le LPTMS en Bretagne...

photo extraite du site `http://www.chateau-du-val.com`

le LPTMS en Bretagne...

Quantum signature of analog Hawking radiation in momentum space

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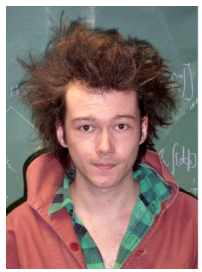
D. Boiron

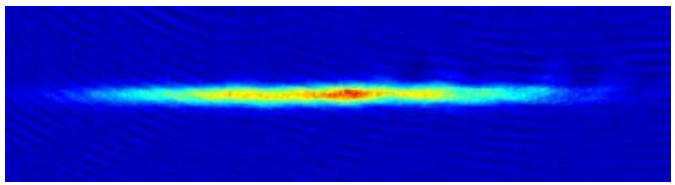
S. Fabbri



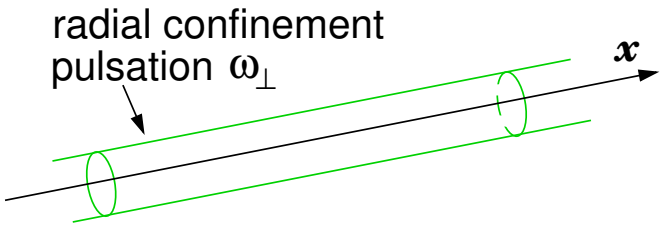
P.-É. Larré

C. Westbrook

P. Zin

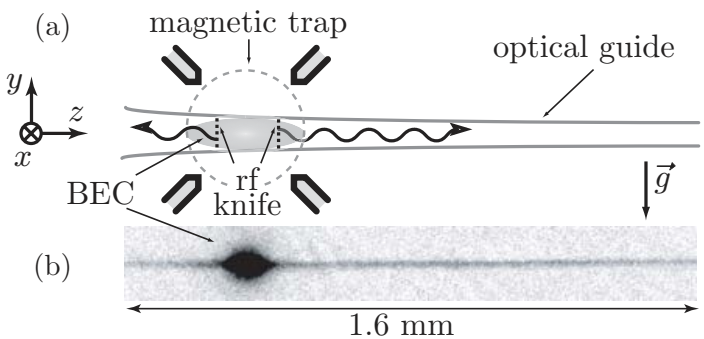


quasi-1D condensate
longitudinal size $\sim 10^2 \mu$m
transverse size $\sim 1\mu$m



harmonic radial confinement :

$$V_\perp(\vec{r}_\perp) = \frac{1}{2} m \omega_\perp^2 r_\perp^2 \ .$$

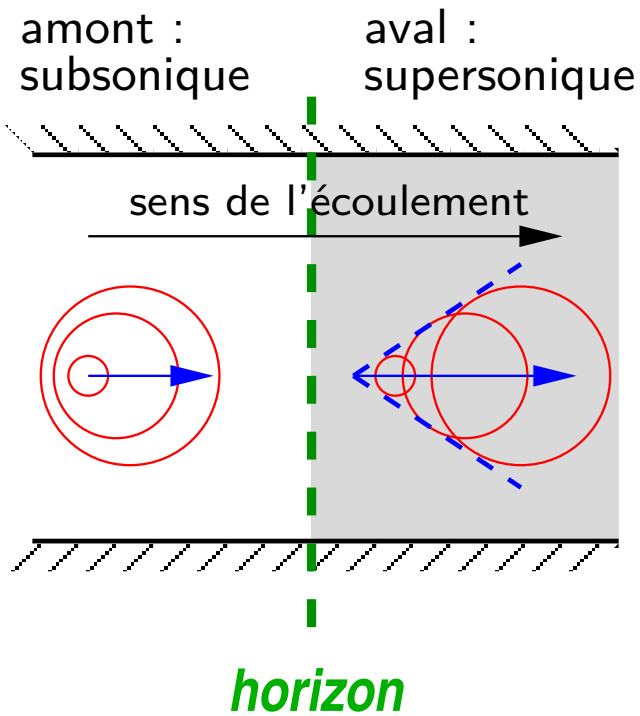


Guerin *et al.*, Phys. Rev. Lett. (2006)

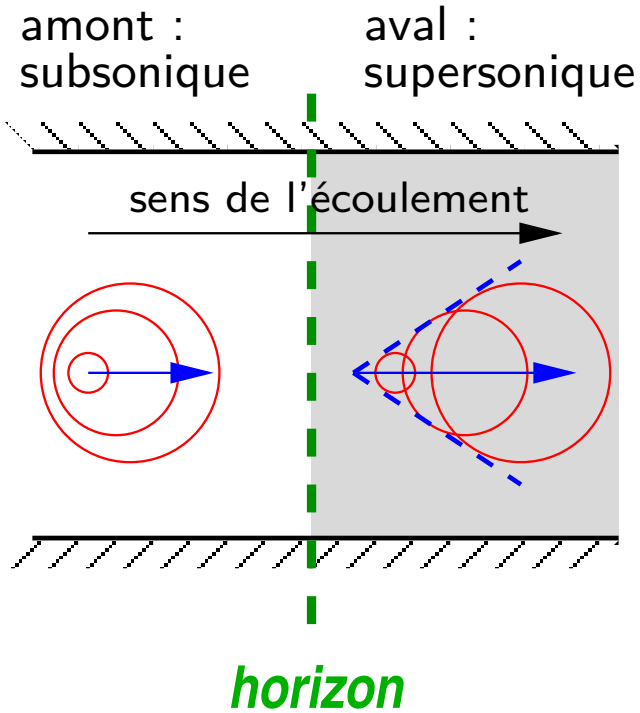
$\rightarrow$ **1D model** : $\psi(x,t)$

