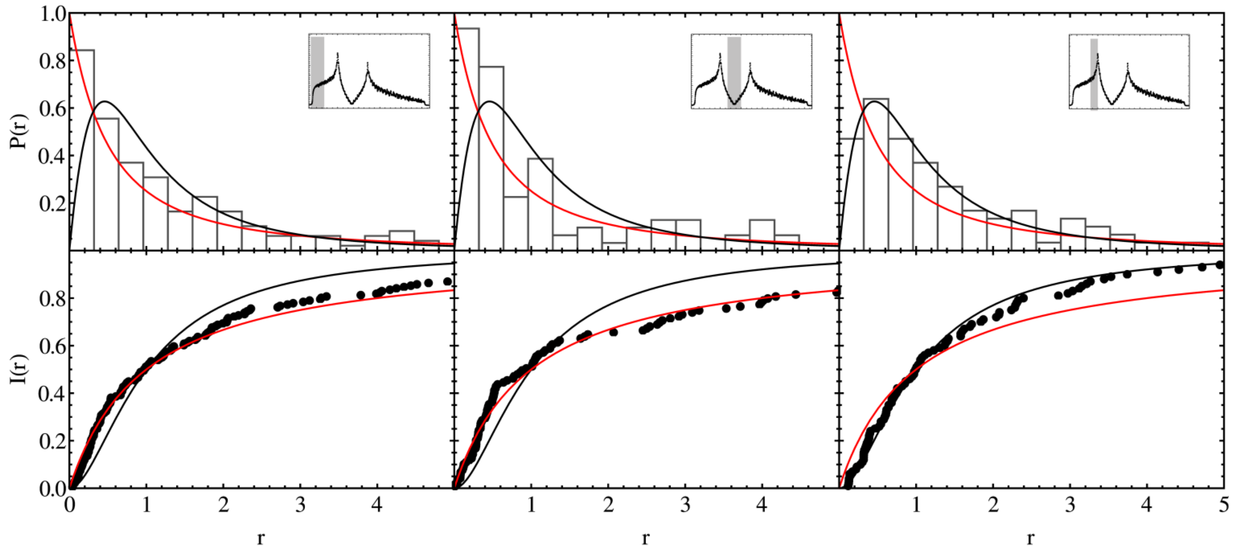




- DOS is unknown around Van Hove singularities
- Ratio of two consecutive spacings $r_i = s_i / s_{i+1}$
- Ratios are independent of the DOS $\rightarrow$ no unfolding necessary
- Analytical prediction for Gaussian RMT ensembles

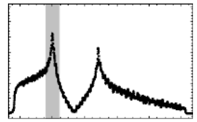
  (Y.Y. Atas, E. Bogomolny, O. Giraud and G. Roux, PRL, **110**, 084101 (2013))