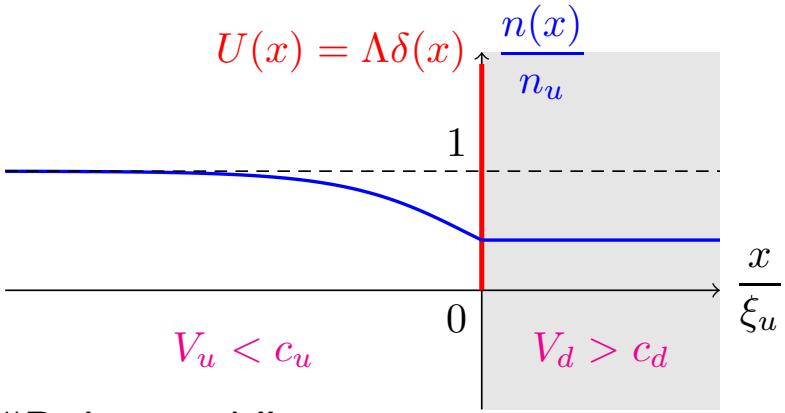
Analogous Hawking radiation

Unruh, Phys. Rev. Lett. (1981)

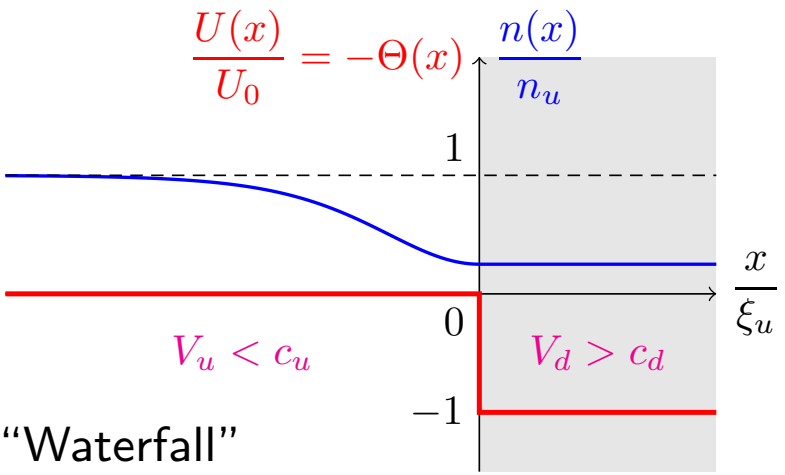


Unruh, Phys. Rev. Lett. (1981)



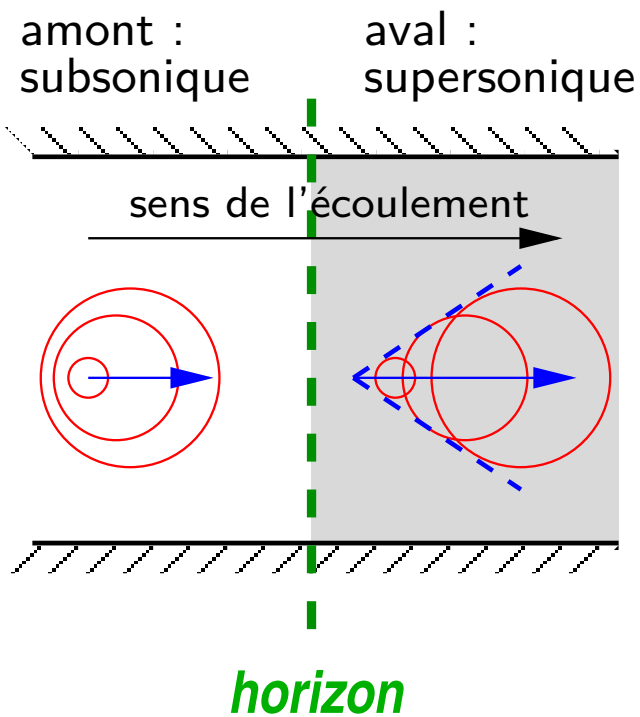


"Delta peak"

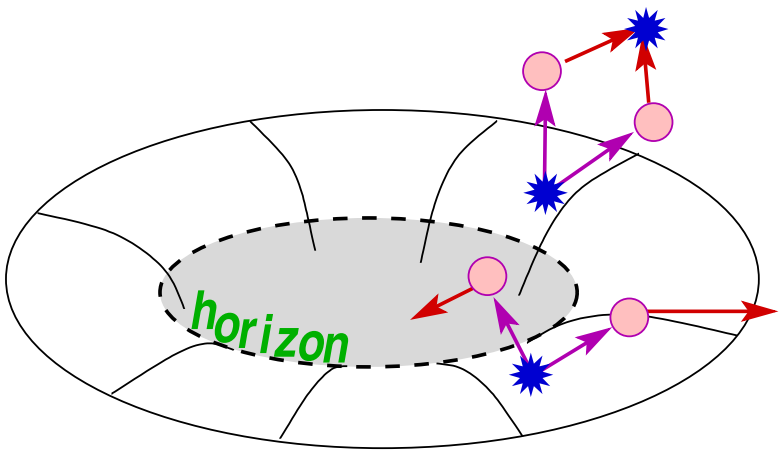


"Waterfall"

Unruh, Phys. Rev. Lett. (1981)

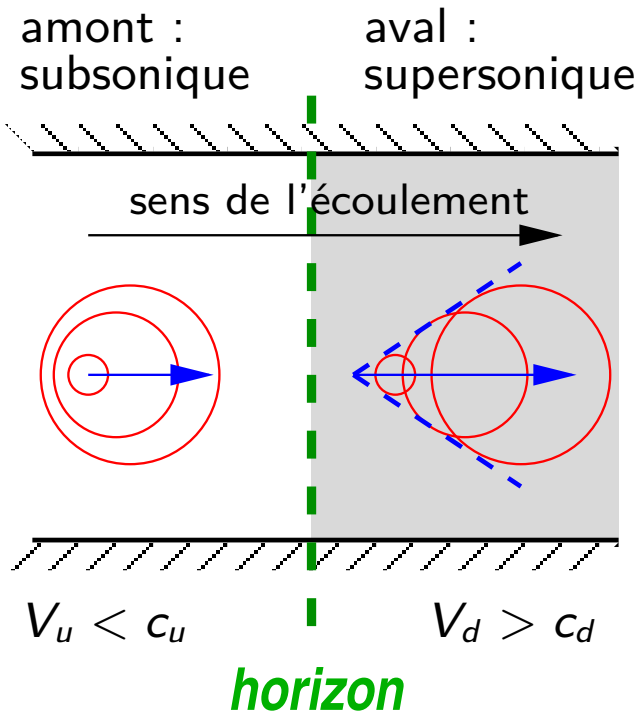


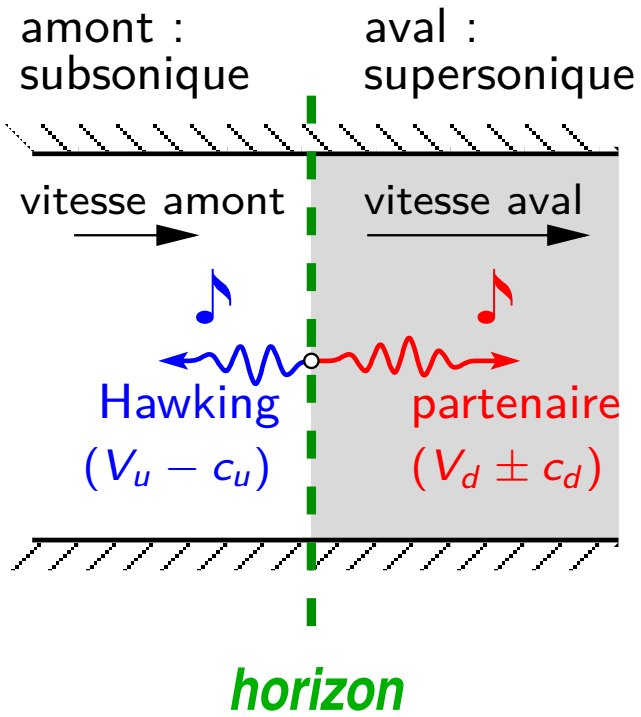
gravitational black hole



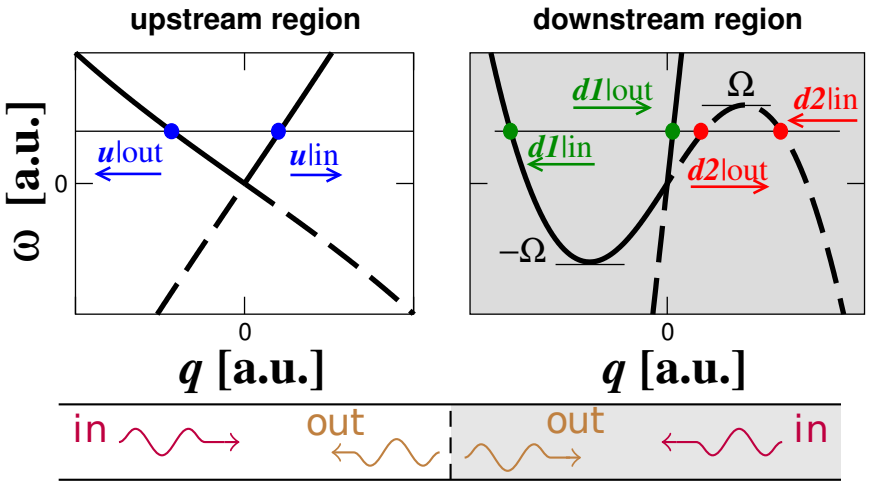
Hawking radiation 75'