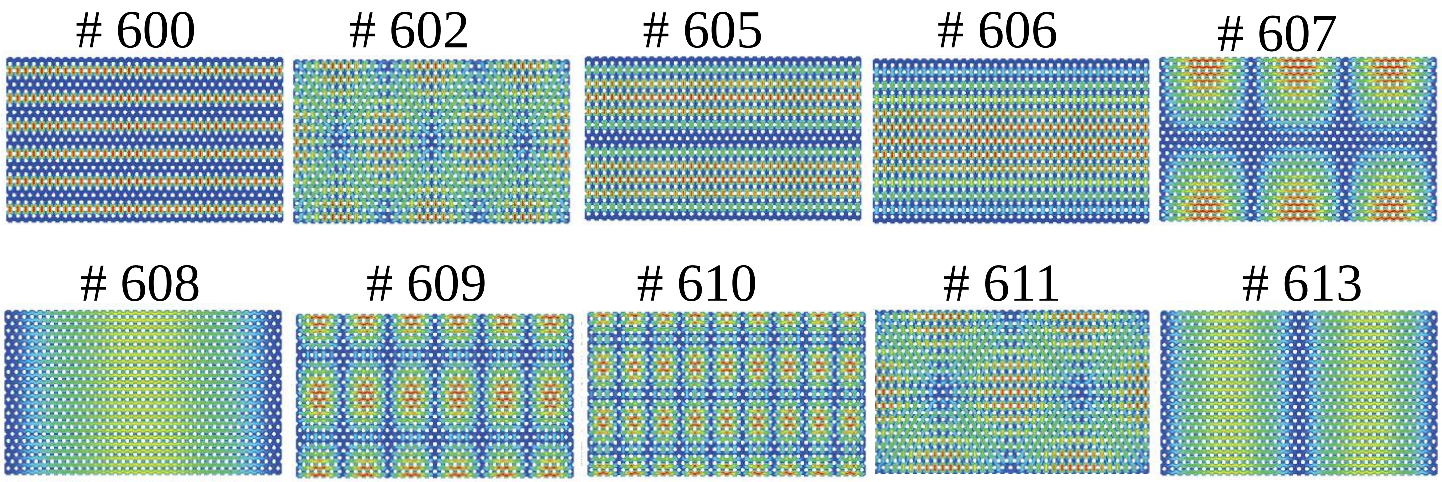
Ratio Distributions for Dirac Billiard

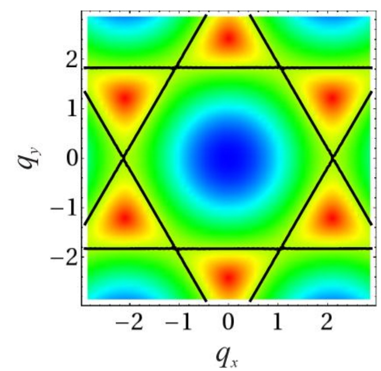


- Poisson: $P(r) = \frac{1}{(1+r)^2}$; GOE: $P(r) = \frac{27}{8}\frac{r+r^2}{(1+r+r^2)^{5/2}}$
- Poisson statistics in the Schrödinger and Dirac regime
- GOE statistics to the left of first van Hove singularity
- Origin ? $\vec{v} = \vec{\nabla}\,\omega(\vec{q}=M) = 0;\ \lambda_{eff} \to 0 \to$ e.m. waves “see the scatterers“