Unruh, Phys. Rev. Lett. (1981)

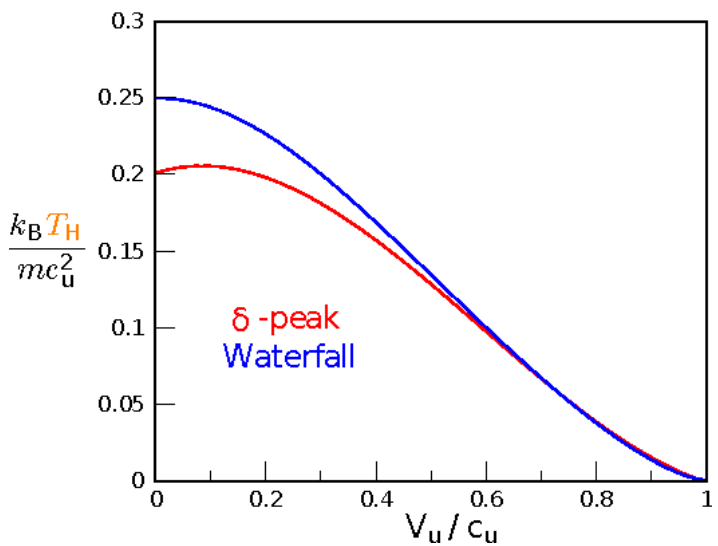




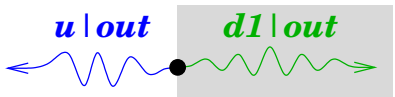
Balbinot, Carusotto, Fabbri, Fagnocchi, Recati, Phys. Rev. A & New J. Phys. (2008)



Hawking radiation in the $u|\text{out}$ channel. Equivalent to a black body radiation of temperature $T_H \simeq 10\%\ \mu$

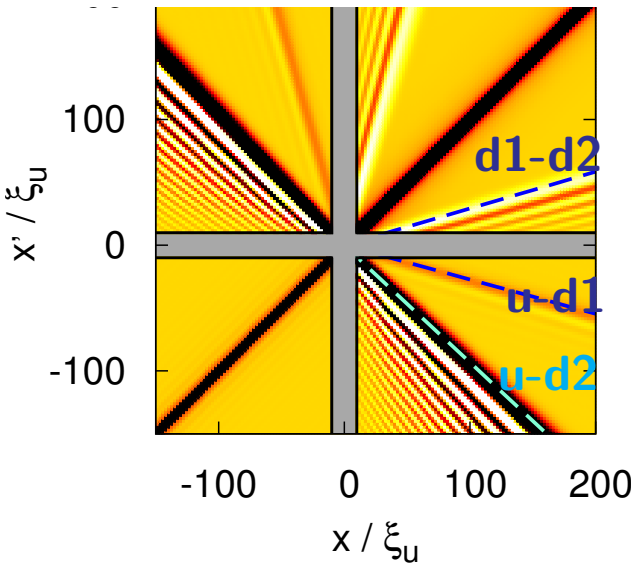


★ example of induced correlation:



$x = (v_d + c_d)t$ correlates with
$x' = (v_u - c_u)t$

★ affects the density correlation pattern $\langle :n(x)n(x'):\rangle$

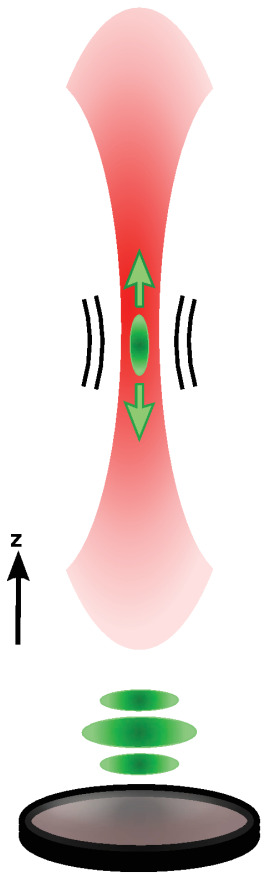


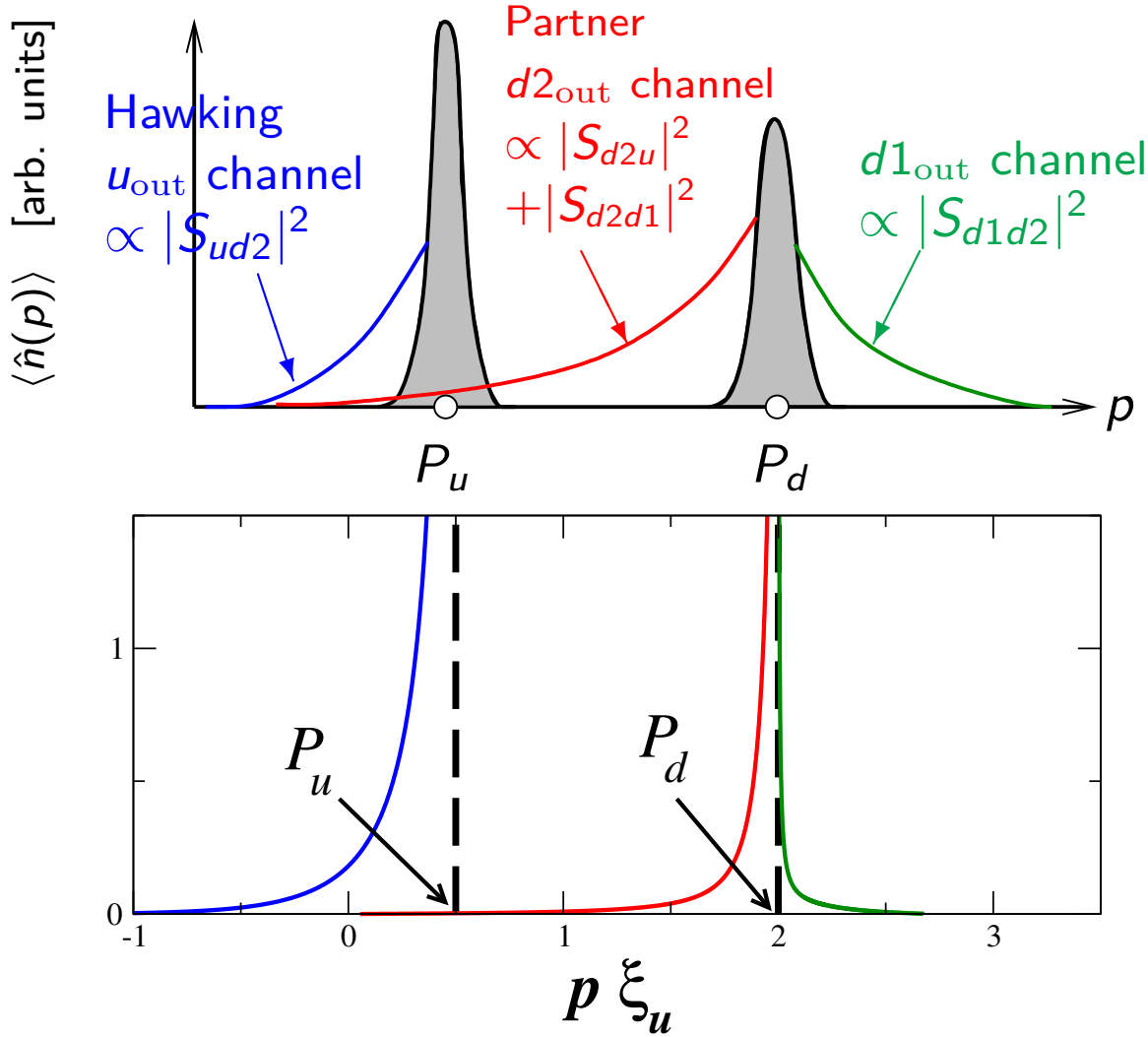
Larré *et al.*, Phys. Rev. A (2012)

One body momentum distribution in the presence of a horizon

$T = 0$, adiabatic opening of the trap

Boiron *et al.* PRL (2015)



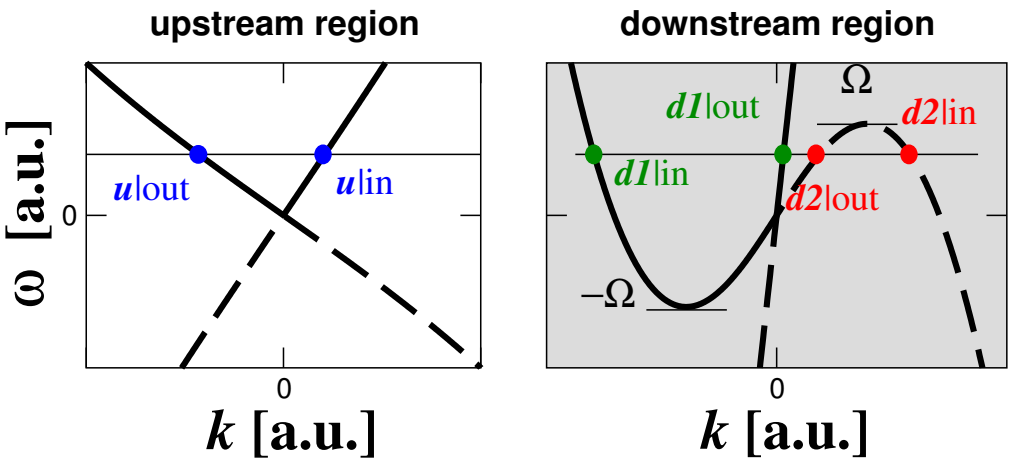


Two body momentum distribution in the presence of a horizon

p, q : absolute momenta in units of ξ_u^{-1}

right plot: $g_2(p,q) \rightarrow$

where $g_2(p,q) = \dfrac{\langle :\hat{n}(p)\hat{n}(q): \rangle}{\langle \hat{n}(p) \rangle \langle \hat{n}(q) \rangle}$