Computed Intensity Distributions around the Lower van Hove Singularity

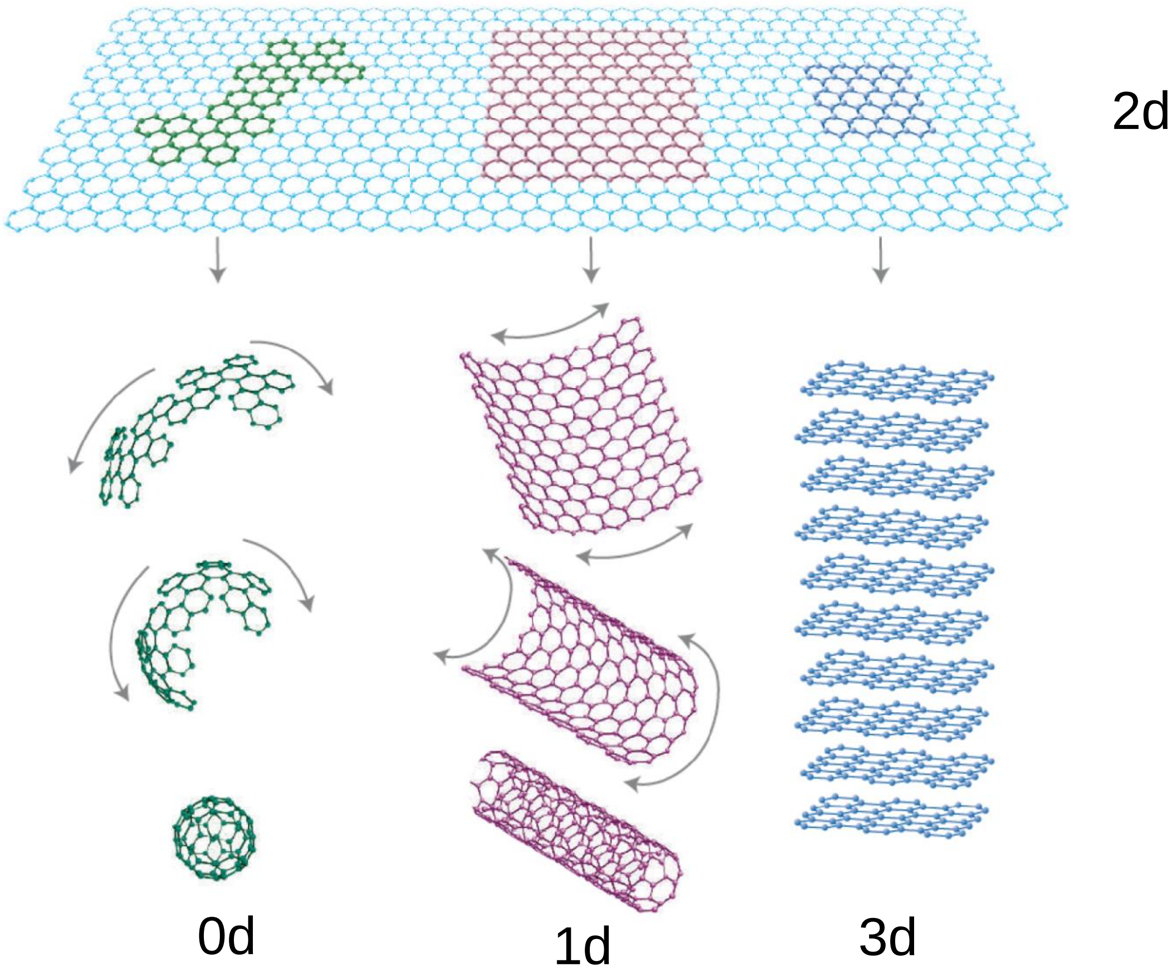






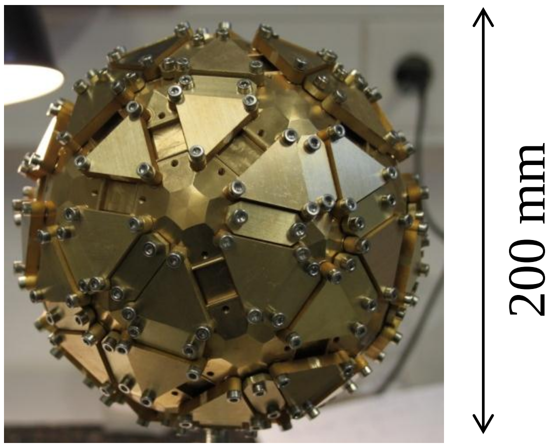
- Some wave function patterns coincide with those of rectangular billiard with Dirichlet / Neumann (#607, 608, 609, 610, 613), Neumann / Dirichlet (#600, 605, 606) or Neumann / Neumann (#602, 611) BCs
- Complexity at the van Hove singularity is due to the fact that the isofrequency line separates different regions in the quasimomentum plane (q_x,q_y)

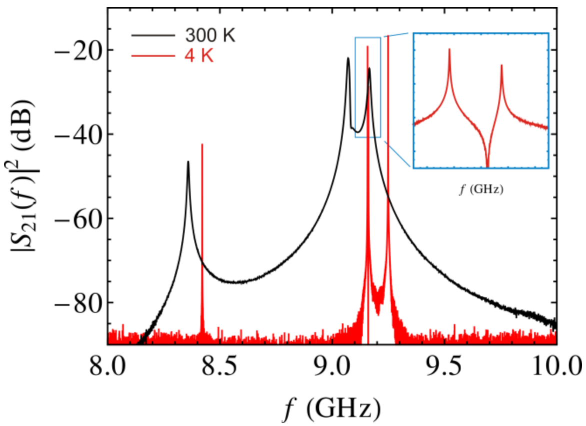
Graphene is the “Mother of all Graphitic Forms“ (Geim and Novoselov (2007))



Outlook

- ”Artificial” Fullerene





- Electronic structure of C_{60} can be described by the Dirac equation on the sphere with a gauge potential → index theorem which connects the topology to the number of zero modes

- Test of quantum chaotic scattering predictions for graphs

  (Gnutzmann, Schanz and Smilansky; Pluhař + Weidenmüller 2013)