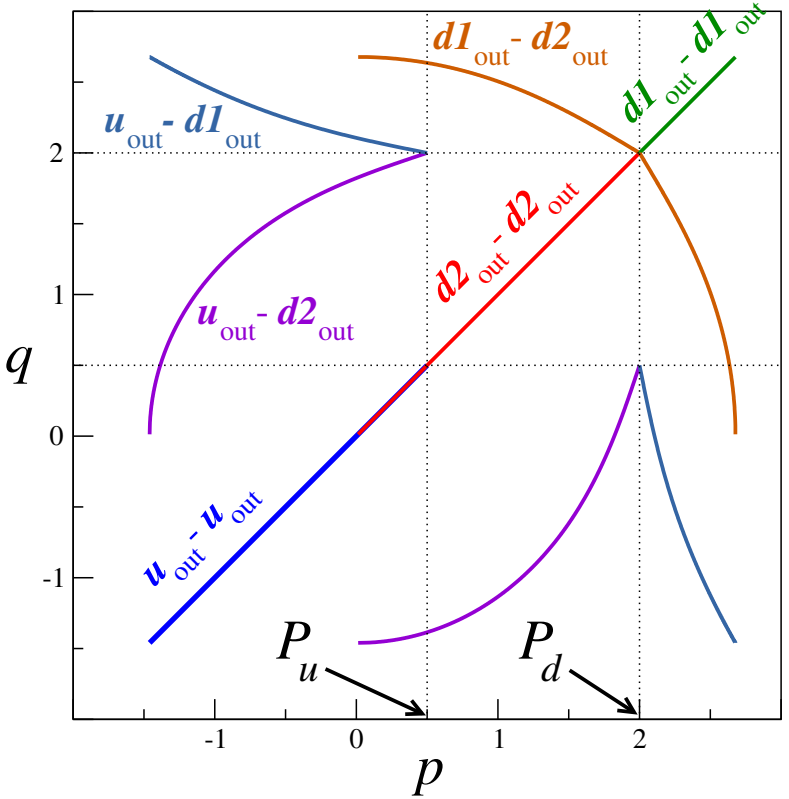


k : momentum relative to the condensate

$p = k + P_{(u/d)}$ where $P_{(u/d)} = mV_{(u/d)}$

$T = 0$ adiabatic opening

Boiron *et al.* PRL (2015)

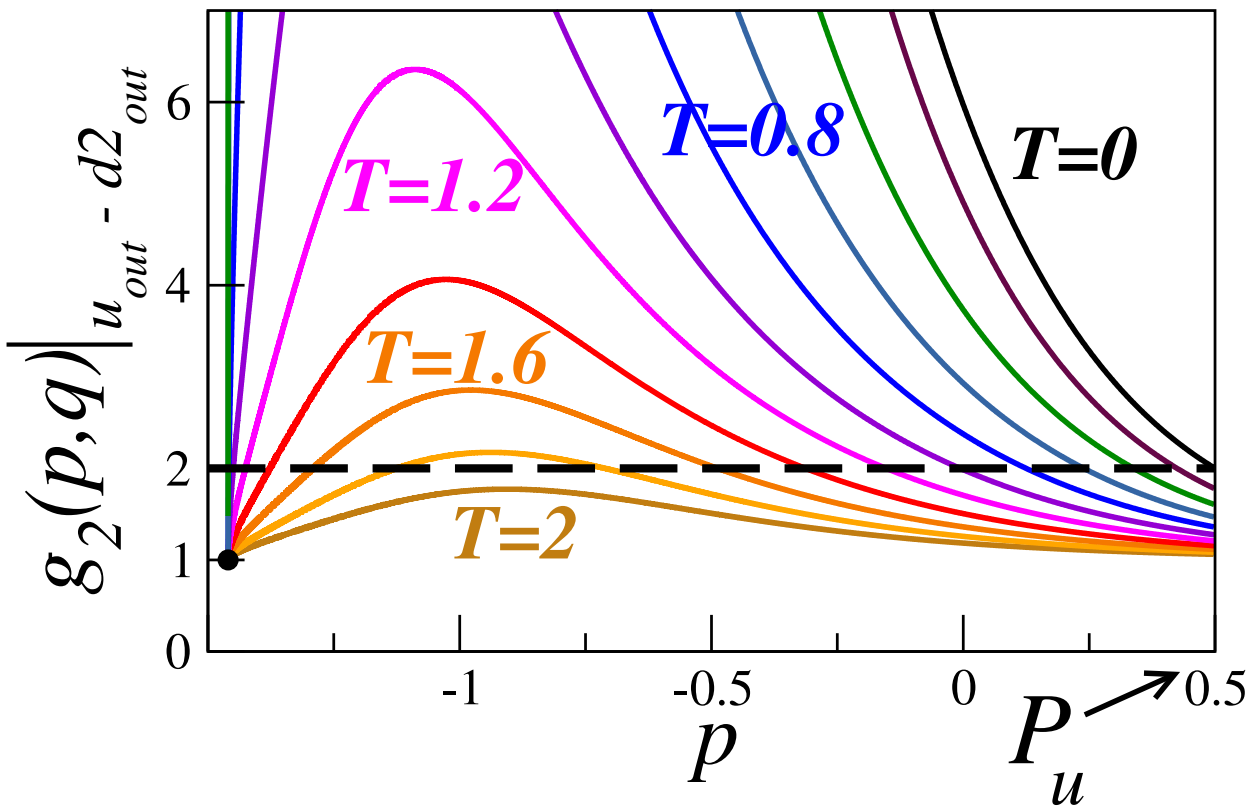


without horizon: $g_2 \equiv 1$

Violation of Cauchy-Schwarz inequality ($T \neq 0$)

C.-S. violation : $$g_2(p,q)\Big|_{u_{\rm out}-d2_{\rm out}} > \sqrt{g_2(p,p)\Big|_{u_{\rm out}} \times g_2(q,q)\Big|_{d2_{\rm out}}} \equiv 2$$

Boiron *et al.* PRL (2015)



T in units of μ

$T_H = 0.13$

$V_u/c_u = 0.5$

$V_d/c_d = 4$

$V_d/V_u = 4$

$n_u/n_d = 4$

$$\langle ... \rangle \stackrel{def}{=} \mathrm{Tr}(\rho ...)$$

$$\begin{aligned}\langle \hat{n}^2 \rangle &= \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle \\ &= \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle + \langle \hat{a}^\dagger 1\, \hat{a} \rangle \\ &= \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle + \langle \hat{n} \rangle\end{aligned}$$

$$\begin{aligned}0 \leq \langle \delta n^2 \rangle &\stackrel{def}{=} \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 \\ &= \underbrace{\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle - \langle \hat{n} \rangle^2}_{sign?} + \langle \hat{n} \rangle\end{aligned}$$

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Cauchy-Schwarz: $|\langle \hat{\boldsymbol{A}} \rangle|^2 \leq \langle \hat{\boldsymbol{A}}^\dagger \hat{\boldsymbol{A}} \rangle$

Hence	$\lvert\langle \hat{a}\hat{a} \rangle\rvert^2 \leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle$
But	$\lvert\langle \hat{a}^\dagger \hat{a} \rangle\rvert^2 \not\leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle$

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stupid theoretical example

average over a number state: $\rho \equiv |n\rangle\langle n|$

$$\begin{aligned}\langle \hat{a}\hat{a} \rangle^2 &= 0 \\ \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle &= n(n-1) \\ \langle \hat{a}^\dagger \hat{a} \rangle^2 &= n^2 \not\leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle\end{aligned}$$

a number state is clearly sub-Poissonian !

$$\langle ... \rangle \stackrel{def}{=} \mathrm{Tr}(\rho ...)$$

$$\begin{aligned}\langle \hat{n}^2 \rangle &= \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle \\ &= \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle + \langle \hat{a}^\dagger 1\, \hat{a} \rangle \\ &= \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle + \langle \hat{n} \rangle\end{aligned}$$

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Cauchy-Schwarz: $|\langle \hat{A} \rangle|^2 \leq \langle \hat{A}^\dagger \hat{A} \rangle$

Hence	$\lvert\langle \hat{a}\hat{a} \rangle\rvert^2 \leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle$
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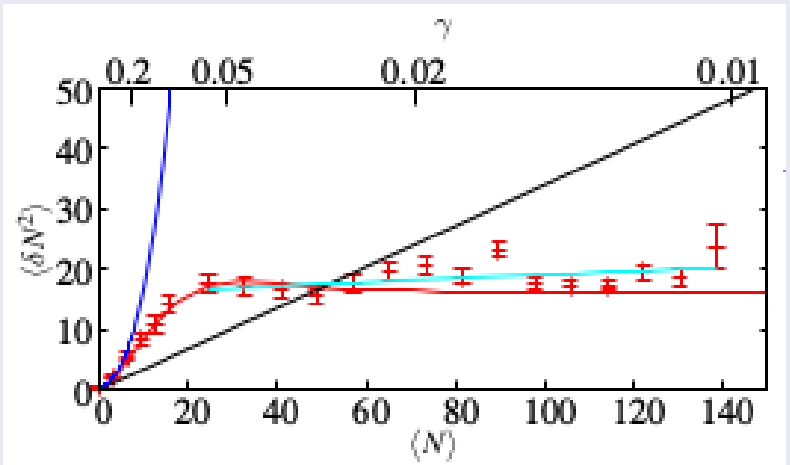
experimental results



Poissonian limit : $\langle \delta N^2 \rangle = 0.34 \langle N \rangle$
Ideal Bose gas
Yang-Yang
Quasi-cond.

Jacqmin *et al.*, *PRL (2011)*

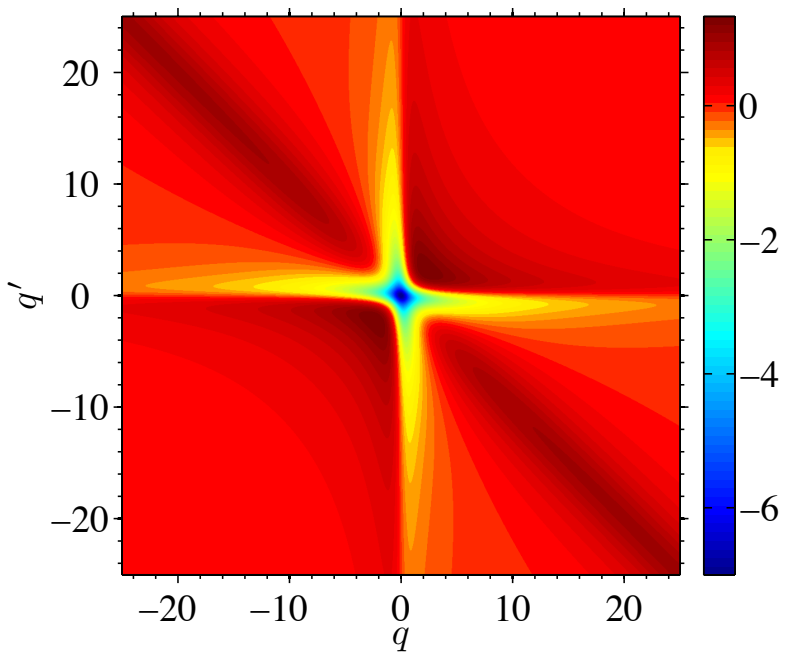
The NLS $\leftrightarrow$ Gross-Pitaevskii eq. is a nonlinear **quantum field** eq. :

$$-\frac{\hbar^2}{2m}\partial_x^2\hat{\psi} + g\,\hat{\psi}^\dagger\hat{\psi}\,\hat{\psi} = i\,\hbar\,\partial_t\hat{\psi}\ , \quad \text{with} \quad \left[\hat{\psi}(x,t), \hat{\psi}^\dagger(y,t)\right] = \delta(x-y)\ .$$

BEC : macroscopic occupation of the lowest quantum state: $\hat{\psi}(x,t) = \psi_{(0)}(x,t) + \hat{\phi}(x,t)$ (Bogoliubov 1947)

$\psi_{(0)}$	: solution of the (classical) NLS
$\hat{\phi}$	: solution of a linearized (quantum) eq.

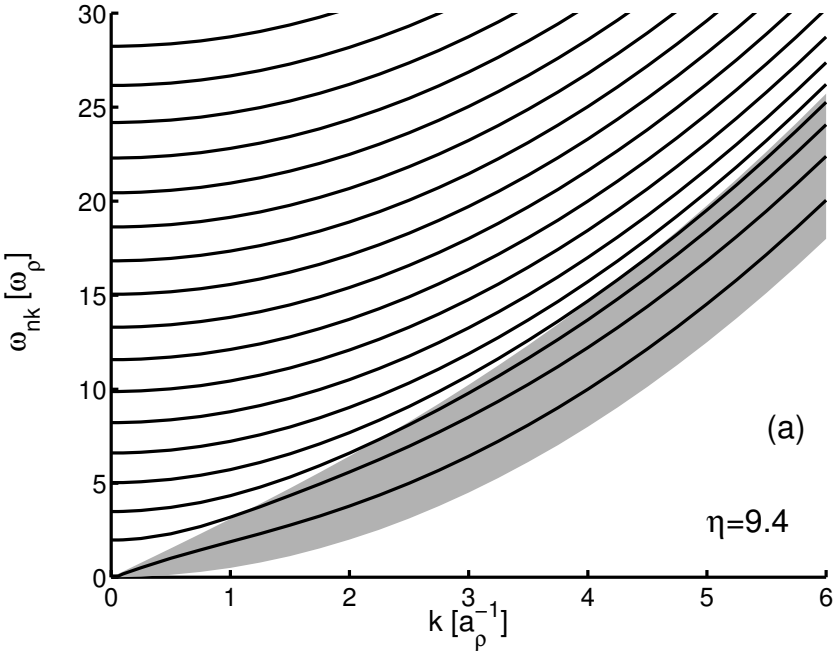
makes it possible to consider vacuum fluctuations. In particular : **Hawking radiation** in a stationary, non uniform setting.



Mathey, Vishwanath, Altman, PRA (2009)

Bouchoule, Arzamasovs, Kheruntsyan, Gangardt, PRA (2012)

when $\hbar\omega_\perp \leq \mu$:



Zaremba, PRA (1998)

Stringari, PRA (1998)

Fedichev & Shlyapnikov, PRA (2001)

Tozzo & Dalfovo, PRA (2002)

modified dispersion relation :

$$\omega_0^2(q) = c_{1d}^2\, q^2 \left(1 - \tfrac{1}{48}(qR_\perp)^2 + \dots\right)$$

new channels :

$$\omega_{n\geq 1}^2(q) = 2n(n+1)\,\omega_\perp^2 + \tfrac{1}{4}(qR_\perp\omega_\perp)^2 + \dots$$

these new channels will be populated

at $T = 0$

mass term $\neq$ Klein-Gordon

$\rightarrow$ new "in" modes

Conclusion

BECs offer interesting prospects to observe Hawking radiation

[Steinhauer, Nat. Phys. (2014)]

general perspective : **quantum effects** with nonlinear matter waves

One- and two-body momentum distributions accessible by present day experimental techniques provide clear direct evidences

- ➪ of the occurrence of a sonic horizon.
- ➥ of the associated acoustic Hawking radiation.
- ☞ of the quantum nature of the Hawking process.
  - ☺ The signature of the quantum behavior persists even at temperatures larger than the chemical potential.